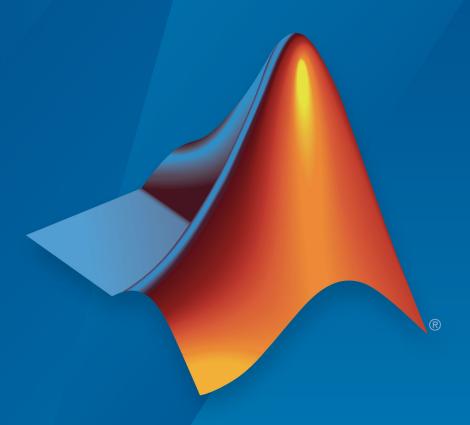
## **Control System Toolbox™**

Reference



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Control System Toolbox™ Reference

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## **Class Reference**

## **TuningGoal.ConicSector class**

Package: TuningGoal

Sector bound for control system tuning

## **Description**

A conic sector bound is a restriction on the output trajectories of a system. If for all nonzero input trajectories u(t), the output trajectory z(t) = (Hu)(t) of a linear system H satisfies:

$$\int_0^T z(t)^{\mathrm{T}} Q \, z(t) dt < 0,$$

for all  $T \ge 0$ , then the output trajectories of H lie in the conic sector described by the symmetric indefinite matrix Q. Selecting different Q matrices imposes different conditions on the system response.

When tuning a control system with systune, use TuningGoal.ConicSector to restrict the output trajectories of the response between specified inputs and outputs to a specified sector. For more information about sector bounds, see "About Sector Bounds and Sector Indices".

#### Construction

Req = TuningGoal.ConicSector(inputname, outputname, Q) creates a tuning goal for restricting the response H(s) from inputs inputname to outputs outputname to the conic sector specified by the symmetric matrix Q. The tuning goal constrains H such that its trajectories z(t) = (Hu)(t) satisfy:

$$\int_0^T z(t)^{\mathrm{T}} Q \, z(t) dt < 0,$$

for all  $T \ge 0$ . (See "About Sector Bounds and Sector Indices".) The matrix  ${\bf Q}$  must have as many negative eigenvalues as there are inputs in H.

To specify frequency-dependent sector bounds, set Q to an LTI model that satisfies  $Q(s)^T = Q(-s)$ .

#### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink® model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - · Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### 0

Sector geometry, specified as:

- A matrix, for constant sector geometry. Q is a symmetric square matrix that is ny on a side, where ny is the number of signals in outputname. The matrix Q must be indefinite to describe a well-defined conic sector. An indefinite matrix has both positive and negative eigenvalues. In particular, Q must have as many negative eigenvalues as there are input channels specified in inputname (the size of the vector input signal u(t)).
- An LTI model, for frequency-dependent sector geometry. Q satisfies  $Q(s)^T = Q(-s)$ . In other words, Q(s) evaluates to a Hermitian matrix at each frequency.

For more information, see "About Sector Bounds and Sector Indices".

## **Properties**

#### SectorMatrix

Sector geometry, specified as a matrix or an LTI model. The Q input argument sets initial value of SectorMatrix when you create the tuning goal, and the same restrictions and characteristics apply to SectorMatrix as apply to Q.

#### Regularization

Regularization parameter, specified as a real nonnegative scalar value.

Given the indefinite factorization of the sector matrix,

$$Q = W_1 W_1^{\mathrm{T}} - W_2 W_2^{\mathrm{T}}, \quad W_1^{\mathrm{T}} W_2 = 0$$

the sector bound

$$H(-j\omega)^{\mathrm{T}}QH(j\omega) < 0$$

is equivalent to

$$H_1(j\omega)^{\mathrm{H}}H_1(j\omega) < H_2(j\omega)^{\mathrm{H}}H_2(j\omega),$$

where  $H_1=W_1^{\rm T}H$ ,  $H_2=W_2^{\rm T}H$ , and  $(\bullet)^{\rm H}$  denotes the Hermitian transpose. Enforcing this condition might become numerically challenging when other tuning goals drive both  $H_1(j\omega)$  and  $H_2(j\omega)$  to zero at some frequencies. This condition is equivalent to controlling the sign of a 0/0 expression, which is intractable in the presence of rounding errors. To avoid this condition, you can regularize the sector bound to

$$H(-j\omega)^{\mathrm{T}}QH(j\omega) < -\varepsilon^{2}I$$

or equivalently,

$$H_1(j\omega)^H H_1(j\omega) + \varepsilon^2 I < H_2(j\omega)^H H_2(j\omega)$$
.

This regularization prevents  $H_2(j\omega)$  from becoming singular, and helps keep evaluation of the tuning goal numerically tractable. Use the Regularization property to set the value of  $\varepsilon$  to a small (but not negligible) fraction of the typical norm of the feedthrough term in H. For example, if you anticipate the norm of the feedthrough term of H to be of order 1 during tuning, try:

Req.Regularization = 1e-3;

#### Default: 0

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply

only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Input

Input signal names, specified as a cell array of character vectors. The input signal names specify the inputs of the constrained response, initially populated by the inputname argument.

#### **Output**

Output signal names, specified as a cell array of character vectors. The output signal names specify the outputs of the constrained response, initially populated by the outputname argument.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default: NaN**

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

#### Conic Sector Goal

Create a tuning goal that restricts the response from an input or analysis point 'u' to an output or analysis point 'y' in a control system to the following sector:

$$S = \left\{ (y, u) : 0 . 1u^2 < uy < 10u^2 \right\}.$$

The *Q* matrix for this sector is given by:

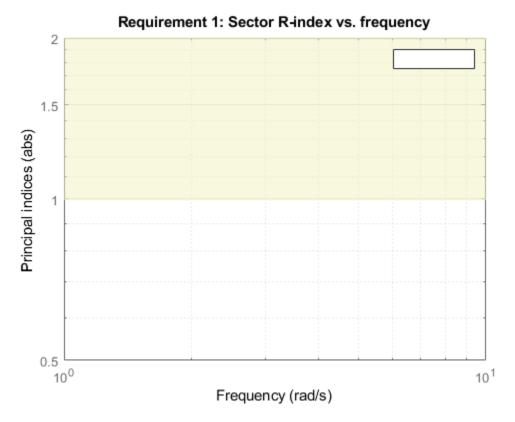
```
a = 0.1;
b = 10;
Q = [1 -(a+b)/2 ; -(a+b)/2 a*b];
Use this Q matrix to create the tuning goal.
TG = TuningGoal.ConicSector('u','y',Q)
TG =
    ConicSector with properties:
        SectorMatrix: [2x2 double]
        Regularization: 0
        Focus: [0 Inf]
        Input: {'u'}
        Output: {'y'}
        Models: NaN
        Openings: {0x1 cell}
        Name: ''
```

Set properties to further configure the tuning goal. For example, suppose the control system model has an analysis point called 'OuterLoop', and you want to enforce the tuning goal with the loop open at that point.

```
TG.Openings = 'OuterLoop';
```

Before or after tuning, use viewGoal to visualize the tuning goal.

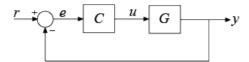
```
viewGoal(TG)
```



The goal is met when the relative sector index R < 1 at all frequencies. The shaded area represents the region where the goal is not met. When you use this requirement to tune a control system CL, viewGoal(TG,CL) shows R for the specified inputs and outputs on this plot, enabling you to identify frequency ranges in which the goal is not met, and by how much.

#### **Constrain Input and Output Trajectories to Conic Sector**

Consider the following control system.



Suppose that the signal u is marked as an analysis point in a Simulink model or genss model of the control system. Suppose also that G is the closed-loop transfer function from u to y. Create a tuning goal that constrains all I/O trajectories  $\{u(t), y(t)\}$  of G to satisfy:

$$\int_{0}^{T} {y(t) \choose u(t)}^{T} Q {y(t) \choose u(t)} dt < 0,$$

for all  $T \ge 0$ . For this example, use sector matrix that imposes input passivity with index 0.5.

$$nu = 0.5;$$
  
 $Q = [0 -1; -1 2*nu];$ 

Constraining the I/O trajectories of G is equivalent to restricting the output trajectories z(t) of H = [G; I] to the sector defined by:

$$\int_0^T z(t)^T Q z(t) dt < 0.$$

(See "About Sector Bounds and Sector Indices" for more details about this equivalence.) To specify this constraint, create a tuning goal that constrains the transfer function H = [G; I], which the transfer function from input u to outputs  $\{y; u\}$ .

```
TG = TuningGoal.ConicSector('u',{'y';'u'},Q);
```

When you specify the same signal 'u' as both input and output, the conic sector tuning goal sets the corresponding transfer function to the identity. Therefore, the transfer function constrained by TG is H = [G; I] as intended. This treatment is specific to the conic sector tuning goal. For other tuning goals, when the same signal appears in both inputs and outputs, the resulting transfer function is zero in the absence of feedback loops, or the complementary sensitivity at that location otherwise. This result occurs because when the software processes analysis points, it assumes the input is injected after the output. See "Mark Signals of Interest for Control System Analysis and Design" for more information about how analysis points work.

## **Tips**

• The conic sector tuning goal requires that  $W_2^T H(s)$  be square and minimum phase, where H(s) is the transfer function between the specified inputs and outputs, and  $W_2$  spans the negative invariant subspace of the sector matrix, Q:

$$Q = W_1 W_1^{\mathsf{T}} - W_2 W_2^{\mathsf{T}}, \quad W_1^{\mathsf{T}} W_2 = 0$$

(See "Algorithms" on page 1-12.) This means that the stabilized dynamics for this goal are not the poles of H, but rather the transmission zeros of  $W_2^{\rm T}H(s)$ . The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

Let

$$Q = W_1 W_1^{\mathrm{T}} - W_2 W_2^{\mathrm{T}}, \quad W_1^{\mathrm{T}} W_2 = 0$$

be an indefinite factorization of Q. When  $W_2^T H(s)$  is square and minimum phase, then the time-domain sector bound on trajectories z(t) = Hu(t),

$$\int_0^T z(t)^{\mathrm{T}} Q \, z(t) dt < 0,$$

is equivalent to the frequency-domain sector condition,

$$H(-j\omega)^{\mathrm{T}}QH(j\omega) < 0$$

for all frequencies. The TuningGoal.ConicSector goal uses this equivalence to convert the time-domain characterization into a frequency-domain condition that systune can handle in the same way it handles gain constraints. To secure this equivalence,

TuningGoal.ConicSector also makes  $W_2^TH(s)$  minimum phase by making all its zeros stable.

For sector bounds, the *R*-index plays the same role as the peak gain does for gain constraints (see "About Sector Bounds and Sector Indices"). The condition

$$H(-j\omega)^{\mathrm{T}}QH(j\omega) < 0$$

is satisfied at all frequencies if and only if the *R*-index is less than one. The viewGoal plot for TuningGoal.ConicSector shows the *R*-index value as a function of frequency (see sectorplot).

When you tune a control system using a TuningGoal object to specify a tuning goal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For the sector bound

$$H(-j\omega)^{\mathrm{T}}QH(j\omega) < 0$$

TuningGoal.ConicSector uses the objective function given by:

$$f(x) = \frac{R}{1 + R/R_{\text{max}}}, \quad R_{\text{max}} = 10^6.$$

R is the sector-bound R-index (see getSectorIndex for details).

The dynamics of H affected by the minimum-phase condition are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

#### See Also

evalGoal|getSectorIndex|slTuner|systume|systume (for slTuner)|
viewGoal

#### **Topics**

"About Sector Bounds and Sector Indices"

"Tuning Control Systems with SYSTUNE"

"Tune Control Systems in Simulink"

#### Introduced in R2016b

## **TuningGoal.ControllerPoles class**

Package: TuningGoal

Constraint on controller dynamics for control system tuning

## **Description**

Use TuningGoal.ControllerPoles to constrain the dynamics of a tunable component in a control system model. Use this tuning goal for constraining the dynamics of tuned blocks identified in a slTuner interface to a Simulink model. If you are tuning a genss model of a control system, use it to constrain tunable elements such as tunableTF or tunableSS. The TuningGoal.ControllerPoles requirement lets you control the minimum decay rate, minimum damping, and maximum natural frequency of the poles of the tunable element, ensuring that the controller is free of fast or resonant dynamics. The tuning goal can also ensure stability of the tuned value of the tunable element.

After you create a requirement object, you can further configure the tuning goal by setting "Properties" on page 1-17 of the object.

#### Construction

Req = TuningGoal.ControllerPoles(blockID,mindecay,mindamping,maxfreq) creates a tuning goal that constrains the dynamics of a tunable component of a control system. The minimum decay rate, minimum damping constant, and maximum natural frequency define a region of the complex plane in which poles of the component must lie. A nonnegative minimum decay ensures stability of the tuned poles. The tuning goal applies to all poles in the block except fixed integrators, such as the *I* term of a PID controller.

#### **Input Arguments**

#### blockID

Tunable component to constrain, specified as a character vector. blockID designates one of the tuned blocks in the control system you are tuning.

- For tuning a Simulink model of a control system, blockID is a tuned block in the slTuner interface to the model. For example, suppose the slTuner interface has a tuned block called Controller. To constrain this block, use 'Controller' for the blockID input argument.
- For tuning a genss model of a control system, blockid is one of the control design blocks of that model. For example, suppose the genss interface has a tunable block with name C1. To constrain this block, use 'C1' for the blockID input argument.

#### mindecay

Minimum decay rate of poles of tunable component, specified as a scalar value in the frequency units of the control system model you are tuning.

Specify  $mindecay \ge 0$  to ensure that the block is stable. If you specify a negative value, the tuned block can include unstable poles.

When you tune the control system using this tuning goal, all poles of the tunable component are constrained to satisfy:

- Re(s) < -mindecay, for continuous-time systems.
- log(|z|) < -mindecay\*Ts, for discrete-time systems with sample time Ts.

#### Default: 0

#### mindamping

Desired minimum damping ratio of poles of the tunable block, specified as a value between 0 and 1.

Poles of the block that depend on the tunable parameters are constrained to satisfy Re(s) < -mindamping\*|s|. In discrete time, the damping ratio is computed using s=log(z)/Ts.

#### **Default:** 0

#### maxfreq

Desired maximum natural frequency of poles of the tunable block, specified as a scalar value in the units of the control system model you are tuning.

Poles of the block are constrained to satisfy |s| < maxfreq for continuous-time blocks, or |log(z)| < maxfreq\*Ts for discrete-time blocks with sample time Ts. This constraint prevents fast dynamics in the tunable block.

Default: Inf

## **Properties**

#### **Block**

Name of tunable component to constrain, specified as a character vector. The blockID input argument sets the value of Block.

#### MinDecay

Minimum decay rate of poles of tunable component, specified as a scalar value in the frequency units of the control system you are tuning. The initial value of this property is set by the mindecay input argument.

 $MinDecay \ge 0$  to ensure that the block is stable. If you specify a negative value, the tuned block can include unstable poles.

When you tune the control system using this tuning goal, all poles of the tunable component are constrained to satisfy Re(s) < -MinDecay for continuous-time systems, or log(|z|) < -MinDecay\*Ts for discrete-time systems with sample time Ts.

You can use dot notation to change the value of this property after you create the tuning goal. For example, suppose Req is a TuningGoal.ControllerPoles tuning goal. Change the minimum decay rate to 0.001:

Req.MinDecay = 0.001;

#### **Default:** 0

#### MinDamping

Desired minimum damping ratio of poles of the tunable block, specified as a value between 0 and 1. The initial value of this property is set by the mindamping input argument.

Poles of the block that depend on the tunable parameters are constrained to satisfy Re(s) < -MinDamping\*|s|. In discrete time, the damping ratio is computed using s=log(z)/Ts.

Default: 0

#### **MaxFrequency**

Desired maximum natural frequency of poles of the tunable block, specified as a scalar value in the frequency units of the control system model you are tuning. The initial value of this property is set by the maxfreq input argument.

Poles of the block are constrained to satisfy  $|s| < \max freq$  for continuous-time blocks, or  $|\log(z)| < \max freq*Ts$  for discrete-time blocks with sample time Ts. This constraint prevents fast dynamics in the tunable block.

You can use dot notation to change the value of this property after you create the tuning goal. For example, suppose Req is a TuningGoal.ControllerPoles tuning goal. Change the maximum frequency to 1000:

```
Req.MaxFrequency = 1000;
```

Default: Inf

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Reg is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

#### **Constrain Dynamics of Tunable Transfer Function**

Create a tuning goal that constrains the dynamics of a tunable transfer function block in a tuned control system.

For this example, suppose that you are tuning a control system that includes a compensator block parametrized as a second-order transfer function. Create a tuning goal that restricts the poles of that transfer function to the region Re(s) < -0.1, |s| < 30.

Create a tunable component that represents the compensator.

```
C = tunableTF('Compensator',2,2);
```

This command creates a Control Design Block named 'Compensator' with two poles and two zeroes. You can construct a tunable control system model, T, by interconnecting this Control Design Block with other tunable and numeric LTI models. If you tune T using systune, the values of these poles and zeroes are unconstrained by default.

Create a tuning requirement to constrain the dynamics of the compensator block. Set the minimum decay rate to 0.1 rad/s, and set the maximum frequency to 30 rad/s.

```
Req = TuningGoal.ControllerPoles('Compensator', 0.1, 0, 30);
```

The mindamping input argument is 0, which imposes no constraint on the damping constant of the poles of the block.

If you tune T using systune and the tuning requirement Req, the poles of the compensator block are constrained satisfy these values. After you tune T, you can use viewGoal to validate the tuned control system against the tuning goal.

## **Tips**

• TuningGoal.ControllerPoles restricts the dynamics of a single tunable component of the control system. To ensure the stability or restrict the overall dynamics of the tuned control system, use TuningGoal.Poles.

## **Algorithms**

When you use a **TuningGoal** object to specify a tuning goal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x), or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.ControllerPoles, f(x) reflects the relative satisfaction or violation of the goal. For example, if you attempt to constrain the pole of a tuned block to a minimum damping of  $\zeta = 0.5$ , then:

- f(x) = 1 means the damping of the pole is  $\zeta = 0.5$  exactly.
- f(x) = 1.1 means the damping is  $\zeta = 0.5/1.1 = 0.45$ , roughly 10% less than the target.
- f(x) = 0.9 means the damping is  $\zeta = 0.5/0.9 = 0.55$ , roughly 10% better than the target.

## **Compatibility Considerations**

#### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox™ license.

### **See Also**

TuningGoal.Poles|evalGoal|looptune|looptune (for slTuner)|systune|
systune (for slTuner)|tunableSS|tunableTF|viewGoal

#### **Topics**

"System Dynamics Specifications"

"Models with Tunable Coefficients"

#### Introduced in R2016a

## **TuningGoal.Gain class**

Package: TuningGoal

Gain constraint for control system tuning

## **Description**

Use the TuningGoal.Gain object to specify a constraint that limits the gain from a specified input to a specified output. Use this tuning goal for control system tuning with tuning commands such as systume or looptune.

When you use TuningGoal.Gain, the software attempts to tune the system so that the gain from the specified input to the specified output does not exceed the specified value. By default, the constraint is applied with the loop closed. To apply the constraint to an open-loop response, use the Openings property of the TuningGoal.Gain object.

You can use a gain constraint to:

- Enforce a design requirement of disturbance rejection across a particular input/output pair, by constraining the gain to be less than 1
- Enforce a custom roll-off rate in a particular frequency band, by specifying a gain profile in that band

#### Construction

Req = TuningGoal.Gain(inputname,outputname,gainvalue) creates a tuning goal that constrains the gain from inputname to outputname to remain below the value gainvalue.

You can specify the inputname or outputname as cell arrays (vector-valued signals). If you do so, then the tuning goal constrains the largest singular value of the transfer matrix from inputname to outputname. See sigma for more information about singular values.

Req = TuningGoal.Gain(inputname,outputname,gainprofile) specifies the maximum gain as a function of frequency. You can specify the target gain profile

(maximum gain across the I/O pair) as a smooth transfer function. Alternatively, you can sketch a piecewise error profile using an frd model.

#### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### gainvalue

Maximum gain (linear). The gain constraint Req specifies that the gain from inputname to outputname is less than gainvalue.

gainvalue is a scalar value. If the signals inputname or outputname are vector-valued signals, then gainvalue constrains the largest singular value of the transfer matrix from inputname to outputname. See sigma for more information about singular values.

#### gainprofile

Gain profile as a function of frequency. The gain constraint Req specifies that the gain from inputname to outputname at a particular frequency is less than gainprofile. You can specify gainprofile as a smooth transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise gain profile using a frd model or the makeweight function. When you do so, the software automatically maps the gain profile onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of the zpk model.

gainprofile is a SISO transfer function. If inputname or outputname are cell arrays, gainprofile applies to all I/O pairs from inputname to outputname

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify gainfprofile as a discrete-time model with the same Ts. If you specify gainfprofile in continuous time, the tuning software discretizes it. Specifying the gain profile in discrete time gives you more control over the gain profile near the Nyquist frequency.

## **Properties**

#### MaxGain

Maximum gain as a function of frequency, expressed as a SISO zpk model.

The software automatically maps the gainvalue or gainprofile input arguments to a zpk model. The magnitude of this zpk model approximates the desired gain profile. The tuning goal derives and is stored in the MaxGain property. Use viewGoal(Req) to plot the magnitude of MaxGain.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Reg. Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Stabilize

Stability requirement on closed-loop dynamics, specified as 1 (true) or 0 (false).

By default, TuningGoal.Gain imposes a stability requirement on the closed-loop transfer function from the specified inputs to outputs, in addition to the gain requirement. If stability is not required or cannot be achieved, set Stabilize to false to remove the stability requirement. For example, if the gain constraint applies to an unstable open-loop transfer function, set Stabilize to false.

Default: 1(true)

#### **InputScaling**

Input signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued input signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

#### Default: []

#### **OutputScaling**

Output signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued output signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

#### Default: []

#### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

#### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

**Default: NaN** 

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

**Default:** {}

Name

Name of the tuning goal, specified as a character vector.

```
For example, if Req is a tuning goal:

Req.Name = 'LoopReq';

Default: []
```

## **Examples**

#### **Disturbance Rejection Goal**

Create a gain constraint that enforces a disturbance rejection requirement from a signal 'du' to a signal 'u'.

```
Reg = TuningGoal.Gain('du','u',1);
```

This requirement specifies that the maximum gain of the response from 'du' to 'u' not exceed 1 (0 dB).

#### **Custom roll-off specification**

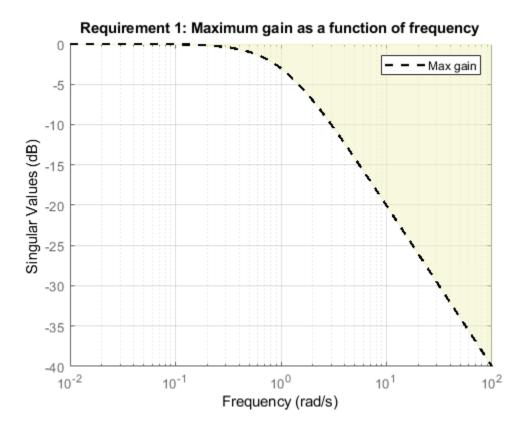
Create a tuning goal that constrains the response from a signal 'du' to a signal 'u' to roll off at 20 dB/decade at frequencies greater than 1. The tuning goal also specifies disturbance rejection (maximum gain of 1) in the frequency range [0,1].

```
gmax = frd([1 1 0.01],[0 1 100]);
Req = TuningGoal.Gain('du','u',gmax);
```

These commands use a frd model to specify the gain profile as a function of frequency. The maximum gain of 1 dB at the frequency 1 rad/s, together with the maximum gain of 0.01 dB at the frequency 100 rad/s, specifies the desired rolloff of 20 dB/decade.

The software converts gmax into a smooth function of frequency that approximates the piecewise specified requirement. Display the gain profile using viewGoal.

```
viewGoal(Req)
```



The dashed line shows the gain profile, and the region indicates where the requirement is violated.

## **Tips**

 This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal object, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Gain, f(x) is given by:

$$f(x) = \|W_F(s)D_o^{-1}T(s,x)D_i\|_{\infty},$$

or its discrete-time equivalent, for discrete-time tuning. Here, T(s,x) is the closed-loop transfer function from Input to Output.  $D_o$  and  $D_i$  are diagonal matrices with the OutputScaling and InputScaling property values on the diagonal, respectively.  $\|\cdot\|_{\infty}$  denotes the  $H_{\infty}$  norm (see getPeakGain).

The frequency weighting function  $W_F$  is the regularized gain profile, derived from the maximum gain profile you specify. The gains of  $W_F$  and 1/MaxGain roughly match inside the frequency band Focus.  $W_F$  is always stable and proper. Because poles of  $W_F$  close to s=0 or s= Inf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify maximum gain profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_F$ , use:

WF = getWeight(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts =  $\theta$  for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

## **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

TuningGoal.LoopShape|TuningGoal.Tracking|looptune|looptune (for slTuner)|makeweight|slTuner|systune|systune (for slTuner)|viewGoal

## **Topics**

"Frequency-Domain Specifications"

"Visualize Tuning Goals"

"Control of a Linear Electric Actuator"

"MIMO Control of Diesel Engine"

#### Introduced in R2016a

# **TuningGoal.LoopShape class**

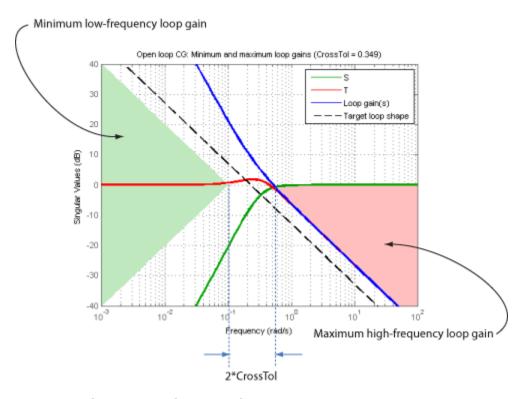
Package: TuningGoal

Target loop shape for control system tuning

## **Description**

Use TuningGoal.LoopShape to specify a target gain profile (gain as a function of frequency) of an open-loop response. TuningGoal.LoopShape constrains the open-loop, point-to-point response (L) at a specified location in your control system. Use this tuning goal for control system tuning with tuning commands, such as systune or looptune.

When you tune a control system, the target open-loop gain profile is converted into constraints on the inverse sensitivity function  $\operatorname{inv}(S) = (I + L)$  and the complementary sensitivity function T = 1-S. These constraints are illustrated for a representative tuned system in the following figure.



Where L is much greater than 1, a minimum gain constraint on  $\operatorname{inv}(S)$  (green shaded region) is equivalent to a minimum gain constraint on L. Similarly, where L is much smaller than 1, a maximum gain constraint on T (red shaded region) is equivalent to a maximum gain constraint on L. The gap between these two constraints is twice the CrossTol parameter, which specifies the frequency band where the loop gain can cross 0 dB.

For multi-input, multi-output (MIMO) control systems, values in the gain profile greater than 1 are interpreted as minimum performance requirements. Such values are lower bounds on the smallest singular value of the open-loop response. Gain profile values less than one are interpreted as minimum roll-off requirements, which are upper bounds on the largest singular value of the open-loop response. For more information about singular values, see sigma.

Use TuningGoal.LoopShape when the loop shape near crossover is simple or well understood (such as integral action). To specify only high gain or low gain constraints in certain frequency bands, use TuningGoal.MinLoopGain and

TuningGoal.MaxLoopGain. When you do so, the software determines the best loop shape near crossover.

## Construction

Req = TuningGoal.LoopShape(location,loopgain) creates a tuning goal for shaping the open-loop response measured at the specified location. The magnitude of the single-input, single-output (SISO) transfer function loopgain specifies the target open-loop gain profile. You can specify the target gain profile (maximum gain across the I/O pair) as a smooth transfer function or sketch a piecewise error profile using an frd model.

Req = TuningGoal.LoopShape(location,loopgain,crosstol) specifies a tolerance on the location of the crossover frequency. crosstol expresses the tolerance in decades. For example, crosstol = 0.5 allows gain crossovers within half a decade on either side of the target crossover frequency specified by loopgain. When you omit crosstol, the tuning goal uses a default value of 0.1 decades. You can increase crosstol when tuning MIMO control systems. Doing so allows more widely varying crossover frequencies for different loops in the system.

Req = TuningGoal.LoopShape(location,wc) specifies just the target gain crossover frequency. This syntax is equivalent to specifying a pure integrator loop shape, loopgain = wc/s.

Req = TuningGoal.LoopShape(location,wcrange) specifies a range for the target gain crossover frequency. The range is a vector of the form wcrange = [wc1,wc2]. This syntax is equivalent to using the geometric mean sqrt(wc1\*wc2) as wc and setting crosstol to the half-width of wcrange in decades. Using a range instead of a single wc value increases the ability of the tuning algorithm to enforce the target loop shape for all loops in a MIMO control system.

## **Input Arguments**

#### location

Location where the open-loop response shape to be constrained is measured, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. What locations are available depends on what kind of system you are tuning:

- If you are tuning a Simulink model of a control system, you can use any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. For example, if the slTuner interface contains an analysis point u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.
- If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

The loop shape requirement applies to the point-to-point open-loop transfer function at the specified location. That transfer function is the open-loop response obtained by injecting signals at the location and measuring the return signals at the same point.

If location specifies multiple locations, then the loop-shape requirement applies to the MIMO open-loop transfer function.

### loopgain

Target open-loop gain profile as a function of frequency.

You can specify loopgain as a smooth SISO transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise gain profile using a frd model or the makeweight function. When you do so, the software automatically maps your specified gain profile to a zpk model whose magnitude approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of that zpk model.

For multi-input, multi-output (MIMO) control systems, values in the gain profile greater than 1 are interpreted as minimum performance requirements. These values are lower bounds on the smallest singular value of L. Gain profile values less than one are interpreted as minimum roll-off requirements, which are upper bounds on the largest singular value of L. For more information about singular values, see sigma.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify loopgain as a discrete-time model with the same Ts. If you

specify loopgain in continuous time, the tuning software discretizes it. Specifying the loop shape in discrete time gives you more control over the loop shape near the Nyquist frequency.

#### crosstol

Tolerance in the location of crossover frequency, in decades. specified as a scalar value. For example, crosstol = 0.5 allows gain crossovers within half a decade on either side of the target crossover frequency specified by loopgain. Increasing crosstol increases the ability of the tuning algorithm to enforce the target loop shape for all loops in a MIMO control system.

#### **Default:** 0.1

#### WC

Target crossover frequency, specified as a positive scalar value. Express wc in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the control system model you are tuning.

#### wcrange

Range for target crossover frequency, specified as a vector of the form [wc1,wc2]. Express wc in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the control system model you are tuning.

## **Properties**

#### LoopGain

Target loop shape as a function of frequency, specified as a SISO zpk model.

The software automatically maps the input argument loopgain onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of the zpk model LoopGain.

#### CrossTol

Tolerance on gain crossover frequency, in decades.

The initial value of CrossTol is set by the crosstol input when you create the tuning goal.

#### Default: 0.1

#### Focus

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0, Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Stabilize

Stability requirement on closed-loop dynamics, specified as 1 (true) or 0 (false).

When Stabilize is true, this requirement stabilizes the specified feedback loop, as well as imposing gain or loop-shape requirements. Set Stabilize to false if stability for the specified loop is not required or cannot be achieved.

#### **Default:** 1 (true)

### LoopScaling

Toggle for automatically scaling loop signals, specified as 'on' or 'off'.

In multi-loop or MIMO control systems, the feedback channels are automatically rescaled to equalize the off-diagonal terms in the open-loop transfer function (loop interaction terms). Set LoopScaling to 'off' to disable such scaling and shape the unscaled open-loop response.

Default: 'on'

#### Location

Location at which the open-loop response shape to be constrained is measured, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location =  $\{'u'\}$ , the tuning goal evaluates the open-loop response measured at an analysis point 'u'. If Location =  $\{'u1', 'u2'\}$ , the tuning goal evaluates the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The initial value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

**Default: NaN** 

### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

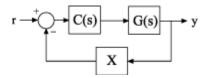
```
Req.Name = 'LoopReq';
```

**Default:** []

# **Examples**

### **Loop Shape and Crossover Tolerance**

Create a target gain profile requirement for the following control system. Specify integral action, gain crossover at 1, and a roll-off requirement of 40 dB/decade.



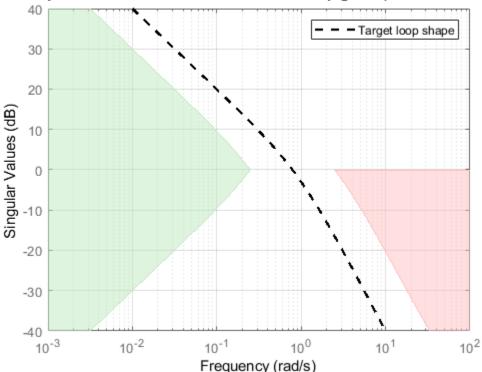
The requirement should apply to the open-loop response measured at the AnalysisPoint block X. Specify a crossover tolerance of 0.5 decades.

```
LS = frd([100 1 0.0001],[0.01 1 100]);
Reg = TuningGoal.LoopShape('X',LS,0.5);
```

The software converts LS into a smooth function of frequency that approximates the piecewise-specified requirement. Display the requirement using viewGoal.

viewGoal(Req)

### Requirement 1: Minimum and maximum loop gains (CrossTol = 0.5)

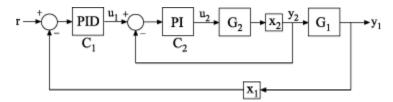


The green and red regions indicate the bounds for the inverse sensitivity, inv(S) = 1-G\*C, and the complementary sensitivity, T = 1-S, respectively. The gap between these regions at 0 dB gain reflects the specified crossover tolerance, which is half a decade to either side of the target loop crossover.

When you use viewGoal (Req,CL) to validate a tuned closed-loop model of this control system, CL, the tuned values of S and T are also plotted.

#### **Specify Different Loop Shapes for Multiple Loops**

Create separate loop shape requirements for the inner and outer loops of the following control system.



For the inner loop, specify a loop shape with integral action, gain crossover at 1, and a roll-off requirement of 40 dB/decade. Additionally, specify that this loop shape requirement should be enforced with the outer loop open.

```
LS2 = frd([100 1 0.0001],[0.01 1 100]);
Req2 = TuningGoal.LoopShape('X2',LS2);
Req2.Openings = 'X1';
```

Specifying 'X2' for the location indicates that Req2 applies to the point-to point, open-loop transfer function at the location X2. Setting Req2.Openings indicates that the loop is opened at the analysis point X1 when Req2 is enforced.

By default, Req2 imposes a stability requirement on the inner loop as well as the loop shape requirement. In some control systems, however, inner-loop stability might not be required, or might be impossible to achieve. In that case, remove the stability requirement from Req2 as follows.

```
Req2.Stabilize = false;
```

For the outer loop, specify a loop shape with integral action, gain crossover at 0.1, and a roll-off requirement of 20 dB/decade.

```
LS1 = frd([10 1 0.01],[0.01 0.1 10]);
Req1 = TuningGoal.LoopShape('X1',LS1);
```

Specifying 'X1' for the location indicates that Req1 applies to the point-to point, open-loop transfer function at the location X1. You do not have to set Req1.Openings because this loop shape is enforced with the inner loop closed.

You might want to tune the control system with both loop shaping requirements Req1 and Req2. To do so, use both requirements as inputs to the tuning command. For example,

suppose CL0 is a tunable genss model of the closed-loop control system. In that case, use [CL,fSoft] = systume(CL0,[Req1,Req2]) to tune the control system to both requirements.

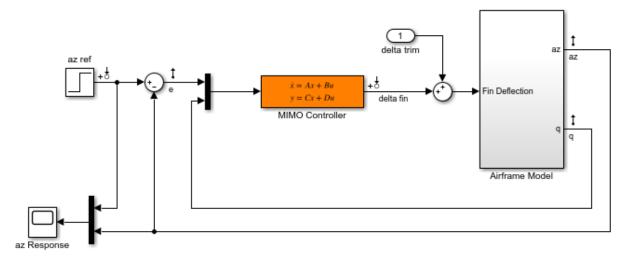
### **Loop Shape for Tuning Simulink Model**

Create a loop-shape requirement for the feedback loop on 'q' in the Simulink model rct\_airframe2. Specify that the loop-shape requirement is enforced with the 'az' loop open.

Open the model.

```
open_system('rct_airframe2')
```

#### Two-loop autopilot for controlling the vertical acceleration of an airframe



Create a loop shape requirement that enforces integral action with a crossover a 2 rad/s for the 'q' loop. This loop shape corresponds to a loop shape of 2/s.

```
s = tf('s');
shape = 2/s;
Req = TuningGoal.LoopShape('q',shape);
```

Specify the location at which to open an additional loop when enforcing the requirement.

```
Req.Openings = 'az';
```

To use this requirement to tune the Simulink model, create an slTuner interface to the model. Identify the block to tune in the interface.

```
ST0 = slTuner('rct_airframe2','MIMO Controller');
```

Designate both az and q as analysis points in the slTuner interface.

```
addPoint(ST0,{'az','q'});
```

This command makes q available as an analysis location. It also allows the tuning requirement to be enforced with the loop open at az.

You can now tune the model using Req and any other tuning requirements. For example:

```
[ST,fSoft] = systume(ST0,Req);
Final: Soft = 0.845, Hard = -Inf, Iterations = 51
```

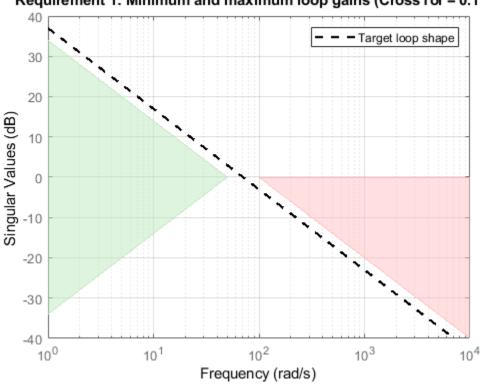
### **Loop Shape Requirement with Crossover Range**

Create a tuning requirement specifying that the open-loop response of loop identified by 'X' cross unity gain between 50 and 100 rad/s.

```
Req = TuningGoal.LoopShape('X',[50,100]);
```

Examine the resulting requirement to see the target loop shape.

```
viewGoal(Req)
```



Requirement 1: Minimum and maximum loop gains (CrossTol = 0.151)

The plot shows that the requirement specifies an integral loop shape, with crossover around 70 rad/s, the geometrical mean of the range [50,100]. The gap at 0 dB between the minimum low-frequency gain (green region) and the maximum high-frequency gain (red region) reflects the allowed crossover range [50,100].

## **Tips**

This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of

systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.LoopShape, f(x) is given by:

$$f(x) = \left\| \frac{W_S S}{W_T T} \right\|_{\infty}.$$

Here,  $S = D^{-1}[I - L(s,x)]^{-1}D$  is the scaled sensitivity function at the specified location, where L(s,x) is the open-loop response being shaped. D is an automatically-computed loop scaling factor. (If the LoopScaling property is set to 'off', then D = I.) T = S - I is the complementary sensitivity function.

 $W_S$  and  $W_T$  are frequency weighting functions derived from the specified loop shape. The gains of these functions roughly match LoopGain and 1/LoopGain, for values ranging from -20 dB to 60 dB. For numerical reasons, the weighting functions level off outside this range, unless the specified loop gain profile changes slope for gains above 60 dB or below -60 dB. Because poles of  $W_S$  or  $W_T$  close to s=0 or s=1nf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify loop shapes with very low-frequency or very high-frequency dynamics.

To obtain  $W_S$  and  $W_T$ , use:

[WS,WT] = getWeights(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts = 0 for continuous time). For more information about the effects of the weighting functions on numeric stability, see "Visualize Tuning Goals".

## **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

TuningGoal.Gain | TuningGoal.MaxLoopGain | TuningGoal.MinLoopGain |
TuningGoal.Tracking | frd | looptune | looptune (for slTuner) | slTuner |
systune | systune (for slTuner) | viewGoal

## **Topics**

"Loop Shape and Stability Margin Specifications"

"Visualize Tuning Goals"

"Tuning Multiloop Control Systems"

"Tuning of a Digital Motion Control System"

#### Introduced in R2016a

# **TuningGoal.LQG class**

Package: TuningGoal

Linear-Quadratic-Gaussian (LQG) goal for control system tuning

## **Description**

Use TuningGoal.LQG to specify a tuning goal that quantifies control performance as an LQG cost. It is applicable to any control structure, not just the classical observer structure of optimal LQG control. You can use this tuning goal for control system tuning with tuning commands, such as systume or looptune.

The LQG cost is given by:

$$J = E(z(t)' QZ z(t)).$$

z(t) is the system response to a white noise input vector w(t). The covariance of w(t) is given by:

$$E(w(t)w(t)') = QW.$$

The vector w(t) typically consists of external inputs to the system such as noise, disturbances, or command. The vector z(t) includes all the system variables that characterize performance, such as control signals, system states, and outputs. E(x) denotes the expected value of the stochastic variable x.

The cost function *J* can also be written as an average over time:

$$J = \lim_{T \to \infty} E\left(\frac{1}{T} \int_0^T z(t)' \, QZ \, z(t) dt\right).$$

After you create a tuning goal, you can further configure it by setting "Properties" on page 1-51 of the object.

## Construction

Req = TuningGoal.LQG(wname, zname, QW, QZ) creates an LQG requirement. wname and zname specify the signals making up w(t) and z(t). The matrices QW and QZ specify

the noise covariance and performance weight. These matrices must be symmetric nonnegative definite. Use scalar values for QW and QZ to specify multiples of the identity matrix.

### **Input Arguments**

#### wname

Noise inputs, w(t), specified as a character vector or a cell array of character vectors, that designate the signals making up w(t) by name, such as 'w' or {'w', 'v'}. The signals available to designate as noise inputs for the tuning goal are as follows.

- If you are using the tuning goal to tune a Simulink model of a control system, then wname can include:
  - · Any model input
  - Any linearization input point in the model
  - Any signal identified as a Controls, Measurements, Switches, or IOs signal in an slTuner interface associated with the Simulink model
- If you are using the tuning goal to tune a generalized state-space model (genss) of a control system using systune, then wname can include:
  - Any input of the control system model
  - Any channel of an AnalysisPoint block in the control system model

For example, if you are tuning a control system model, T, then wname can be an input name contained in T. InputName. Also, if T contains an AnalysisPoint block with a location named X, then wname can include X.

- If you are using the tuning goal to tune a controller model, C0 for a plant G0, using looptune, then wname can include:
  - Any input of C0 or G0
  - Any channel of an AnalysisPoint block in CO or GO

If wname is a channel of an AnalysisPoint block of a generalized model, the noise input for the tuning goal is the implied input associated with the switch:



#### zname

Performance outputs, z(t), specified as a character vector or a cell array of character vectors, that designate the signals making up z(t) by name, such as 'y' or {'y', 'u'}. The signals available to designate as performance outputs for the tuning goal are as follows.

- If you are using the tuning goal to tune a Simulink model of a control system, then zname can include:
  - Any model output
  - Any linearization output point in the model
  - Any signal identified as a Controls, Measurements, Switches, or IOs signal in an slTuner interface associated with the Simulink model
- If you are using the tuning goal to tune a generalized state-space model (genss) of a control system using systume, then zname can include:
  - Any output of the control system model
  - Any channel of an AnalysisPoint block in the control system model

For example, if you are tuning a control system model, T, then zname can be an output name contained in T.OutputName. Also, if T contains an AnalysisPoint block with a channel named X, then zname can include X.

- If you are using the tuning goal to tune a controller model, C0 for a plant G0, using looptune, then zname can include:
  - Any input of C0 or G0
  - Any channel of an AnalysisPoint block in CO or GO

If zname is a channel of an AnalysisPoint block of a generalized model, the performance output for the tuning goal is the implied output associated with the switch:



#### **OW**

Covariance of the white noise input vector w(t), specified as a scalar or a matrix. Use a scalar value to specify a multiple of the identity matrix. Otherwise specify a symmetric nonnegative definite matrix with as many rows as there are entries in the vector w(t). A diagonal matrix means the entries of w(t) are uncorrelated.

The covariance of w(t) is given by:

$$E(w(t)w(t)') = QW.$$

When you are tuning a control system in discrete time, the LQG tuning goal assumes:

$$E(w[k]w[k]') = QW/T_s.$$

 $T_s$  is the model sample time. This assumption ensures consistent results with tuning in the continuous-time domain. In this assumption, w[k] is discrete-time noise obtained by sampling continuous white noise w(t) with covariance QW. If in your system w[k] is a truly discrete process with known covariance QWd, use the value  $T_s*QWd$  for the QW value when creating the LQG goal.

#### Default: I

#### QZ

Performance weights, specified as a scalar or a matrix. Use a scalar value to specify a multiple of the identity matrix. Otherwise specify a symmetric nonnegative definite matrix. Use a diagonal matrix to independently scale or penalize the contribution of each variable in z.

The performance weights contribute to the cost function according to:

$$J = E(z(t)') QZ \qquad z(t).$$

When you use the LQG tuning goal as a hard goal, the software tries to drive the cost function J < 1. When you use it as a soft goal, the cost function J is minimized subject to

any hard goals and its value is contributed to the overall objective function. Therefore, select QZ values to properly scale the cost function so that driving it below 1 or minimizing it yields the performance you require.

**Default:** I

## **Properties**

#### **NoiseCovariance**

Covariance matrix of the noise inputs w(t), specified as a matrix. The value of the NoiseCovariance property is set by the WZ input argument when you create the LQG tuning goal.

#### **PerformanceWeight**

Weights for the performance signals z(t), specified as a matrix. The value of the PerformanceWeight property is set by the QZ input argument when you create the LQG tuning goal.

### Input

Noise input signal names, specified as a cell array of character vectors. The input signal names specify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the wname input argument when you construct the tuning goal.

### **Output**

Performance output signal names, specified as a cell array of character vectors. The output signal names specify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the zname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systume, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model

array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if **Req** is a tuning goal:

```
Req.Name = 'LoopReq';
```

#### Default: []

## **Tips**

• When you use this tuning goal to tune a continuous-time control system, systune attempts to enforce zero feedthrough (D=0) on the transfer that the tuning goal constrains. Zero feedthrough is imposed because the  $H_2$  norm, and therefore the value of the tuning goal, is infinite for continuous-time systems with nonzero feedthrough.

systune enforces zero feedthrough by fixing to zero all tunable parameters that contribute to the feedthrough term. systune returns an error when fixing these tunable parameters is insufficient to enforce zero feedthrough. In such cases, you must modify the tuning goal or the control structure, or manually fix some tunable parameters of your system to values that eliminate the feedthrough term.

When the constrained transfer function has several tunable blocks in series, the software's approach of zeroing all parameters that contribute to the overall feedthrough might be conservative. In that case, it is sufficient to zero the feedthrough term of one of the blocks. If you want to control which block has feedthrough fixed to zero, you can manually fix the feedthrough of the tuned block of your choice.

To fix parameters of tunable blocks to specified values, use the Value and Free properties of the block parametrization. For example, consider a tuned state-space block:

```
C = tunableSS('C',1,2,3);
```

To enforce zero feedthrough on this block, set its D matrix value to zero, and fix the parameter.

```
C.D.Value = 0;
C.D.Free = false;
```

For more information on fixing parameter values, see the Control Design Block reference pages, such as tunableSS.

This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from wname to zname, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x), or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.LQG, f(x) is given by the cost function J:

$$J = E(z(t)' QZ z(t)).$$

When you use the LQG requirement as a hard goal, the software tries to drive the cost function J < 1. When you use it as a soft goal, the cost function J is minimized subject to any hard goals and its value is contributed to the overall objective function. Therefore, select QZ values to properly scale the cost function so that driving it below 1 or minimizing it yields the performance you require.

## **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Variance | TuningGoal.WeightedVariance | evalGoal | slTuner |
systume | systume (for slTuner) | viewGoal

### **Topics**

"Vibration Control in Flexible Beam" "Time-Domain Specifications"

#### Introduced in R2016a

# **TuningGoal.Margins class**

Package: TuningGoal

Stability margin requirement for control system tuning

## **Description**

Use TuningGoal.Margins to specify a tuning goal for the gain and phase margins of a SISO or MIMO feedback loop. You can use this tuning goal for validating a tuned control system with viewGoal. You can also use the tuning goal for control system tuning with tuning commands such as systume or looptune.

After you create a tuning goal, you can configure it further by setting "Properties" on page 1-57 of the object.

After using the tuning goal to tune a control system, you can visualize the tuning goal and the tuned value using the viewGoal command. For information about interpreting the margins goal, see "Stability Margins in Control System Tuning".

## Construction

Req = TuningGoal.Margins(location,gainmargin,phasemargin) creates a tuning goal that specifies the minimum gain and phase margins at the specified location in the control system.

### **Input Arguments**

#### location

Location in the control system at which the minimum gain and phase margins apply, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. What locations are available depends on what kind of system you are tuning:

• If you are tuning a Simulink model of a control system, you can use any linear analysis point marked in the model, or any linear analysis point in an slTuner interface

associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. For example, if the slTuner interface contains an analysis point u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

• If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

The margin requirements apply to the point-to-point, open-loop transfer function at the specified loop-opening location. That transfer function is the open-loop response obtained by injecting signals at the specified location, and measuring the return signals at the same point.

If location is a cell array, then the margin requirement applies to the MIMO open-loop transfer function.

### gainmargin

Required minimum gain margin for the feedback loop, specified as a scalar value in dB.

For MIMO feedback loops, the gain margin is based upon the notion of disk margins, which guarantee stability for concurrent gain and phase variations of <code>±gainmargin</code> and <code>±phasemargin</code> in all feedback channels. See "Stability Analysis Using Disk Margins" (Robust Control Toolbox) for more information about disk margins.

### phasemargin

Required minimum phase margin for the feedback loop, specified as a scalar value in degrees.

For MIMO feedback loops, the phase margin is based upon the notion of disk margins, which guarantee stability for concurrent gain and phase variations of <code>±gainmargin</code> and <code>±phasemargin</code> in all feedback channels. See "Stability Analysis Using Disk Margins" (Robust Control Toolbox) for more information about disk margins.

## **Properties**

#### GainMargin

Required minimum gain margin for the feedback loop, specified as a scalar value in decibels (dB).

The value of the GainMargin property is set by the gainmargin input argument when you create the tuning goal.

#### **PhaseMargin**

Required minimum phase margin for the feedback loop, specified as a scalar value in degrees.

The value of the PhaseMargin property is set by the phasemargin input argument when you create the tuning goal.

### ScalingOrder

Controls the order (number of states) of the scalings involved in computing MIMO stability margins. Static scalings (ScalingOrder = 0) are used by default. Increasing the order may improve results at the expense of increased computations. Use viewGoal to assess the gap between optimized and actual margins. If this gap is too large, consider increasing the scaling order. See "Stability Margins in Control System Tuning".

**Default:** 0 (static scaling)

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. For best results with stability margin requirements, pick a frequency band extending about one decade on each side of the gain crossover frequencies. For example, suppose Req is a TuningGoal.Margins object that you are using to tune a system with approximately 10 rad/s bandwidth. To limit the enforcement of the tuning goal, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Location

Location at which the minimum gain and phase margins apply, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location = {'u'}, the tuning goal enforces the minimum gain and phase margins at an analysis point 'u'.

The value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### Default: NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Reg is a tuning goal:

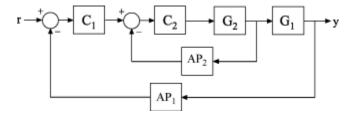
Req.Name = 'LoopReq';

**Default:** []

## **Examples**

### SISO Margin Requirement Evaluated with Additional Loop Opening

Create a margin requirement for the inner loop of the following control system. The requirement imposes a minimum gain margin of  $5\ dB$  and a minimum phase margin of  $40\ degrees$ .



Create a model of the system. To do so, specify and connect the numeric plant models G1 and G2, and the tunable controllers C1 and C2. Also specify and connect the AnalysisPoint blocks AP1 and AP2 that mark points of interest for analysis and tuning.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
AP1 = AnalysisPoint('AP1');
AP2 = AnalysisPoint('AP2');
T = feedback(G1*feedback(G2*C2,AP2)*C1,AP1);
Create a tuning requirement object.
```

Req = TuningGoal.Margins('AP2',5,40);

```
This requirement imposes the specified stability margins on the feedback loop identified by the AnalysisPoint channel 'AP2', which is the inner loop.
```

Specify that these margins are evaluated with the outer loop of the control system open.

```
Req.Openings = {'AP1'};
```

Adding 'AP1' to the Openings property of the tuning requirements object ensures that systume evaluates the requirement with the loop open at that location.

Use systume to tune the free parameters of T to meet the tuning requirement specified by Req. You can then use viewGoal to validate the tuned control system against the requirement.

### **MIMO Margin Requirement in Frequency Band**

Create a requirement that sets minimum gain and phase margins for the loop defined by three loop-opening locations in a control system to tune. Because this loop is defined by three loop-opening locations, it is a MIMO loop.

The requirement sets a minimum gain margin of 10 dB and a minimum phase margin of 40 degrees, within the band between 0.1 and 10 rad/s.

```
Req = TuningGoal.Margins({'r','theta','phi'},10,40);
```

The names 'r', 'theta', and 'phi' must specify valid loop-opening locations in the control system that you are tuning.

Limit the requirement to the frequency band between 0.1 and 10 rad/s.

```
Req.Focus = [0.1 \ 10];
```

## **Tips**

• This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Margins, f(x) is given by:

$$f(x) = \|2\alpha S - \alpha I\|_{\infty}.$$

 $S = D^{-1}[I - L(s,x)]^{-1}D$  is the scaled sensitivity function.

L(s,x) is the open-loop response being shaped.

D is an automatically-computed loop scaling factor.

 $\boldsymbol{\alpha}$  is a scalar parameter computed from the specified gain and phase margin.

# **Compatibility Considerations**

## Functionality moved from Robust Control Toolbox

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

 $eval Goal \mid loop tune \mid loop tune \; (for \; slTuner) \mid systume \mid systume \; (for \; slTuner) \mid view Goal$ 

## **Topics**

"Loop Shape and Stability Margin Specifications"

"Tuning Control Systems with SYSTUNE"

"Digital Control of Power Stage Voltage"

"Tuning of a Two-Loop Autopilot"

"Fixed-Structure Autopilot for a Passenger Jet"

"Stability Margins in Control System Tuning"

#### Introduced in R2016a

# TuningGoal.MinLoopGain class

Package: TuningGoal

Minimum loop gain constraint for control system tuning

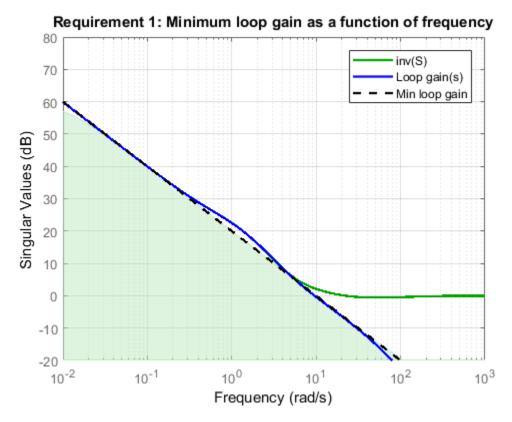
## **Description**

Use the TuningGoal.MinLoopGain object to enforce a minimum loop gain in a particular frequency band. Use this tuning goal with control system tuning commands such as systume or looptune.

This tuning goal imposes a minimum gain on the open-loop frequency response (L) at a specified location in your control system. You specify the minimum open-loop gain as a function of frequency (a minimum gain profile). For MIMO feedback loops, the specified gain profile is interpreted as a lower bound on the smallest singular value of L.

When you tune a control system, the minimum gain profile is converted to a minimum gain constraint on the inverse of the sensitivity function, inv(S) = (I + L).

The following figure shows a typical specified minimum gain profile (dashed line) and a resulting tuned loop gain, L (blue line). The shaded region represents gain profile values that are forbidden by this tuning goal. The figure shows that when L is much larger than 1, imposing a minimum gain on inv(S) is a good proxy for a minimum open-loop gain.



TuningGoal.MinLoopGain and TuningGoal.MaxLoopGain specify only low-gain or high-gain constraints in certain frequency bands. When you use these tuning goals, systune and looptune determine the best loop shape near crossover. When the loop shape near crossover is simple or well understood (such as integral action), you can use TuningGoal.LoopShape to specify that target loop shape.

## **Construction**

Req = TuningGoal.MinLoopGain(location,loopgain) creates a tuning goal for boosting the gain of a SISO or MIMO feedback loop. The tuning goal specifies that the open-loop frequency response (L) measured at the specified locations exceeds the minimum gain profile specified by loopgain.

You can specify the minimum gain profile as a smooth transfer function or sketch a piecewise error profile using an frd model or the makeweight command. Only gain values greater than 1 are enforced.

For MIMO feedback loops, the specified gain profile is interpreted as a lower bound on the smallest singular value of L.

Req = TuningGoal.MinLoopGain(location, fmin, gmin) specifies a minimum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmin at the specified frequency, fmin.

### **Input Arguments**

#### location

Location at which the maximum open-loop gain is constrained, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. What loop-opening locations are available depends on what kind of system you are tuning:

- If you are tuning a Simulink model of a control system, you can use any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. For example, if the slTuner interface contains an analysis point u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.
- If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

If location is a cell array of loop-opening locations, then the minimum gain goal applies to the resulting MIMO loop.

#### loopgain

Minimum open-loop gain as a function of frequency.

You can specify loopgain as a smooth SISO transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise gain profile using a frd model or the makeweight command. For example, the following frd model specifies a minimum gain of 100 (40 dB) below 0.1 rad/s, rolling off at a rate of -20 dB/dec at higher frequencies.

```
loopgain = frd([100 \ 100 \ 10],[0 \ 1e-1 \ 1]);
```

When you use an frd model to specify loopgain, the software automatically maps your specified gain profile to a zpk model. The magnitude of this model approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of that zpk model.

Only gain values larger than 1 are enforced. For multi-input, multi-output (MIMO) feedback loops, the gain profile is interpreted as a lower bound on the smallest singular value of L. For more information about singular values, see sigma.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify loopgain as a discrete-time model with the same Ts. If you specify loopgain in continuous time, the tuning software discretizes it. Specifying the loop gain in discrete time gives you more control over the loop gain near the Nyquist frequency.

#### fmin

Frequency of minimum gain gmin, specified as a scalar value in rad/s.

Use this argument to specify a minimum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmin at the specified frequency, fmin.

#### gmin

Value of minimum gain occurring at fmin, specified as a scalar absolute value.

Use this argument to specify a minimum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmin at the specified frequency, fmin.

# **Properties**

#### MinGain

Minimum open-loop gain as a function of frequency, specified as a SISO zpk model.

The software automatically maps the input argument loopgain onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Alternatively, if you use the fmin and gmin arguments to specify the gain profile, this property is set to K/s. The software chooses K such that the gain value is gmin at the specified frequency, fmin.

Use viewGoal (Req) to plot the magnitude of the open-loop minimum gain profile.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Stabilize

Stability requirement on closed-loop dynamics, specified as 1 (true) or 0 (false).

When Stabilize is true, this requirement stabilizes the specified feedback loop, as well as imposing gain or loop-shape requirements. Set Stabilize to false if stability for the specified loop is not required or cannot be achieved.

Default: 1 (true)

## LoopScaling

Toggle for automatically scaling loop signals, specified as 'on' or 'off'.

In multi-loop or MIMO control systems, the feedback channels are automatically rescaled to equalize the off-diagonal terms in the open-loop transfer function (loop interaction terms). Set LoopScaling to 'off' to disable such scaling and shape the unscaled open-loop response.

Default: 'on'

#### Location

Location at which minimum loop gain is constrained, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location = {'u'}, the tuning goal evaluates the open-loop response measured at an analysis point 'u'. If Location = {'u1', 'u2'}, the tuning goal evaluates the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

**Default:** NaN

### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear

analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

## **Minimum Loop Gain Tuning Goal**

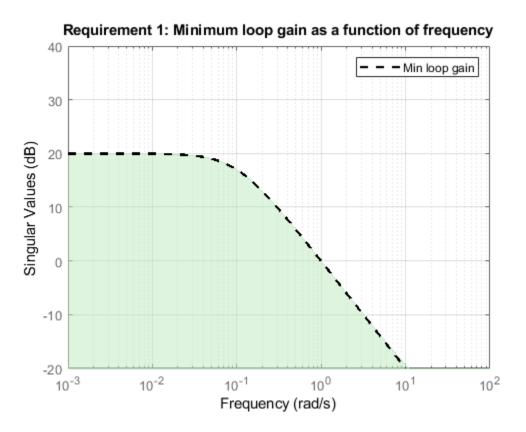
Create a tuning goal that boosts the open-loop gain of a feedback loop to at least a specified profile.

Suppose that you are tuning a control system that has a loop-opening location identified by PILoop. Specify that the open-loop gain measured at that location exceeds a minimum gain of 10 (20 dB) below 0.1 rad/s, rolling off at a rate of -20 dB/dec at higher frequencies. Use an frd model to sketch this gain profile.

```
loopgain = frd([10 10 0.1],[0 1e-1 10]);
Req = TuningGoal.MinLoopGain('PILoop',loopgain);
```

The software converts loopgain into a smooth function of frequency that approximates the piecewise-specified gain profile. Display the tuning goal using viewGoal.

viewGoal(Req)



The dashed line shows the specified the gain profile. The shaded region indicates where the tuning goal is violated, except that gain values less than 1 are not enforced. Therefore, this tuning goal only specifies a minimum gain at frequencies below 1 rad/s.

You can use Req as an input to looptune or systune when tuning the control system. Then use viewGoal(Req,T) to compare the tuned loop gain to the minimum gain specified in the tuning goal, where T represents the tuned control system.

### Integral Minimum Gain Specified as Gain Value at Single Frequency

Create a tuning goal that specifies a minimum loop gain profile of the form L = K / s. The gain profile attains the value of -20 dB (0.01) at 100 rad/s.

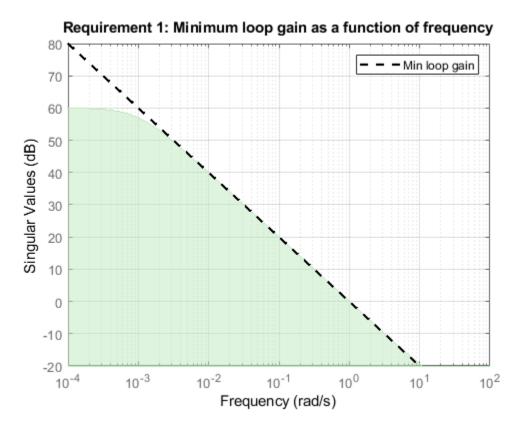
Req = TuningGoal.MinLoopGain('X',100,0.01);
viewGoal(Req)

# Requirement 1: Minimum loop gain as a function of frequency 50 Min loop gain 40 30 Singular Values (dB) 20 0 -10 -20 10<sup>-2</sup> 10<sup>0</sup> 10<sup>2</sup> 10<sup>1</sup> $10^{-1}$ Frequency (rad/s)

viewGoal confirms that the tuning goal is correctly specified. You can use this tuning goal to tune a control system that has a loop-opening location identified as 'X'. Since loop gain values less than 1 are ignored, this tuning goal specifies minimum gain only below 1 rad/s, with no restriction on loop gain at higher frequency.

Although the specified gain profile (dashed line) is a pure integrator, for numeric reasons, the gain profile enforced during tuning levels off at very low frequencies, as described in "Algorithms" on page 1-76. To see the regularized gain profile, expand the axes of the tuning-goal plot.

xlim([10^-4,10^2]) ylim([-20,80])

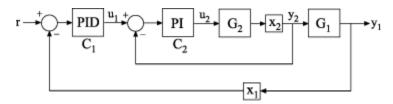


The shaded region reflects the modified gain profile.

### Minimum Loop Gain as Constraint on Sensitivity Function

Examine a minimum loop gain tuning goal against the tuned loop gain. A minimum loop gain tuning goal is converted to a constraint on the gain of the sensitivity function at the location specified in the tuning goal.

To see this relationship between the minimum loop gain and the sensitivity function, tune the following closed-loop system with analysis points at X1 and X2. The control system has tunable PID controllers C1 and C2.



Create a model of the control system.

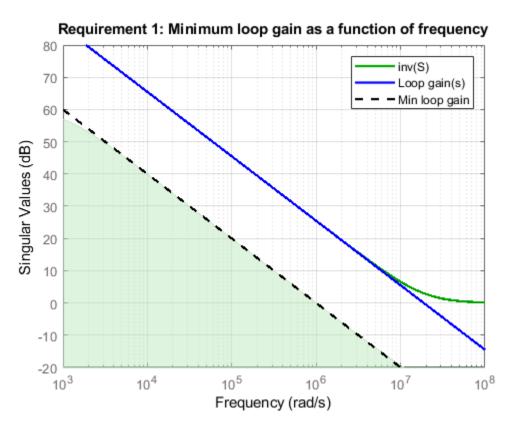
```
G2 = zpk([],-2,3);
G1 = zpk([],[-1 -1 -1],10);
C20 = tunablePID('C2','pi');
C10 = tunablePID('C1','pid');
X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
InnerLoop = feedback(X2*G2*C20,1);
CL0 = feedback(G1*InnerLoop*C10,X1);
CL0.InputName = 'r';
CL0.OutputName = 'y';
```

Specify some tuning goals, including a minimum loop gain. Tune the control system to these requirements.

```
Rtrack = TuningGoal.Tracking('r','y',10,0.01);
Rreject = TuningGoal.Gain('X2','y',0.1);
Rgain = TuningGoal.MinLoopGain('X2',100,10000);
Rgain.Openings = 'X1';

opts = systuneOptions('RandomStart',2);
rng('default'); % for reproducibility
[CL,fSoft] = systune(CL0,[Rtrack,Rreject,Rgain]);
Final: Soft = 2.81, Hard = -Inf, Iterations = 80
```

Examine the TuningGoal.MinLoopGain goal against the corresponding tuned response. viewGoal(Rgain,CL)

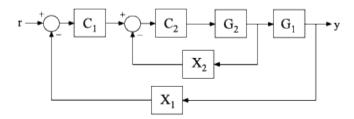


The plot shows the achieved loop gain for the loop at X2 (blue line). The plot also shows the inverse of the achieved sensitivity function, S, at the location X2 (green line). The inverse sensitivity function at this location is given by inv(S) = I+L. Here, L is the open-loop point-to-point loop transfer measured at X2.

The minimum loop gain goal Rgain is constraint on inv(S), represented in the plot by the green shaded region. The constraint on inv(S) can be thought of as a minimum gain constraint on L that applies where the gain of L (or the smallest singular value of L, for MIMO loops) is greater than 1.

### Loop-Gain Requirement without Stability Constraint on Inner Loop

Create requirements that specify a minimum loop gain of 20 dB (100) at 50 rad/s and a maximum loop gain of -20 dB (0.01) at 1000 rad/s on the inner loop of the following control system.



Create the maximum and minimum loop gain requirements.

```
RMinGain = TuningGoal.MinLoopGain('X2',50,100);
RMaxGain = TuningGoal.MaxLoopGain('X2',1000,0.01);
```

Configure the requirements to apply to the loop gain of the inner loop measured with the outer loop open.

```
RMinGain.Openings = 'X1';
RMaxGain.Openings = 'X1';
```

Setting Req.Openings tells the tuning algorithm to enforce the requirements with a loop open at the specified location. With the outer loop open, the requirements apply only to the inner loop.

By default, tuning using TuningGoal.MinLoopGain or TuningGoal.MaxLoopGain imposes a stability requirement as well as the minimum or maximum loop gain. Practically, in some control systems it is not possible to achieve a stable inner loop. In that case, remove the stability requirement for the inner loop by setting the Stabilize property to false.

```
RMinGain.Stabilize = false;
RMaxGain.Stabilize = false;
```

When you tune using either of these requirements, the tuning algorithm still imposes a stability requirement on the overall tuned control system, but not on the inner loop alone.

# **Tips**

• This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). Here, x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.MinLoopGain, f(x) is given by:

$$f(x) = \left\| W_S \left( D^{-1} S D \right) \right\|_{\infty}.$$

Here, D is a diagonal scaling (for MIMO loops). S is the sensitivity function at Location.  $W_S$  is a frequency-weighting function derived from the minimum loop gain profile, MinGain. The gain of this function roughly matches MaxGain for values ranging from -20 dB to 60 dB. For numerical reasons, the weighting function levels off outside this range, unless the specified gain profile changes slope outside this range. This adjustment is called regularization. Because poles of  $W_S$  close to S=0 or S=1 inf might lead to poor numeric conditioning of the systume optimization problem, it is not recommended to specify gain profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_S$ , use:

WS = getWeight(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts = 0 for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

Although S is a closed-loop transfer function, driving f(x) < 1 is equivalent to enforcing a lower bound on the open-loop transfer function, L, in a frequency band where the gain of L is greater than 1. To see why, note that S = 1/(1 + L). For SISO loops, when |L| >> 1,  $|S| \approx 1/|L|$ . Therefore, enforcing the open-loop minimum gain requirement,  $|L| > |W_S|$ , is roughly equivalent to enforcing  $|W_SS| < 1$ . For MIMO loops, similar reasoning applies, with  $||S|| \approx 1/\sigma_{\min}(L)$ , where  $\sigma_{\min}$  is the smallest singular value.

For an example illustrating the constraint on S, see "Minimum Loop Gain as Constraint on Sensitivity Function" on page 1-72.

# **Compatibility Considerations**

## Functionality moved from Robust Control Toolbox

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

TuningGoal.Gain | TuningGoal.LoopShape | TuningGoal.Margins |
TuningGoal.MaxLoopGain | evalGoal | looptune | looptune (for slTuner) |
sigma | slTuner | systune | systune (for slTuner) | viewGoal

## **Topics**

"Loop Shape and Stability Margin Specifications"

"Visualize Tuning Goals"

"PID Tuning for Setpoint Tracking vs. Disturbance Rejection"

### Introduced in R2016a

# TuningGoal.MaxLoopGain class

Package: TuningGoal

Maximum loop gain constraint for control system tuning

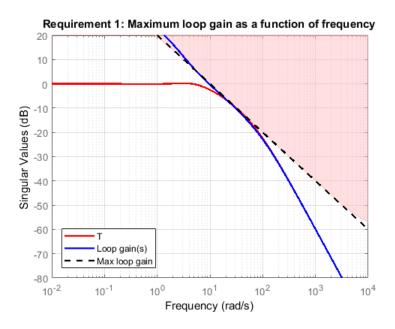
# **Description**

Use TuningGoal.MaxLoopGain to enforce a maximum loop gain and desired roll-off in a particular frequency band. Use this tuning goal with control system tuning commands such as systume or looptune.

This tuning goal imposes a maximum gain on the open-loop frequency response (L) at a specified location in your control system. You specify the maximum open-loop gain as a function of frequency (a maximum gain profile). For MIMO feedback loops, the specified gain profile is interpreted as an upper bound on the largest singular value of L.

When you tune a control system, the maximum gain profile is converted to a maximum gain constraint on the complementary sensitivity function, T) = L/(I + L).

The following figure shows a typical specified maximum gain profile (dashed line) and a resulting tuned loop gain, L (blue line). The shaded region represents gain profile values that are forbidden by this tuning goal. The figure shows that when L is much smaller than 1, imposing a maximum gain on T is a good proxy for a maximum open-loop gain.



TuningGoal.MaxLoopGain and TuningGoal.MinLoopGain specify only high-gain or low-gain constraints in certain frequency bands. When you use these tuning goals, systune and looptune determine the best loop shape near crossover. When the loop shape near crossover is simple or well understood (such as integral action), you can use TuningGoal.LoopShape to specify that target loop shape.

## Construction

Req = TuningGoal.MaxLoopGain(location,loopgain) creates a tuning goal for limiting the gain of a SISO or MIMO feedback loop. The tuning goal limits the open-loop frequency response measured at the specified locations to the maximum gain profile specified by loopgain. You can specify the maximum gain profile as a smooth transfer function or sketch a piecewise error profile using an frd model or the makeweight command. Only gain values smaller than 1 are enforced.

Req = TuningGoal.MaxLoopGain(location, fmax, gmax) specifies a maximum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmax at the specified frequency, fmax.

## **Input Arguments**

#### location

Location at which the maximum open-loop gain is constrained, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. What loop-opening locations are available depends on what kind of system you are tuning:

- If you are tuning a Simulink model of a control system, you can use any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. For example, if the slTuner interface contains an analysis point u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.
- If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

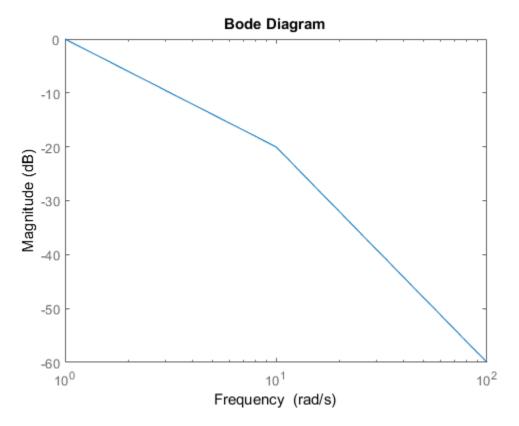
If location is a cell array of loop-opening locations, then the maximum gain requirement applies to the resulting MIMO loop.

## loopgain

Maximum open-loop gain as a function of frequency.

You can specify loopgain as a smooth SISO transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise gain profile using a frd model or the makeweight command. For example, the following frd model specifies a maximum gain of 1 (0 dB) at 1 rad/s, rolling off at a rate of -20 dB/dec up to 10 rad/s, and a rate of -40 dB/dec at higher frequencies.

```
loopgain = frd([1 1e-1 1e-3],[1 10 100]);
bodemag(loopgain)
```



When you use an frd model to specify loopgain, the software automatically maps your specified gain profile to a zpk model. The magnitude of this model approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of that zpk model.

Only gain values smaller than 1 are enforced. For multi-input, multi-output (MIMO) feedback loops, the gain profile is interpreted as a minimum roll-off requirement, which is an upper bound on the largest singular value of L. For more information about singular values, see sigma.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify loopgain as a discrete-time model with the same Ts. If you specify loopgain in continuous time, the tuning software discretizes it. Specifying the

loop gain in discrete time gives you more control over the loop gain near the Nyquist frequency.

#### fmax

Frequency of maximum gain gmax, specified as a scalar value in rad/s.

Use this argument to specify a maximum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmax at the specified frequency, fmax.

### gmax

Value of maximum gain occurring at fmax, specified as a scalar absolute value.

Use this argument to specify a maximum gain profile of the form loopgain = K/s (integral action). The software chooses K such that the gain value is gmax at the specified frequency, fmax.

# **Properties**

#### MaxGain

Maximum open-loop gain as a function of frequency, specified as a SISO zpk model.

The software automatically maps the input argument loopgain onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Alternatively, if you use the fmax and gmax arguments to specify the gain profile, this property is set to K/s. The software chooses K such that the gain value is gmax at the specified frequency, fmax.

Use viewGoal(Req) to plot the magnitude of the open-loop maximum gain profile.

### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply

only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Stabilize

Stability requirement on closed-loop dynamics, specified as 1 (true) or 0 (false).

When Stabilize is true, this requirement stabilizes the specified feedback loop, as well as imposing gain or loop-shape requirements. Set Stabilize to false if stability for the specified loop is not required or cannot be achieved.

**Default:** 1 (true)

### LoopScaling

Toggle for automatically scaling loop signals, specified as 'on' or 'off'.

In multi-loop or MIMO control systems, the feedback channels are automatically rescaled to equalize the off-diagonal terms in the open-loop transfer function (loop interaction terms). Set LoopScaling to 'off' to disable such scaling and shape the unscaled open-loop response.

Default: 'on'

#### Location

Location at which minimum loop gain is constrained, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location = {'u'}, the tuning goal evaluates the open-loop response measured at an analysis point 'u'. If Location = {'u1', 'u2'}, the tuning goal evaluates the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default: NaN**

### **Openings**

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then <code>Openings</code> can include any linear analysis point marked in the model, or any linear analysis point in an <code>slTuner</code> interface associated with the Simulink model. Use <code>addPoint</code> to add analysis points and loop openings to the <code>slTuner</code> interface. Use <code>getPoints</code> to get the list of analysis points available in an <code>slTuner</code> interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

### **Maximum Loop Gain Tuning Goal**

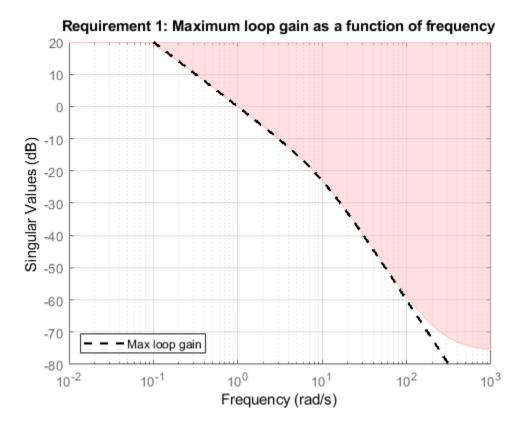
Create a tuning goal that limits the maximum open-loop gain of a feedback loop to a specified profile.

Suppose that you are tuning a control system that has a loop-opening location identified by PILoop. Limit the open-loop gain measured at that location to 1 (0 dB) at 1 rad/s, rolling off at a rate of -20 dB/dec up to 10 rad/s, and a rate of -40 dB/dec at higher frequencies. Use an frd model to sketch this gain profile.

```
loopgain = frd([1 le-1 le-3],[1 10 100]);
Req = TuningGoal.MaxLoopGain('PILoop',loopgain);
```

The software converts loopgain into a smooth function of frequency that approximates the piecewise-specified gain profile. Display the tuning goal using viewGoal.

```
viewGoal(Req)
```



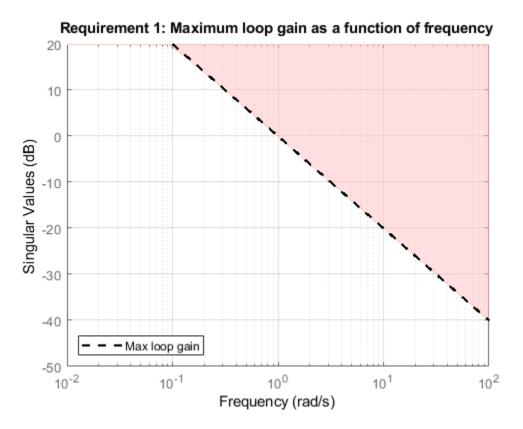
The dashed line shows the specified gain profile. The shaded region indicates where the tuning goal is violated, except that gain values greater than 1 are not enforced. Therefore, this tuning goal only specifies minimum roll-off rates at frequencies above 1 rad/s.

You can use Req as an input to looptune or systume when tuning the control system. Then use viewGoal(Req,T) to compare the tuned loop gain to the minimum gain specified in the tuning goal, where T represents the tuned control system.

### Integral Maximum Gain Specified as Gain Value at Single Frequency

Create a tuning goal that specifies a maximum loop gain of the form L = K/s. The maximum gain attains the value of -20 dB (0.01) at 100 rad/s.

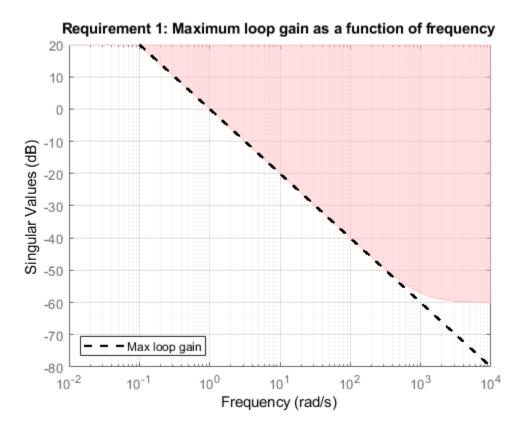
Req = TuningGoal.MaxLoopGain('X',100,0.01);
viewGoal(Req)



viewGoal confirms that the tuning goal is correctly specified. You can use this tuning goal to tune a control system that has a loop-opening location identified as 'X'. Since loop gain values greater than 1 are ignored, this requirement specifies a rolloff of 20 dB/decade above 1 rad/s, with no restriction on loop gain below that frequency.

Although the specified gain profile (dashed line) is a pure integrator, for numeric reasons, the gain profile enforced during tuning levels off at very high frequencies, as described in "Algorithms" on page 1-76. To see the regularized gain profile, expand the axes of the tuning-goal plot.

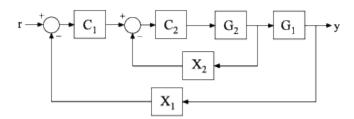
xlim([10^-2,10^4]) ylim([-80,20])



The shaded region reflects the modified gain profile.

### Loop-Gain Requirement without Stability Constraint on Inner Loop

Create requirements that specify a minimum loop gain of 20 dB (100) at 50 rad/s and a maximum loop gain of -20 dB (0.01) at 1000 rad/s on the inner loop of the following control system.



Create the maximum and minimum loop gain requirements.

```
RMinGain = TuningGoal.MinLoopGain('X2',50,100);
RMaxGain = TuningGoal.MaxLoopGain('X2',1000,0.01);
```

Configure the requirements to apply to the loop gain of the inner loop measured with the outer loop open.

```
RMinGain.Openings = 'X1';
RMaxGain.Openings = 'X1';
```

Setting Req.Openings tells the tuning algorithm to enforce the requirements with a loop open at the specified location. With the outer loop open, the requirements apply only to the inner loop.

By default, tuning using TuningGoal.MinLoopGain or TuningGoal.MaxLoopGain imposes a stability requirement as well as the minimum or maximum loop gain. Practically, in some control systems it is not possible to achieve a stable inner loop. In that case, remove the stability requirement for the inner loop by setting the Stabilize property to false.

```
RMinGain.Stabilize = false;
RMaxGain.Stabilize = false;
```

When you tune using either of these requirements, the tuning algorithm still imposes a stability requirement on the overall tuned control system, but not on the inner loop alone.

# **Tips**

• This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). Here, x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.MaxLoopGain, f(x) is given by:

$$f(x) = \left\| W_T \left( D^{-1} T D \right) \right\|_{\infty}.$$

Here, D is a diagonal scaling (for MIMO loops). T is the complementary sensitivity function at Location.  $W_T$  is a frequency-weighting function derived from the maximum loop gain profile, MaxGain. The gain of this function roughly matches 1/MaxGain for values ranging from -60 dB to 20 dB. For numerical reasons, the weighting function levels off outside this range, unless the specified gain profile changes slope outside this range. This adjustment is called regularization. Because poles of  $W_T$  close to S = 0 or S = Inf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify gain profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_T$ , use:

WT = getWeight(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts = 0 for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

Although T is a closed-loop transfer function, driving f(x) < 1 is equivalent to enforcing an upper bound on the open-loop transfer, L, in a frequency band where the gain of L is less than one. To see why, note that T = L/(I + L). For SISO loops, when |L| << 1,  $|T| \approx |L|$ . Therefore, enforcing the open-loop maximum gain requirement,  $|L| < 1/|W_T|$ , is roughly equivalent to enforcing  $|W_T T| < 1$ . For MIMO loops, similar reasoning applies, with  $||T|| \approx \sigma_{\max}(L)$ , where  $\sigma_{\max}$  is the largest singular value.

# **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## **See Also**

TuningGoal.Gain | TuningGoal.LoopShape | TuningGoal.Margins |
TuningGoal.MinLoopGain | evalGoal | looptune | looptune (for slTuner) |
sigma | slTuner | systune | systune (for slTuner) | viewGoal

## **Topics**

"Loop Shape and Stability Margin Specifications"

"Visualize Tuning Goals"

"PID Tuning for Setpoint Tracking vs. Disturbance Rejection"

"MIMO Control of Diesel Engine"

"Tuning of a Two-Loop Autopilot"

#### Introduced in R2016a

# **TuningGoal.Overshoot class**

Package: TuningGoal

Overshoot constraint for control system tuning

# **Description**

Use TuningGoal.Overshoot to limit the overshoot in the step response from specified inputs to specified outputs of a control system. Use this tuning goal for control system tuning with tuning commands such as systume or looptune.

## Construction

Req = TuningGoal.Overshoot(inputname,outputname,maxpercent) creates a tuning goal for limiting the overshoot in the step response between the specified signal locations. The scalar maxpercent specifies the maximum overshoot as a percentage.

When you use TuningGoal.Overshoot for tuning, the software maps overshoot constraints to peak gain constraints assuming second-order system characteristics. Therefore, the mapping is only approximate for higher-order systems. In addition, this tuning goal cannot reliably reduce the overshoot below 5%.

## **Input Arguments**

### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - · Any model input.
  - Any linear analysis point marked in the model.

 Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.

- Any linear analysis point marked in the model.
- Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points yl and y2. Use 'yl' to designate that point as an output signal when creating tuning goals. Use  $\{'yl', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

### maxpercent

Maximum percent overshoot, specified as a scalar value. For example, the following code specifies a maximum 5% overshoot in the step response from 'r' to 'y'.

Req = TuningGoal.Overshoot('r','y',5);

TuningGoal.OverShoot cannot reliably reduce the overshoot below 5%.

# **Properties**

#### MaxOvershoot

Maximum percent overshoot, specified as a scalar value. For example, the scalar value 5 means the overshoot should not exceed 5%. The initial value of the MaxOvershoot property is set by the maxpercent input argument when you construct the tuning goal.

### **InputScaling**

Reference signal scaling, specified as a vector of positive real values.

For a MIMO tracking requirement, when the choice of units results in a mix of small and large signals in different channels of the response, use this property to specify the relative amplitude of each entry in the vector-valued step input. This information is used to scale the off-diagonal terms in the transfer function from reference to tracking error. This scaling ensures that cross-couplings are measured relative to the amplitude of each reference signal.

For example, suppose that Req is a tuning goal that signals  $\{'y1', 'y2'\}$  track reference signals  $\{'r1', 'r2'\}$ . Suppose further that you require the outputs to track the references with less than 10% cross-coupling. If r1 and r2 have comparable amplitudes, then it is sufficient to keep the gains from r1 to y2 and r2 and y1 below 0.1. However, if r1 is 100 times larger than r2, the gain from r1 to y2 must be less than 0.001 to ensure that r1 changes y2 by less than 10% of the r2 target. To ensure this result, set the InputScaling property as follows.

```
Req.InputScaling = [100,1];
```

This tells the software to take into account that the first reference signal is 100 times greater than the second reference signal.

The default value, [], means no scaling.

## Default: []

### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the Outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### Default: NaN

### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then <code>Openings</code> can include any linear analysis point marked in the model, or any linear analysis point in an <code>slTuner</code> interface associated with the Simulink model. Use <code>addPoint</code> to add analysis points and loop openings to the <code>slTuner</code> interface. Use <code>getPoints</code> to get the list of analysis points available in an <code>slTuner</code> interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

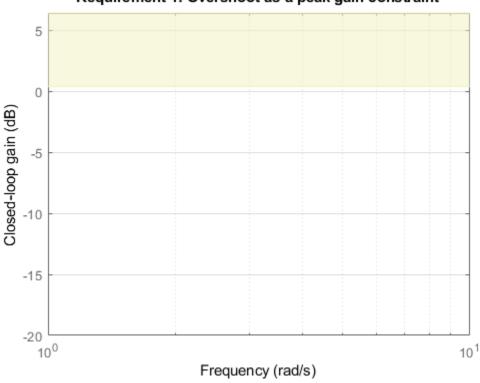
### **Overshoot Constraint**

Create a tuning goal that limits the overshoot of the step response from signals named 'r' to 'y' in a control system to 10 percent.

```
Req = TuningGoal.Overshoot('r','y',10);
```

The overshoot tuning goal is evaluated as a constraint on the peak system gain, assuming second-order model characteristics (see "Algorithms" on page 1-99). Visualizing the tuning goal shows a shaded area where the target peak gain is exceeded.

```
viewGoal(Reg)
```



Requirement 1: Overshoot as a peak gain constraint

If you visualize the tuning goal with a tuned system, the plot includes the corresponding system response.

Configure other characteristics of the tuning goal by setting properties. For instance, configure the tuning goal to apply only to the second model in a model array to tune. Also, configure it to be evaluated with a loop open at an analysis point in the control system called OuterLoop.

```
Req.Models = 2;
Req.Openings = 'OuterLoop';
```

# **Tips**

 This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x), or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Overshoot, f(x) reflects the relative satisfaction or violation of the goal. The percent deviation from f(x) = 1 roughly corresponds to the percent deviation from the specified overshoot target. For example, f(x) = 1.2 means the actual overshoot exceeds the target by roughly 20%, and f(x) = 0.8 means the actual overshoot is about 20% less than the target.

TuningGoal.Overshoot uses  $\|T\|_{\infty}$  as a proxy for the overshoot, based on second-order model characteristics. Here, T is the closed-loop transfer function that the tuning goal constrains. The overshoot is tuned in the range from 5% ( $\|T\|_{\infty} = 1$ ) to 100% ( $\|T\|_{\infty}$ ). TuningGoal.Overshoot is ineffective at forcing the overshoot below 5%.

# **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

TuningGoal.Gain | TuningGoal.Sensitivity | evalGoal | looptune | looptune
(for slTuner) | slTuner | systume | systume (for slTuner) | viewGoal

## **Topics**

"Time-Domain Specifications"
"Multi-Loop PI Control of a Robotic Arm"

### Introduced in R2016a

# **TuningGoal.Passivity class**

Package: TuningGoal

Passivity constraint for control system tuning

# **Description**

A system is passive if all its I/O trajectories (u(t),y(t)) satisfy:

$$\int_0^T y(t)^T u(t) dt > 0,$$

for all T > 0. Equivalently, a system is passive if its frequency response is positive real, which means that for all  $\omega > 0$ ,

$$G(j\omega) + G(j\omega)^H > 0$$

Use TuningGoal.Passivity to enforce passivity of the response between specified inputs and outputs, when using a control system tuning command such as systume. You can also use TuningGoal.Passivity to ensure a particular excess or shortage of passivity (see getPassiveIndex).

## Construction

Req = TuningGoal.Passivity(inputname, outputname) creates a tuning goal for enforcing passivity of the response from the specified inputs to the specified outputs.

Req = TuningGoal.Passivity(inputname,outputname,nu,rho) creates a tuning
goal for enforcing:

$$\int_0^T y(t)^T u(t) dt > \nu \int_0^T u(t)^T u(t) dt + \rho \int_0^T y(t)^T y(t) dt,$$

for all T > 0. This tuning goal enforces an excess of passivity at the inputs or outputs when nu > 0 or rho > 0, respectively. The tuning goal allows for a shortage of input passivity when nu < 0. See getPassiveIndex for more information about these indices.

## **Input Arguments**

### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use {'y1', 'y2'} to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### nu

Target passivity at the inputs listed in inputname, specified as a scalar value. The input passivity index is defined as the largest value of  $\nu$  for which:

$$\int_{0}^{T} y(t)^{T} u(t) dt > \nu \int_{0}^{T} u(t)^{T} u(t) dt,$$

for all T > 0. Equivalently, nu is the largest  $\nu$  for which:

$$G(j\omega) + G(j\omega)^H > 2\nu I$$

When you set a target nu in a TuningGoal.Passivity goal, the tuning software:

- Ensures that the specified response is input strictly passive when nu > 0. The
  magnitude of nu sets the required excess of passivity.
- Allows the response to be not input strictly passive when nu < 0. The magnitude of nu sets the permitted shortage of passivity.</li>

#### **Default:** 0

#### rho

Target passivity at the outputs listed in outputname, specified as a scalar value. The output passivity index is defined as the largest value of  $\rho$  for which:

$$\int_{0}^{T} y(t)^{T} u(t) dt > \rho \int_{0}^{T} y(t)^{T} y(t) dt,$$

for all T > 0.

When you set a target rho in a TuningGoal.Passivity goal, the tuning software:

- Ensures that the specified response is output strictly passive when rho > 0. The
  magnitude of rho sets the required excess of passivity.
- Allows the response to be not output strictly passive when rho < 0. The magnitude of rho sets the permitted shortage of passivity.

#### **Default:** 0

## **Properties**

#### IPX

Target passivity at the inputs, stored as a scalar value. This value specifies the required amount of passivity at the inputs listed in inputname. The initial value of this property is set by the input argument nu when you create the TuningGoal.Passivity goal.

**Default:** 0

#### 0PX

Target passivity at the outputs, stored as a scalar value. This value specifies the required amount of passivity at the outputs listed in outputname The initial value of this property is set by the input argument rho when you create the TuningGoal.Passivity goal.

#### **Default:** 0

#### Focus

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Input

Input signal names, specified as a cell array of character vectors. The input signal names specify the input locations for determining passivity, initially populated by the inputname argument.

#### **Output**

Output signal names, specified as a cell array of character vectors. The output signal names specify the output locations for determining passivity, initially populated by the outputname argument.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

**Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

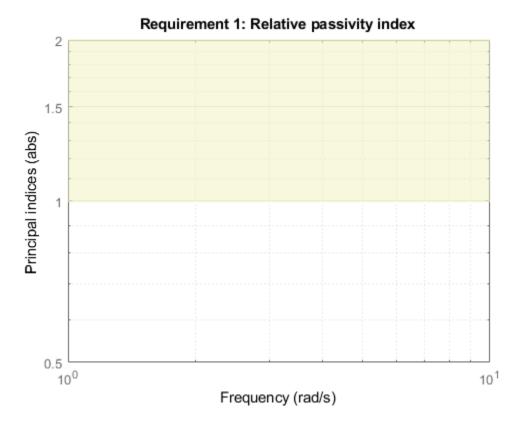
### **Passivity Requirement**

Create a requirement that ensures passivity in the response from an input or analysis point 'u' to an output or analysis point 'y' in a control system.

```
TG = TuningGoal.Passivity('u','y');
```

Use viewGoal to visualize the tuning goal.

```
viewGoal(TG)
```



The requirement is satisfied when the relative passivity index R < 1 at all frequencies. The shaded area represents the region where the requirement is violated. When you use this requirement to tune a control system CL, viewGoal (TG,CL) shows R for the specified inputs and outputs on this plot, enabling you to identify frequency ranges in which the passivity requirement is violated.

### Input Passivity in Specified Frequency Range

Create a requirement that ensures that the response from an input 'u' to an output 'y' is input strictly passive, with an excess of passivity of 2.

```
TGi = TuningGoal.Passivity('u','y',2,0);
```

Restrict the requirement to apply only within the frequency range between 0 and 10 rad/s.

```
TGi.Focus = [0 10];
```

## **Tips**

• Use viewGoal to visualize this tuning goal. For enforcing passivity with nu = 0 and rho = 0, viewGoal plots the relative passivity indices as a function of frequency (see passiveplot). These are the singular values of  $(I - G(j\omega))(I - G(j\omega))^{-1}$ . The transfer function G from inputname to outputname (evaluated with loops open as specified in Openings) is passive when the largest singular value is less than 1 at all frequencies.

For nonzero nu or rho, viewGoal plots the relative index as described in "Algorithms" on page 1-109.

 This tuning goal imposes an implicit minimum-phase constraint on the transfer function G + I. The transmission zeros of G + I are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For the TuningGoal.Passivity goal, for a closed-loop transfer function G(s,x) from inputname to outputname, f(x) is given by:

$$f(x) = \frac{R}{1 + R/R_{\text{max}}}, \quad R_{\text{max}} = 10^6.$$

R is the relative sector index (see <code>getSectorIndex</code>) of <code>[G(s,x);I]</code>, for the sector represented by:

$$Q = \begin{pmatrix} 2\rho & -I \\ -I & 2\nu \end{pmatrix},$$

using the values of the OPX and IPX properties for  $\rho$  and  $\nu$ , respectively.

## **See Also**

TuningGoal.WeightedPassivity | evalGoal | getPassiveIndex | looptune | looptune (for slTuner) | passiveplot | slTuner | systune | systune (for slTuner) | viewGoal

### **Topics**

"About Passivity and Passivity Indices"
"Tuning Control Systems with SYSTUNE"

## **TuningGoal.Poles class**

Package: TuningGoal

Constraint on control system dynamics

## **Description**

Use TuningGoal.Poles to constrain the closed-loop dynamics of a control system or of specific feedback loops within the control system. You can use this tuning goal for control system tuning with tuning commands, such as systune or looptune. A TuningGoal.Poles goal can ensure a minimum decay rate or minimum damping of the poles of the control system or loop. It can also eliminate fast dynamics in the tuned system.

### Construction

Req = TuningGoal.Poles(mindecay,mindamping,maxfreq) creates a default template for constraining the closed-loop pole locations. The minimum decay rate, minimum damping constant, and maximum natural frequency define a region of the complex plane in which poles of the component must lie. Set mindecay = 0, mindamping = 0, or maxfreq = Inf to skip any of the three constraints.

Req = TuningGoal.Poles(location,mindecay,mindamping,maxfreq) constrains the poles of the sensitivity function measured at a specified location in the control system. (See getSensitivity for information about sensitivity functions.) Use this syntax to narrow the scope of the tuning goal to a particular feedback loop.

If you want to constrain the poles of the system with one or more feedback loops opened, set the <code>Openings</code> property. To limit the enforcement of this tuning goal to poles having natural frequency within a specified frequency range, set the <code>Focus</code> property. (See "Properties" on page 1-113.)

## **Input Arguments**

#### mindecay

Minimum decay rate of poles of tunable component, specified as a nonnegative scalar value in the frequency units of the control system model you are tuning.

When you tune the control system using this tuning goal, the closed-loop poles of the control system are constrained to satisfy:

- Re(s) < -mindecay, for continuous-time systems.
- log(|z|) < -mindecay\*Ts, for discrete-time systems with sample time Ts.

Set mindecay = 0 to impose no constraint on the decay rate.

#### mindamping

Desired minimum damping ratio of the closed-loop poles, specified as a value between 0 and 1.

Poles that depend on the tunable parameters are constrained to satisfy Re(s) < -mindamping\*|s|. In discrete time, the damping ratio is computed using s=log(z)/Ts.

Set mindamping = 0 to impose no constraint on the damping ratio.

### maxfreq

Desired maximum natural frequency of closed-loop poles, specified as a scalar value in the frequency units of the control system model you are tuning.

Poles are constrained to satisfy |s| < maxfreq for continuous time, or |log(z)| < maxfreq\*Ts for discrete-time systems with sample time Ts. This constraint prevents fast dynamics in the closed-loop system.

Set maxfreq = Inf to impose no constraint on the natural frequency.

#### location

Location at which poles are assessed, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. When you use this input, the tuning goal constrains the poles of the sensitivity function

measured at this location. (See getSensitivity for information about sensitivity functions.) What locations are available depends on what kind of system you are tuning:

- If you are tuning a Simulink model of a control system, you can use any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. For example, if the slTuner interface contains an analysis point u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.
- If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

If location specifies multiple locations, then the poles constraint applies to the sensitivity of the MIMO loop.

## **Properties**

### **MinDecay**

Minimum decay rate of closed-loop poles of tunable component, specified as a positive scalar value in the frequency units of the control system you are tuning. The initial value of this property is set by the mindecay input argument.

When you tune the control system using this tuning goal, closed-loop poles are constrained to satisfy Re(s) < -MinDecay for continuous-time systems, or log(|z|) < -MinDecay\*Ts for discrete-time systems with sample time Ts.

You can use dot notation to change the value of this property after you create the tuning goal. For example, suppose Req is a TuningGoal.Poles tuning goal. Change the minimum decay rate to 0.001:

```
Req.MinDecay = 0.001;
```

#### **Default:** 0

#### MinDamping

Desired minimum damping ratio of closed-loop poles, specified as a value between 0 and 1. The initial value of this property is set by the mindamping input argument.

Poles that depend on the tunable parameters are constrained to satisfy Re(s) < -MinDamping\*|s|. In discrete time, the damping ratio is computed using s=log(z)/Ts.

#### **Default:** 0

#### **MaxFrequency**

Desired maximum natural frequency of closed-poles, specified as a scalar value in the frequency units of the control system model you are tuning. The initial value of this property is set by the maxfreq input argument.

Poles of the block are constrained to satisfy |s| < maxfreq for continuous-time systems, or |log(z)| < maxfreq\*Ts for discrete-time systems with sample time Ts. This constraint prevents fast dynamics in the tuned control system.

You can use dot notation to change the value of this property after you create the tuning goal. For example, suppose Req is a TuningGoal.ControllerPoles tuning goal. Change the maximum frequency to 1000:

```
Req.MaxFrequency = 1000;
```

#### Default: Inf

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Location

Location at which poles are assessed, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location =  $\{'u'\}$ , the tuning goal evaluates the open-loop response measured at an analysis point 'u'. If Location =  $\{'u1', 'u2'\}$ , the tuning goal evaluates the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The initial value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then <code>Openings</code> can include any linear analysis point marked in the model, or any linear analysis point in an <code>slTuner</code> interface associated with the Simulink model. Use <code>addPoint</code> to add analysis points and loop openings to the <code>slTuner</code> interface. Use <code>getPoints</code> to get the list of analysis points available in an <code>slTuner</code> interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

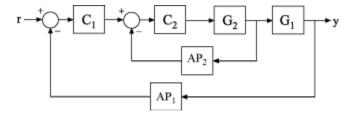
Req.Name = 'LoopReq';

Default: []

## **Examples**

### Constrain Closed-Loop Dynamics of Specified Loop of System to Tune

Create a requirement that constrains the inner loop of the following control system to be stable and free of fast dynamics. Specify that the constraint is evaluated with the outer loop open.



Create a model of the system. To do so, specify and connect the numeric plant models, G1 and G2, and the tunable controllers C1 and C2. Also, create and connect the AnalysisPoint blocks, AP1 and AP2, which mark points of interest for analysis and tuning.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
AP1 = AnalysisPoint('AP1');
AP2 = AnalysisPoint('AP2');
T = feedback(G1*feedback(G2*C2,AP2)*C1,AP1);
```

Create a tuning requirement that constrains the dynamics of the closed-loop poles. Restrict the poles of the inner loop to the region Re(s) < -0.1, |s| < 30.

```
Req = TuningGoal.Poles(0.1,0,30);
```

Setting the minimum damping to zero imposes no constraint on the damping constants for the poles.

Specify that the constraint on the tuned system poles is applied with the outer loop open.

```
Reg.Openings = 'AP1';
```

When you tune T using this requirement, the constraint applies to the poles of the entire control system evaluated with the loop open at 'AP1'. In other words, the poles of the inner loop plus the poles of C1 and G1 are all considered.

After you tune T, you can use viewGoal to validate the tuned control system against the requirement.

### **Constrain Dynamics of Specified Feedback Loop**

Create a requirement that constrains the inner loop of the system of the previous example to be stable and free of fast dynamics. Specify that the constraint is evaluated with the outer loop open.

Create a tuning requirement that constrains the dynamics of the inner feedback loop, the loop identified by AP2. Restrict the poles of the inner loop to the region Re(s) < -0.1, |s| < 30.

```
Req = TuningGoal.Poles('AP2',0.1,0,30);
```

Specify that the constraint on the tuned system poles is applied with the outer loop open.

```
Req.Openings = 'AP1';
```

When you tune T using this requirement, the constraint applies only to the poles of the inner loop, evaluated with the outer loop open. In this case, since G1 and C1 do not contribute to the sensitivity function at AP2 when the outer loop is open, the requirement constrains only the poles of G2 and C2.

After you tune T, you can use viewGoal to validate the tuned control system against the requirement.

## **Tips**

• TuningGoal.Poles restricts the closed-loop dynamics of the tuned control system. To constrain the dynamics or ensure the stability of a single tunable component, use TuningGoal.ControllerPoles.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x), or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Poles, f(x) reflects the relative satisfaction or violation of the goal. For example, if you attempt to constrain the closed-loop poles of a feedback loop to a minimum damping of  $\zeta = 0.5$ , then:

- f(x) = 1 means the smallest damping among the constrained poles is  $\zeta = 0.5$  exactly.
- f(x) = 1.1 means the smallest damping  $\zeta = 0.5/1.1 = 0.45$ , roughly 10% less than the target.
- f(x) = 0.9 means the smallest damping  $\zeta = 0.5/0.9 = 0.55$ , roughly 10% better than the target.

## **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.ControllerPoles|evalGoal|looptune|looptune (for slTuner)
| systume | systume (for slTuner) | tunableSS | tunableTF | viewGoal

### **Topics**

"System Dynamics Specifications"

"Digital Control of Power Stage Voltage"

"Multiloop Control of a Helicopter"

### Introduced in R2016a

## **TuningGoal.Rejection class**

Package: TuningGoal

Disturbance rejection requirement for control system tuning

## **Description**

Use TuningGoal.Rejection to specify the minimum attenuation of a disturbance injected at a specified location in a control system. This tuning goal helps you tune control systems with tuning commands such as systume or looptune.

When you use TuningGoal.Rejection, the software attempts to tune the system so that the attenuation of a disturbance at the specified location exceeds the minimum attenuation factor you specify. This attenuation factor is the ratio between the open- and closed-loop sensitivities to the disturbance and is a function of frequency. You can achieve disturbance attenuation only inside the control bandwidth. The loop gain must be larger than one for the disturbance to be attenuated (attenuation factor > 1).

### Construction

Req = TuningGoal.Rejection(distloc,attfact) creates a tuning goal for rejecting a disturbance entering at distloc. This tuning goal constrains the minimum disturbance attenuation factor to the frequency-dependent value, attfact.

### **Input Arguments**

#### distloc

Disturbance location, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

• If you are using the tuning goal to tune a Simulink model of a control system, then distloc can include any signal identified as an analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as the disturbance input when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel disturbance input.

• If you are using the tuning goal to tune a generalized state-space model (genss) of a control system, then inputname can include any AnalysisPoint channel in the model. For example, if you are tuning a control system model, T, which contains an AnalysisPoint block with a location named AP\_u, then distloc can include 'AP\_u'. (Use getPoints to get a list of analysis points available in a genss model.) The constrained disturbance location is injected at the implied input associated with the analysis point, and measured at the implied output:

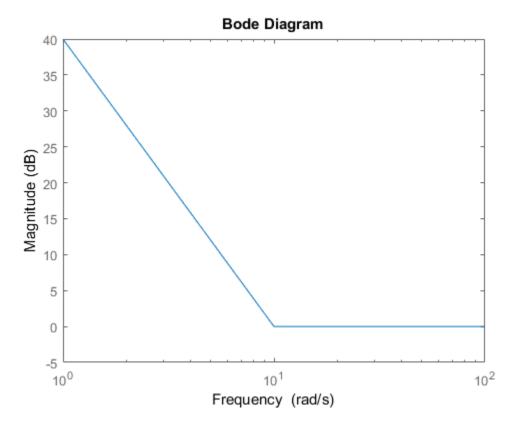


#### attfact

Attenuation factor as a function of frequency, specified as a numeric LTI model.

TuningGoal.Rejection constrains the minimum disturbance attenuation to the frequency-dependent value attfact. You can specify attfact as a smooth transfer function (tf, zpk, or ss model). Alternatively, you can specify a piecewise gain profile using a frd model. For example, the following code specifies an attenuation factor of 100 (40 dB) below 1 rad/s, gradually dropping to 1 (0 dB) past 10 rad/s, for a disturbance injected at u.

```
attfact = frd([100 100 1 1],[0 1 10 100]);
Req = TuningGoal.Rejection('u',attfact);
bodemag(attfact)
ylim([-5,40])
```



When you use an frd model to specify attfact, the gain profile is automatically mapped onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Use viewGoal(Req) to visualize the resulting attenuation profile.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify attfact as a discrete-time model with the same Ts. If you specify attfact in continuous time, the tuning software discretizes it. Specifying the attenuation profile in discrete time gives you more control over the profile near the Nyquist frequency.

## **Properties**

#### **MinAttenuation**

Minimum disturbance attenuation as a function of frequency, expressed as a SISO zpk model.

The software automatically maps the attfact input argument to a zpk model. The magnitude of this zpk model approximates the desired attenuation factor and is stored in the MinAttenuation property. Use viewGoal(Req) to plot the magnitude of MinAttenuation.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

### LoopScaling

Toggle for automatically scaling loop signals, specified as 'on' or 'off'.

For multiloop or MIMO disturbance rejection tuning goals, the feedback channels are automatically rescaled to equalize the off-diagonal (loop interaction) terms in the open-loop transfer function. Set LoopScaling to 'off' to disable such scaling and shape the unscaled open-loop response.

Default: 'on'

#### Location

Location of disturbance, specified as a specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if

Location =  $\{'u'\}$ , the tuning goal evaluates disturbance rejection at an analysis point 'u'. If Location =  $\{'u1', 'u2'\}$ , the tuning goal evaluates the rejection at based on the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The initial value of the Location property is set by the distloc input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### Default: NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then <code>Openings</code> can include any linear analysis point marked in the model, or any linear analysis point in an <code>slTuner</code> interface associated with the Simulink model. Use <code>addPoint</code> to add analysis points and loop openings to the <code>slTuner</code> interface. Use <code>getPoints</code> to get the list of analysis points available in an <code>slTuner</code> interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

### **Constant Minimum Attenuation in Frequency Band**

Create a tuning goal that enforces a attenuation of at least a factor of 10 between 0 and 5 rad/s. The tuning goal applies to a disturbance entering a control system at a point identified as 'u'.

```
Req = TuningGoal.Rejection('u',10);
Req.Name = 'Rejection spec';
Req.Focus = [0 5]
```

### **Frequency-Dependent Attenuation Profile**

Create a tuning goal that enforces an attenuation factor of at least 100 (40 dB) below 1 rad/s, gradually dropping to 1 (0 dB) past 10 rad/s. The tuning goal applies to a disturbance entering a control system at a point identified as 'u'.

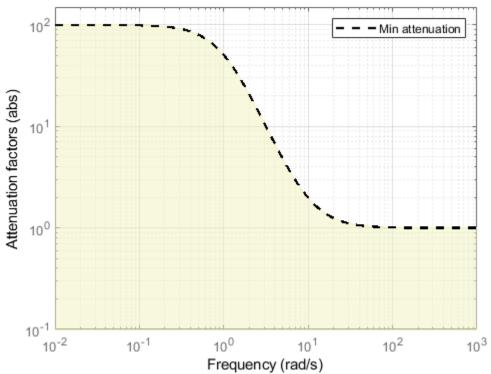
```
attfact = frd([100 100 1 1],[0 1 10 100]);
Req = TuningGoal.Rejection('u',attfact);
```

These commands use a frd model to specify the minimum attenuation profile as a function of frequency. The minimum attenuation of 100 below 1 rad/s, together with the minimum attenuation of 1 at the frequencies of 10 and 100 rad/s, specifies the desired rolloff.

attfact is converted into a smooth function of frequency that approximates the piecewise specified profile. Display the gain profile using viewGoal.

viewGoal(Req)





The shaded region indicates where the tuning goal is violated.

## **Tips**

 This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). In this case, x is the vector of free (tunable) parameters in the control system. The parameter values are adjusted automatically to minimize f(x) or drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Rejection, f(x) is given by:

$$f(x) = \max_{\omega \in \Omega} \|W_S(j\omega)S(j\omega, x)\|_{\infty},$$

or its discrete-time equivalent. Here,  $S(j\omega,x)$  is the closed-loop sensitivity function measured at the disturbance location.  $\Omega$  is the frequency interval over which the tuning goal is enforced, specified in the Focus property.  $W_S$  is a frequency weighting function derived from the specified attenuation profile. The gains of  $W_S$  and MinAttenuation roughly match for gain values ranging from -20 dB to 60 dB. For numerical reasons, the weighting function levels off outside this range, unless the specified attenuation profile changes slope outside this range. This adjustment is called regularization. Because poles of  $W_S$  close to s=0 or s=1nf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify attenuation profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_S$ , use:

WS = getWeight(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts =  $\theta$  for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

## **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### **See Also**

TuningGoal.LoopShape | TuningGoal.Tracking | looptune | looptune (for slTuner) | slTuner | systume | systume (for slTuner) | viewGoal

### **Topics**

"Time-Domain Specifications"

"Visualize Tuning Goals"

"Decoupling Controller for a Distillation Column"

"Tuning of a Two-Loop Autopilot"

#### Introduced in R2016a

## **TuningGoal.Sensitivity class**

Package: TuningGoal

Sensitivity requirement for control system tuning

## **Description**

Use TuningGoal.Sensitivity to limit the sensitivity of a feedback loop to disturbances. Constrain the sensitivity to be smaller than one at frequencies where you need good disturbance rejection. Use this tuning goal for control system tuning with tuning commands such as systume or looptune.

### Construction

Req = TuningGoal.Sensitivity(location, maxsens) creates a tuning goal for limiting the sensitivity to disturbances entering a feedback loop at the specified location. maxsens specifies the maximum sensitivity as a function of frequency. You can specify the maximum sensitivity profile as a smooth transfer function or sketch a piecewise error profile using an frd model or the makeweight command.

See getSensitivity for more information about sensitivity functions.)

### **Input Arguments**

#### location

Location at which the sensitivity to disturbances is constrained, specified as a character vector or cell array of character vectors that identify one or more locations in the control system to tune. What locations are available depends on what kind of system you are tuning:

If you are tuning a Simulink model of a control system, you can use any linear analysis
point marked in the model, or any linear analysis point in an slTuner interface
associated with the Simulink model. Use addPoint to add analysis points to the
slTuner interface. For example, if the slTuner interface contains an analysis point

u, you can use 'u' to refer to that point when creating tuning goals. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

• If you are tuning a generalized state-space (genss) model of a control system, you can use any AnalysisPoint location in the control system model. For example, the following code creates a PI loop with an analysis point at the plant input 'u'.

```
AP = AnalysisPoint('u');
G = tf(1,[1 2]);
C = tunablePID('C','pi');
T = feedback(G*AP*C,1);
```

When creating tuning goals, you can use 'u' to refer to the analysis point at the plant input. Use getPoints to get the list of analysis points available in a genss model.

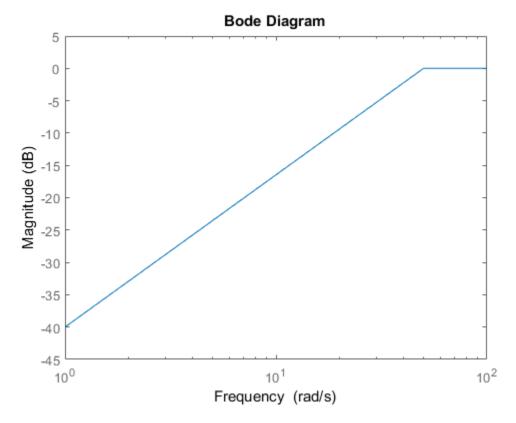
If location is a cell array, then the sensitivity requirement applies to the MIMO loop.

#### maxsens

Maximum sensitivity to disturbances as a function of frequency.

You can specify maxsens as a smooth SISO transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise gain profile using a frd model or the makeweight command. For example, the following frd model specifies a maximum sensitivity of 0.01 (-40 dB) at 1 rad/s, increasing to 1 (0 dB) past 50 rad/s.

```
maxsens = frd([0.01 1 1],[1 50 100]);
bodemag(maxsens)
ylim([-45,5])
```



When you use an frd model to specify maxsens, the software automatically maps your specified gain profile to a zpk model whose magnitude approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of that zpk model.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify maxsens as a discrete-time model with the same Ts. If you specify maxsens in continuous time, the tuning software discretizes it. Specifying the maximum sensitivity profile in discrete time gives you more control over the profile near the Nyquist frequency.

## **Properties**

#### **MaxSensitivity**

Maximum sensitivity as a function of frequency, specified as a SISO zpk model.

The software automatically maps the input argument maxsens onto a zpk model. The magnitude of this zpk model approximates the desired gain profile. Use viewGoal(Req) to plot the magnitude of the zpk model MaxSensitivity.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

### LoopScaling

Toggle for automatically scaling loop signals, specified as 'on' or 'off'.

In multi-loop or MIMO control systems, the feedback channels are automatically rescaled to equalize the off-diagonal terms in the open-loop transfer function (loop interaction terms). Set LoopScaling to 'off' to disable such scaling and shape the unscaled sensitivity function.

Default: 'on'

#### Location

Location of disturbance, specified as a cell array of character vectors that identify one or more analysis points in the control system to tune. For example, if Location =  $\{'u'\}$ , the tuning goal evaluates the open-loop response measured at an analysis point 'u'. If

Location =  $\{'u1', 'u2'\}$ , the tuning goal evaluates the MIMO open-loop response measured at analysis points 'u1' and 'u2'.

The initial value of the Location property is set by the location input argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

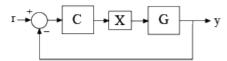
```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

### **Disturbance Sensitivity at Plant Input**

Create a tuning goal that limits the sensitivity to disturbance at the plant input of the following control system. The control system contains an analysis point named 'X' at the plant input.

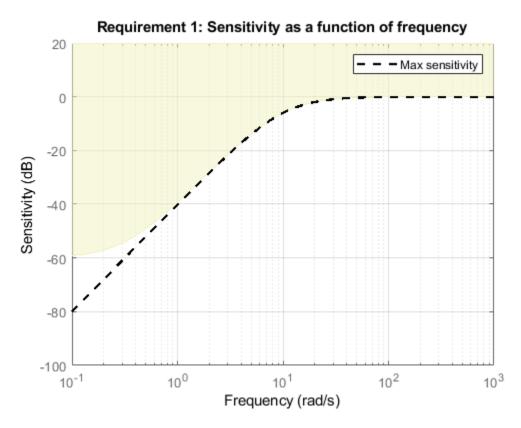


Specify a maximum sensitivity of 0.01 (-40 dB) at 1 rad/s, increasing to 1 (0 dB) past 10 rad/s. Use an frd model to sketch this target sensitivity.

```
maxsens = frd([0.01 1 1],[1 10 100]);
Req = TuningGoal.Sensitivity('X',maxsens);
```

The software converts maxsens into a smooth function of frequency that approximates the piecewise-specified gain profile. Visualize this function using viewGoal.

```
viewGoal(Req)
```



The shaded region indicates regions where the tuning goal is violated. The shaded region differs from the specified gain profile (dashed line) at very low frequencies because of modifications that the software introduces for numeric stability, as described in "Algorithms" on page 1-136.

# Sensitivity Goal with Limited Frequency Range and Model Application

Create a tuning goal that specifies a maximum sensitivity of 0.1 (10%) at frequencies below 5 rad/s. Configure the tuning goal to apply only to the second and third plant models.

```
Req = TuningGoal.Sensitivity('u',0.1);
Req.Focus = [0 5];
Req.Models = [2 3];
```

You can use Req as an input to looptune or systune when tuning a control system that has an analysis point called 'u'. Setting the Focus property limits the application of the tuning goal to frequencies between 0 and 5 rad/s. Setting the Models property restricts application of the tuning goal to the second and third models in an array, when you use the tuning goal to tune an array of control system models.

## **Tips**

• This tuning goal imposes an implicit stability constraint on the closed-loop sensitivity function measured at Location, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

## **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Sensitivity, f(x) is given by:

```
f(x) = ||W_S(s)S(s, x)||_{\infty}
```

or its discrete-time equivalent. Here, S(s,x) is the closed-loop sensitivity function measured at the location specified in the tuning goal.  $W_S$  is a frequency weighting function derived from the specified sensitivity profile. The gains of  $W_S$  and 1/MaxSensitivity roughly match for gain values ranging from -20~dB to 60~dB. For numerical reasons, the weighting function levels off outside this range, unless the specified sensitivity profile changes slope outside this range. This adjustment is called regularization. Because poles of  $W_S$  close to s=0~or~s=Inf might lead to poor numeric

conditioning of the systune optimization problem, it is not recommended to specify sensitivity profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_S$ , use:

WS = getWeight(Req,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts =  $\theta$  for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

## **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Gain | TuningGoal.LoopShape | TuningGoal.MaxLoopGain |
TuningGoal.MinLoopGain | TuningGoal.Rejection | evalGoal | looptune |
looptune (for slTuner) | slTuner | systume | systume (for slTuner) |
viewGoal

### **Topics**

"Frequency-Domain Specifications"

"Visualize Tuning Goals"

#### Introduced in R2016a

## **TuningGoal.StepRejection class**

Package: TuningGoal

Step disturbance rejection requirement for control system tuning

## **Description**

Use TuningGoal.StepRejection to specify how a step disturbance injected at a specified location in your control system affects the signal at a specified output location. Use this tuning goal with control system tuning commands such as systume or looptune.

You can specify the desired response in time-domain terms of peak value, settling time, and damping ratio. Alternatively, you can specify the response as a stable reference model having DC-gain. In that case, the tuning goal is to reject the disturbance as well as or better than the reference model.

To specify disturbance rejection in terms of a frequency-domain attenuation profile, use TuningGoal.Rejection.

### Construction

Req = TuningGoal.StepRejection(inputname, outputname, refsys) creates a tuning goal that constrains how a step disturbance injected at a location inputname affects the response at outputname. The tuning goal is that the disturbance be rejected as well as or better than the reference system. inputname and outputname can describe a SISO or MIMO response of your control system. For MIMO responses, the number of inputs must equal the number of outputs.

Req = TuningGoal.StepRejection(inputname,outputname,peak,tSettle) specifies an oscillation-free response in terms of a peak value and a settling time.

Req = TuningGoal.StepRejection(inputname,outputname,peak,tSettle, zeta) allows for damped oscillations with a damping ratio of at least zeta.

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - · Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use {'y1', 'y2'} to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

### refsys

Reference system for target step rejection, specified as a SISO dynamic system model, such as a tf, zpk, or ss model. refsys must be stable and proper, and must have zero DC gain. This restriction ensures perfect rejection of the steady-state disturbance.

refsys can be continuous or discrete. If refsys is discrete, it can include time delays which are treated as poles at z = 0.

For best results, refsys and the open-loop response from the disturbance to the output should have similar gains at the frequency where the reference model gain peaks. You can check the peak gain and peak frequency using getPeakGain. For example:

```
[gmax,fmax] = getPeakGain(refsys);
```

Use getIOTransfer to extract the corresponding open-loop response from the system you are tuning.

#### peak

Peak absolute value of target response to disturbance, specified as a scalar value.

#### **tSettle**

Target settling time of the response to disturbance, specified as a positive scalar value, in the time units of the control system you are tuning.

#### zeta

Minimum damping ratio of oscillations in the response to disturbance, specified as a value between 0 and 1.

#### **Default:** 1

# **Properties**

#### ReferenceModel

Reference system for target response to step disturbance, specified as a SISO (zpk) model. The step response of this model specifies how the output signals specified by outputname should respond to the step disturbance at inputname.

If you use the refsys input argument to create the tuning goal, then the value of ReferenceModel is zpk(refsys).

If you use the peak, tSample, and zeta input arguments, then ReferenceModel is a zpk representation of the first-order or second-order transfer function whose step response has the specified characteristics.

### **InputScaling**

Input signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued input signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

### Default: []

### **OutputScaling**

Output signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued output signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

#### Default: []

#### Input

Names of disturbance input locations, specified as a cell array of character vectors. This property is initially populated by the inputname argument when you create the tuning goal.

#### **Output**

Names of locations at which response to step disturbance is measured, specified as a cell array of character vectors. This property is initially populated by the outputname argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default: NaN**

### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

## **Examples**

### **Specify First-Order or Second-Order Step Disturbance Response Characteristics**

Create a requirement that specifies the step disturbance response in terms of peak time-domain response, settling time, and damping of oscillations.

Suppose you want the response at 'y' to a disturbance injected at 'd' to never exceed an absolute value of 0.25, and to settle within 5 seconds. Create a TuningGoal.StepRejection requirement that captures these specifications and also specifies non-oscillatory response.

```
Req1 = TuningGoal.StepRejection('d','y',0.25,5);
```

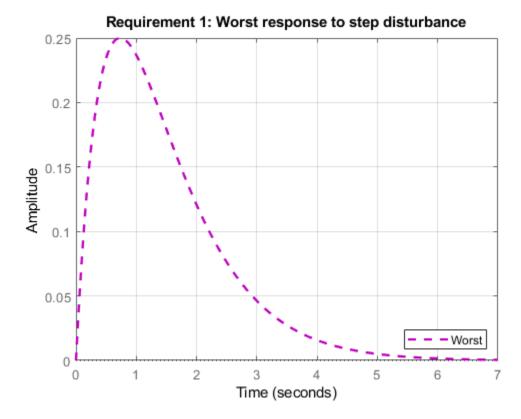
Omitting an explicit value for the damping ratio, zeta, is equivalent to setting zeta = 1. Therefore, Req specifies a non-oscillatory response. The software converts the peak value and settling time into a reference transfer function whose step response has the desired time-domain profile. This transfer function is stored in the ReferenceModel property of Req.

#### Req1.ReferenceModel

```
ans =
    0.92883 s
    (s+1.367)^2
Continuous-time zero/pole/gain model.
```

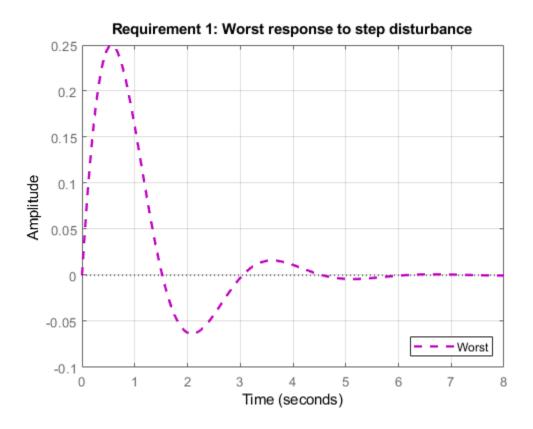
Confirm the target response by displaying Req.

```
figure()
viewGoal(Req1)
```



Suppose your application can tolerate oscillations provided the damping ratio is less than 0.4. Create a requirement that specifies this disturbance response.

```
Req2 = TuningGoal.StepRejection('d','y',0.25,5,0.4);
figure()
viewGoal(Req2)
```



### Step Disturbance Rejection with Custom Reference Model

Create a requirement that specifies the step disturbance response as a transfer function.

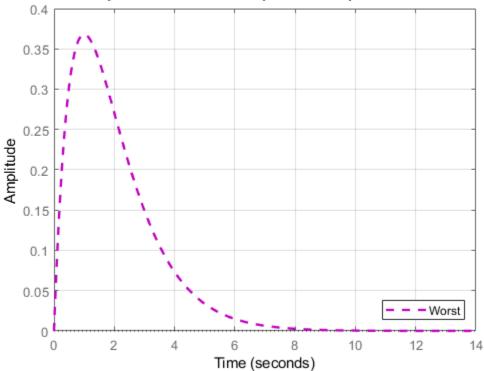
Suppose you want the response to a disturbance injected at an analysis point d in your control system and measured at a point 'y' to be rejected at least as well as the transfer function

$$H(s) = \frac{s}{s^2 + 2s + 1}.$$

Create a TuningGoal.StepRejection requirement.

```
H = tf([1 0],[1 2 1]);
Req = TuningGoal.StepRejection('d','y',H);
Display the requirement.
viewGoal(Req)
```

### Requirement 1: Worst response to step disturbance



The plot displayed by viewGoal shows the step response of the specified transfer function. This response is the target time-domain response to disturbance.

# **Tips**

• This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

**TuningGoal.StepRejection** aims to keep the gain from disturbance to output below the gain of the reference model. The scalar value of the tuning goal f(x) is given by:

$$f(x) = \|W_F(s)T_{dy}(s,x)\|_{\infty},$$

or its discrete-time equivalent. Here,  $T_{dy}(s,x)$  is the closed-loop transfer function from Input to Output, and  $\|\cdot\|_{\infty}$  denotes the  $H_{\infty}$  norm (see norm).  $W_F$  is a frequency weighting function derived from the step-rejection profile you specify in the tuning goal. The gains of  $W_F$  and 1/ReferenceModel roughly match for gain values within 60 dB of the peak gain. For numerical reasons, the weighting function levels off outside this range, unless you specify a reference model that changes slope outside this range. This adjustment is called regularization. Because poles of  $W_F$  close to s=0 or s=1nf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify reference models with very low-frequency or very high-frequency dynamics.

To obtain  $W_F$ , use:

WF = getWeight(Reg,Ts)

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts = 0 for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

# **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Gain | TuningGoal.LoopShape | evalGoal | looptune | looptune
(for slTuner) | slTuner | systume | systume (for slTuner) | viewGoal

### **Topics**

"Time-Domain Specifications"

"Visualize Tuning Goals"

"Tuning Control Systems with SYSTUNE"

"Tune Control Systems in Simulink"

#### Introduced in R2016a

# TuningGoal.StepTracking class

Package: TuningGoal

Step response requirement for control system tuning

# **Description**

Use TuningGoal.StepTracking to specify a target step response from specified inputs to specified outputs of a control system. Use this tuning goal with control system tuning commands such as systume or looptune.

### Construction

Req = TuningGoal.StepTracking(inputname, outputname, refsys) creates a tuning goal that constrains the step response between the specified signal locations to match the step response of a stable reference system, refsys. The constraint is satisfied when the relative difference between the tuned and target responses falls within a tolerance specified by the RelGap property of the tuning goal (see "Properties" on page 1-155). inputname and outputname can describe a SISO or MIMO response of your control system. For MIMO responses, the number of inputs must equal the number of outputs.

Req = TuningGoal.StepTracking(inputname,outputname,tau) specifies the desired step response as a first-order response with time constant tau:

Req.ReferenceModel = 
$$\frac{1/\text{tau}}{s + 1/\text{tau}}$$
.

Req = TuningGoal.StepTracking(inputname,outputname,tau,overshoot) specifies the desired step response as a second-order response with natural period tau, natural frequency 1/tau, and percent overshoot overshoot:

Req.ReferenceModel = 
$$\frac{(1/\tan^2)}{s^2 + 2(\text{zeta/tau})s + (1/\tan^2)}.$$

The damping is given by zeta = cos(atan2(pi, -log(overshoot/100))).

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### refsys

Reference system for target step response, specified as a dynamic system model, such as a tf, zpk, or ss model. refsys must be stable and must have DC gain of 1 (zero steady-state error).

refsys can be continuous or discrete. If refsys is discrete, it can include time delays which are treated as poles at z = 0.

refsys can be MIMO, provided that it is square and that its DC singular value (sigma) is 1. If refsys is a MIMO model, then its number of inputs and outputs must match the dimensions of inputname and outputname.

For best results, refsys should also include intrinsic system characteristics such as non-minimum-phase zeros (undershoot).

#### tau

Time constant or natural period of target step response, specified as a positive scalar.

If you use the syntax Req =

TuningGoal.StepTracking(inputname,outputname,tau) to specify a first-order target response, then tau is the time constant of the response decay. In that case, the target is the step response of the system given by:

$$\mbox{Req.ReferenceModel} = \frac{1/\mbox{tau}}{s+1/\mbox{tau}} \, .$$

If you use the syntax Req =

TuningGoal.StepTracking(inputname,outputname,tau,overshoot) to specify a second-order target response, then tau is the inverse of the natural frequency of the response. In that case, the target is the step response of the system given by:

Req.ReferenceModel = 
$$\frac{(1/\tan^2)}{s^2 + 2(\text{zeta/tau})s + (1/\tan^2)}.$$

The damping of the system is given by zeta = cos(atan2(pi, -log(overshoot/100))).

#### overshoot

Percent overshoot of target step response, specified as a scalar value in the range (0,100).

# **Properties**

#### ReferenceModel

Reference system for target step response, specified as a SISO or MIMO state-space (ss) model. When you use the tuning goal to tune a control system, the step response from inputname to outputname is tuned to match this target response to within the tolerance specified by the RelGap property.

If you use the refsys input argument to create the tuning goal, then the value of ReferenceModel is ss(refsys).

If you use the tau or tau and overshoot input arguments, thenReferenceModel is a state-space representation of the corresponding first-order or second-order transfer function.

ReferenceModel must be stable and have unit DC gain (zero steady-state error). For best results, ReferenceModel should also include intrinsic system characteristics such as non-minimum-phase zeros (undershoot).

### RelGap

Maximum relative matching error, specified as a positive scalar value. This property specifies the matching tolerance as the maximum relative gap between the target and actual step responses. The relative gap is defined as:

$$gap = \frac{\|y(t) - y_{ref}(t)\|_2}{\|1 - y_{ref}(t)\|_2}.$$

y(t) –  $y_{ref}(t)$  is the response mismatch, and 1 –  $y_{ref}(t)$  is the step-tracking error of the target model.  $\|\cdot\|_2$  denotes the signal energy (2-norm).

Increase the value of RelGap to loosen the matching tolerance.

**Default:** 0.1

#### **InputScaling**

Reference signal scaling, specified as a vector of positive real values.

For a MIMO tracking requirement, when the choice of units results in a mix of small and large signals in different channels of the response, use this property to specify the relative amplitude of each entry in the vector-valued step input. This information is used to scale the off-diagonal terms in the transfer function from reference to tracking error. This scaling ensures that cross-couplings are measured relative to the amplitude of each reference signal.

For example, suppose that Req is a tuning goal that signals  $\{'y1', 'y2'\}$  track reference signals  $\{'r1', 'r2'\}$ . Suppose further that you require the outputs to track the references with less than 10% cross-coupling. If r1 and r2 have comparable amplitudes, then it is sufficient to keep the gains from r1 to y2 and r2 and y1 below 0.1. However, if r1 is 100 times larger than r2, the gain from r1 to y2 must be less than 0.001 to ensure that r1 changes y2 by less than 10% of the r2 target. To ensure this result, set the InputScaling property as follows.

```
Req.InputScaling = [100,1];
```

This tells the software to take into account that the first reference signal is 100 times greater than the second reference signal.

The default value, [], means no scaling.

### Default: []

### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

**Default: NaN** 

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

**Default:** {}

Name

#### ordare. (

Name of the tuning goal, specified as a character vector.

```
For example, if Req is a tuning goal:

Req.Name = 'LoopReq';

Default: []
```

# **Examples**

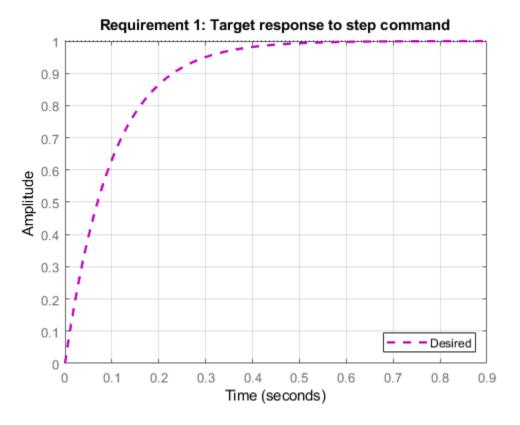
### **Step Response Requirement with Specified Tolerance**

Create a requirement for the step response from a signal named 'r' to a signal named 'y'. Constrain the step response to match the transfer function H = 10/(s+10), but allow 20% relative variation between the target the tuned responses.

```
H = tf(10,[1 10]);
Req = TuningGoal.StepResp('r','y',H);
```

By default, this requirement allows a relative gap of 0.1 between the target and tuned responses. To change the relative gap to 20%, set the RelGap property of the requirement.

```
Req.RelGap = 0.2;
Examine the requirement.
viewGoal(Req);
```



The dashed line shows the target step response specified by this requirement. You can use this requirement to tune a control system model, T, that contains valid input and output locations named 'r' and 'y'. If you do so, the command viewGoal(Req,T) plots the achieved step response from 'r' to 'y' for comparison to the target response.

## First-Order Step Response With Known Time Constant

Create a requirement that specifies a first-order step response with time constant of 5 seconds. Create the requirement for the step response from a signal named 'r' to a signal named 'y'.

Req = TuningGoal.StepResp('r','y',5);

When you use this requirement to tune a control system model, T, the time constant 5 is taken to be expressed in the prevailing units of the control system. For example, if T is a genss model and the property T.TimeUnit is 'seconds', then this requirement specifies a target time constant of 5 seconds for the response from the input 'r' to the output 'y' of 'T'.

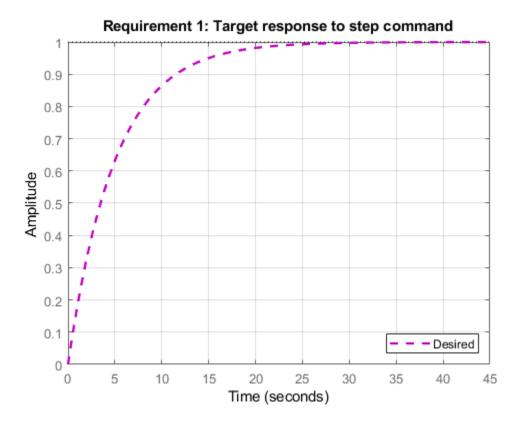
The specified time constant is converted into a reference state-space model stored in the ReferenceModel property of the requirement.

```
refsys = tf(Req.ReferenceModel)
refsys =
    0.2
    ...
s + 0.2
Continuous-time transfer function.
```

As expected, refsys is a first-order model.

Examine the requirement. The viewGoal command displays the target response, which is the step response of the reference model.

```
viewGoal(Req);
```



The dashed line shows the target step response specified by this requirement, a first-order response with a time constant of five seconds.

# **Second-Order Step Response With Known Natural Period and Overshoot**

Create a requirement that specifies a second-order step response with a natural period of 5 seconds, and a 10% overshoot. Create the requirement for the step response from a signal named 'r' to a signal named 'y'.

Req = TuningGoal.StepResp('r','y',5,10);

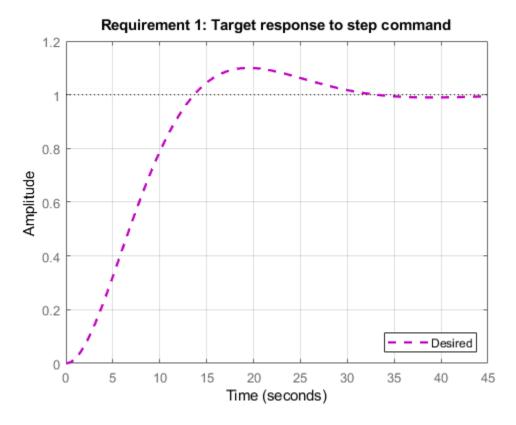
When you use this requirement to tune a control system model, T, the natural period 5 is taken to be expressed in the prevailing units of the control system. For example, if T is a genss model and the property T. TimeUnit is 'seconds', then this requirement specifies a target natural period of 5 seconds for the response from the input 'r' to the output 'y' of 'T'.

The specified parameters of the response is converted into a reference state-space model stored in the ReferenceModel property of the requirement.

As expected, refsys is a second-order model.

Examine the requirement. The viewGoal command displays the target response, which is the step response of the reference model.

```
viewGoal(Req);
```



The dashed line shows the target step response specified by this requirement, a secondorder response with 10% overshoot and a natural period of five seconds.

# Tracking Goal with Limited Model Application and Additional Loop Openings

Create a tuning goal that specifies a first-order step response with time constant of 5 seconds. Set the Models and Openings properties to further configure the tuning goal's applicability.

```
Req = TuningGoal.StepTracking('r','y',5);
Req.Models = [2 3];
Req.Openings = 'OuterLoop'
```

When tuning a control system that has an input 'r', an output 'y', and an analysis-point location 'OuterLoop', you can use Req as an input to looptune or systune. Setting the Openings property specifies that the step response from 'r' to 'y' is measured with the loop opened at 'OuterLoop'. When tuning an array of control system models, setting the Models property restricts how the tuning goal is applied. In this example, the tuning goal applies only to the second and third models in an array.

# **Tips**

 This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). Here, x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.StepTracking, f(x) is given by:

$$f(x) = \frac{\left\|\frac{1}{s}\left(T(s, x) - T_{ref}(s)\right)\right\|_{2}}{\operatorname{RelGap}\left\|\frac{1}{s}\left(T_{ref}(s) - I\right)\right\|_{2}}.$$

T(s,x) is the closed-loop transfer function from Input to Output with parameter values x, and  $T_{ref}(s)$  is the reference model specified in the ReferenceModel property.  $\|\cdot\|_2$  denotes the  $H_2$  norm (see norm).

# **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Overshoot | TuningGoal.Tracking | evalGoal | looptune |
looptune (for slTuner) | systume | systume (for slTuner) | viewGoal

### **Topics**

"Time-Domain Specifications"
"PID Tuning for Setneint Treeling vs. Dis

"PID Tuning for Setpoint Tracking vs. Disturbance Rejection"

#### Introduced in R2016a

# **TuningGoal.Tracking class**

Package: TuningGoal

Tracking requirement for control system tuning

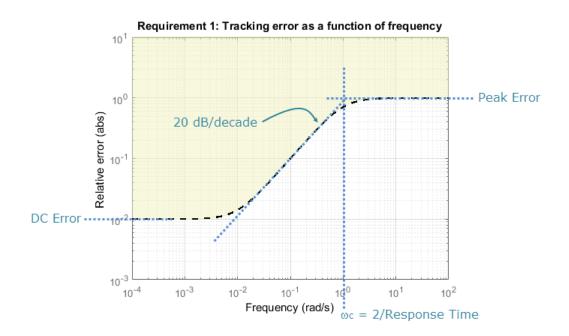
# **Description**

Use TuningGoal.Tracking to specify a frequency-domain tracking requirement between specified inputs and outputs. This tuning goal specifies the maximum relative error (gain from reference input to tracking error) as a function of frequency. Use this tuning goal for control system tuning with tuning commands such as systume or looptune.

You can specify the maximum error profile directly by providing a transfer function. Alternatively, you can specify a target DC error, peak error, and response time. These parameters are converted to the following transfer function that describes the maximum frequency-domain tracking error:

$$\text{MaxError} = \frac{(\text{PeakError})s + \omega_c(\text{DCError})}{s + \omega_c}.$$

Here,  $\omega_c$  is 2/(response time). The following plot illustrates these relationships for an example set of values.



### Construction

Req = TuningGoal.Tracking(inputname,outputname,responsetime,dcerror, peakerror) creates a tuning goal Req that constrains the tracking performance from inputname to outputname in the frequency domain. This tuning goal specifies a maximum error profile as a function of frequency given by:

$$\mbox{MaxError} = \frac{(\mbox{PeakError})s + \omega_c(\mbox{DCError})}{s + \omega_c} \,. \label{eq:maxerror}$$

The tracking bandwidth  $\omega_c = 2/\text{responsetime}$ . The maximum relative steady-state error is given by dcerror, and peakerror gives the peak relative error across all frequencies.

You can specify a MIMO tracking requirement by specifying signal names or a cell array of multiple signal names for inputname or outputname. For MIMO tracking requirements, use the InputScaling property to help limit cross-coupling. See "Properties" on page 1-171.

Req = TuningGoal.Tracking(inputname,outputname,maxerror) specifies the maximum relative error as a function of frequency. You can specify the target error profile (maximum gain from reference signal to tracking error signal) as a smooth transfer function. Alternatively, you can sketch a piecewise error profile using an frd model.

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - · Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - · Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### responsetime

Target response time, specified as a positive scalar value. The tracking bandwidth is given by  $\omega_c = 2/\text{responsetime}$ . Express the target response time in the time units of the models to be tuned. For example, when tuning a model T, if T.TimeUnit is 'minutes', then express the target response time in minutes.

#### dcerror

Maximum steady-state fractional tracking error, specified as a positive scalar value. For example, dcerror = 0.01 sets a maximum steady-state error of 1%.

If inputname or outputname are vector-valued, dcerror applies to all I/O pairs from inputname to outputname.

#### **Default:** 0.001

#### peakerror

Maximum fractional tracking error across all frequencies, specified as a positive scalar value greater than 1.

#### Default: 1

#### maxerror

Target tracking error profile as a function of frequency, specified as a SISO numeric LTI model.

maxerror is the maximum gain from reference signal to tracking error signal. You can specify maxerror as a smooth transfer function (tf, zpk, or ss model). Alternatively, you can sketch a piecewise error profile using an frd model. When you do so, the software automatically maps the error profile to a zpk model. The magnitude of the zpk model approximates the desired error profile. Use show(Req) to plot the magnitude of the zpk model.

maxerror must be a SISO LTI model. If inputname or outputname are cell arrays, maxerror applies to all I/O pairs from inputname to outputname.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify maxerror as a discrete-time model with the same Ts. If you specify maxerror in continuous time, the tuning software discretizes it. Specifying the error profile in discrete time gives you more control over the error profile near the Nyquist frequency.

# **Properties**

#### MaxError

Maximum error as a function of frequency, expressed as a SISO zpk model. This property stores the maximum tracking error as a function of frequency (maximum gain from reference signal to tracking error signal).

If you use the syntax Req =

TuningGoal.Tracking(inputname,outputname,maxerror), then the MaxError property is the zpk equivalent or approximation of the LTI model you supplied as the maxerror input argument.

If you use the syntax Req =

TuningGoal.Tracking(inputname, outputname, resptime, dcerror, peakerror), then the MaxError is a zpk transfer function given by:

$$\text{MaxError} = \frac{(\text{PeakError})s + \omega_c(\text{DCError})}{s + \omega_c}.$$

MaxError is a SISO LTI model. If inputname or outputname are cell arrays, MaxError applies to all I/O pairs from inputname to outputname.

Use show(Req) to plot the magnitude of MaxError.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### **InputScaling**

Reference signal scaling, specified as a vector of positive real values.

For a MIMO tracking requirement, when the choice of units results in a mix of small and large signals in different channels of the response, use this property to specify the relative amplitude of each entry in the vector-valued step input. This information is used to scale the off-diagonal terms in the transfer function from reference to tracking error. This scaling ensures that cross-couplings are measured relative to the amplitude of each reference signal.

For example, suppose that Req is a tuning goal that signals  $\{'y1', 'y2'\}$  track reference signals  $\{'r1', 'r2'\}$ . Suppose further that you require the outputs to track the references with less than 10% cross-coupling. If r1 and r2 have comparable amplitudes, then it is sufficient to keep the gains from r1 to y2 and r2 and y1 below 0.1. However, if r1 is 100 times larger than r2, the gain from r1 to y2 must be less than 0.001 to ensure that r1 changes y2 by less than 10% of the r2 target. To ensure this result, set the InputScaling property as follows.

```
Req.InputScaling = [100,1];
```

This tells the software to take into account that the first reference signal is 100 times greater than the second reference signal.

The default value, [], means no scaling.

```
Default: []
```

#### Input

Reference signal names, specified as a character vector or cell array of character vectors specifying the names of the signals to be tracked, populated by the inputname argument.

#### **Output**

Output signal names, specified as a character vector or cell array of character vectors specifying the names of the signals that must track the reference signals, populated by the outputname argument.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control

system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'ul', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

### Tracking Goal With Response Time and Maximum Steady-State Tracking Error

Create a tracking goal specifying that a signal 'theta' track a signal 'theta\_ref'. The required response time is 2, in the time units of the control system you are tuning. The maximum steady-state error is 0.1%.

```
Req = TuningGoal.Tracking('theta ref','theta',2,0.001);
```

Since peakerror is unspecified, this tuning goal uses the default value, 1.

# Tracking Goal With Maximum Tracking Error as a Function of Frequency

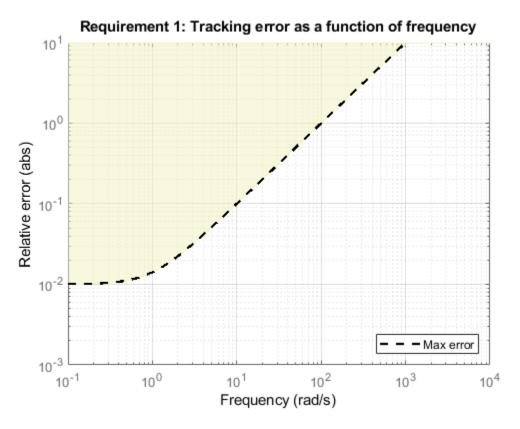
Create a tracking goal specifying that a signal 'theta' track a signal 'theta\_ref'. The maximum relative error is 0.01 (1%) in the frequency range [0,1]. The relative error increases to 1 (100%) at the frequency 100.

Use an frd model to specify the error profile as a function of frequency.

```
err = frd([0.01 0.01 1],[0 1 100]);
Req = TuningGoal.Tracking('theta ref','theta',err);
```

The software converts err into a smooth function of frequency that approximates the piecewise specified profile. Display this function using viewGoal.

viewGoal(Req)



The dashed line is the target error profile stored in MaxError, and the shaded region indicates where the tuning goal is violated.

# **Tips**

 This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Tracking, f(x) is given by:

$$f(x) = ||W_F(s)(T(s, x) - I)||_{\infty}$$

or its discrete-time equivalent. Here, T(s,x) is the closed-loop transfer function from Input to Output, and  $\|\cdot\|_{\infty}$  denotes the  $H_{\infty}$  norm (see getPeakGain).  $W_F$  is a frequency weighting function derived from the error profile you specify in the tuning goal. The gains of  $W_F$  and 1/MaxError roughly match for gain values between -20 dB and 60 dB. For numerical reasons, the weighting function levels off outside this range, unless you specify a reference model that changes slope outside this range. This adjustment is called regularization. Because poles of  $W_F$  close to s=0 or s=1nf might lead to poor numeric conditioning of the systune optimization problem, it is not recommended to specify error profiles with very low-frequency or very high-frequency dynamics.

To obtain  $W_F$ , use:

```
WF = getWeight(Req,Ts)
```

where Req is the tuning goal, and Ts is the sample time at which you are tuning (Ts = 0 for continuous time). For more information about regularization and its effects, see "Visualize Tuning Goals".

# **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Gain | TuningGoal.LoopShape | evalGoal | looptune | looptune
(for slTuner) | slTuner | systume | systume (for slTuner) | viewGoal

### **Topics**

"Time-Domain Specifications"

"Visualize Tuning Goals"

"Tuning Control Systems with SYSTUNE"

"Tune Control Systems in Simulink"

"PID Tuning for Setpoint Tracking vs. Disturbance Rejection"

"Decoupling Controller for a Distillation Column"

"Digital Control of Power Stage Voltage"

"Tuning of a Two-Loop Autopilot"

#### Introduced in R2016a

# **TuningGoal.Transient class**

Package: TuningGoal

Transient matching requirement for control system tuning

# **Description**

Use the TuningGoal.Transient object to constrain the transient response from specified inputs to specified outputs. This tuning goal specifies that the transient response closely match the response of a reference model. Specify the closeness of the required match using the RelGap property of the tuning goal (see "Properties" on page 1-182). You can constrain the response to an impulse, step, or ramp input signal. You can also constrain the response to an input signal given by the impulse response of an input filter you specify.

# Construction

Req = TuningGoal.Transient(inputname,outputname,refsys) requires that the impulse response from inputname to outputname closely matches the impulse response of the reference model refsys. Specify the closeness of the required match using the RelGap property of the tuning goal (see "Properties" on page 1-182). inputname and outputname can describe a SISO or MIMO response of your control system. For MIMO responses, the number of inputs must equal the number of outputs.

Req = TuningGoal.Transient(inputname,outputname,refsys,inputtype) specifies whether the input signal that generates the constrained transient response is and impulse, step, or ramp signal.

Req = TuningGoal.Transient(inputname,outputname,refsys,inputfilter) specifies the input signal for generating the transient response that the tuning goal constrains. Specify the input signal as a SISO transfer function, inputfilter, that is the Laplace transform of the desired time-domain input signal. The impulse response of inputfilter is the desired input signal.

## **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - · Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use {'y1', 'y2'} to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### refsys

Reference system for target transient response, specified as a dynamic system model, such as a tf, zpk, or ss model. The desired transient response is the response of this model to the input signal specified by inputtype or inputfilter. The reference model must be stable, and the series connection of the reference model with the input shaping filter must have no feedthrough term.

#### inputtype

Type of input signal that generates the constrained transient response, specified as one of the following values:

- 'impulse' Constrain the response at outputname to a unit impulse applied at inputname.
- 'step' Constrain the response to a unit step. Using 'step' is equivalent to using the TuningGoal.StepTracking design goal.
- 'ramp' Constrain the response to a unit ramp, u = t.

Default: 'impulse'

### inputfilter

Custom input signal for generating the transient response, specified as a SISO transfer function (tf or zpk) model that represents the Laplace transform of the desired input signal. inputfilter must be continuous, and can have no poles in the open right-half plane.

The frequency response of inputfilter gives the signal spectrum of the desired input signal, and the impulse response of inputfilter is the time-domain input signal.

For example, to constrain the transient response to a unit-amplitude sine wave of frequency w, set inputfilter to tf(w,[1,0,w^2]). This transfer function is the Laplace transform of  $\sin(wt)$ .

The series connection of refsys with inputfilter must have no feedthrough term.

# **Properties**

#### ReferenceModel

Reference system for target transient response, specified as a SISO or MIMO state-space (ss) model. When you use the tuning goal to tune a control system, the transient response from inputname to outputname is tuned to match this target response to within the tolerance specified by the RelGap property.

The refsys argument to TuningGoal. Transient sets the value of ReferenceModel to ss(refsys).

### **InputShaping**

Input signal for generating the transient response, specified as a SISO zpk model that represents the Laplace transform of the time-domain input signal. InputShaping must be continuous, and can have no poles in the open right-half plane. The value of this property is populated using the inputtype or inputfilter arguments used when creating the tuning goal.

For tuning goals created using the inputtype argument, InputShaping takes the following values:

inputtype	InputShaping
'impulse'	1
'step'	1/s
'ramp'	$1/s^2$

For tuning goals created using an inputfilter transfer function, InputShaping takes the value zpk(inputfilter).

The series connection of ReferenceModel with InputShaping must have no feedthrough term.

#### **Default:** 1

#### RelGap

Maximum relative matching error, specified as a positive scalar value. This property specifies the matching tolerance as the maximum relative gap between the target and actual transient responses. The relative gap is defined as:

$$\mathrm{gap} = \frac{\|y(t) - y_{ref}(t)\|_2}{\|y_{ref(tr)}(t)\|_2}\,.$$

y(t) –  $y_{ref}(t)$  is the response mismatch, and 1 –  $y_{ref(tr)}(t)$  is the transient portion of  $y_{ref}$  (deviation from steady-state value or trajectory).  $\|\cdot\|_2$  denotes the signal energy (2-norm). The gap can be understood as the ratio of the root-mean-square (RMS) of the mismatch to the RMS of the reference transient

Increase the value of RelGap to loosen the matching tolerance.

#### **Default:** 0.1

#### **InputScaling**

Input signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued input signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

### Default: []

### **OutputScaling**

Output signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued output signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

Default: []

#### Input

Input signal names, specified as a as a cell array of character vectors that indicate the inputs for the transient responses that the tuning goal constrains. The initial value of the Input property is populated by the inputname argument when you create the tuning goal.

#### **Output**

Output signal names, specified as a cell array of character vectors that indicate the outputs where transient responses that the tuning goal constrains are measured. The initial value of the Output property is populated by the outputname argument when you create the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default: NaN**

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear

analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

# Transient Response Requirement with Specified Input Type and Tolerance

Create a requirement for the transient response from a signal named 'r' to a signal named 'u'. Constrain the impulse response to match the response of transfer function refsys = 1/(s+1), but allow 20% relative variation between the target and tuned responses.

```
refsys = tf(1,[1 1]);
Req1 = TuningGoal.Transient('r','u',refsys);
```

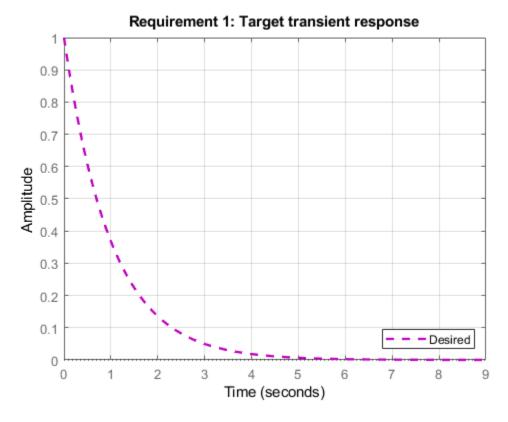
When you do not specify a response type, the requirement constrains the transient response. By default, the requirement allows a relative gap of 0.1 between the target and

tuned responses. To change the relative gap to 20%, set the RelGap property of the requirement.

Req1.RelGap = 0.2;

Examine the requirement.

viewGoal(Req1)



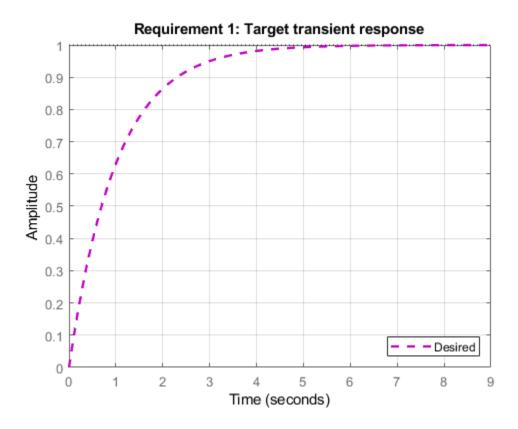
The dashed line shows the target impulse response specified by this requirement. You can use this requirement to tune a control system model, T, that contains valid input and output locations named 'r' and 'u'. If you do so, the command viewGoal (Req1,T) plots the achieved impulse response from 'r' to 'u' for comparison to the target response.

Create a requirement that constrains the response to a step input, instead of the impulse response.

Req2 = TuningGoal.Transient('r','u',refsys,'step');

Examine this requirement.

viewGoal(Req2)



Req2 is equivalent to the following step tracking requirement:

Req3 = TuningGoal.StepTracking('r','u',refsys);

### **Constrain Transient Response to Custom Input Signal**

Create a requirement for the transient response from 'r' to 'u'. Constrain the response to a sinusoidal input signal, rather than to an input, step, or ramp.

To specify a custom input signal, set the input filter to the Laplace transform of the desired signal. For example, suppose you want to constrain the response to a signal of  $\sin \omega t$ . The Laplace transform of this signal is given by:

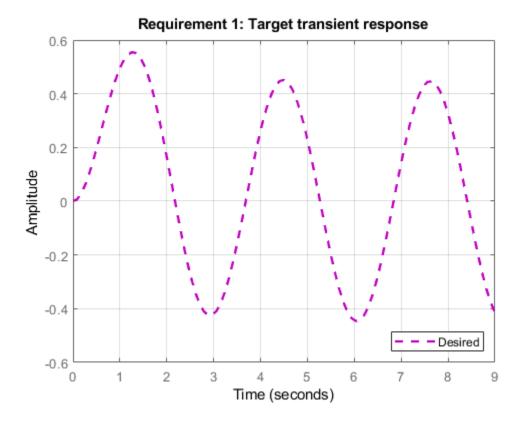
inputfilter = 
$$\frac{\omega}{s^2 + \omega^2}$$
.

Create a requirement that constrains the response at 'u' to a sinusoidal input of natural frequency 2 rad/s at 'r'. The response should match that of the reference system refsys = 1/(s+1).

```
refsys = tf(1,[1 1]);
w = 2;
inputfilter = tf(w,[1 0 w^2]);
Req = TuningGoal.Transient('u','r',refsys,inputfilter);
```

Examine the requirement to see the shape of the target response.

```
viewGoal(Req)
```



# Transient Response Goal with Limited Model Application and Additional Loop Openings

Create a tuning goal that constrains the impulse response. Set the Models and Openings properties to further configure the tuning goal's applicability.

```
refsys = tf(1,[1 1]);
Req = TuningGoal.Transient('r','u',refsys);
Req.Models = [2 3];
Req.Openings = 'OuterLoop'
```

When tuning a control system that has an input (or analysis point) 'r', an output (or analysis point) 'u', and another analysis point at location 'OuterLoop', you can use Req as an input to looptune or systune. Setting the Openings property specifies that the impulse response from 'r' to 'y' is computed with the loop opened at 'OuterLoop'. When tuning an array of control system models, setting the Models property restricts how the tuning goal is applied. In this example, the tuning goal applies only to the second and third models in an array.

# **Tips**

• When you use this tuning goal to tune a continuous-time control system, systune attempts to enforce zero feedthrough (D=0) on the transfer that the tuning goal constrains. Zero feedthrough is imposed because the  $H_2$  norm, and therefore the value of the tuning goal (see "Algorithms" on page 1-191), is infinite for continuous-time systems with nonzero feedthrough.

systune enforces zero feedthrough by fixing to zero all tunable parameters that contribute to the feedthrough term. systune returns an error when fixing these tunable parameters is insufficient to enforce zero feedthrough. In such cases, you must modify the tuning goal or the control structure, or manually fix some tunable parameters of your system to values that eliminate the feedthrough term.

When the constrained transfer function has several tunable blocks in series, the software's approach of zeroing all parameters that contribute to the overall feedthrough might be conservative. In that case, it is sufficient to zero the feedthrough term of one of the blocks. If you want to control which block has feedthrough fixed to zero, you can manually fix the feedthrough of the tuned block of your choice.

To fix parameters of tunable blocks to specified values, use the Value and Free properties of the block parametrization. For example, consider a tuned state-space block:

```
C = tunableSS('C',1,2,3);
```

To enforce zero feedthrough on this block, set its D matrix value to zero, and fix the parameter.

```
C.D.Value = 0;
C.D.Free = false;
```

- For more information on fixing parameter values, see the Control Design Block reference pages, such as tunableSS.
- This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.Transient, f(x) is based upon the relative gap between the tuned response and the target response:

$$gap = \frac{\|y(t) - y_{ref}(t)\|_2}{\|y_{ref(tr)}(t)\|_2}.$$

y(t) –  $y_{ref}(t)$  is the response mismatch, and 1 –  $y_{ref(tr)}(t)$  is the transient portion of  $y_{ref}$  (deviation from steady-state value or trajectory).  $\|\cdot\|_2$  denotes the signal energy (2-norm). The gap can be understood as the ratio of the root-mean-square (RMS) of the mismatch to the RMS of the reference transient

# **Compatibility Considerations**

# Functionality moved from Robust Control Toolbox

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

# See Also

TuningGoal.StepRejection | TuningGoal.StepTracking | evalGoal | looptune |
looptune (for slTuner) | slTuner | systume (for slTuner) |
viewGoal

### **Topics**

 $\hbox{``Time-Domain Specifications''}$ 

"Tuning Control Systems with SYSTUNE"

"Tune Control Systems in Simulink"

#### Introduced in R2016a

# **TuningGoal.Variance class**

Package: TuningGoal

Noise amplification constraint for control system tuning

# **Description**

Use TuningGoal.Variance to specify a tuning goal that limits the noise amplification from specified inputs to outputs. The noise amplification is defined as either:

- The square root of the output variance, for a unit-variance white-noise input
- The root-mean-square of the output, for a unit-variance white-noise input
- The  $H_2$  norm of the transfer function from the specified inputs to outputs, which equals the total energy of the impulse response

These definitions are different interpretations of the same quantity. TuningGoal.Variance imposes the same limit on these quantities.

You can use TuningGoal.Variance for control system tuning with tuning commands, such as systume or looptune. Specifying this tuning goal allows you to tune the system response to white-noise inputs. For stochastic inputs with a nonuniform spectrum (colored noise), use TuningGoal.WeightedVariance instead.

After you create a tuning goal, you can further configure the tuning goal by setting "Properties" on page 1-196 of the object.

### Construction

Req = TuningGoal.Variance(inputname,outputname,maxamp) creates a tuning goal that limits the noise amplification of the transfer function from inputname to outputname to the scalar value maxamp.

When you tune a control system in discrete time, this tuning goal assumes that the physical plant and noise process are continuous. To ensure that continuous-time and discrete-time tuning give consistent results, maxamp is interpreted as a constraint on the

continuous-time  $H_2$  norm. If the plant and noise processes are truly discrete and you want to constrain the discrete-time  $H_2$  norm instead, multiply maxamp by  $\sqrt{T_s}$ .  $T_s$  is the sample time of the model you are tuning.

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - · Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - · Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### maxamp

Maximum noise amplification from inputname to outputname, specified as a positive scalar value. This value specifies the maximum value of the output variance at the signals specified in outputname, for unit-variance white noise signal at inputname. This value corresponds to the maximum  $H_2$  norm from inputname to outputname.

When you tune a control system in discrete time, this tuning goal assumes that the physical plant and noise process are continuous, and interprets maxamp as a bound on the continuous-time  $H_2$  norm. This ensures that continuous-time and discrete-time tuning give consistent results. If the plant and noise processes are truly discrete, and you want to bound the discrete-time  $H_2$  norm instead, specify the value  $\max mp/\sqrt{T_s}$ .  $T_s$  is the sample time of the model you are tuning.

# **Properties**

### **MaxAmplification**

Maximum noise amplification, specified as a positive scalar value. This property specifies the maximum value of the output variance at the signals specified in Output, for unit-variance white noise signal at Input. This value corresponds to the maximum  $H_2$  norm from Input to Output. The initial value of MaxAmplification is set by the maxamp input argument when you construct the tuning goal.

### **InputScaling**

Input signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued input signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

#### Default: []

#### **OutputScaling**

Output signal scaling, specified as a vector of positive real values.

Use this property to specify the relative amplitude of each entry in vector-valued output signals when the choice of units results in a mix of small and large signals. This information is used to scale the closed-loop transfer function from Input to Output when the tuning goal is evaluated.

Suppose T(s) is the closed-loop transfer function from Input to Output. The tuning goal is evaluated for the scaled transfer function  $D_o^{-1}T(s)D_i$ . The diagonal matrices  $D_o$  and  $D_i$  have the OutputScaling and InputScaling values on the diagonal, respectively.

The default value, [], means no scaling.

### Default: []

#### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

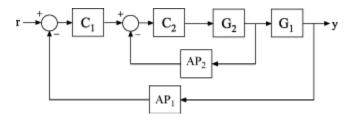
For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
Default: []
```

# **Examples**

#### Constrain Noise Amplification Evaluated with a Loop Opening

Create a requirement that constrains the amplification of the variance from the analysis point AP2 to the output y of the following control system, measured with the outer loop open.



Create a model of the system. To do so, specify and connect the numeric plant models G1 and G2, and the tunable controllers C1 and C2. Also specify and connect the AnalysisPoint blocks AP1 and AP2 that mark points of interest for analysis and tuning.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
AP1 = AnalysisPoint('AP1');
AP2 = AnalysisPoint('AP2');
T = feedback(G1*feedback(G2*C2,AP2)*C1,AP1);
```

Create a tuning requirement that constrains the noise amplification from the implicit input associated with the analysis point, AP2, to the output y.

```
Req = TuningGoal.Variance('AP2','y',0.1);
```

This constraint limits the amplification to a factor of 0.1.

Specify that the transfer function from AP2 to y is evaluated with the outer loop open when tuning to this constraint.

```
Req.Openings = {'AP1'};
```

Use systume to tune the free parameters of T to meet the tuning requirement specified by Req. You can then validate the tuned control system against the requirement using viewGoal(Req,T).

# **Tips**

• When you use this tuning goal to tune a continuous-time control system, systune attempts to enforce zero feedthrough (D=0) on the transfer that the tuning goal constrains. Zero feedthrough is imposed because the  $H_2$  norm, and therefore the value of the tuning goal (see "Algorithms" on page 1-201), is infinite for continuous-time systems with nonzero feedthrough.

systune enforces zero feedthrough by fixing to zero all tunable parameters that contribute to the feedthrough term. systune returns an error when fixing these tunable parameters is insufficient to enforce zero feedthrough. In such cases, you must modify the tuning goal or the control structure, or manually fix some tunable parameters of your system to values that eliminate the feedthrough term.

When the constrained transfer function has several tunable blocks in series, the software's approach of zeroing all parameters that contribute to the overall feedthrough might be conservative. In that case, it is sufficient to zero the feedthrough term of one of the blocks. If you want to control which block has feedthrough fixed to zero, you can manually fix the feedthrough of the tuned block of your choice.

To fix parameters of tunable blocks to specified values, use the Value and Free properties of the block parametrization. For example, consider a tuned state-space block:

```
C = tunableSS('C',1,2,3);
```

To enforce zero feedthrough on this block, set its D matrix value to zero, and fix the parameter.

```
C.D.Value = 0;
C.D.Free = false;
```

- For more information on fixing parameter values, see the Control Design Block reference pages, such as tunableSS.
- This tuning goal imposes an implicit stability constraint on the closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

# **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). The vector x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal. Variance, f(x) is given by:

$$f(x) = \left\| \frac{1}{\text{MaxAmplification}} T(s, x) \right\|_{2}.$$

T(s,x) is the closed-loop transfer function from Input to Output.  $\|\cdot\|_2$  denotes the  $H_2$  norm (see norm).

For tuning discrete-time control systems, f(x) is given by:

$$f(x) = \left\| \frac{1}{\text{MaxAmplification}\sqrt{T_s}} T(z, x) \right\|_2.$$

 $T_s$  is the sample time of the discrete-time transfer function T(z,x).

# **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

# See Also

TuningGoal.WeightedVariance | evalGoal | looptune | looptune (for slTuner) | norm | slTuner | systune (for slTuner) | viewGoal

### **Topics**

"Frequency-Domain Specifications"
"Active Vibration Control in Three-Story Building"
"Fault-Tolerant Control of a Passenger Jet"

#### Introduced in R2016a

# **TuningGoal.WeightedPassivity class**

Package: TuningGoal

Frequency-weighted passivity constraint

# **Description**

A system is passive if all its I/O trajectories (u(t),y(t)) satisfy:

$$\int_0^T y(t)^T u(t)dt > 0,$$

for all T > 0. TuningGoal.WeightedPassivity enforces the passivity of the transfer function:

$$H(s) = W_L(s)T(s)W_R(s),$$

where  $T_s$  is a closed-loop response in the control system being tuned.  $W_L$  and  $W_R$  are weighting functions used to emphasize particular frequency bands. Use TuningGoal.WeightedPassivity with control system tuning commands such as systume.

### Construction

Req = TuningGoal.WeightedPassivity(inputname,outputname,WL,WR) creates a tuning goal for enforcing passivity of the transfer function:

$$H(s) = W_L(s)T(s)W_R(s), \label{eq:hamiltonian}$$

where  $T_s$  is the closed-loop transfer function from the specified inputs to the specified outputs. The weights WL and WR can be matrices or LTI models.

By default, the tuning goal enforces passivity of the weighted transfer function H. You can also enforce input and output passivity indices, with a specified excess or shortage of passivity. (See <code>getPassiveIndex</code> for more information about passivity indices.) To do so, set the IPX and OPX properties of the tuning goal. See "Weighted Passivity and Input Passivity" on page 1-210.

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use {'y1', 'y2'} to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### WL, WR

Input and output weighting functions, specified as scalars, matrices, or SISO or MIMO numeric LTI models.

The functions WL and WR provide the weights for the tuning goal. The tuning goal ensures passivity of the weighted transfer function:

$$H(s) = W_L(s)T(s)W_R(s),$$

where T(s) is the transfer function from inputname to outputname. The function WL provides the weighting for the output channels of T(s), and WR provides the weighting for the input channels. You can specify:

- Scalar weighting use a scalar or numeric matrix.
- Frequency-dependent weighting use a SISO or MIMO numeric LTI model. For example:

```
WL = tf(1,[1 0.01]);

WR = 10;
```

If WL or WR is a matrix or a MIMO model, then inputname and outputname must be vector signals. The dimensions of the vector signals must be such that the dimensions of T(s) are commensurate with the dimensions of WL and WR. For example, if you specify WR = diag([1 10]), then inputname must include two signals. Scalar values and SISO LTI models, however, automatically expand to any input or output dimension.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify the weighting functions as discrete-time models with the same Ts. If you specify the weighting functions in continuous time, the tuning software discretizes them. Specifying the weighting functions in discrete time gives you more control over the weighting functions near the Nyquist frequency.

```
A value of WL = [] or WR = [] is interpreted as the identity.
```

#### Default: []

# **Properties**

#### WL

Frequency-weighting function for the output channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WL input argument when you construct the tuning goal.

#### WR

Frequency-weighting function for the input channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WR input argument when you construct the tuning goal.

#### IPX

Target passivity at the inputs listed in inputname, specified as a scalar value. The input passivity index is defined as the largest value of  $\nu$  for which the trajectories  $\{u(t), y(t)\}$  of the weighted transfer function H satisfy:

$$\int_0^T y(t)^T u(t)dt > \nu \int_0^T u(t)^T u(t)dt,$$

for all T > 0.

By default, the tuning goal enforces strict passivity of the weighted transfer function. To enforce an input passivity index with a specified excess or shortage of passivity, set the IPX property of the tuning goal. When you do so, the tuning software:

- Ensures that the weighted response is input strictly passive when IPX > 0. The
  magnitude of IPX sets the required excess of passivity.
- Allows the weighted response to be not input strictly passive when IPX < 0. The magnitude of IPX sets the permitted shortage of passivity.

See "Weighted Passivity and Input Passivity" on page 1-210 for an example. See getPassiveIndex for more information about passivity indices.

#### **Default:** 0

#### **OPX**

Target passivity at the outputs listed in outputname, specified as a scalar value. The output passivity index is defined as the largest value of  $\rho$  for which the trajectories  $\{u(t), y(t)\}$  of the weighted transfer function H satisfy:

$$\int_{0}^{T} y(t)^{T} u(t) dt > \rho \int_{0}^{T} y(t)^{T} y(t) dt,$$

for all T > 0.

By default, the tuning goal enforces strict passivity of the weighted transfer function. To enforce an output passivity index with a specified excess or shortage of passivity, set the OPX property of the tuning goal. When you do so, the tuning software:

- Ensures that the weighted response is output strictly passive when OPX > 0. The magnitude of IPX sets the required excess of passivity.
- Allows the weighted response to be not output strictly passive when OPX < 0. The
  magnitude of IPX sets the permitted shortage of passivity.</li>

See "Weighted Passivity and Input Passivity" on page 1-210 for an example. See getPassiveIndex for more information about passivity indices.

#### **Default:** 0

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

Req.Focus = [1,100];

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Input

Input signal names, specified as a cell array of character vectors. The input signal names specify the input locations for determining passivity, initially populated by the inputname argument.

#### **Output**

Output signal names, specified as a cell array of character vectors. The output signal names specify the output locations for determining passivity, initially populated by the outputname argument.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

Req.Models = 2:4;

When Models = NaN, the tuning goal applies to all models.

#### **Default:** NaN

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control

system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

# **Examples**

### **Weighted Passivity and Input Passivity**

Create a tuning goal that enforces the passivity of the transfer function:

$$H(s) = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} T(s) \left(\frac{1}{s}\right),$$

where T(s) is the transfer function from an input 'd' to outputs ['y'; 'z'] in a control system model.

```
WL = tf(1,[1 0]);
WR = diag([1 10]);
TG = TuningGoal.WeightedPassivity('d',{'y','z'},WL,WR);
```

Use TG with systune to enforce that weighted passivity requirement.

Suppose that instead of enforcing overall passivity of the weighted transfer function H, you want to ensure that H is input strictly passive with an input feedforward passivity index of at least 0.1. To do so, set the IPX property of TG.

```
TG.IPX = 0.1;
```

### **Tips**

- Use viewGoal to visualize this tuning goal. For enforcing passivity with IPX = 0 and OPX = 0, viewGoal plots the relative passivity indices as a function of frequency (see passiveplot). These are the singular values of  $(I H(j\omega))(I H(j\omega))^{-1}$ . The weighted transfer function H is passive when the largest singular value is less than 1 at all frequencies.
  - For nonzero IPX or OPX, viewGoal plots the relative index as described in "Algorithms" on page 1-211.
- This tuning goal imposes an implicit minimum-phase constraint on the transfer function H + I, where H is the weighted closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The transmission zeros of H + I are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

### **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x), where x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.WeightedPassivity, for a closed-loop transfer function T(s,x) from inputname to outputname, and the weighted transfer function H(s,x) = WL\*T(s,x)\*WR, f(x) is given by:

$$f(x) = \frac{R}{1 + R/R_{\text{max}}}, \quad R_{\text{max}} = 10^6.$$

R is the relative sector index (see <code>getSectorIndex</code>) of [H(s,x);I], for the sector represented by:

$$Q = \begin{pmatrix} 2\rho & -I \\ -I & 2\nu \end{pmatrix},$$

using the values of the OPX and IPX properties for  $\rho$  and  $\nu$ , respectively.  $R_{\text{max}}$  is fixed at  $10^6$ , included to avoid numerical errors for very large R.

### **See Also**

TuningGoal.Passivity | evalGoal | getPassiveIndex | looptune | looptune
(for slTuner) | passiveplot | slTuner | systume | systume (for slTuner) |
viewGoal

### **Topics**

"About Passivity and Passivity Indices"

"Vibration Control in Flexible Beam"

"Tuning Control Systems with SYSTUNE"

"Tune Control Systems in Simulink"

### TuningGoal.WeightedGain class

Package: TuningGoal

Frequency-weighted gain constraint for control system tuning

### **Description**

Use TuningGoal.WeightedGain to limit the weighted gain from specified inputs to outputs. The weighted gain is the maximum across frequency of the gain from input to output, multiplied by weighting functions that you specify. You can use the TuningGoal.WeightedGain tuning goal for control system tuning with tuning commands such as systume or looptune.

After you create a tuning goal, you can configure it further by setting "Properties" on page 1-216 of the object.

### Construction

Req = TuningGoal.WeightedGain(inputname,outputname,WL,WR) creates a tuning goal that specifies that the closed-loop transfer function, H(s), from the specified input to output meets the requirement:

$$||W_L(s)H(s)W_R(s)||_{\infty}$$

The notation  $|| \cdot ||_{\infty}$  denotes the maximum gain across frequency (the  $H_{\infty}$  norm).

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

• If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:

- Any model input.
- Any linear analysis point marked in the model.
- Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

• If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:

- · Any model output.
- Any linear analysis point marked in the model.
- Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### WL,WR

Frequency-weighting functions, specified as scalars, matrices, or SISO or MIMO numeric LTI models.

The functions WL and WR provide the weights for the tuning goal. The tuning goal ensures that the gain H(s) from the specified input to output satisfies the inequality:

```
||WL(s)H(s)WR(s)||_{\infty} \qquad \qquad < \qquad \qquad 1.
```

WL provides the weighting for the output channels of H(s), and WR provides the weighting for the input channels. You can specify scalar weights or frequency-dependent weighting. To specify a frequency-dependent weighting, use a numeric LTI model. For example:

```
WL = tf(1,[1 0.01]);

WR = 10;
```

If you specify MIMO weighting functions, then inputname and outputname must be vector signals. The dimensions of the vector signals must be such that the dimensions of H(s) are commensurate with the dimensions of WL and WR. For example, if you specify WR = diag([1 10]), then inputname must include two signals. Scalar values, however, automatically expand to any input or output dimension.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify the weighting functions as discrete-time models with the same Ts. If you specify the weighting functions in continuous time, the tuning software discretizes them. Specifying the weighting functions in discrete time gives you more control over the weighting functions near the Nyquist frequency.

A value of WL = [] or WR = [] is interpreted as the identity.

### **Properties**

#### WL

Frequency-weighting function for the output channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WL input argument when you construct the tuning goal.

#### WR

Frequency-weighting function for the input channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WR input argument when you construct the tuning goal.

#### **Focus**

Frequency band in which tuning goal is enforced, specified as a row vector of the form [min,max].

Set the Focus property to limit enforcement of the tuning goal to a particular frequency band. Express this value in the frequency units of the control system model you are tuning (rad/TimeUnit). For example, suppose Req is a tuning goal that you want to apply only between 1 and 100 rad/s. To restrict the tuning goal to this band, use the following command:

```
Req.Focus = [1,100];
```

**Default:** [0,Inf] for continuous time; [0,pi/Ts] for discrete time, where Ts is the model sample time.

#### Stabilize

Stability requirement on closed-loop dynamics, specified as 1 (true) or 0 (false).

By default, TuningGoal.Gain imposes a stability requirement on the closed-loop transfer function from the specified inputs to outputs, in addition to the gain requirement. If stability is not required or cannot be achieved, set Stabilize to false to remove the stability requirement. For example, if the gain constraint applies to an unstable open-loop transfer function, set Stabilize to false.

**Default:** 1(true)

#### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

#### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

#### **Default: NaN**

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

## Examples

Default: []

#### **Constrain Weighted Gain of Closed-Loop System**

Create a tuning goal requirement that constrains the gain of a closed-loop SISO system from its input, r, to its output, y. Weight the gain at its input by a factor of 10 and at its output by the frequency-dependent weight 1/(s + 0.01).

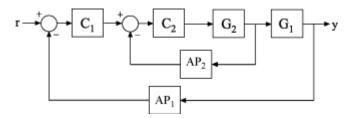
```
WL = tf(1,[1 0.01]);
WR = 10;
Req = TuningGoal.WeightedGain('r','y',WL,WR);
```

You can use the requirement Req with systune to tune the free parameters of any control system model that has an input signal named 'r' and an output signal named 'y'.

You can then use viewGoal to validate the tuned control system against the requirement.

#### **Constrain Weighted Gain Evaluated with a Loop Opening**

Create a requirement that constrains the gain of the outer loop of the following control system, evaluated with the inner loop open.



Create a model of the system. To do so, specify and connect the numeric plant models, G1 and G2, the tunable controllers C1 and C2. Also, create and connect the AnalysisPoint blocks that mark points of interest for analysis or tuning, AP1 and AP2.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
AP1 = AnalysisPoint('AP1');
AP2 = AnalysisPoint('AP2');
T = feedback(G1*feedback(G2*C2,AP2)*C1,AP1);
T.InputName = 'r';
T.OutputName = 'y';
```

Create a tuning requirement that constrains the gain of this system from r to y. Weight the gain at the output by s/(s+0.5).

```
WL = tf([1 0],[1 0.5]);
Req = TuningGoal.WeightedGain('r','y',WL,[]);
```

This requirement is equivalent to Req = TuningGoal.Gain('r', 'y', 1/WL). However, for MIMO systems, you can use TuningGoal.WeightedGain to create channel-specific weightings that cannot be expressed as TuningGoal.Gain requirements.

Specify that the transfer function from r to y be evaluated with the outer loop open for the purpose of tuning to this constraint.

```
Req.Openings = 'AP1';
```

By default, tuning using TuningGoal.WeightedGain imposes a stability requirement as well as the gain requirement. Practically, in some control systems it is not possible to achieve a stable inner loop. When this occurs, remove the stability requirement for the inner loop by setting the Stabilize property to false.

```
Req.Stabilize = false;
```

The tuning algorithm still imposes a stability requirement on the overall tuned control system, but not on the inner loop alone.

Use systune to tune the free parameters of T to meet the tuning requirement specified by Req. You can then validate the tuned control system against the requirement using the command viewGoal(Req,T).

### **Tips**

• This tuning goal imposes an implicit stability constraint on the weighted closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

### **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.WeightedGain, f(x) is given by:

$$f(x) = \|W_L T(s, x) W_R\|_{\infty}.$$

T(s,x) is the closed-loop transfer function from Input to Output.  $\|\cdot\|_{\infty}$  denotes the  $H_{\infty}$  norm (see getPeakGain).

### **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

evalGoal|looptune|looptune (for slTuner)|slTuner|systume|systume
(for slTuner)|viewGoal

## **Topics**

"Frequency-Domain Specifications"

Introduced in R2016a

### TuningGoal.WeightedVariance class

Package: TuningGoal

Frequency-weighted H<sub>2</sub> norm constraint for control system tuning

### **Description**

Use TuningGoal.WeightedVariance to limit the weighted  $H_2$  norm of the transfer function from specified inputs to outputs. The  $H_2$  norm measures:

- The total energy of the impulse response, for deterministic inputs to the transfer function.
- The square root of the output variance for a unit-variance white-noise input, for stochastic inputs to the transfer function. Equivalently, the  $H_2$  norm measures the root-mean-square of the output for such input.

You can use TuningGoal.WeightedVariance for control system tuning with tuning commands, such as systume or looptune. By specifying this tuning goal, you can tune the system response to stochastic inputs with a nonuniform spectrum such as colored noise or wind gusts. You can also use TuningGoal.WeightedVariance to specify LQG-like performance objectives.

After you create a tuning goal object, you can configure it further by setting "Properties" on page 1-227 of the object.

### Construction

Req = TuningGoal.Variance(inputname,outputname,WL,WR) creates a tuning goal Req. This tuning goal specifies that the closed-loop transfer function H(s) from the specified input to output meets the requirement:

$$||W_{L}(s)H(s)W_{R}(s)||_{2}$$
 < 1.

The notation  $||\cdot||_2$  denotes the  $H_2$  norm.

When you are tuning a discrete-time system, Req imposes the following constraint:

$$\frac{1}{\sqrt{T_s}} \|W_L(z)T(z, x)W_R(z)\|_2 < 1.$$

The  $H_2$  norm is scaled by the square root of the sample time  $T_s$  to ensure consistent results with tuning in continuous time. To constrain the true discrete-time  $H_2$  norm, multiply either  $W_L$  or  $W_R$  by  $\sqrt{T_s}$ .

### **Input Arguments**

#### inputname

Input signals for the tuning goal, specified as a character vector or, for multiple-input tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then inputname can include:
  - Any model input.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points u1 and u2. Use 'u1' to designate that point as an input signal when creating tuning goals. Use {'u1', 'u2'} to designate a two-channel input.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then inputname can include:
  - Any input of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then inputname can be any input name in T. InputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then inputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If inputname is an AnalysisPoint location of a generalized model, the input signal for the tuning goal is the implied input associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### outputname

Output signals for the tuning goal, specified as a character vector or, for multiple-output tuning goals, a cell array of character vectors.

- If you are using the tuning goal to tune a Simulink model of a control system, then outputname can include:
  - Any model output.
  - Any linear analysis point marked in the model.
  - Any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

For example, suppose that the slTuner interface contains analysis points y1 and y2. Use 'y1' to designate that point as an output signal when creating tuning goals. Use  $\{'y1', 'y2'\}$  to designate a two-channel output.

- If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then outputname can include:
  - Any output of the genss model
  - Any AnalysisPoint location in the control system model

For example, if you are tuning a control system model, T, then outputname can be any output name in T.OutputName. Also, if T contains an AnalysisPoint block with a location named AP\_u, then outputname can include 'AP\_u'. Use getPoints to get a list of analysis points available in a genss model.

If outputname is an AnalysisPoint location of a generalized model, the output signal for the tuning goal is the implied output associated with the AnalysisPoint block:



For more information about analysis points in control system models, see "Mark Signals of Interest for Control System Analysis and Design".

#### WL, WR

Frequency-weighting functions, specified as scalars, matrices, or SISO or MIMO numeric LTI models.

The functions WL and WR provide the weights for the tuning goal. The tuning goal ensures that the gain H(s) from the specified input to output satisfies the inequality:

WL provides the weighting for the output channels of H(s), and WR provides the weighting for the input channels. You can specify scalar weights or frequency-dependent weighting. To specify a frequency-dependent weighting, use a numeric LTI model. For example:

$$WL = tf(1,[1 0.01]);$$
  
 $WR = 10;$ 

If you specify MIMO weighting functions, then inputname and outputname must be vector signals. The dimensions of the vector signals must be such that the dimensions of H(s) are commensurate with the dimensions of WL and WR. For example, if you specify WR = diag([1 10]), then inputname must include two signals. Scalar values, however, automatically expand to any input or output dimension.

If you are tuning in discrete time (that is, using a genss model or slTuner interface with nonzero Ts), you can specify the weighting functions as discrete-time models with the same Ts. If you specify the weighting functions in continuous time, the tuning software discretizes them. Specifying the weighting functions in discrete time gives you more control over the weighting functions near the Nyquist frequency.

A value of WL = [] or WR = [] is interpreted as the identity.

### **Properties**

#### WL

Frequency-weighting function for the output channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WL input argument when you construct the tuning goal.

#### WR

Frequency-weighting function for the input channels of the transfer function to constrain, specified as a scalar, a matrix, or a SISO or MIMO numeric LTI model. The initial value of this property is set by the WR input argument when you construct the tuning goal.

#### Input

Input signal names, specified as a cell array of character vectors that identify the inputs of the transfer function that the tuning goal constrains. The initial value of the Input property is set by the inputname input argument when you construct the tuning goal.

#### **Output**

Output signal names, specified as a cell array of character vectors that identify the outputs of the transfer function that the tuning goal constrains. The initial value of the Output property is set by the outputname input argument when you construct the tuning goal.

#### Models

Models to which the tuning goal applies, specified as a vector of indices.

Use the Models property when tuning an array of control system models with systune, to enforce a tuning goal for a subset of models in the array. For example, suppose you want to apply the tuning goal, Req, to the second, third, and fourth models in a model array passed to systune. To restrict enforcement of the tuning goal, use the following command:

```
Req.Models = 2:4;
```

When Models = NaN, the tuning goal applies to all models.

**Default: NaN** 

#### Openings

Feedback loops to open when evaluating the tuning goal, specified as a cell array of character vectors that identify loop-opening locations. The tuning goal is evaluated against the open-loop configuration created by opening feedback loops at the locations you identify.

If you are using the tuning goal to tune a Simulink model of a control system, then Openings can include any linear analysis point marked in the model, or any linear analysis point in an slTuner interface associated with the Simulink model. Use addPoint to add analysis points and loop openings to the slTuner interface. Use getPoints to get the list of analysis points available in an slTuner interface to your model.

If you are using the tuning goal to tune a generalized state-space (genss) model of a control system, then Openings can include any AnalysisPoint location in the control system model. Use getPoints to get the list of analysis points available in the genss model.

For example, if  $Openings = \{'u1', 'u2'\}$ , then the tuning goal is evaluated with loops open at analysis points u1 and u2.

#### **Default:** {}

#### Name

Name of the tuning goal, specified as a character vector.

For example, if Req is a tuning goal:

```
Req.Name = 'LoopReq';
```

Default: []

### **Examples**

#### Weighted Constraint on H2 Norm

Create a constraint for a transfer function with one input, r, and two outputs, e and y, that limits the  $H_2$  norm as follows:

$$\left\| \frac{\frac{1}{s+0.001} T_{re}}{\frac{s}{0.001s+1} T_{ry}} \right\|_{2} < 1.$$

 $T_{re}$  is the closed-loop transfer function from r to e, and  $T_{ry}$  is the closed-loop transfer function from r to y .

```
s = tf('s');
WL = blkdiag(1/(s+0.001),s/(0.001*s+1));
Req = TuningGoal.WeightedVariance('r',{'e','y'},WL,[]);
```

### **Tips**

• When you use this tuning goal to tune a continuous-time control system, systune attempts to enforce zero feedthrough (D=0) on the transfer that the tuning goal constrains. Zero feedthrough is imposed because the  $H_2$  norm, and therefore the value of the tuning goal (see "Algorithms" on page 1-230), is infinite for continuous-time systems with nonzero feedthrough.

systune enforces zero feedthrough by fixing to zero all tunable parameters that contribute to the feedthrough term. systune returns an error when fixing these tunable parameters is insufficient to enforce zero feedthrough. In such cases, you must modify the tuning goal or the control structure, or manually fix some tunable parameters of your system to values that eliminate the feedthrough term.

When the constrained transfer function has several tunable blocks in series, the software's approach of zeroing all parameters that contribute to the overall feedthrough might be conservative. In that case, it is sufficient to zero the feedthrough term of one of the blocks. If you want to control which block has feedthrough fixed to zero, you can manually fix the feedthrough of the tuned block of your choice.

To fix parameters of tunable blocks to specified values, use the Value and Free properties of the block parametrization. For example, consider a tuned state-space block:

```
C = tunableSS('C',1,2,3);
```

To enforce zero feedthrough on this block, set its D matrix value to zero, and fix the parameter.

```
C.D.Value = 0;
C.D.Free = false;
```

For more information on fixing parameter values, see the Control Design Block reference pages, such as tunableSS.

• This tuning goal imposes an implicit stability constraint on the weighted closed-loop transfer function from Input to Output, evaluated with loops opened at the points identified in Openings. The dynamics affected by this implicit constraint are the stabilized dynamics for this tuning goal. The MinDecay and MaxRadius options of systuneOptions control the bounds on these implicitly constrained dynamics. If the optimization fails to meet the default bounds, or if the default bounds conflict with other requirements, use systuneOptions to change these defaults.

### **Algorithms**

When you tune a control system using a TuningGoal, the software converts the tuning goal into a normalized scalar value f(x). x is the vector of free (tunable) parameters in the control system. The software then adjusts the parameter values to minimize f(x) or to drive f(x) below 1 if the tuning goal is a hard constraint.

For TuningGoal.WeightedVariance, f(x) is given by:

$$f(x) = ||W_L T(s, x) W_R||_2$$
.

T(s,x) is the closed-loop transfer function from Input to Output.  $\|\cdot\|_2$  denotes the  $H_2$  norm (see norm).

For tuning discrete-time control systems, f(x) is given by:

$$f(x) = \frac{1}{\sqrt{T_s}} \|W_L(z)T(z, x)W_R(z)\|_2.$$

 $T_s$  is the sample time of the discrete-time transfer function T(z,x).

### **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

TuningGoal.Gain | TuningGoal.LoopShape | TuningGoal.Variance | looptune |
looptune (for slTuner) | norm | slTuner | systume | systume (for slTuner)

### **Topics**

"Frequency-Domain Specifications"
"Fault-Tolerant Control of a Passenger Jet"

#### Introduced in R2016a

# **Functions — Alphabetical List**

### abs

Entrywise magnitude of frequency response

### **Syntax**

absfrd = abs(sys)

### **Description**

absfrd = abs(sys) computes the magnitude of the frequency response contained in the FRD model sys. For MIMO models, the magnitude is computed for each entry. The output absfrd is an FRD object containing the magnitude data across frequencies.

### See Also

bodemag | fnorm | sigma

Introduced in R2006a

### absorbDelay

Replace time delays by poles at z = 0 or phase shift

### **Syntax**

```
sysnd = absorbDelay(sysd)
[sysnd,G] = absorbDelay(sysd)
```

### **Description**

sysnd = absorbDelay(sysd) absorbs all time delays of the dynamic system model
sysd into the system dynamics or the frequency response data.

For discrete-time models (other than frequency response data models), a delay of k sampling periods is replaced by k poles at z=0. For continuous-time models (other than frequency response data models), time delays have no exact representation with a finite number of poles and zeros. Therefore, use pade to compute a rational approximation of the time delay.

For frequency response data models in both continuous and discrete time, absorbDelay absorbs all time delays into the frequency response data as a phase shift.

[sysnd,G] = absorbDelay(sysd) returns the matrix G that maps the initial states of the ss model sysd to the initial states of the sysnd.

### **Examples**

#### **Absorb Time Delay into System Dynamics**

Create a discrete-time transfer function that has a time delay.

```
z = tf('z',-1);

sysd = (-0.4*z -0.1)/(z^2 + 1.05*z + 0.08);

sysd.InputDelay = 3
```

The display of sysd represents the InputDelay as a factor of  $z^{(-3)}$ , separate from the system poles that appear in the transfer function denominator.

Absorb the time delay into the system dynamics as poles at z = 0.

The display of sysnd shows that the factor of  $z^{-}(-3)$  has been absorbed as additional poles in the denominator.

Verify that sysnd has no input delay.

```
sysnd.InputDelay ans = 0
```

#### Convert Leading Structural Zeros of Polynomial Model to Regular Coefficients

Create a discrete-time polynomial model.

```
m = idpoly(1,[0 0 0 2 3]);
```

Convert m to a transfer function model.

```
sys = tf(m)
```

```
sys =
  z^(-2) * (2 z^-1 + 3 z^-2)
Sample time: unspecified
Discrete-time transfer function.
```

The numerator of the transfer function, sys, is [0 2 3] and the transport delay, sys.IODelay, is 2. This is because the value of the B polynomial, m.B, has 3 leading zeros. The first fixed zero shows lack of feedthrough in the model. The two zeros after that are treated as input-output delays.

Use absorbDelay to treat the leading zeros as regular B coefficients.

```
m2 = absorbDelay(m);
sys2 = tf(m2)

sys2 =
   2 z^-3 + 3 z^-4

Sample time: unspecified
Discrete-time transfer function.
```

The numerator of sys2 is [0 0 0 2 3] and transport delay is 0. The model m2 treats the leading zeros as regular coefficients by freeing their values. m2.Structure.B.Free(2:3) is TRUE while m.Structure.B.Free(2:3) is FALSE.

### See Also

hasdelay | pade | totaldelay

Introduced in R2011b

### allmargin

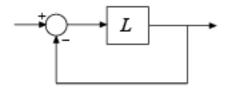
Gain margin, phase margin, delay margin, and crossover frequencies

### **Syntax**

```
S = allmargin(L)
S = allmargin(mag,phase,w,ts)
```

### **Description**

S = allmargin(L) computes the gain margin, phase margin, delay margin, and the corresponding crossover frequencies for the SISO or MIMO negative feedback loop with open-loop response L. The negative feedback loop is computed as feedback(L,eye(M)), where M is the number of inputs and outputs in L.



For a MIMO system, allmargin returns loop-at-a-time stability margins for the negative-feedback closed loop system. Use allmargin to find classical margins of any SISO or MIMO model, including models with delays.

S = allmargin(mag,phase,w,ts) computes the stability margins from the frequency response data mag, phase, w, and the sample time, ts.

### **Examples**

#### **Stability Margins of a Transfer Function**

For this example, consider a SISO open-loop transfer function L given by,

$$L = \frac{25}{s^3 + 10s^2 + 10s + 10}$$

L = tf(25,[1 10 10 10]);

Find the stability margins of L.

```
S = allmargin(L)
```

```
S = struct with fields:
    GainMargin: 3.6000
    GMFrequency: 3.1623
    PhaseMargin: 29.1104
    PMFrequency: 1.7844
    DelayMargin: 0.2847
    DMFrequency: 1.7844
    Stable: 1
```

The output S is a structure with the classical margins and their respective crossover frequencies of the negative feedback loop of L.

### Stability Margins of a MIMO System

For this example, consider a MIMO state-space model L with 2 inputs and 2 outputs.

Load the data.

```
load('mimoStateSpaceModel.mat','L')
```

Find the classical margins for the MIMO system.

```
S = allmargin(L)
```

```
S=2×7 struct
GainMargin
GMFrequency
PhaseMargin
```

```
PMFrequency
DelayMargin
DMFrequency
Stable
```

The output S is a 2-by-1 structure array of classical margins and their respective crossover frequencies. For instance, S(1) refers to the stability margins of the first I/O feedback channel with all other loops closed.

#### Stability Margins of Models in an Array

For this example, load invertedPendulumArray.mat, which contains a 3-by-3 array of inverted pendulum SISO models. The mass of the pendulum varies as you move from model to model along a single column of sys, and the length of the pendulum varies as you move along a single row. The mass values used are 100g, 200g and 300g, and the pendulum lengths used are 3m, 2m and 1m respectively.

```
Column 1 Column 2 Column 3

Row 1 100g,3m 100g,2m 100g,1m

Row 2 200g,3m 200g,2m 200g,1m

Row 3 300g,3m 300g,2m 300g,1m

load('invertedPendulumArray.mat','sys');
size(sys)

3x3 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Find stability margins for all models in the array.

```
S = allmargin(sys)
S=3×3 struct
    GainMargin
    GMFrequency
    PhaseMargin
    PMFrequency
    DelayMargin
    DMFrequency
```

Stable

allmargin returns a 3-by-3 structure array S, in which each entry is a structure containing the stability margins of the corresponding entry in S . For instance, the stability margins of the model with 100g pendulum weight and S length is contained in S (1,2).

#### **Stability Margins from Frequency Response Data**

For this example, load the frequency response data of an open loop system, consisting of magnitudes m and phase values p measured at the frequencies in w.

```
load('openLoopFRData.mat', 'm', 'p', 'w', 'ts');
```

Compute stability margins using the frequency response data.

```
S = allmargin(m,p,w,ts)

S = struct with fields:
    GainMargin: 0.6249
    GMFrequency: 1.2732
    PhaseMargin: [-90.0000 48.9853]
    PMFrequency: [1.0000 1.5197]
    DelayMargin: [4.7124 0.5626]
    DMFrequency: [1.0000 1.5197]
    Stable: NaN
```

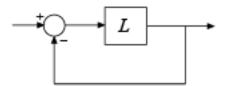
The output S is a structure with the classical margins and their respective crossover frequencies. Since allmargin cannot assess the stability for frequency response data models, S.Stable = NaN.

### **Input Arguments**

#### L — Open-loop response

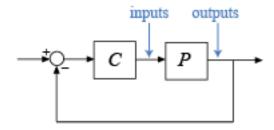
dynamic system model | model array

Open-loop response, specified as a dynamic system model. L can be SISO or MIMO, as long as it has the same number of inputs and outputs. allmargin computes the classical stability margins for the negative-feedback closed-loop system feedback(L,eye(M)):



To compute the stability margins of the positive feedback system feedback(L, eye(M), +1), use allmargin(-L).

When you have a controller P and a plant C, you can compute the stability margins for gain and phase variations at the plant inputs or outputs. From the following diagram:



- To compute margins at the plant outputs, set L = P\*C.
- To compute margins at the plant inputs, set L = C\*P.

L can be continuous time or discrete time. If L is a generalized state-space model (genss or uss), then allmargin uses the current or nominal value of all control design blocks in L.

If L is a frequency-response data model (such as frd), then allmargin computes the margins at each frequency represented in the model. The function returns the margins at the frequency with the smallest stability margin.

If L is a model array, then  ${\tt allmargin}$  computes margins for each model in the array.

#### mag — Magnitude of system response

3-D array

Magnitude of the system response in absolute units, specified as a 3-D array. mag is an M-by-M-by-N array, where M is the number of inputs or outputs, and N is the number of frequency points. For more information on obtaining mag, see "Obtain Magnitude and Phase Data" on page 2-70 and "Magnitude and Phase of MIMO System" on page 2-71.

#### phase — Phase of system response

3-D array

Phase of the system response in degrees, specified as a 3-D array. phase is an M-by-M-by-N array, where M is the number of inputs or outputs, and N is the number of frequency points. For more information on obtaining phase, see "Obtain Magnitude and Phase Data" on page 2-70 and "Magnitude and Phase of MIMO System" on page 2-71.

#### w — Frequencies at which the magnitude and phase values of system response are obtained

column vector

Frequencies at which the magnitude and phase values of system response are obtained, specified as a column vector. You can provide the frequency vector w in any units; allmargin returns frequencies in the same units. allmargin interpolates between frequency points to approximate the true stability margins.

#### ts — Sample time

integer

Sample time, specified as an integer. allmargin uses ts to find the stability margins from frequency response data.

- For continuous-time models, set ts = 0.
- For discrete-time models, ts is a positive integer representing the sampling period. To
  denote a discrete-time model with unspecified sample time, set ts = -1.

### **Output Arguments**

#### S — Gain, phase, and delay margins

structure | structure array

Gain, phase, and delay margins, returned as a structure array.

The output S is a structure with the following fields:

- GMFrequency: All-180° (modulo 360°) crossover frequencies in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property of L.
- GainMargin: Corresponding gain margins, defined as 1/G, where G is the gain at the -180° crossover frequency. Gain margins are in absolute units.
- PMFrequency: All 0-dB crossover frequencies in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property of L.
- PhaseMargin: Corresponding phase margins in degrees.
- DMFrequency and DelayMargin: DelayMargin is the maximum amount of delay that the system can tolerate before it loses stability. DMFrequency contains critical frequencies corresponding to the delay margins. Delay margins are specified in the time units of the system for continuous-time systems and multiples of the sample time for discrete-time systems.
- Stable: 1 if the nominal closed-loop system is stable, 0 if unstable, and NaN if stability cannot be assessed. In general, allmargin cannot assess the stability of an frd system.

When L is an M-by-M MIMO system, S is an M-by-1 structure array. For instance, S(j) gives the stability margins for the j-th feedback channel with all other loops closed (one-loop-at-a-time margins).

### **Tips**

- allmargin assumes that the system with open-loop response L is a negative-feedback system. To compute the classical stability margins of the positive feedback system feedback(L,eye(M),+1), use allmargin(-L).
- To compute classical margins for a system modeled in Simulink, first linearize the model to obtain the open-loop response at a particular operating point. Then, use allmargin to compute classical stability margins for the linearized system. For more information, see "Stability Margins of a Simulink Model" (Robust Control Toolbox).

### **See Also**

Linear System Analyzer | diskmargin | margin

**Topics**"Stability Margins of a Simulink Model" (Robust Control Toolbox)

**Introduced before R2006a** 

### **AnalysisPoint**

Points of interest for linear analysis

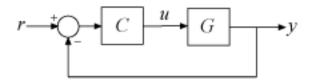
### **Syntax**

AP = AnalysisPoint(name)
AP = AnalysisPoint(name,N)

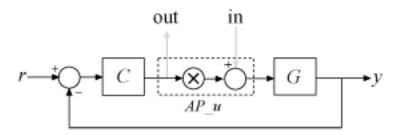
### **Description**

AnalysisPoint is a Control Design Block for marking a location in a control system model as a point of interest for linear analysis and controller tuning. You can combine an AnalysisPoint block with numeric LTI models, tunable LTI models, and other Control Design Blocks to build tunable models of control systems. AnalysisPoint locations are available for analysis with commands such as getIOTransfer or getLoopTransfer. Such locations are also available for specifying design goals for control system tuning.

For example, consider the following control system.



Suppose that you are interested in the effects of disturbance injected at u in this control system. Inserting an AnalysisPoint block at the location u associates an implied input, implied output, and the option to open the loop at that location, as in the following diagram.



Suppose that T is a model of the control system including the AnalysisPoint block, AP\_u. In this case, the command getIOTransfer(T, 'AP\_u', 'y') returns a model of the closed-loop transfer function from u to y. Likewise, the command getLoopTransfer(T, 'AP\_u', -1) returns a model of the negative-feedback open-loop response, CG, measured at the location u.

AnalysisPoint blocks are also useful when tuning a control system using tuning commands such as systume. You can use an AnalysisPoint block to mark a loop-opening location for open-loop tuning requirements such as TuningGoal.LoopShape or TuningGoal.Margins. You can also use an AnalysisPoint block to mark the specified input or output for tuning requirements such as TuningGoal.Gain. For example, Req = TuningGoal.Margins('AP\_u',5,40) constrains the gain and phase margins at the location u.

You can create AnalysisPoint blocks explicitly using the AnalysisPoint command and connect them with other block diagram components using model interconnection commands. For example, the following code creates a model of the system illustrated above. (See "Construction" on page 2-16 and "Examples" on page 2-0 below for more information.)

```
G = tf(1,[1 2]);
C = tunablePID('C','pi');
AP_u = AnalysisPoint('u');
T = feedback(G*AP_u*C,1); % closed loop r->y
```

You can also create analysis points implicitly, using the connect command. The following syntax creates a dynamic system model with analysis points, by interconnecting multiple models sys1,sys2,...,sysN:

```
sys = connect(sys1,sys2,...,sysN,inputs,outputs,APs);
```

APs lists the signal locations at which to insert analysis points. The software automatically creates and inserts an AnalysisPoint block with channels corresponding to these locations. See connect for more information.

## Construction

AP = AnalysisPoint(name) creates a single-channel analysis point. Insert AP anywhere in the generalized model of your control system to mark a point of interest for linear analysis or controller tuning. name specifies the block name.

AP = AnalysisPoint(name, N) creates a multi-channel analysis point with N channels. Use this block to mark a vector-valued signal as a point of interest or to bundle together several points of interest.

## **Input Arguments**

#### name

Analysis point name, specified as a character vector such as 'AP'. This input argument sets the value of the Name property of the AnalysisPoint block. (See "Properties" on page 2-16.) When you build a control system model using the block, the Name property is what appears in the Blocks list of the resulting genss model.

#### N

Number of channels for a multichannel analysis point, specified as a scalar integer.

# **Properties**

#### Location

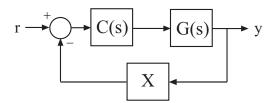
Names of channels in the AnalysisPoint blocks, specified as a character vector or a cell array of character vectors.

By default, the analysis-point channels are named after the name argument. For example, if you have a single-channel analysis point, AP, that has name 'AP', then AP.Location = 'AP' by default. If you have a multi-channel analysis point, then AP.Location = {'AP(1)', 'AP(2)',...} by default. Set AP.Location to a different value if you want to customize the channel names.

### 0pen

Loop-opening state, specified as a logical value or vector of logical values. This property tracks whether the loop is open or closed at the analysis point.

For example, consider the feedback loop of the following illustration.



You can model this feedback loop as follows.

```
G = tf(1,[1 2]);
C = tunablePID('C','pi');
X = AnalysisPoint('X');
T = feedback(G*C,X);
```

You can get the transfer function from r to y with the feedback loop open at X as follows.

```
Try = getIOTransfer(T, 'r', 'y', 'X');
```

In the resulting generalized state-space (genss) model, the AnalysisPoint block 'X' is marked open. In other words, Try.Blocks.X.Open = 1.

For a multi-channel analysis point, then **Open** is a logical vector with as many entries as the analysis point has channels.

**Default:** 0 for all channels

#### Ts

Sample time. For AnalysisPoint blocks, the value of this property is automatically set to the sample time of other blocks and models you connect it with.

### **Default:** 0 (continuous time)

#### TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- · 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

### Default: 'seconds'

### **InputName**

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to  $\{'controls(1)'; 'controls(2)'\}$ .

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- · Specifying connection points when interconnecting models

**Default:** ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector For single-output models. For example, 'seconds'.
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

## OutputGroup

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system 1'.

Default: ''

### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

## **Default:** [0×1 string]

#### UserData

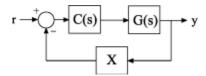
Any type of data you want to associate with system, specified as any MATLAB® data type.

### Default: []

# **Examples**

### **Feedback Loop with Analysis Point**

Create a model of the following feedback loop with an analysis point in the feedback path.



For this example, the plant model is G = 1/(s + 2). C is a tunable PI controller, and X is the analysis point.

```
G = tf(1,[1 2]);
C = tunablePID('C','pi');
X = AnalysisPoint('X');
T = feedback(G*C,X);
T.InputName = 'r';
T.OutputName = 'y';
```

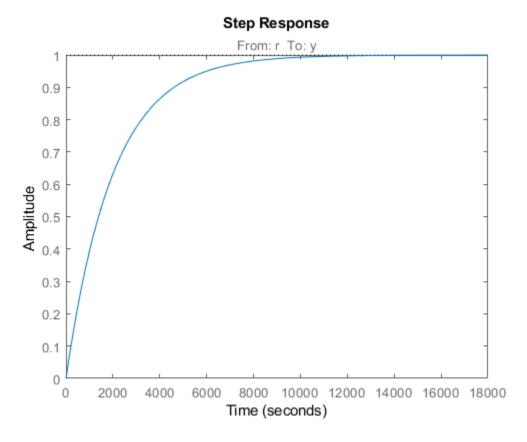
T is a tunable genss model. T.Blocks contains the Control Design Blocks of the model, which are the controller, C, and the analysis point, X.

#### T.Blocks

```
ans = struct with fields:
    C: [1x1 tunablePID]
    X: [1x1 AnalysisPoint]
```

Examine the step response of T.

```
stepplot(T)
```



The presence of the AnalysisPoint block does not change the dynamics of the model.

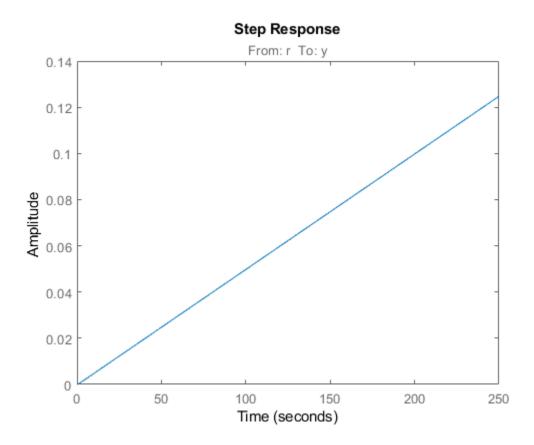
You can use the analysis point for linear analysis of the system. For instance, extract the system response at 'y' to a disturbance injected at the analysis point.

The AnalysisPoint block also allows you to temporarily open the feedback loop at that point. For example, compute the open-loop response from 'r' to 'y'.

```
Try_open = getIOTransfer(T, 'r', 'y', 'X');
```

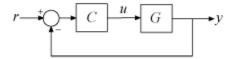
Specifying the analysis point name as the last argument to getIOTransfer extracts the response with the loop open at that point. Examine the step response of Try\_open to verify that it is the open-loop response.

stepplot(Try\_open);



## Feedback Loop With Analysis Point Inserted by connect

Create a model of the following block diagram from r to y. Insert an analysis point at an internal location, u.



Create C and G, and name the inputs and outputs.

```
C = pid(2,1);
C.InputName = 'e';
C.OutputName = 'u';
G = zpk([],[-1,-1],1);
G.InputName = 'u';
G.OutputName = 'y';
```

Create the summing junction.

```
Sum = sumblk('e = r - y');
```

Combine C, G, and the summing junction to create the aggregate model, with an analysis point at u.

```
T = connect(G,C,Sum,'r','y','u')
T =
```

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 3 states, and AnalysisPoints\_: Analysis point, 1 channels, 1 occurrences.

```
Type "ss(T)" to see the current value, "get(T)" to see all properties, and "T.Blocks"
```

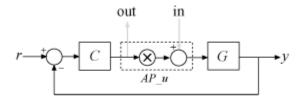
The resulting T is a genss model. The connect command creates the AnalysisPoint block, AnalysisPoints\_, and inserts it into T. To see the name of the analysis point channel in AnalysisPoints\_, use getPoints.

```
getPoints(T)
ans = 1x1 cell array
{'u'}
```

The analysis point channel is named 'u'. You can use this analysis point to extract system responses. For example, the following commands extract the open-loop transfer at u and the closed-loop response at v to a disturbance injected at u.

```
L = getLoopTransfer(T, 'u', -1);
Tuy = getIOTransfer(T, 'u', 'y');
```

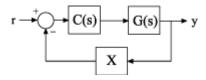
T is equivalent to the following block diagram, where  $AP_u$  designates the AnalysisPoint block AnalysisPoints\_ with channel name u.



### **Multi-Channel Analysis Points**

Create a block for marking two analysis points in a MIMO model.

In the control system of the following illustration, consider each signal a vector-valued signal of size 2. In other words, the signal r represents  $\{r(1), r(2)\}$ , y represents  $\{y(1), y(2)\}$ , and so on.



The feedback signal is therefore also a vector-valued signal of size 2. Create a block for marking the two analysis points in the feedback path.

```
AP = AnalysisPoint('X',2)
AP =
Multi-channel analysis point at locations:
```

```
X(1)
X(2)
```

Type "ss(AP)" to see the current value and "get(AP)" to see all properties.

The AnalysisPoint block is stored as a variable in the MATLAB® workspace called AP. In addition, the Name property of the block is set to X. When you interconnect the block with numeric LTI models or other Control Design Blocks, this analysis-point block is identified in the Blocks property of the resulting genss model as X. The block name X is automatically expanded to generate the channel names X(1) and X(2).

It is sometimes convenient to change the channel names to match the names of the signals they correspond to in a block diagram of your model. For example, suppose the points of interest you want to mark in your model are signals named L and V. Change the Location property of AP to make the names match those signals.

```
AP.Location = {'L';'V'}
AP =
Multi-channel analysis point at locations:
    L
    V

Type "ss(AP)" to see the current value and "get(AP)" to see all properties.
```

Although the channel names have changed, the block name remains X.

#### AP.Name

```
ans =
```

Therefore, the Blocks property of a genss model you build with this block still identifies the block as X. Use getPoints to find the channel names of available analysis points in a genss model.

## **See Also**

connect | genss | getPoints

## **Topics**

- "Control System with Multichannel Analysis Points"
- "Control Design Blocks"
- "Models with Tunable Coefficients"
- "Mark Signals of Interest for Control System Analysis and Design"

### Introduced in R2014b

# append

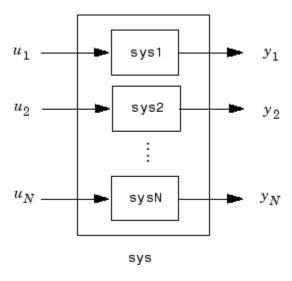
Group models by appending their inputs and outputs

## **Syntax**

sys = append(sys1,sys2,...,sysN)

# **Description**

sys = append(sys1, sys2, ..., sysN) appends the inputs and outputs of the models sys1,...,sysN to form the augmented model sys depicted below.



For systems with transfer functions  $H_1(s), \ldots, H_N(s)$ , the resulting system sys has the block-diagonal transfer function

$$\begin{bmatrix} H_1(s) & 0 & \dots & 0 \\ 0 & H_2(s) & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & H_N(s) \end{bmatrix}$$

For state-space models sys1 and sys2 with data  $(A_1, B_1, C_1, D_1)$  and  $(A_2, B_2, C_2, D_2)$ , append(sys1,sys2) produces the following state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

# **Arguments**

The input arguments sys1,..., sysN can be model objects s of any type. Regular matrices are also accepted as a representation of static gains, but there should be at least one model in the input list. The models should be either all continuous, or all discrete with the same sample time. When appending models of different types, the resulting type is determined by the precedence rules (see "Rules That Determine Model Type" for details).

There is no limitation on the number of inputs.

# **Examples**

## **Append Inputs and Outputs of Models**

Create a SISO transfer function.

```
sys1 = tf(1,[1 0]);
size(sys1)
```

Transfer function with 1 outputs and 1 inputs.

Create a SISO continuous-time state-space model.

```
sys2 = ss(1,2,3,4);
size(sys2)
State-space model with 1 outputs, 1 inputs, and 1 states.
```

Append the inputs and outputs of sys1, a SISO static gain system, and sys2. The resulting model should be a 3-input, 3-output state-space model.

```
sys = append(sys1, 10, sys2)
sys =
  A =
       x1
           x2
   x1
        0
             0
   x2
        (-)
            1
  B =
           u2
       u1
               u3
   x1
             0
                 0
        1
   x2
        (-)
             (-)
                 2
  C =
       x1
           x2
   ٧1
        1
             0
        0
             0
   y2
   у3
             3
  D =
       u1
           u2
                u3
        0
            0
                 (-)
   у1
        0
           10
                 0
   y2
                 4
   у3
Continuous-time state-space model.
size(sys)
State-space model with 3 outputs, 3 inputs, and 2 states.
```

## See Also

connect | feedback | parallel | series

## **Introduced before R2006a**

# augstate

Append state vector to output vector

# **Syntax**

```
asys = augstate(sys)
```

# **Description**

asys = augstate(sys) appends the state vector to the outputs of a state-space model.

Given a state-space model sys with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(or their discrete-time counterpart), augstate appends the states x to the outputs y to form the model

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} x + \begin{bmatrix} D \\ 0 \end{bmatrix} u$$

This command prepares the plant so that you can use the feedback command to close the loop on a full-state feedback u = -Kx.

## Limitation

Because augstate is only meaningful for state-space models, it cannot be used with TF, ZPK or FRD models.

## See Also

feedback | parallel | series

## **Introduced before R2006a**

## balreal

Gramian-based input/output balancing of state-space realizations

# **Syntax**

```
[sysb,g] = balreal(sys)
[sysb,g,T,Ti] = balreal(sys)
[       ] = balreal(sys,opts)
```

# **Description**

[sysb,g] = balreal(sys) computes a balanced realization sysb for the stable portion of the LTI model sys. balreal handles both continuous and discrete systems. If sys is not a state-space model, it is first and automatically converted to state space using ss.

For stable systems, sysb is an equivalent realization for which the controllability and observability Gramians are equal and diagonal, their diagonal entries forming the vector g of Hankel singular values. Small entries in g indicate states that can be removed to simplify the model (use modred to reduce the model order).

If sys has unstable poles, its stable part is isolated, balanced, and added back to its unstable part to form sysb. The entries of g corresponding to unstable modes are set to Inf.

[sysb, g, T, Ti] = balreal(sys) also returns the vector g containing the diagonal of the balanced Gramian, the state similarity transformation  $x_b = Tx$  used to convert sys to sysb, and the inverse transformation  $Ti = T^{-1}$ .

If the system is normalized properly, the diagonal g of the joint Gramian can be used to reduce the model order. Because g reflects the combined controllability and observability of individual states of the balanced model, you can delete those states with a small g(i) while retaining the most important input-output characteristics of the original system. Use modred to perform the state elimination.

[\_\_\_\_] = balreal(sys,opts) computes the balanced realization using options that you specify using hsvdOptions. Options include offset and tolerance options for

computing the stable-unstable decompositions. The options also allow you to limit the Gramian computation to particular time and frequency intervals. See hsvd0ptions for details.

# **Examples**

### **Balanced Realization of Stable System**

Consider the following zero-pole-gain model, with near-canceling pole-zero pairs:

A state-space realization with balanced gramians is obtained by

```
[sysb,g] = balreal(sys);
```

The diagonal entries of the joint gramian are

```
g' ans = 1 \times 3 0.1006 0.0001 0.0000
```

This indicates that the last two states of sysb are weakly coupled to the input and output. You can then delete these states by

```
sysr = modred(sysb,[2 3],'del');
```

This yields the following first-order approximation of the original system.

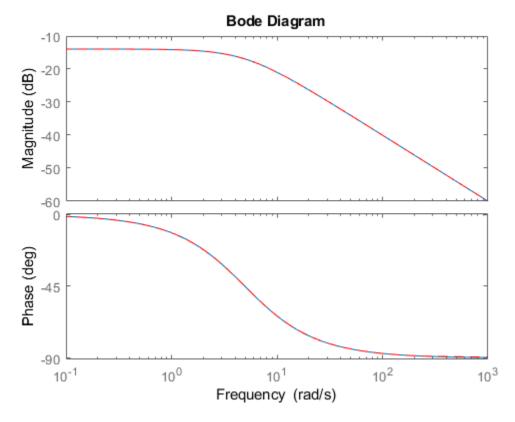
```
zpk(sysr)
```

```
ans =
1.0001
(s+4.97)
```

Continuous-time zero/pole/gain model.

Compare the Bode responses of the original and reduced-order models.

bodeplot(sys,sysr,'r--')



The plots shows that removing the second and third states does not have much effect on system dynamics.

## **Balanced Realization of Unstable System**

Create an unstable system.

Continuous-time transfer function.

Apply balreal to create a balanced-gramian realization.

[sysbal,g] = balreal(sys)

```
sysbal =
 A =
      x1 x2
  x1
      1 0
  x2
     0 -1
 B =
          u1
  x1 0.7071
  x2 0.7071
 C =
           x1
     0.7071 -0.7071
  у1
 D =
      u1
  y1 0
```

Continuous-time state-space model.

 $g = 2 \times 1$ 

The unstable pole shows up as Inf in the vector g.

# **Algorithms**

Consider the model

$$\dot{x} = Ax + Bu$$
$$v = Cx + Du$$

with controllability and observability Gramians  $W_c$  and  $W_o$ . The state coordinate transformation  $\bar{x} = Tx$  produces the equivalent model

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$

$$v = CT^{-1}\bar{x} + Du$$

and transforms the Gramians to

$$\overline{W}_C = TW_CT^T$$
,  $\overline{W}_O = T^{-T}W_OT^{-1}$ 

The function balreal computes a particular similarity transformation T such that

$$\overline{W}_c = \overline{W}_o = diag(g)$$

See [1], [2] for details on the algorithm.

If you use the TimeIntervals or FreqIntervals options of hsvdOptions, then balreal bases the balanced realization on time-limited or frequency-limited controllability and observability Gramians. For information about calculating time-limited and frequency-limited Gramians, see gram and [4].

## References

- [1] Laub, A.J., M.T. Heath, C.C. Paige, and R.C. Ward, "Computation of System Balancing Transformations and Other Applications of Simultaneous Diagonalization Algorithms," *IEEE*<sup>®</sup> *Trans. Automatic Control*, AC-32 (1987), pp. 115-122.
- [2] Moore, B., "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 17-31.
- [3] Laub, A.J., "Computation of Balancing Transformations," *Proc. ACC*, San Francisco, Vol.1, paper FA8-E, 1980.
- [4] Gawronski, W. and J.N. Juang. "Model Reduction in Limited Time and Frequency Intervals." *International Journal of Systems Science*. Vol. 21, Number 2, 1990, pp. 349–376.

## See Also

balred | gram | hsvd | hsvdOptions | modred

**Introduced before R2006a** 

## **balred**

Model order reduction

# **Syntax**

```
rsys = balred(sys,ORDERS)
rsys = balred(sys,ORDERS,BALDATA)
rsys = balred(____,opts)
```

# **Description**

rsys = balred(sys, ORDERS) computes a reduced-order approximation rsys of the LTI model sys. The desired order (number of states) for rsys is specified by ORDERS. You can try multiple orders at once by setting ORDERS to a vector of integers, in which case rsys is a vector of reduced-order models. balred uses implicit balancing techniques to compute the reduced-order approximation rsys. Use hsvd to plot the Hankel singular values and pick an adequate approximation order. States with relatively small Hankel singular values can be safely discarded.

When sys has unstable poles, it is first decomposed into its stable and unstable parts using stabsep, and only the stable part is approximated. Use balredOptions to specify additional options for the stable/unstable decomposition.

When you have System Identification  $Toolbox^{m}$  software installed, sys can only be an identified state-space model (idss). The reduced-order model is also an idss model.

rsys = balred(sys, ORDERS, BALDATA) uses balancing data returned by hsvd. Because hsvd does most of the work needed to compute rsys, this syntax is more efficient when using hsvd and balred jointly.

rsys = balred(\_\_\_\_, opts) computes the model reduction using options that you specify using balredOptions. Options include offset and tolerance options for computing the stable-unstable decompositions. There also options for emphasizing particular time or frequency intervals. See balredOptions for details.

**Note** The order of the approximate model is always at least the number of unstable poles and at most the minimal order of the original model (number NNZ of nonzero Hankel singular values using an eps-level relative threshold)

# **Examples**

### **Reduced-Order Approximation with Offset Option**

Compute a reduced-order approximation of the system given by:

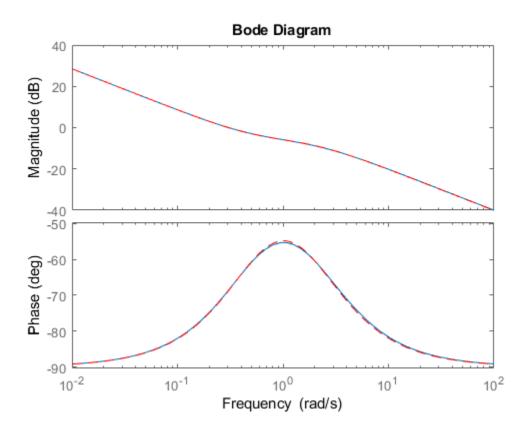
$$G(s) = \frac{(s+0.5)(s+1.1)(s+2.9)}{(s+10^{-6})(s+1)(s+2)(s+3)}.$$

Use the Offset option to exclude the pole at  $s = 10^{-6}$  from the stable term of the stable/ unstable decomposition.

```
sys = zpk([-.5 -1.1 -2.9],[-le-6 -2 -1 -3],1);
% Create balredOptions
opt = balredOptions('Offset',.001,'StateElimMethod','Truncate');
% Compute second-order approximation
rsys = balred(sys,2,opt);
```

Compare the responses of the original and reduced-order models.

```
bodeplot(sys,rsys,'r--')
```

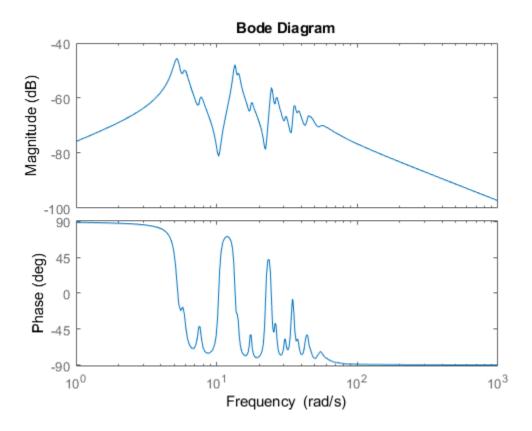


## **Model Reduction in a Particular Frequency Band**

Reduce a high-order model with a focus on the dynamics in a particular frequency range.

Load a model and examine its frequency response.

```
load(fullfile(matlabroot, 'examples', 'control', 'build.mat'), 'G')
bodeplot(G)
```

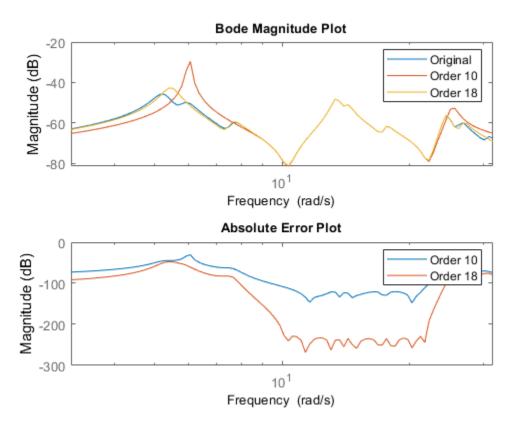


G is a 48th-order model with several large peak regions around 5.2 rad/s, 13.5 rad/s, and 24.5 rad/s, and smaller peaks scattered across many frequencies. Suppose that for your application you are only interested in the dynamics near the second large peak, between 10 rad/s and 22 rad/s. Focus the model reduction on the region of interest to obtain a good match with a low-order approximation. Use balredOptions to specify the frequency interval for balred.

```
bopt = balredOptions('StateElimMethod','Truncate','FreqIntervals',[10,22]);
GLim10 = balred(G,10,bopt);
GLim18 = balred(G,18,bopt);
```

Examine the frequency responses of the reduced-order models. Also, examine the difference between those responses and the original response (the absolute error).

```
subplot(2,1,1);
bodemag(G,GLim10,GLim18,logspace(0.5,1.5,100));
title('Bode Magnitude Plot')
legend('Original','Order 10','Order 18');
subplot(2,1,2);
bodemag(G-GLim10,G-GLim18,logspace(0.5,1.5,100));
title('Absolute Error Plot')
legend('Order 10','Order 18');
```



With the frequency-limited energy computation, even the 10th-order approximation is quite good in the region of interest.

# **Alternative Functionality**

## App

**Model Reducer** 

### **Live Editor Task**

Reduce Model Order

# **Compatibility Considerations**

# MatchDC option honored when specified frequency or time intervals exclude DC

Behavior changed in R2017b

When you use balred for model reduction, you can use balredOptions to restrict the computation to specified frequency or time intervals. If the StateElimMethod option of balredOptions is set to 'MatchDC' (the default value), then balred attempts to match the DC gain of the original and reduced models, even if the specified intervals exclude DC (frequency = 0 or time = Inf).

Prior to R2017b, if you specified time or frequency intervals that excluded DC, balred did not attempt to match the DC gain of the original and reduced models, even if StateElimMethod = 'MatchDC'.

## References

[1] Varga, A., "Balancing-Free Square-Root Algorithm for Computing Singular Perturbation Approximations," Proc. of 30th IEEE CDC, Brighton, UK (1991), pp. 1062-1065.

## See Also

## **Functions**

balredOptions|hsvd

# **Apps Model Reducer**

**Live Editor Tasks** Reduce Model Order

## **Topics**

"Balanced Truncation Model Reduction"
"Model Reduction Basics"

Introduced before R2006a

# balredOptions

Create option set for model order reduction

# **Syntax**

```
opts = balredOptions
opts = balredOptions('OptionName', OptionValue)
```

# **Description**

opts = balredOptions returns the default option set for the balred command.

opts = balredOptions('OptionName', OptionValue) accepts one or more comma-separated name/value pairs. Specify OptionName inside single quotes.

# **Input Arguments**

## **Name-Value Pair Arguments**

## **FreqIntervals**

Frequency intervals for computing frequency-limited Hankel singular values, specified as a matrix with two columns. Each row specifies a frequency interval [fmin fmax], where fmin and fmax are nonnegative frequencies, expressed in the frequency unit of the model. When identifying low-energy states to truncate, the software computes state contributions to system behavior in these frequency ranges only. For example:

- To restrict the computation to the range between 3 rad/s and 15 rad/s, assuming the frequency unit of the model is rad/s, set FreqIntervals to [3 15].
- To restrict the computation to two frequency intervals, 3-15 rad/s and 40-60 rad/s, use [3 15; 40 60].
- To specify all frequencies below a cutoff frequency fcut, use [0 fcut].

• To specify all frequencies above the cutoff, use [fcut Inf] in continuous time, or [fcut pi/Ts] in discrete time, where Ts is the sample time of the model.

The default value, [], imposes no frequency limitation and is equivalent to [0 Inf] in continuous time or [0 pi/Ts] in discrete time. However, if you specify a TimeIntervals value other than [], then this limit overrides FreqIntervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

If StateElimMethod = 'MatchDC' (the default value), then balred attempts to match the DC gain of the original and reduced models, even if the specified frequency intervals exclude 0. This behavior might reduce the quality of the match in the specified intervals. To improve the match within frequency intervals that exclude 0, set StateElimMethod = 'Truncate'.

If both the frequency and time intervals do include DC, you can still set StateElimMethod = 'Truncate' to improve the match at other frequencies and times.

### Default: []

#### **TimeIntervals**

Time intervals for computing time-limited Hankel singular values, specified as a matrix with two columns. Each row specifies a time interval [tmin tmax], where tmin and tmax are nonnegative times, expressed in the time unit of the model. When identifying low-energy states to truncate, the software computes state contributions to the system's impulse response in these time intervals only. For example:

- To restrict the computation to the range between 3 s and 15 s, assuming the time unit of the model is seconds, set TimeIntervals to [3 15].
- To restrict the computation to two time intervals, 3-15 s and 40-60 s, use [3 15; 40 60].
- To specify all times from zero up to a cutoff time tcut, use [0 tcut]. To specify all times after the cutoff, use [tcut Inf].

The default value, [], imposes no time limitation and is equivalent to [0 Inf]. However, if you specify a FreqIntervals value other than [], then this limit overrides Timeintervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

If StateElimMethod = 'MatchDC' (the default value), then balred attempts to match the DC gain of the original and reduced models, even if the specified time intervals exclude Inf. This behavior might reduce the quality of the match in the specified intervals. To improve the match within time intervals that exclude Inf, set StateElimMethod = 'Truncate'.

If both the frequency and time intervals do include DC, you can still set StateElimMethod = 'Truncate' to improve the match at other frequencies and times.

### Default: []

### **StateElimMethod**

State elimination method. Specifies how to eliminate the weakly coupled states (states with smallest Hankel singular values). Specified as one of the following values:

'MatchDC' Discards the specified states and alters the remaining states to

preserve the DC gain.

'Truncate' Discards the specified states without altering the remaining states.

This method tends to product a better approximation in the frequency domain, but the DC gains are not guaranteed to match.

Default: 'MatchDC'

### AbsTol, RelTol

Absolute and relative error tolerance for stable/unstable decomposition. Positive scalar values. For an input model G with unstable poles, balred first extracts the stable dynamics by computing the stable/unstable decomposition  $G \to GS + GU$ . The AbsTol and RelTol tolerances control the accuracy of this decomposition by ensuring that the frequency responses of G and GS + GU differ by no more than AbsTol + RelTol\*abs(G). Increasing these tolerances helps separate nearby stable and unstable modes at the expense of accuracy. See stabsep for more information.

**Default:** AbsTol = 0; RelTol = 1e-8

#### Offset

Offset for the stable/unstable boundary. Positive scalar value. In the stable/unstable decomposition, the stable term includes only poles satisfying

- Re(s) < -Offset \* max(1, |Im(s)|) (Continuous time)
- |z| < 1 Offset (Discrete time)

Increase the value of Offset to treat poles close to the stability boundary as unstable.

#### Default: 1e-8

For additional information on the options and how to use them, see the balred reference page.

### **Examples**

#### **Reduced-Order Approximation with Offset Option**

Compute a reduced-order approximation of the system given by:

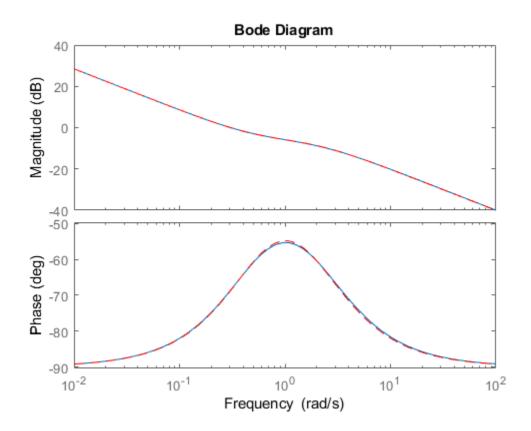
$$G(s) = \frac{(s+0.5)(s+1.1)(s+2.9)}{(s+10^{-6})(s+1)(s+2)(s+3)}.$$

Use the Offset option to exclude the pole at  $s = 10^{-6}$  from the stable term of the stable/unstable decomposition.

```
sys = zpk([-.5 -1.1 -2.9],[-le-6 -2 -1 -3],1);
% Create balredOptions
opt = balredOptions('Offset',.001,'StateElimMethod','Truncate');
% Compute second-order approximation
rsys = balred(sys,2,opt);
```

Compare the responses of the original and reduced-order models.

```
bodeplot(sys,rsys,'r--')
```

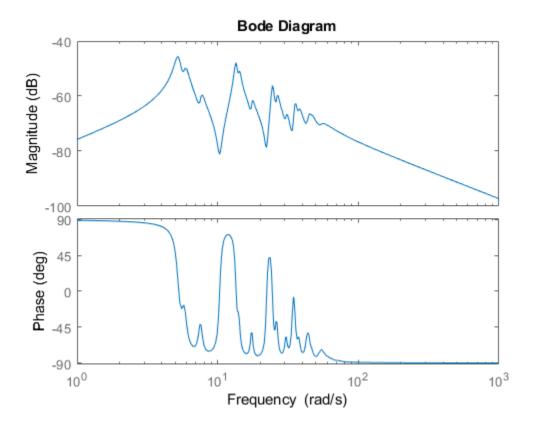


#### **Model Reduction in a Particular Frequency Band**

Reduce a high-order model with a focus on the dynamics in a particular frequency range.

Load a model and examine its frequency response.  $\,$ 

```
load(fullfile(matlabroot,'examples','control','build.mat'),'G')
bodeplot(G)
```

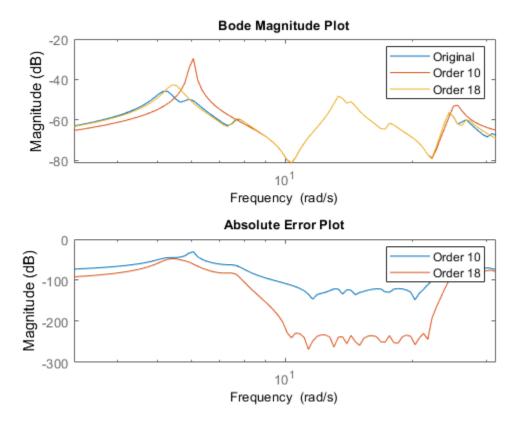


G is a 48th-order model with several large peak regions around 5.2 rad/s, 13.5 rad/s, and 24.5 rad/s, and smaller peaks scattered across many frequencies. Suppose that for your application you are only interested in the dynamics near the second large peak, between 10 rad/s and 22 rad/s. Focus the model reduction on the region of interest to obtain a good match with a low-order approximation. Use balredOptions to specify the frequency interval for balred.

```
bopt = balredOptions('StateElimMethod','Truncate','FreqIntervals',[10,22]);
GLim10 = balred(G,10,bopt);
GLim18 = balred(G,18,bopt);
```

Examine the frequency responses of the reduced-order models. Also, examine the difference between those responses and the original response (the absolute error).

```
subplot(2,1,1);
bodemag(G,GLim10,GLim18,logspace(0.5,1.5,100));
title('Bode Magnitude Plot')
legend('Original','Order 10','Order 18');
subplot(2,1,2);
bodemag(G-GLim10,G-GLim18,logspace(0.5,1.5,100));
title('Absolute Error Plot')
legend('Order 10','Order 18');
```



With the frequency-limited energy computation, even the 10th-order approximation is quite good in the region of interest.

### **Compatibility Considerations**

# MatchDC option honored when specified frequency or time intervals exclude DC

Behavior changed in R2017b

When you use balred for model reduction, you can use balredOptions to restrict the computation to specified frequency or time intervals. If the StateElimMethod option of balredOptions is set to 'MatchDC' (the default value), then balred attempts to match the DC gain of the original and reduced models, even if the specified intervals exclude DC (frequency = 0 or time = Inf).

Prior to R2017b, if you specified time or frequency intervals that excluded DC, balred did not attempt to match the DC gain of the original and reduced models, even if StateElimMethod = 'MatchDC'.

#### See Also

balred | gramOptions | hsvdOptions | stabsep

### **Topics**

"Balanced Truncation Model Reduction"

Introduced in R2010a

### bandwidth

Frequency response bandwidth

### **Syntax**

```
fb = bandwidth(sys)
fb = bandwidth(sys,dbdrop)
```

### **Description**

fb = bandwidth(sys) returns the bandwidth of the SISO dynamic system model sys. The bandwidth is the first frequency where the gain drops below 70.79% (-3 dB) of its DC value. The bandwidth is expressed in rad/TimeUnit, where TimeUnit is the TimeUnit property of sys.

fb = bandwidth(sys,dbdrop) returns the bandwidth for a specified gain drop.

### **Examples**

#### **Compute System Bandwidth**

Compute the bandwidth of the transfer function sys = 1/(s+1).

```
sys = tf(1,[1 1]);
fb = bandwidth(sys)
fb = 0.9976
```

This result shows that the gain of sys drops to 3 dB below its DC value at around 1 rad/s.

#### Find Bandwidth of System with Custom Gain Drop

Compute the frequency at which the gain of a system drops to 3.5 dB below its DC value. Create a state-space model.

```
A = [-2,-1;1,0];
B = [1;0];
C = [1,2];
D = 1;
sys = ss(A,B,C,D);
Find the 3.5 dB bandwidth of sys.
dbdrop = -3.5;
fb = bandwidth(sys,dbdrop)
fb = 0.8348
```

#### **Find Bandwidth of Model Array**

Find the bandwidth of each entry in a 5-by-1 array of transfer function models. Use a for loop to create the array, and confirm its dimensions.

```
sys = tf(zeros(1,1,5));
s = tf('s');
for m = 1:5
    sys(:,:,m) = m/(s^2+s+m);
end
size(sys)

5x1 array of transfer functions.
Each model has 1 outputs and 1 inputs.
Find the bandwidths.
fb = bandwidth(sys)
fb = 5x1
    1.2712
    1.9991
    2.5298
    2.9678
```

3.3493

bandwidth returns an array in which each entry is the bandwidth of the corresponding entry in sys. For instance, the bandwidth of sys(:,:,2) is fb(2).

### **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO dynamic system model or an array of SISO dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.
- Frequency-response data models such as frd models. For such models, bandwidth uses the first frequency point to approximate the DC gain.

If sys is an array of models, bandwidth returns an array of the same size, where each entry is the bandwidth of the corresponding model in sys. For more information on model arrays, see "Model Arrays".

#### dbdrop — Gain drop

-3 (default) | negative scalar

Gain drop in dB, specified as a real negative scalar.

### **Output Arguments**

#### fb — Frequency response bandwidth

scalar | array

Frequency response bandwidth, returned as a scalar or an array. If sys is:

- A single model, then fb is the bandwidth of sys.
- A model array, then fb is an array of the same size as the model array sys. Each entry is the bandwidth of the corresponding entry in sys.

fb is expressed in rad/TimeUnit, where TimeUnit is the TimeUnit property of sys.

# **See Also**

bodeplot | dcgain | issiso

Introduced before R2006a

### bdschur

Block-diagonal Schur factorization

### **Syntax**

```
[T,B,BLKS] = bdschur(A,CONDMAX)
[T,B] = bdschur(A,[],BLKS)
```

### **Description**

[T,B,BLKS] = bdschur(A,CONDMAX) computes a transformation matrix T such that  $B = T \setminus A * T$  is block diagonal and each diagonal block is a quasi upper-triangular Schur matrix.

[T,B] = bdschur(A,[],BLKS) pre-specifies the desired block sizes. The input matrix A should already be in Schur form when you use this syntax.

### **Input Arguments**

- A: Matrix for block-diagonal Schur factorization.
- CONDMAX: Specifies an upper bound on the condition number of *T*. By default, CONDMAX = 1/sqrt(eps). Use CONDMAX to control the tradeoff between block size and conditioning of *T* with respect to inversion. When CONDMAX is a larger value, the blocks are smaller and T becomes more ill-conditioned.

### **Output Arguments**

- T: Transformation matrix.
- B: Matrix  $B = T \setminus A * T$ .
- BLKS: Vector of block sizes.

# **See Also**

ordschur|schur

Introduced in R2008a

# blkdiag

Block-diagonal concatenation of models

### **Syntax**

```
sys = blkdiag(sys1,sys2,...,sysN)
```

### **Description**

sys = blkdiag(sys1,sys2,...,sysN) produces the aggregate system

blkdiag is equivalent to append.

### **Examples**

#### **Perform Block-Diagonal Concatenation**

Perform block-diagonal concatenation of a transfer function model and a state-space model.

Create the SISO continuous-time transfer function model, 1/s.

```
sys1 = tf(1,[1 0]);
```

Create a SISO continuous-time state-space model with state-space matrices 1,2,3, and 4.

```
sys2 = ss(1,2,3,4);
```

Concatenate sys1, a SISO static gain system, and sys2. The resulting model is a 3-input, 3-output state-space model.

```
sys = blkdiag(sys1,10,sys2)
sys =
  A =
       x1 x2
            0
   x1
        0
   x2
        0
            1
  B =
       u1
           u2
               u3
       1
            0
                0
   x1
   x2
        0
            0
                2
  C =
       x1
           x2
        1
            0
   у1
        0
            0
   у2
   у3
            3
  D =
       u1
           u2 u3
   у1
           0
                0
   у2
        0
           10
                 0
                4
   у3
```

Continuous-time state-space model.

Alternatively, use the append command.

```
sys = append(sys1, 10, sys2);
```

### See Also

append | feedback | parallel | series

#### Introduced in R2009a

### bode

Bode plot of frequency response, or magnitude and phase data

### **Syntax**

```
bode(sys)
bode(sys1,sys2,...,sysN)
bode(sys1,LineSpec1,...,sysN,LineSpecN)
bode(____,w)

[mag,phase,wout] = bode(sys)
[mag,phase,wout] = bode(sys,w)
[mag,phase,wout,sdmag,sdphase] = bode(sys,w)
```

### **Description**

bode(sys) creates a Bode plot of the frequency response of a dynamic system model sys. The plot displays the magnitude (in dB) and phase (in degrees) of the system response as a function of frequency. bode automatically determines frequencies to plot based on system dynamics.

If sys is a multi-input, multi-output (MIMO) model, then bode produces an array of Bode plots, each plot showing the frequency response of one I/O pair.

bode(sys1,sys2,...,sysN) plots the frequency response of multiple dynamic systems on the same plot. All systems must have the same number of inputs and outputs.

bode(sys1,LineSpec1,...,sysN,LineSpecN) specifies a color, linestyle, and marker for each system in the plot.

bode ( \_\_\_\_, w) plots system responses for frequencies specified by w.

- If w is a cell array of the form {wmin,wmax}, then bode plots the response at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then bode plots the response at each specified frequency.

You can use w with any of the input-argument combinations in previous syntaxes.

[mag,phase,wout] = bode(sys) returns the magnitude and phase of the response at each frequency in the vector wout. The function automatically determines frequencies in wout based on system dynamics. This syntax does not draw a plot.

[mag, phase, wout] = bode(sys, w) returns the response data at the frequencies specified by w.

- If w is a cell array of the form {wmin, wmax}, then wout contains frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then wout = w.

[mag,phase,wout,sdmag,sdphase] = bode(sys,w) also returns the estimated standard deviation of the magnitude and phase values for the identified model sys. If you omit w, then the function automatically determines frequencies in wout based on system dynamics.

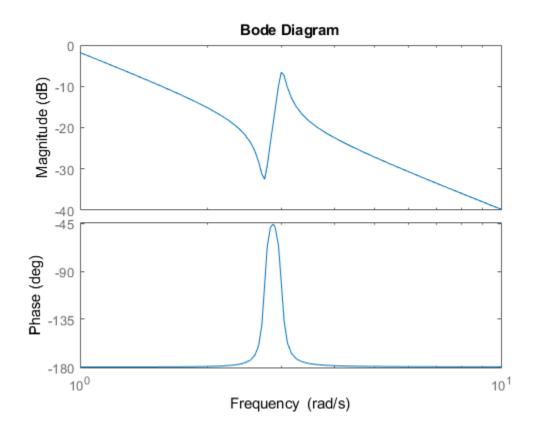
### **Examples**

#### **Bode Plot of Dynamic System**

Create a Bode plot of the following continuous-time SISO dynamic system.

$$H(s) = \frac{s^2 + 0.1s + 7.5}{s^4 + 0.12s^3 + 9s^2}.$$

 $H = tf([1 \ 0.1 \ 7.5],[1 \ 0.12 \ 9 \ 0 \ 0]);$ bode(H)

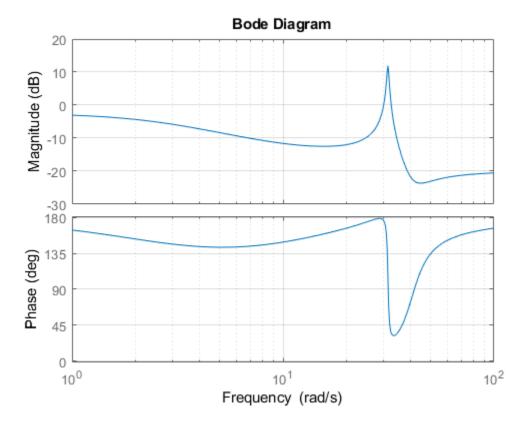


bode automatically selects the plot range based on the system dynamics.

#### **Bode Plot at Specified Frequencies**

Create a Bode plot over a specified frequency range. Use this approach when you want to focus on the dynamics in a particular range of frequencies.

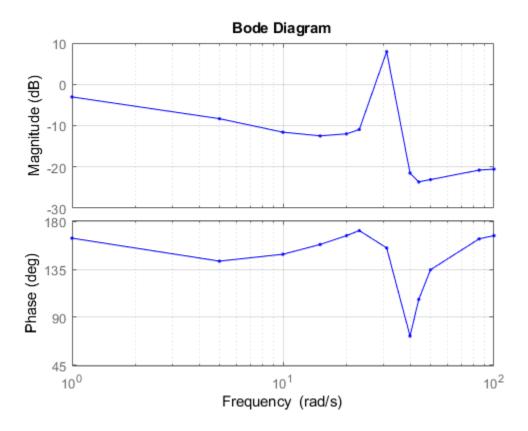
```
\label{eq:Hamiltonian} \begin{array}{ll} H = \mbox{tf([-0.1,-2.4,-181,-1950],[1,3.3,990,2600]);} \\ \mbox{bode(H,\{1,100\})} \\ \mbox{grid on} \end{array}
```



The cell array  $\{1,100\}$  specifies the minimum and maximum frequency values in the Bode plot. When you provide frequency bounds in this way, the function selects intermediate points for frequency response data.

Alternatively, specify a vector of frequency points to use for evaluating and plotting the frequency response.

```
w = [1 5 10 15 20 23 31 40 44 50 85 100]; bode(H,w,'.-') grid on
```



bode plots the frequency response at the specified frequencies only.

#### **Compare Bode Plots of Several Dynamic Systems**

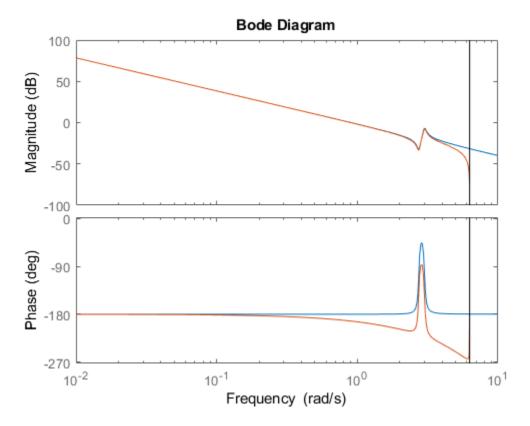
 $\label{lem:continuous-time} Create\ continuous-time\ and\ discrete-time\ dynamic\ systems.$ 

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);

Hd = c2d(H,0.5,'zoh');
```

Create a Bode plot that displays both systems.

bode(H,Hd)



The Bode plot of a discrete-time system includes a vertical line marking the Nyquist frequency of the system.

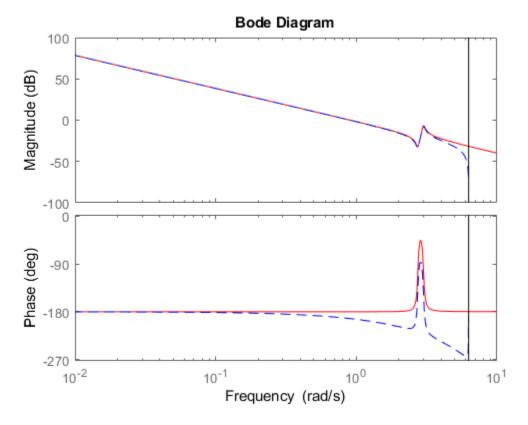
#### **Bode Plot with Specified Line Attributes**

Specify the line style, color, or marker for each system in a Bode plot using the LineSpec input argument.

```
H = tf([1 \ 0.1 \ 7.5],[1 \ 0.12 \ 9 \ 0 \ 0]);

Hd = c2d(H,0.5,'zoh');

bode(H,'r',Hd,'b--')
```



The first LineSpec, 'r', specifies a solid red line for the response of H. The second LineSpec, 'b--', specifies a dashed blue line for the response of Hd.

#### **Obtain Magnitude and Phase Data**

Compute the magnitude and phase of the frequency response of a SISO system.

If you do not specify frequencies, **bode** chooses frequencies based on the system dynamics and returns them in the third output argument.

```
H = tf([1 \ 0.1 \ 7.5],[1 \ 0.12 \ 9 \ 0 \ 0]);
[mag,phase,wout] = bode(H);
```

Because H is a SISO model, the first two dimensions of mag and phase are both 1. The third dimension is the number of frequencies in wout.

Thus, each entry along the third dimension of mag gives the magnitude of the response at the corresponding frequency in wout.

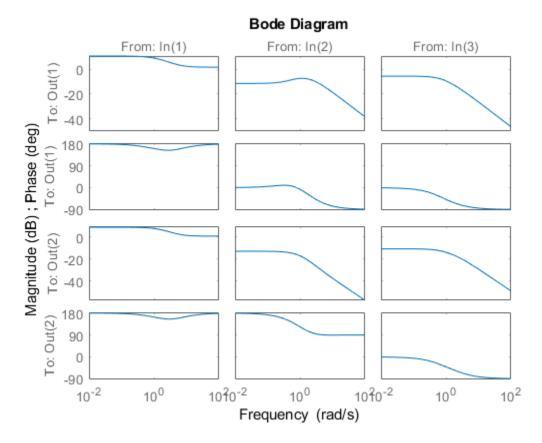
#### Magnitude and Phase of MIMO System

For this example, create a 2-output, 3-input system.

```
rng(0,'twister'); % For reproducibility
H = rss(4,2,3);
```

For this system, bode plots the frequency responses of each I/O channel in a separate plot in a single figure.

```
bode(H)
```



Compute the magnitude and phase of these responses at 20 frequencies between 1 and 10 radians.

```
w = logspace(0,1,20);
[mag,phase] = bode(H,w);
```

mag and phase are three-dimensional arrays, in which the first two dimensions correspond to the output and input dimensions of H, and the third dimension is the number of frequencies. For instance, examine the dimensions of mag.

```
size(mag) ans = 1 \times 3
```

```
2 3 20
```

Thus, for example, mag(1,3,10) is the magnitude of the response from the third input to the first output, computed at the 10th frequency in w. Similarly, phase(1,3,10) contains the phase of the same response.

#### **Bode Plot of Identified Model**

Compare the frequency response of a parametric model, identified from input/output data, to a nonparametric model identified using the same data.

Identify parametric and nonparametric models based on data.

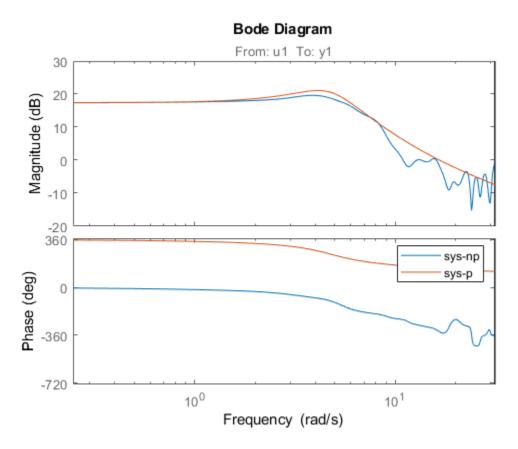
```
load iddata2 z2;
w = linspace(0,10*pi,128);
sys_np = spa(z2,[],w);
sys_p = tfest(z2,2);
```

Using the spa and tfest commands requires System Identification Toolbox™ software.

 $\verb|sys_np| is a nonparametric identified model. \verb|sys_p| is a parametric identified model. \\$ 

Create a Bode plot that includes both systems.

```
bode(sys_np,sys_p,w);
legend('sys-np','sys-p')
```



You can display the confidence region on the Bode plot by right-clicking the plot and selecting **Characteristics > Confidence Region**.

### Obtain Magnitude and Phase Standard Deviation Data of Identified Model

Compute the standard deviation of the magnitude and phase of an identified model. Use this data to create a  $3\sigma$  plot of the response uncertainty.

Identify a transfer function model based on data. Obtain the standard deviation data for the magnitude and phase of the frequency response.

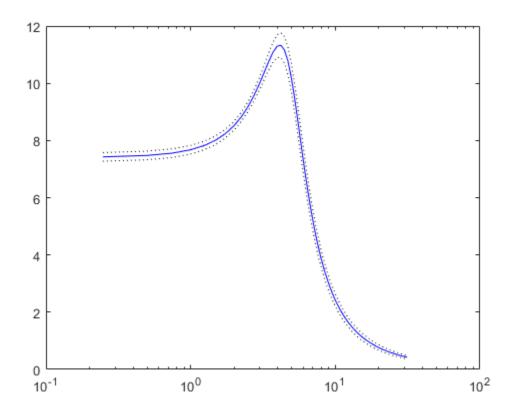
```
load iddata2 z2;
sys_p = tfest(z2,2);
w = linspace(0,10*pi,128);
[mag,ph,w,sdmag,sdphase] = bode(sys_p,w);
```

Using the tfest command requires System Identification Toolbox™ software.

sys\_p is an identified transfer function model. sdmag and sdphase contain the standard deviation data for the magnitude and phase of the frequency response, respectively.

Use the standard deviation data to create a  $3\sigma$  plot corresponding to the confidence region.

```
mag = squeeze(mag);
sdmag = squeeze(sdmag);
semilogx(w,mag,'b',w,mag+3*sdmag,'k:',w,mag-3*sdmag,'k:');
```



# **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems that you can use include:  $\frac{1}{2}$ 

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

- For tunable control design blocks, the function evaluates the model at its current value for both plotting and returning frequency response data.
- For uncertain control design blocks, the function plots the nominal value and random samples of the model. When you use output arguments, the function returns frequency response data for the nominal model only.
- Frequency-response data models such as frd models. For such models, the function plots the response at frequencies defined in the model.
- Identified LTI models, such as idtf, idss, or idproc models. For such models, the function can also plot confidence intervals and return standard deviations of the frequency response. See "Bode Plot of Identified Model" on page 2-73. (Using identified models requires System Identification Toolbox software.)

If sys is an array of models, the function plots the frequency responses of all models in the array on the same axes.

#### LineSpec — Line style, marker, and color

character vector | string

Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.

```
Example: 'r--' specifies a red dashed line
```

Example: '\*b' specifies blue asterisk markers

Example: 'y' specifies a yellow line

#### w - Frequencies

{wmin,wmax} | vector

Frequencies at which to compute and plot frequency response, specified as the cell array {wmin,wmax} or as a vector of frequency values.

- If w is a cell array of the form {wmin,wmax}, then the function computes the index at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then the function computes the index at each specified frequency. For example, use logspace to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the model.

### **Output Arguments**

#### mag — Magnitude of system response

3-D array

Magnitude of the system response in absolute units, returned as a 3-D array. The dimensions of this array are (number of system outputs)  $\times$  (number of system inputs)  $\times$  (number of frequency points).

- For SISO systems, mag(1,1,k) gives the magnitude of the response at the kth frequency in w or wout. For an example, see "Obtain Magnitude and Phase Data" on page 2-70.
- For MIMO systems, mag(i,j,k) gives the magnitude of the response at the kth frequency from the jth input to the ith output. For an example, see "Magnitude and Phase of MIMO System" on page 2-71.

To convert the magnitude from absolute units to decibels, use:

```
magdb = 20*log10(mag)
```

#### phase — Phase of system response

3-D array

Phase of the system response in degrees, returned as a 3-D array. The dimensions of this array are (number of outputs)  $\times$  (number of inputs)  $\times$  (number of frequency points).

- For SISO systems, mag(1,1,k) gives the phase of the response at the kth frequency in w or wout. For an example, see "Obtain Magnitude and Phase Data" on page 2-70.
- For MIMO systems, mag(i,j,k) gives the phase of the response at the kth frequency from the jth input to the ith output. For an example, see "Magnitude and Phase of MIMO System" on page 2-71.

#### wout — Frequencies

vector

Frequencies at which the function returns the system response, returned as a column vector. The function chooses the frequency values based on the model dynamics, unless you specify frequencies using the input argument w.

Frequency values are in radians per TimeUnit, where TimeUnit is the value of the TimeUnit property of sys.

#### sdmag — Standard deviation of magnitude

Estimated standard deviation of the magnitude of the response at each frequency point, returned as a 3-D array. sdmag has the same dimensions as mag.

If sys is not an identified LTI model, sdmag is [].

#### sdphase — Standard deviation of phase

Estimated standard deviation of the phase of the response at each frequency point, returned as a 3-D array. sdphase has the same dimensions as phase.

If sys is not an identified LTI model, sdphase is [].

### **Tips**

When you need additional plot customization options, use bodeplot instead.

### **Algorithms**

bode computes the frequency response as follows:

- 1 Compute the zero-pole-gain (zpk) representation of the dynamic system.
- **2** Evaluate the gain and phase of the frequency response based on the zero, pole, and gain data for each input/output channel of the system.
  - For continuous-time systems, **bode** evaluates the frequency response on the imaginary axis  $s = j\omega$  and considers only positive frequencies.
  - For discrete-time systems, **bode** evaluates the frequency response on the unit circle. To facilitate interpretation, the command parameterizes the upper half of the unit circle as:

$$z=e^{j\omega T_S}, \quad 0\leq \omega \leq \omega_N=rac{\pi}{T_S},$$

where  $T_s$  is the sample time and  $\omega_N$  is the Nyquist frequency. The equivalent continuous-time frequency  $\omega$  is then used as the x-axis variable. Because  $H(e^{j\omega T_S})$  is periodic with period  $2\omega_N$ , bode plots the response only up to the Nyquist frequency  $\omega_N$ . If sys is a discrete-time model with unspecified sample time, bode uses  $T_s=1$ .

### See Also

bodeplot | freqresp | nichols | nyquist | step

#### **Topics**

"Frequency-Domain Responses"

"Dynamic System Models"

Introduced before R2006a

# bodemag

Bode magnitude response of LTI models

### **Syntax**

```
bodemag(sys)
bodemag(sys,{wmin,wmax})
bodemag(sys,w)
bodemag(sys1,sys2,...,sysN,w)
```

### **Description**

bodemag(sys) plots the magnitude of the frequency response of the dynamic system model sys (Bode plot without the phase diagram). The frequency range and number of points are chosen automatically.

bodemag(sys, {wmin, wmax}) draws the magnitude plot for frequencies between wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys).

bodemag(sys,w) uses the user-supplied vector W of frequencies, in rad/TimeUnit, at
which the frequency response is to be evaluated.

bodemag(sys1,sys2,...,sysN,w) shows the frequency response magnitude of several models sys1,sys2,...,sysN on a single plot. The frequency vector w is optional. You can also specify a color, line style, and marker for each model. For example:

```
bodemag(sys1,'r',sys2,'y--',sys3,'gx')
```

### **See Also**

Linear System Analyzer | bode

Introduced before R2006a

# bodeoptions

Create list of Bode plot options

### **Syntax**

```
P = bodeoptions
P = bodeoptions('cstprefs')
```

### **Description**

P = bodeoptions returns a default set of plot options for use with the bodeplot. You can use these options to customize the Bode plot appearance using the command line. This syntax is useful when you want to write a script to generate plots that look the same regardless of the preference settings of the MATLAB session in which you run the script.

P = bodeoptions('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor". This syntax is useful when you want to change a few plot options but otherwise use your default preferences. A script that uses this syntax may generate results that look different when run in a session with different preferences.

The following table summarizes the Bode plot options.

Option	Description
Title, XLabel, YLabel	Label text and style, specified as a structure with the following fields:
	• String — Label text, specified as a character vector, for example 'Amplitude'.
	• FontSize — <b>Default</b> : 8
	• FontWeight — <b>Default</b> : 'Normal'
	• Font Angle — <b>Default</b> : 'Normal'
	• Color — Vector of RGB values ranging from 0 to 1. <b>Default</b> : [0,0,0]
	• Interpreter — <b>Default</b> : 'tex'
TickLabel	Tick label style, specified as a structure with the following fields:
	• FontSize <b>Default</b> : 8
	• FontWeight — <b>Default</b> : 'Normal'
	• Font Angle — <b>Default</b> : 'Normal'
	• Color — Vector of RGB values ranging from 0 to 1. <b>Default</b> : [0,0,0]
Grid	Show or hide the grid Specified as one of the following values: 'off'   'on' Default: 'off'
GridColor	Color of the grid lines Specified as one of the following: Vector of RGB values in the range [0,1]   character vector of color name   'none'. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'.  Default: [0.15,0.15,0.15]
XlimMode, YlimMode	Axis limit modes. <b>Default</b> : 'auto'
Xlim, Ylim	Axes limits, specified as an array of the form [min,max]
IOGrouping	Grouping of input-output pairs Specified as one of the following values: 'none'  'inputs' 'outputs' 'all' Default: 'none'

Option	Description
InputLabels, OutputLabels	Input and output label styles
InputVisible, OutputVisible	Visibility of input and output channels
ConfidenceRegionNum berSD	Number of standard deviations to use to plotting the response confidence region (identified models only).
	Default: 1.

Option	Description
FreqUnits	Frequency units, specified as one of the following values:
	• 'Hz'
	• 'rad/second'
	• 'rpm'
	• 'kHz'
	• 'MHz'
	• 'GHz'
	• 'rad/nanosecond'
	• 'rad/microsecond'
	• 'rad/millisecond'
	• 'rad/minute'
	• 'rad/hour'
	• 'rad/day'
	• 'rad/week'
	• 'rad/month'
	• 'rad/year'
	• 'cycles/nanosecond'
	• 'cycles/microsecond'
	• 'cycles/millisecond'
	• 'cycles/hour'
	• 'cycles/day'
	• 'cycles/week'
	• 'cycles/month'
	• 'cycles/year'
FreqScale	Frequency scale Specified as one of the following values: 'linear'   'log' Default: 'log'

Option	Description
MagUnits	Magnitude units Specified as one of the following values: 'dB'   'abs' Default: 'dB'
MagScale	Magnitude scale Specified as one of the following values: 'linear'   'log' Default: 'linear'
MagVisible	Magnitude plot visibility Specified as one of the following values: 'on'   'off' Default: 'on'
MagLowerLimMode	Enables a lower magnitude limit Specified as one of the following values: 'auto'   'manual' Default: 'auto'
MagLowerLim	Specifies the lower magnitude limit
PhaseUnits	Phase units Specified as one of the following values: 'deg'   'rad' Default: 'deg'
PhaseVisible	Phase plot visibility Specified as one of the following values: 'on'   'off' Default: 'on'
PhaseWrapping	Enables phase wrapping Specified as one of the following values: 'on'   'off' When you set PhaseWrapping to 'on', the plot wraps accumulated phase at the value specified by the PhaseWrappingBranch property.  Default: 'off'
PhaseWrappingBranch	Phase value at which the plot wraps accumulated phase when PhaseWrapping is set to 'on'. <b>Default</b> : -180 (phase wraps into the interval [-180º,180º))
PhaseMatching	Enables phase matching Specified as one of the following values: 'on'   'off' Default: 'off'
PhaseMatchingFreq	Frequency for matching phase
PhaseMatchingValue	The value to which phase responses are matched closely

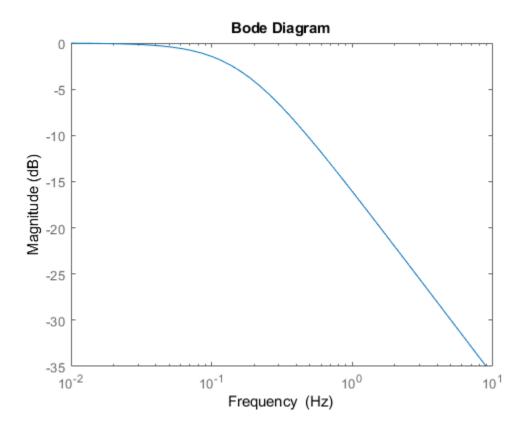
# **Examples**

### **Create Bode Plot with Custom Settings**

Create a Bode plot that suppresses the phase plot and uses frequency units Hz instead of the default radians/second. Otherwise, the plot uses the settings that are saved in the toolbox preferences.

First, create an options set based on the toolbox preferences.

```
opts = bodeoptions('cstprefs');
Change properties of the options set.
opts.PhaseVisible = 'off';
opts.FreqUnits = 'Hz';
Create a plot using the options.
h = bodeplot(tf(1,[1,1]),opts);
```



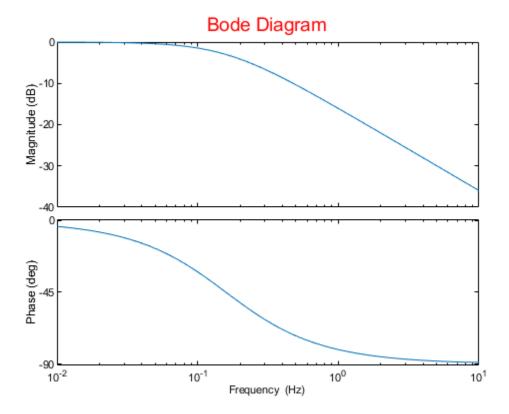
Depending on your own toolbox preferences, the plot you obtain might look different from this plot. Only the properties that you set explicitly, in this example PhaseVisible and FreqUnits, override the toolbox preferences.

### **Custom Plot Settings Independent of Preferences**

Create a Bode plot that uses 14-point red text for the title. This plot should look the same, regardless of the preferences of the MATLAB session in which it is generated.

First, create a default options set.

```
opts = bodeoptions;
Change properties of the options set.
opts.Title.FontSize = 14;
opts.Title.Color = [1 0 0];
opts.FreqUnits = 'Hz';
Create a plot using the options.
h = bodeplot(tf(1,[1,1]),opts);
```



Because opts begins with a fixed set of options, the plot result is independent of the toolbox preferences of the MATLAB session.

# **See Also**

bode | bodeplot | getoptions | setoptions

Introduced in R2008a

# bodeplot

Plot Bode frequency response with additional plot customization options

## **Syntax**

```
h = bodeplot(sys)
bodeplot(sys)
bodeplot(sys1,sys2,...)
bodeplot(AX,...)
bodeplot(..., plotoptions)
bodeplot(sys,w)
```

# **Description**

h = bodeplot(sys) plot the Bode magnitude and phase of the dynamic system model sys and returns the plot handle h to the plot. You can use this handle to customize the plot with the getoptions and setoptions commands.

bodeplot(sys) draws the Bode plot of the model sys. The frequency range and number of points are chosen automatically.

bodeplot(sys1,sys2,...) graphs the Bode response of multiple models sys1,sys2,... on a single plot. You can specify a color, line style, and marker for each model, as in

```
bodeplot(sys1,'r',sys2,'y--',sys3,'gx')
```

bodeplot(AX,...) plots into the axes with handle AX.

bodeplot(..., plotoptions) plots the Bode response with the options specified in plotoptions. Type

help bodeoptions

for a list of available plot options. See "Match Phase at Specified Frequency" on page 2-94 for an example of phase matching using the PhaseMatchingFreq and PhaseMatchingValue options.

bodeplot(sys,w) draws the Bode plot for frequencies specified by w. When w =
{wmin,wmax}, the Bode plot is drawn for frequencies between wmin and wmax (in rad/ TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys.). When w is a user-supplied vector w of frequencies, in rad/TimeUnit, the Bode response is drawn for the specified frequencies.

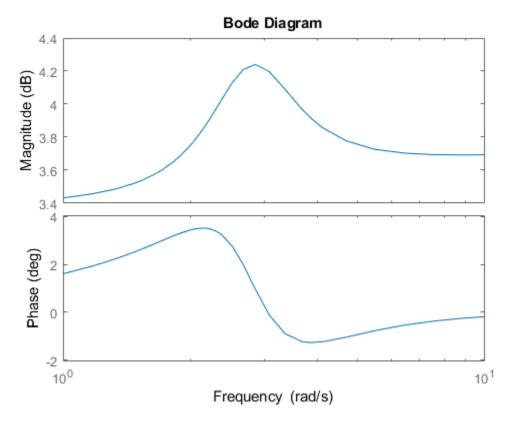
See logspace to generate logarithmically spaced frequency vectors.

## **Examples**

### **Change Bode Plot Options with Plot Handle**

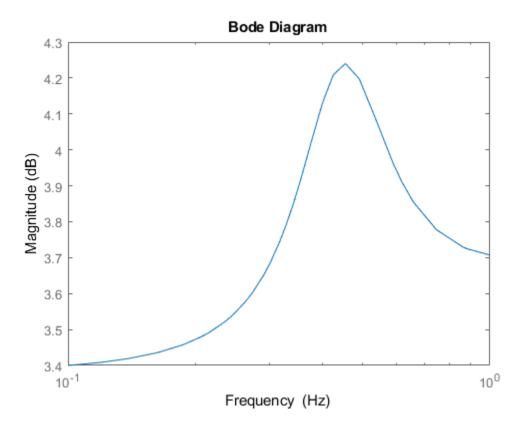
```
Generate a Bode plot.
```

```
sys = rss(5);
h = bodeplot(sys);
```



Change the units to Hz and suppress the phase plot. To do so, edit properties of the plot handle, h.

setoptions(h,'FreqUnits','Hz','PhaseVisible','off');

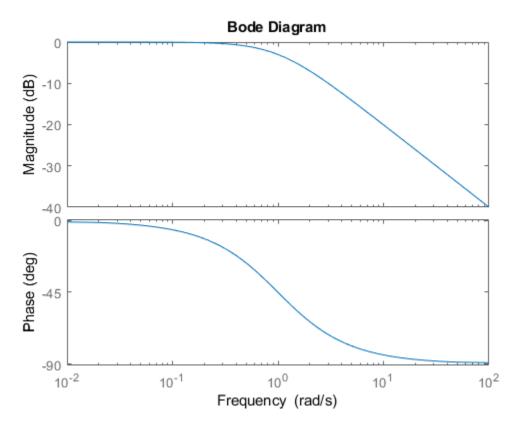


The plot automatically updates when you call setoptions.

### **Match Phase at Specified Frequency**

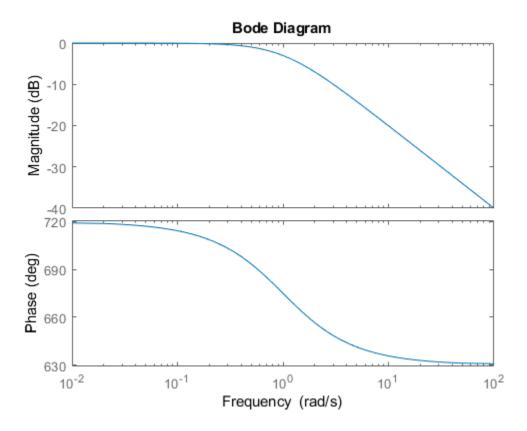
Create a Bode plot of a dynamic system.

```
sys = tf(1,[1 1]);
h = bodeplot(sys);
```



Fix the phase at 1 rad/s to 750 degrees. To do so, get the plot properties. Then alter the properties PhaseMatchingFreq and PhaseMatchingValue to match a phase to a specified frequency.

```
p = getoptions(h);
p.PhaseMatching = 'on';
p.PhaseMatchingFreq = 1;
p.PhaseMatchingValue = 750;
Update the plot.
setoptions(h,p);
```



The first bode plot has a phase of -45 degrees at a frequency of 1 rad/s. Setting the phase matching options so that at 1 rad/s the phase is near 750 degrees yields the second Bode plot. Note that, however, the phase can only be -45 + N\*360, where N is an integer, and so the plot is set to the nearest allowable phase, namely 675 degrees (or 2\*360 - 45 = 675).

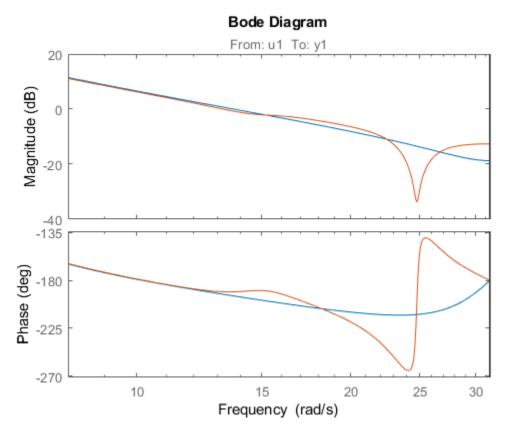
### **Display Confidence Regions of Identified Models**

Compare the frequency responses of identified state-space models of order 2 and 6 along with their 2  $\sigma$  confidence regions.

```
load iddata1
sys1 = n4sid(z1, 2);
sys2 = n4sid(z1, 6);
```

Both models produce about 70% fit to data. However, sys2 shows higher uncertainty in its frequency response, especially close to Nyquist frequency as shown by the plot:

```
w = linspace(8,10*pi,256);
h = bodeplot(sys1,sys2,w);
setoptions(h,'PhaseMatching','on','ConfidenceRegionNumberSD',2);
```



Right-click the plot and select **Characteristics > Confidence Region** to turn on the confidence region characteristic. Alternatively, type showConfidence(h) to plot the confidence region.

### Frequency Response of Identified Parametric and Nonparametric Models

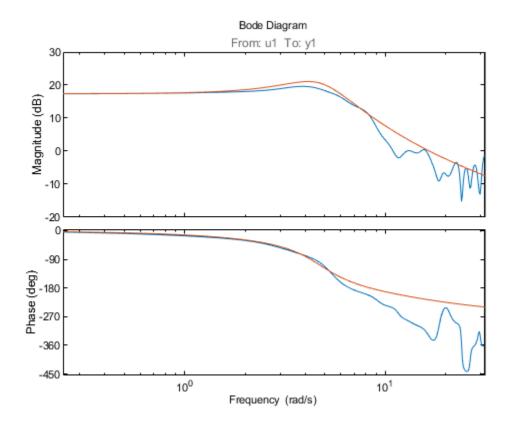
Compare the frequency response of a parametric model, identified from input/output data, to a nonparametric model identified using the same data. Identify parametric and non-parametric models based on data.

```
load iddata2 z2;
w = linspace(0,10*pi,128);
sys_np = spa(z2,[],w);
sys_p = tfest(z2,2);
```

spa and tfest require System Identification Toolbox $^{\text{\tiny TM}}$  software. sys\_np is a nonparametric identified model. sys p is a parametric identified model.

Create a Bode plot that includes both systems.

```
opt = bodeoptions;
opt.PhaseMatching = 'on';
bodeplot(sys_np,sys_p,w,opt);
```



# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

## **See Also**

bode | bodeoptions | getoptions | setoptions

### **Introduced before R2006a**

### c2d

Convert model from continuous to discrete time

## **Syntax**

```
sysd = c2d(sysc,Ts)
sysd = c2d(sysc,Ts,method)
sysd = c2d(sysc,Ts,opts)
[sysd,G] = c2d( )
```

# **Description**

sysd = c2d(sysc,Ts) discretizes the continuous-time dynamic system model sysc using zero-order hold on the inputs and a sample time of Ts.

sysd = c2d(sysc, Ts, method) specifies the discretization method.

sysd = c2d(sysc, Ts, opts) specifies additional options for the discretization.

[sysd,G] = c2d( $\underline{\phantom{a}}$ ), where sysc is a state-space model, returns a matrix, G that maps the continuous initial conditions  $x_0$  and  $u_0$  of the state-space model to the discrete-time initial state vector x[0].

## **Examples**

#### **Discretize a Transfer Function**

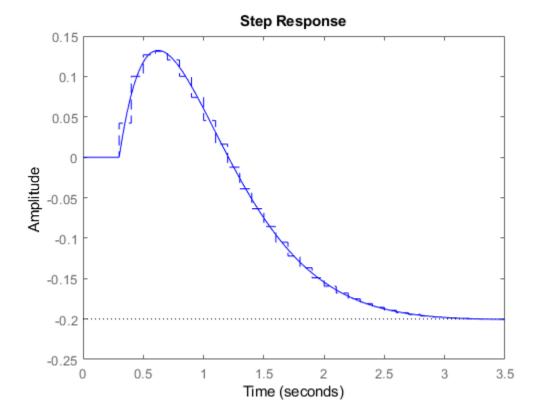
Discretize the following continuous-time transfer function:

$$H(s) = e^{-0.3s} \frac{s-1}{s^2 + 4s + 5}.$$

This system has an input delay of 0.3 s. Discretize the system using the triangle (first-order-hold) approximation with sample time Ts = 0.1 s.

```
H = tf([1 -1],[1 4 5],'InputDelay', 0.3);
Hd = c2d(H,0.1,'foh');
```

Compare the step responses of the continuous-time and discretized systems.



### **Discretize Model with Fractional Delay Absorbed into Coefficients**

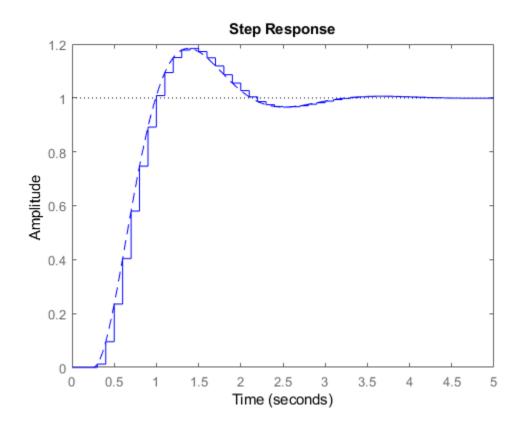
Discretize the following delayed transfer function using zero-order hold on the input, and a 10-Hz sampling rate.

```
H(s) = e^{-0.25s} \frac{10}{s^2 + 3s + 10}. h = tf(10,[1\ 3\ 10],'IODelay',0.25); hd = c2d(h,0.1) hd = \frac{0.01187\ z^2 + 0.06408\ z + 0.009721}{z^2 - 1.655\ z + 0.7408} Sample time: 0.1 seconds Discrete-time transfer function.
```

In this example, the discretized model hd has a delay of three sampling periods. The discretization algorithm absorbs the residual half-period delay into the coefficients of hd.

Compare the step responses of the continuous-time and discretized models.

```
step(h,'--',hd,'-')
```



### **Discretize Model With Approximated Fractional Delay**

Create a continuous-time state-space model with two states and an input delay.

```
x2
        1
            0
  B =
       u1
   x1
        2
        0
   х2
  C =
        x1
              х2
       0.5
              1
   у1
  D =
       u1
        0
   у1
  Input delays (seconds): 2.7
Continuous-time state-space model.
```

Discretize the model using the Tustin discretization method and a Thiran filter to model fractional delays. The sample time Ts = 1 second.

```
opt = c2dOptions('Method','tustin','FractDelayApproxOrder',3);
sysd1 = c2d(sys,1,opt)
sysd1 =
  A =
               x1
                           x2
                                       х3
                                                   х4
                                                               x5
          -0.4286
                      -0.5714
                                 -0.00265
                                              0.06954
                                                            2,286
   x1
   x2
           0.2857
                      0.7143
                               -0.001325
                                             0.03477
                                                            1.143
                                  -0.2432
                                               0.1449
                                                          -0.1153
   х3
                0
                            0
   x4
                0
                            0
                                     0.25
                                                                0
                0
                                                                0
   х5
                            0
                                        0
                                                0.125
  B =
              u1
   x1
       0.002058
   x2
       0.001029
   хЗ
               8
   x4
               0
   х5
               0
  C =
               x1
                           x2
                                       х3
                                                   x4
                                                               x5
```

The discretized model now contains three additional states x3, x4, and x5 corresponding to a third-order Thiran filter. Since the time delay divided by the sample time is 2.7, the third-order Thiran filter ('FractDelayApproxOrder' = 3) can approximate the entire time delay.

#### **Discretize Identified Model**

Estimate a continuous-time transfer function, and discretize it.

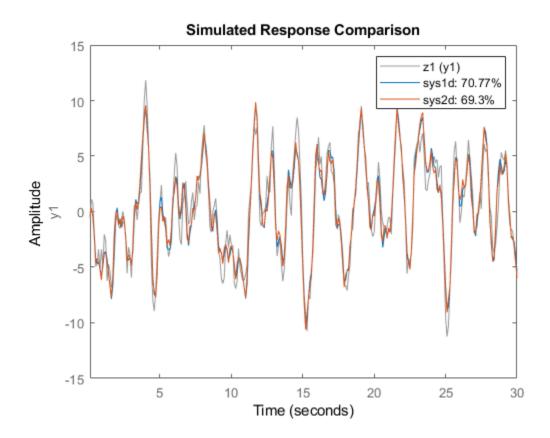
```
load iddata1
sys1c = tfest(z1,2);
sys1d = c2d(sys1c,0.1,'zoh');
```

Estimate a second order discrete-time transfer function.

```
sys2d = tfest(z1,2,'Ts',0.1);
```

Compare the response of the discretized continuous-time transfer function model, sysld, and the directly estimated discrete-time model, sys2d.

```
compare(z1,sys1d,sys2d)
```



The two systems are almost identical.

#### **Build Predictor Model**

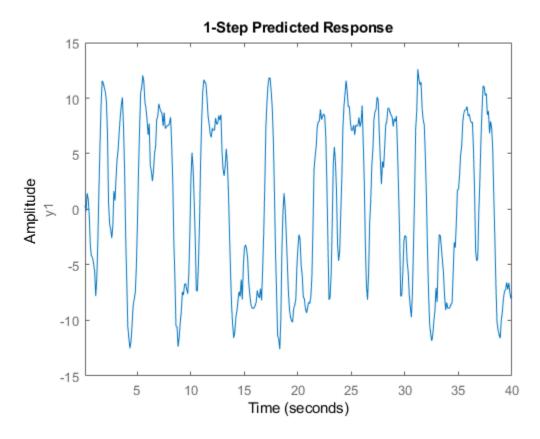
Discretize an identified state-space model to build a one-step ahead predictor of its response.  $\$ 

Create a continuous-time identified state-space model using estimation data.

```
load iddata2
sysc = ssest(z2,4);
```

Predict the 1-step ahead predicted response of sysc.

predict(sysc,z2)



Discretize the model.

```
sysd = c2d(sysc,0.1,'zoh');
```

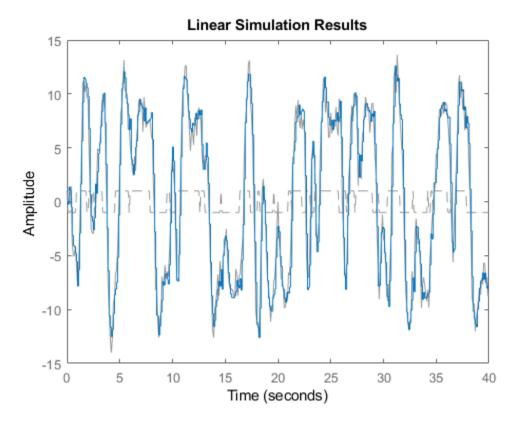
Build a predictor model from the discretized model, sysd.

```
[A,B,C,D,K] = idssdata(sysd);
Predictor = ss(A-K*C,[K B-K*D],C,[0 D],0.1);
```

Predictor is a two-input model which uses the measured output and input signals ([z1.y z1.u]) to compute the 1-step predicted response of sysc.

Simulate the predictor model to get the same response as the predict command.

lsim(Predictor,[z2.y,z2.u])



The simulation of the predictor model gives the same response as predict(sysc,z2).

# **Input Arguments**

 ${\it sysc-Continuous-time\ dynamic\ system}$ 

dynamic system model

Continuous-time model, specified as a dynamic system model such as tf, ss, or zpk. sysc cannot be a frequency response data model. sysc can be a SISO or MIMO system, except that the 'matched' discretization method supports SISO systems only.

sysc can have input/output or internal time delays; however, the 'matched',
'impulse', and 'least-squares' methods do not support state-space models with
internal time delays.

The following identified linear systems cannot be discretized directly:

- idgrey models whose FunctionType is 'c'. Convert to idss model first.
- idproc models. Convert to idtf or idpoly model first.

### Ts — Sample time

positive scalar

Sample time, specified as a positive scalar that represents the sampling period of the resulting discrete-time system. Ts is in TimeUnit, which is the sysc.TimeUnit property.

#### method — Discretization method

```
'zoh' (default) | 'foh' | 'impulse' | 'tustin' | 'matched' | 'least-squares'
```

Discretization method, specified as one of the following values:

- 'zoh' Zero-order hold (default). Assumes the control inputs are piecewise constant over the sample time Ts.
- 'foh' Triangle approximation (modified first-order hold). Assumes the control inputs are piecewise linear over the sample time Ts.
- 'impulse' Impulse invariant discretization
- 'tustin' Bilinear (Tustin) method. To specify this method with frequency prewarping (formerly known as the 'prewarp' method), use the PrewarpFrequency option of c2dOptions.
- $\bullet \quad \text{'matched'} Zero\text{-pole matching method} \\$
- $\bullet \quad \text{'least-squares'} \text{Least-squares method} \\$

For information about the algorithms for each conversion method, see "Continuous-Discrete Conversion Methods".

### opts — Discretization options

c2d0ptions object

Discretization options, specified as a c2dOptions object. For example, specify the prewarp frequency, order of the Thiran filter or discretization method as an option.

## **Output Arguments**

### sysd — Discrete-time model

dynamic system model

Discrete-time model, returned as a dynamic system model of the same type as the input system sysc.

When sysc is an identified (IDLTI) model, sysd:

- Includes both measured and noise components of sysc. The innovations variance  $\lambda$  of the continuous-time identified model sysc, stored in its NoiseVarianceproperty, is interpreted as the intensity of the spectral density of the noise spectrum. The noise variance in sysd is thus  $\lambda/Ts$ .
- Does not include the estimated parameter covariance of sysc. If you want to translate the covariance while discretizing the model, use translatecov.

### G — Mapping of continuous initial conditions of state-space model to discretetime initial state vector

matrix

Mapping of continuous-time initial conditions  $x_0$  and  $u_0$  of the state-space model sysc to the discrete-time initial state vector x[0], returned as a matrix. The mapping of initial conditions to the initial state vector is as follows:

$$x[0] = G \cdot \begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$$

For state-space models with time delays, c2d pads the matrix G with zeroes to account for additional states introduced by discretizing those delays. See "Continuous-Discrete Conversion Methods" for a discussion of modeling time delays in discretized systems.

### See Also

Convert Model Rate | c2dOptions | d2c | d2d | thiran | translatecov

## **Topics**

"Dynamic System Models"
"Discretize a Compensator"

"Continuous-Discrete Conversion Methods"

### **Introduced before R2006a**

# c2dOptions

Create option set for continuous- to discrete-time conversions

## **Syntax**

```
opts = c2dOptions
opts = c2dOptions('OptionName', OptionValue)
```

## **Description**

opts = c2d0ptions returns the default options for c2d.

opts = c2dOptions('OptionName', OptionValue) accepts one or more commaseparated name/value pairs that specify options for the c2d command. Specify OptionName inside single quotes.

## **Input Arguments**

### **Name-Value Pair Arguments**

#### Method

Discretization method, specified as one of the following values:

'zoh'	Zero-order hold, where c2d assumes the control inputs are piecewise
	constant over the sample time Ts.

'foh' Triangle approximation (modified first-order hold), where c2d

assumes the control inputs are piecewise linear over the sample time

Ts. (See [1] on page 2-115, p. 228.)

'impulse' Impulse-invariant discretization.

'tustin' Bilinear (Tustin) approximation. By default, c2d discretizes with no

> prewarp and rounds any fractional time delays to the nearest multiple of the sample time. To include prewarp, use the

PrewarpFrequency option. To approximate fractional time delays,

use the Fract Delay Approx Order option.

'matched' Zero-pole matching method. (See [1] on page 2-115, p. 224.) By

> default, c2d rounds any fractional time delays to the nearest multiple of the sample time. To approximate fractional time delays, use the

FractDelayApproxOrder option.

'least-Least-squares method. Minimize the error between the frequency squares'

responses of the continuous-time and discrete-time systems up to the

Nyquist frequency.

For information about the algorithms for each conversion method, see "Continuous-Discrete Conversion Methods"

Default: 'zoh'

### **PrewarpFrequency**

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the discretized system. Takes positive scalar values. A value of 0 corresponds to the standard 'tustin' method without prewarp.

**Default:** 0

### FractDelayApproxOrder

Maximum order of the Thiran filter used to approximate fractional delays in the 'tustin' and 'matched' methods. Takes integer values. A value of 0 means that c2d rounds fractional delays to the nearest integer multiple of the sample time.

Default: 0

# **Examples**

### **Discretize Two Models Using Tustin Discretization Method**

Generate two random continuous-time state-space models.

```
sys1 = rss(3,2,2);

sys2 = rss(4,4,1);
```

Create an option set for c2d to use the Tustin discretization method and 3.4 rad/s prewarp frequency.

```
opt = c2d0ptions('Method', 'tustin', 'PrewarpFrequency',3.4);
```

Discretize the models, sys1 and sys2, using the same option set, but different sample times.

```
dsys1 = c2d(sys1,0.1,opt);
dsys2 = c2d(sys2,0.2,opt);
```

### References

[1] Franklin, G.F., Powell, D.J., and Workman, M.L., *Digital Control of Dynamic Systems* (3rd Edition), Prentice Hall, 1997.

## **See Also**

c2d

Introduced in R2010a

### canon

Canonical state-space realization

## **Syntax**

```
csys = canon(sys,type)
csys = canon(sys,'modal',condt)
[csys,T]= canon(____)
```

## **Description**

csys = canon(sys, type) transforms the linear model sys into a canonical state-space
model csys. type specifies whether csys is in modal or companion form.

For information on controllable and observable canonical forms, see "Canonical State-Space Realizations".

csys = canon(sys,'modal',condt) specifies an upper bound condt on the
condition number of the block-diagonalizing transformation. Use condt if you have close
lying eigenvalues in csys.

[csys,T]= canon( $\underline{\phantom{a}}$ ) also returns the state-coordinate transformation matrix T that relates the states of the state-space model sys to the states of csys.

# **Examples**

### **Convert State-Space Model to Companion Canonical Form**

aircraftPitchModel.mat contains the state-space matrices of an aircraft where the input is elevator deflection angle  $\delta$  and the output is the aircraft pitch angle  $\theta$ .

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} [\delta]$$
$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} [\delta]$$

Load the model data to the workspace and create the state-space model sys.

Continuous-time state-space model.

Convert the resultant state-space model sys to companion canonical form.

```
csys = canon(sys,'companion')
csys =
   A =
```

Continuous-time state-space model.

csys is the companion canonical form of sys.

### **Convert State-Space Model to Modal Canonical Form**

pendulumModelCart.mat contains the state-space model of an inverted pendulum on a cart where the outputs are the cart displacement x and the pendulum angle  $\theta$ . The control input u is the horizontal force on the cart.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.5 & 30 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 5 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

First, load the state-space model sys to the workspace.

```
load('pendulumCartModel.mat','sys');
```

Convert sys to modal canonical form and extract the transformation matrix.

```
[csys,T] = canon(sys,'modal')
csys =
  A =
            x1
                     x2
                              х3
                                       x4
             (-)
                      0
                               (-)
                                        (-)
   x1
   x2
             0
                 5.453
                               0
                                        0
   хЗ
             0
                      0
                         -5.503
                                        0
             0
                      0
                               0
                                   -0.05
   х4
  B =
            u1
   x1
        1.875
       -6.009
   x2
   х3
        6.386
   x4
       -2.409
  C =
                                                    х4
               x1
                           x2
                                        хЗ
   у1
                               -0.007411
               16 -0.007321
                                                 12.45
   у2
                0
                     -0.07388
                                -0.07344
                                             -0.01038
  D =
       u1
   у1
        0
   у2
        0
Continuous-time state-space model.
T = 4 \times 4
```

```
0.0625
          1.2500
                    -0.0000
                               -0.1250
     0
          0.1123
                    -6.7981
                               -1.2468
     0
          0.1140
                    -6.7768
                               1.2316
     0
         -1.6061
                    -0.0080
                               0.1606
```

csys is the modal canonical form of sys, while T represents the transformation between the state vectors of sys and csys.

#### **Convert Zero-Pole-Gain Model to Modal Canonical Form**

For this example, consider the following system with doubled poles and clusters of close poles:

$$sys(s) = 100 \frac{(s-1)(s+1)}{s(s+10)(s+10.0001)(s-(1+i))^2(s-(1-i))^2}$$

Create a zpk model of this system and convert it to modal canonical form using the string 'modal'.

```
sys = zpk([1 -1], [0 -10 -10.0001 1+1i 1-1i 1+1i 1-1i], 100);
csys1 = canon(sys,'modal');
csys1.A
ans = 7 \times 7
        0 0 0 0 0 0
0 1.0000 1.0000 0 0
0 -1.0000 1.0000 2.0548 0
                                                              0
                                                              (-)
                                                              0
                    0
                             1.0000 1.0000
               0
                        0 -1.0000 1.0000
        0
                 0
                0
                        0
                                 0
                                        0 -10.0000 8.0573
                0
                        (-)
                                   0
                                           0 -10.0001
```

#### csys1.B

sys has a pair of poles at s = -10 and s = -10.0001, and two complex poles of multiplicity 2 at s = 1+i and s = 1-i. As a result, the modal form csys1 is a state-space model with a block of size 2 for the two poles near s = -10, and a block of size 4 for the complex eigenvalues.

Now, separate the two poles near s = -10 by increasing the value of the condition number of the block-diagonalizing transformation. Use a value of 1e10 for this example.

```
csys2 = canon(sys,'modal',1e10);
csys2.A
ans = 7 \times 7
          0
                      0
                                  0
                                              0
                                                          0
                                                                      0
                                                                                 0
          0
                1.0000
                            1.0000
                                              0
                                                                      0
                                                                                 0
                                                          0
          0
               -1.0000
                            1.0000
                                        2.0548
                                                          0
                                                                      (-)
                                                                                 0
          (-)
                                        1.0000
                                                    1.0000
                                                                      (-)
                                                                                 0
                      0
                                  0
          0
                      0
                                  0
                                       -1.0000
                                                    1.0000
                                                                      0
                                                                                 0
          0
                      0
                                  0
                                                             -10.0000
                                                                                 0
                                              0
                                                          0
          0
                      0
                                  0
                                              0
                                                          0
                                                                      0 -10.0001
```

```
format shortE
csys2.B
```

ans =  $7 \times 1$ 

- 3.2000e-01
- -6.5691e-03
- 5.4046e-02
- -1.9502e-01
- 1.0637e+00
- 3.2533e+05
- 3.2533e+05

The A matrix of csys2 includes separate diagonal elements for the poles near s = -10. Increasing the condition number results in some very large values in the B matrix.

### **Convert System to Companion Canonical Form**

The file icEngine.mat contains one data set with 1500 input-output samples collected at the a sampling rate of 0.04 seconds. The input u(t) is the voltage (V) controlling the By-Pass Idle Air Valve (BPAV), and the output y(t) is the engine speed (RPM/100).

Use the data in icEngine.mat to create a state-space model with identifiable parameters.

```
load icEngine.mat
z = iddata(y,u,0.04);
sys = n4sid(z,4,'InputDelay',2);
```

Convert the identified state-space model sys to companion canonical form.

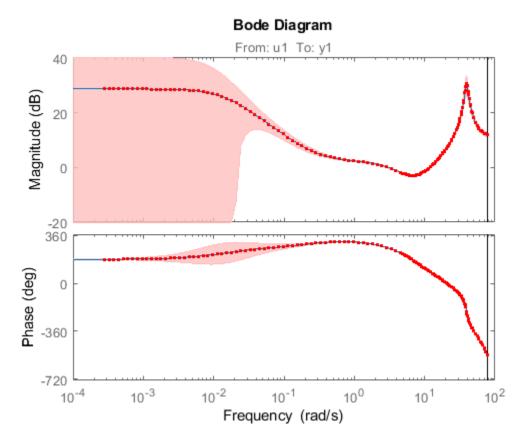
```
csys = canon(sys,'companion');
```

Obtain the covariance of the resulting form by running a zero-iteration update to model parameters.

```
opt = ssestOptions;
opt.SearchOptions.MaxIterations = 0;
csys = ssest(z,csys,opt);
```

Compare frequency response confidence bounds of sys to csys.

```
h = bodeplot(sys,csys,'r.');
showConfidence(h)
```



The frequency response confidence bounds are identical.

## **Input Arguments**

#### sys — Dynamic system

dynamic system model

Dynamic system, specified as a SISO, or MIMO dynamic system model. Dynamic systems that you can use include:

• Continuous-time or discrete-time numeric LTI models, such as tf, zpk, ss, or pid models.

• Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

The resulting canonical state-space model assumes

- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.
- Identified LTI models, such as idtf, idss, idproc, idpoly, and idgrey models. (Using identified models requires System Identification Toolbox software.)

You cannot use frequency-response data models such as frd models.

#### type — Transformation type

```
'modal' (default) | 'companion'
```

Transformation type, specified as either 'modal' or 'companion'. If type is unspecified, then canon converts the specified dynamic system model to modal canonical form by default.

The companion canonical form is the same as the observable canonical form. For information on controllable and observable canonical forms, see "Canonical State-Space Realizations".

Modal Form

In modal form, *A* is a block-diagonal matrix. The block size is typically 1-by-1 for real eigenvalues and 2-by-2 for complex eigenvalues. However, if there are repeated eigenvalues or clusters of nearby eigenvalues, the block size can be larger.

For example, for a system with eigenvalues  $(\lambda_1, \sigma \pm j\omega, \lambda_2)$ , the modal A matrix is of the form

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \sigma & \omega & 0 \\ 0 & -\omega & \sigma & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix}$$

· Companion Form

In the companion realization, the characteristic polynomial of the system appears explicitly in the rightmost column of the A matrix. For a system with characteristic polynomial

$$P(s) = s^n + \alpha_1 s^{n-1} + ... + \alpha_{n-1} s + \alpha_n$$

the corresponding companion A matrix is

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & -\alpha_n \\ 1 & 0 & 0 & \dots & 0 & -\alpha_{n-1} \\ 0 & 1 & 0 & \dots & 0 & -\alpha_{n-2} \\ 0 & 0 & 1 & \dots & 0 & -\alpha_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -\alpha_1 \end{bmatrix}$$

The companion transformation requires that the system is controllable from the first input. The transformation to companion form is based on the controllability matrix which is almost always numerically singular for mid-range orders. Hence, avoid using it when possible.

The companion canonical form is the same as the observable canonical form. For more information on observable and controllable canonical forms, see "Canonical State-Space Realizations".

# condt — Upper bound on the condition number of the block-diagonalizing transformation

1e8 (default) | positive scalar

Upper bound on the condition number of the block-diagonalizing transformation, specified as a positive scalar. This argument is available only when type is set to 'modal'.

Increase condt to reduce the size of the eigenvalue clusters in the A matrix of csys. Setting condt = Inf diagonalizes matrix A.

## **Output Arguments**

### csys — Canonical state-space form of the dynamic model

ss model object

Canonical state-space form of the dynamic model, returned as an ss model object. csys is a state-space realization of sys in the canonical form specified by type.

#### T — Transformation matrix

matrix

Transformation matrix, returned as an n-by-n matrix, where n is the number of states. T is the transformation between the state vector x of the state-space model sys and the state vector  $x_c$  of csys:

$$x_c = Tx$$

.

This argument is available only when sys is an ss model object.

### Limitations

- You cannot use frequency-response data models to convert to canonical state-space form.
- The companion form is poorly conditioned for most state-space computations, that is, the transformation to companion form is based on the controllability matrix which is almost always numerically singular for mid-range orders. Hence, avoid using it when possible.

## **Algorithms**

The canon command uses the bdschur command to convert sys into modal form and to compute the transformation T. If sys is not a state-space model, canon first converts it to state space using ss.

The reduction to companion form uses a state similarity transformation based on the controllability matrix [1].

#### References

[1] Kailath, T. Linear Systems, Prentice-Hall, 1980.

### See Also

ctrb|ctrbf|genss|idgrey|idpoly|idproc|idss|idtf|pid|ss|ss2ss|tf
|uss|zpk

### **Topics**

"Canonical State-Space Realizations"

#### **Introduced before R2006a**

#### care

(Not recommended) Continuous-time algebraic Riccati equation solution

**Note** care is not recommended. Use icare instead. For more information, see "Compatibility Considerations".

## **Syntax**

```
[X,L,G] = care(A,B,Q)
[X,L,G] = care(A,B,Q,R,S,E)
[X,L,G,report] = care(A,B,Q,...)
[X1,X2,D,L] = care(A,B,Q,..., 'factor')
```

## **Description**

[X,L,G] = care(A,B,Q) computes the unique solution X of the continuous-time algebraic Riccati equation

$$A^TX + XA - XBB^TX + O = 0$$

The care function also returns the gain matrix,  $G = R^{-1}B^{T}XE$ .

[X,L,G] = care(A,B,Q,R,S,E) solves the more general Riccati equation

$$A^{T}XE + E^{T}XA - (E^{T}XB + S)R^{-1}(B^{T}XE + S^{T}) + Q = 0$$

When omitted, R, S, and E are set to the default values R=I, S=0, and E=I. Along with the solution X, care returns the gain matrix  $G = R^{-1}(B^TXE + S^T)$  and a vector L of closed-loop eigenvalues, where

```
L=eig(A-B*G,E)
```

[X,L,G,report] = care(A,B,Q,...) returns a diagnosis report with:

- -1 when the associated Hamiltonian pencil has eigenvalues on or very near the imaginary axis (failure)
- -2 when there is no finite stabilizing solution X
- The Frobenius norm of the relative residual if X exists and is finite.

This syntax does not issue any error message when X fails to exist.

[X1, X2, D, L] = care(A, B, Q, ..., 'factor') returns two matrices X1, X2 and a diagonal scaling matrix D such that  $X = D^*(X2/X1)*D$ .

The vector L contains the closed-loop eigenvalues. All outputs are empty when the associated Hamiltonian matrix has eigenvalues on the imaginary axis.

## **Examples**

#### **Example 1**

#### **Solve Algebraic Riccati Equation**

Given

$$A = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad R = 3$$

you can solve the Riccati equation

$$A^TX + XA - XBR^{-1}B^TX + C^TC = 0$$

by

This yields the solution

Х

```
x = 0.5895 1.8216
1.8216 8.8188
```

You can verify that this solution is indeed stabilizing by comparing the eigenvalues of a and a-b\*g.

```
[eig(a) eig(a-b*g)]
ans =
    -3.4495    -3.5026
    1.4495    -1.4370
```

Finally, note that the variable 1 contains the closed-loop eigenvalues eig(a-b\*g).

```
l =
-3.5026
-1.4370
```

#### **Example 2**

#### Solve H-infinity ( $H_{\infty}$ )-like Riccati Equation

To solve the  $H_{\infty}$ -like Riccati equation

$$A^{T}X + XA + X(\gamma^{-2}B_{1}B_{1}^{T} - B_{2}B_{2}^{T})X + C^{T}C = 0$$

rewrite it in the care format as

$$A^{T}X + XA - X \left[B_{1}B_{2}\right] \begin{bmatrix} -\gamma^{2}I & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} B_{1}^{T} \\ B_{2}^{T} \end{bmatrix} X + C^{T}C = 0$$

You can now compute the stabilizing solution X by

```
B = [B1 , B2]
m1 = size(B1,2)
m2 = size(B2,2)
R = [-g^2*eye(m1) zeros(m1,m2) ; zeros(m2,m1) eye(m2)]
X = care(A,B,C'*C,R)
```

### Limitations

The (A,B) pair must be stabilizable (that is, all unstable modes are controllable). In addition, the associated Hamiltonian matrix or pencil must have no eigenvalue on the imaginary axis. Sufficient conditions for this to hold are (Q,A) detectable when S=0 and R>0, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

## **Algorithms**

care implements the algorithms described in [1]. It works with the Hamiltonian matrix when R is well-conditioned and E = I; otherwise it uses the extended Hamiltonian pencil and QZ algorithm.

## **Compatibility Considerations**

#### care is not recommended

Not recommended starting in R2019a

Starting in R2019a, use the icare command to solve continuous-time Riccati equations. This approach has improved accuracy through better scaling and the computation of K is more accurate when R is ill-conditioned relative to care. Furthermore, icare includes an optional info structure to gather the implicit solution data of the Riccati equation.

The following table shows some typical uses of care and how to update your code to use icare instead.

Not Recommended	Recommended
[X,L,G] = care(A,B,Q,R,S,E)	[X,K,L] = icare(A,B,Q,R,S,E,G) computes the stabilizing solution X, the state-feedback gain K and the closed-loop eigenvalues L of the continuous-time algebraic Riccati equation. For more information, see icare.
<pre>[X,L,G,report] = care(A,B,Q,R,S,E)</pre>	[X,K,L,info] = icare(A,B,Q,R,S,E,G) computes the stabilizing solution X, the state-feedback gain K, the closed-loop eigenvalues L of the continuous-time algebraic Riccati equation. The info structure contains the implicit solution data. For more information, see icare.

There are no plans to remove care at this time.

### References

[1] Arnold, W.F., III and A.J. Laub, "Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations," *Proc. IEEE*, 72 (1984), pp. 1746-1754

### **See Also**

icare

**Introduced before R2006a** 

# chgFreqUnit

Change frequency units of frequency-response data model

## **Syntax**

sys new = chgFreqUnit(sys,newfrequnits)

## **Description**

sys\_new = chgFreqUnit(sys,newfrequnits) changes units of the frequency points
in sys to newfrequnits. Both Frequency and FrequencyUnit properties of sys
adjust so that the frequency responses of sys and sys\_new match.

## **Input Arguments**

#### sys

Frequency-response data (frd, idfrd, or genfrd) model

#### newfrequnits

New units of frequency points, specified as one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

rad/TimeUnit and cycles/TimeUnit express frequency units relative to the system
time units specified in the TimeUnit property.

Default: 'rad/TimeUnit'

## **Output Arguments**

```
sys_new
```

Frequency-response data model of the same type as sys with new units of frequency points. The frequency response of sys new is same as sys.

## **Examples**

#### **Change Frequency Units of Frequency-Response Data Model**

Create a frequency-response data model.

```
load(fullfile(matlabroot, 'examples', 'controls_id', 'AnalyzerData'));
sys = frd(resp,freq);
```

The data file AnalyzerData has column vectors freq and resp. These vectors contain 256 test frequencies and corresponding complex-valued frequency response points, respectively. The default frequency units of sys is rad/TimeUnit, where TimeUnit is the system time units.

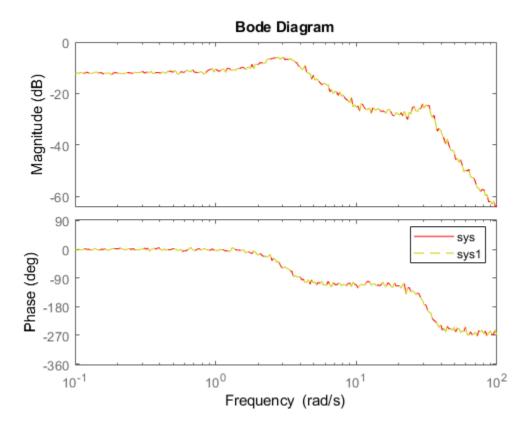
Change the frequency units.

```
sys1 = chgFreqUnit(sys,'rpm');
```

The FrequencyUnit property of sys1 is rpm.

Compare the Bode responses of sys and sys1.

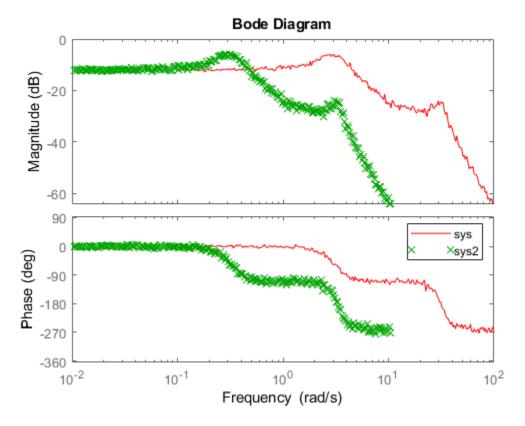
```
bodeplot(sys,'r',sys1,'y--');
legend('sys','sys1')
```



The magnitude and phase of sys and sys1 match because chgFreqUnit command changes the units of frequency points in sys without modifying system behavior.

Change the FrequencyUnit property of sys to compare the Bode response with the original system.

```
sys2 = sys;
sys2.FrequencyUnit = 'rpm';
bodeplot(sys,'r',sys2,'gx');
legend('sys','sys2');
```



Changing the FrequencyUnit property changes the system behavior. Therefore, the Bode responses of sys and sys2 do not match. For example, the original corner frequency at about 2 rad/s changes to approximately 2 rpm (or 0.2 rad/s).

# **Tips**

• Use chgFreqUnit to change the units of frequency points without modifying system behavior.

### See Also

chgTimeUnit|frd

# **Topics**

"Specify Frequency Units of Frequency-Response Data Model"

Introduced in R2011a

# chgTimeUnit

Change time units of dynamic system

## **Syntax**

sys new = chgTimeUnit(sys,newtimeunits)

## **Description**

sys\_new = chgTimeUnit(sys,newtimeunits) changes the time units of sys to newtimeunits. The time- and frequency-domain characteristics of sys and sys\_new match.

## **Input Arguments**

#### sys

Dynamic system model

#### newtimeunits

New time units, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'

- 'months'
- 'years'

Default: 'seconds'

## **Output Arguments**

#### sys\_new

Dynamic system model of the same type as sys with new time units. The time response of sys new is same as sys.

If sys is an identified linear model, both the model parameters as and their minimum and maximum bounds are scaled to the new time units.

## **Examples**

#### **Change Time Units of Dynamic System Model**

Create a transfer function model.

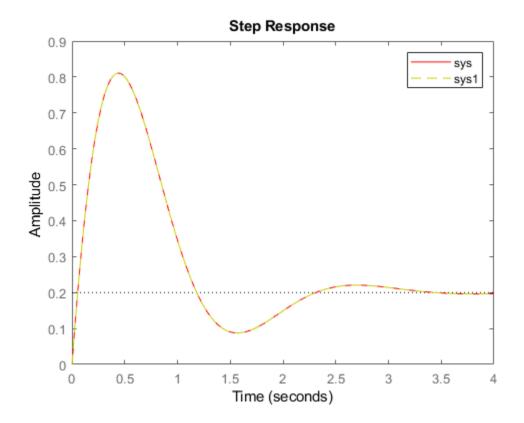
```
num = [4 2];
den = [1 3 10];
sys = tf(num,den);
```

By default, the time unit of sys is 'seconds'. Create a new model with the time units changed to minutes.

```
sys1 = chgTimeUnit(sys,'minutes');
```

This command sets the TimeUnit property of sys1 to 'minutes', without changing the dynamics. To confirm that the dynamics are unchanged, compare the step responses of sys and sys1.

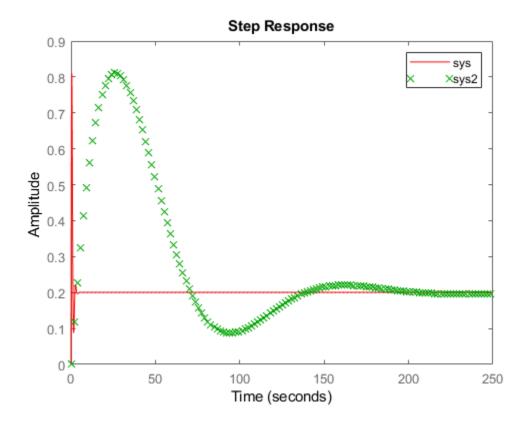
```
stepplot(sys,'r',sys1,'y--');
legend('sys','sys1');
```



The step responses are the same.

If you change the TimeUnit property of the system instead of using chgTimeUnit, the dynamics of the system do change. To see this, change the TimeUnit property of a copy of sys and compare the step response with the original system.

```
sys2 = sys;
sys2.TimeUnit = 'minutes';
stepplot(sys,'r',sys2,'gx');
legend('sys','sys2');
```



The step responses of sys and sys2 do not match. For example, the original rise time of 0.04 seconds changes to 0.04 minutes.

# **Tips**

• Use chgTimeUnit to change the time units without modifying system behavior.

### See Also

chgFreqUnit|frd|pid|ss|tf|zpk

# **Topics**

"Specify Model Time Units"

Introduced in R2011a

### clone

Copy online state estimation object

## **Syntax**

```
obj clone = clone(obj)
```

## **Description**

obj\_clone = clone(obj) creates a copy of the online state estimation object obj with
the same property values.

If you want to copy an existing object and then modify properties of the copied object, use the clone command. Do not create additional objects using syntax obj2 = obj. Any changes made to the properties of the new object created in this way (obj2) also change the properties of the original object (obj).

## **Examples**

#### **Clone an Online State Estimation Object**

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. To create the object, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. Specify the initial state values for the two states as [2;0].

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0])
obj =
   extendedKalmanFilter with properties:
        HasAdditiveProcessNoise: 1
        StateTransitionFcn: @vdpStateFcn
        HasAdditiveMeasurementNoise: 1
```

```
MeasurementFcn: @vdpMeasurementFcn
StateTransitionJacobianFcn: []
MeasurementJacobianFcn: []
State: [2x1 double]
StateCovariance: [2x2 double]
ProcessNoise: [2x2 double]
MeasurementNoise: 1
```

Use clone to generate an object with the same properties as the original object.

Modify the MeasurementNoise property of obj2.

```
obj2.MeasurementNoise = 2;
```

Verify that MeasurementNoise property of original object obj remains unchanged and equals 1.

```
obj.MeasurementNoise
```

```
ans = 1
```

## **Input Arguments**

#### obj — Object for online state estimation

extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter object

Object for online state estimation of a nonlinear system, created using one of the following commands:

- extendedKalmanFilter
- unscentedKalmanFilter
- particleFilter

## **Output Arguments**

#### obj\_clone — Clone of online state estimation object

extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter object

Clone of online state estimation object obj, returned as an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object with the same properties as obj.

### See Also

correct | extendedKalmanFilter | initialize | particleFilter | predict |
unscentedKalmanFilter

#### Introduced in R2016b

## codegen

Generate MATLAB code for tunable gain surfaces

## **Syntax**

```
code = codegen(GS)
```

## **Description**

code = codegen(GS) generates MATLAB code for the tunable surface GS. The generated code is a function that takes the scalar values of the scheduling variables and returns the scalar-valued or matrix-valued gain, depending on GS.

## **Examples**

#### **Generate Code from Tunable Surface**

Create a tunable surface that represents a scalar gain with a bilinear dependence on two scheduling variables. Suppose that the scheduling variables are alpha, ranging from 0-15 degrees, and V, ranging from 300-600 m/s. The tunable surface covers a linearly spaced grid in this operating range.

```
[alpha,V] = ndgrid(0:3:15,300:50:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
GS0 = tunableSurface('K',1,domain,shapefcn);
```

Usually, you use GS0 to parameterize a scheduled gain and tune the surface coefficients with systume. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS0, [100, 28, 40, 10]);
```

Generate MATLAB code that computes the scalar gain as a function of scheduling variables.

```
code = codegen(GS)
code =
    'function Gain = fcn(alpha ,V )
    %#codegen
    % Type casting
     ZER0 = zeros(1,1,'like',alpha +V);
     alpha_ = cast(alpha_,'like',ZERO);
     V_{-} = cast(V_{-}, 'like', ZER0);
     % Tuned gain surface coefficients
     Coeffs = cast([100 28 40 10], 'like', ZERO);
     Offsets = cast([7.5 450], 'like', ZERO);
     Scalings = cast([7.5 150], 'like', ZER0);
     % Normalization
     alpha = (alpha - Offsets(1))/Scalings(1);
     V = (V - Offsets(2))/Scalings(2);
    % Compute weighted sum of terms
     Y = [alpha, V, alpha*V];
     Gain = Coeffs(1);
     for i=1:numel(Y)
        Gain = Gain + Coeffs(i+1) * Y(i);
     end
```

The resulting code is a function, fcn, that takes two scheduling variables and returns a scalar gain. The function includes the **%#codegen** directive, so that it can be used for further code generation, such as implementing a tuned gain schedule in hardware.

The function includes four sections. The first section ensures that the scheduling variables are cast to the same type. The second section encodes the gain coefficients and the offsets and scalings that the software extracts from GS. These values are hard-coded into fcn, which can compute the gain surface without reference to GS. The third section uses these values to compute the normalized scheduling variables. (See tunableSurface for more information about normalization.)

The last section computes the gain by summing up all the terms in the polynomial expression for the gain surface.

## **Input Arguments**

#### **GS** — Tunable gain surface

tunableSurface object

Tunable gain surface, specified as a tunableSurface object.

## **Output Arguments**

#### code — Generated code for gain surface

character array

Generated code for the gain surface, returned as a character array. The code contains a function,  $Gain_{=} fcn(x1_,x2_,...,xN_{-})$ , that computes the gain from the scheduling variables  $x1_,x2_,...,xN_{-}$  of GS. The expression relating the gain to the scheduling variables, the coefficients of the expression, and the normalization of the scheduling variables are all taken from GS, and the code can run without further reference to TS. The returned function includes the %#codegen directive so that it can be used for further code generation, such as implementing a tuned gain schedule in hardware.

When you use writeBlockValue to write tuned gain-surface coefficients from an slTuner interface to a MATLAB Function block, it uses this generated MATLAB code.

### See Also

tunableSurface

#### **Topics**

"Model Gain-Scheduled Control Systems in Simulink"

#### Introduced in R2017b

<sup>&</sup>quot;Parameterize Gain Schedules"

# conj

Form model with complex conjugate coefficients

## **Syntax**

```
sysc = conj(sys)
```

## **Description**

sysc = conj(sys) constructs a complex conjugate model sysc by applying complex conjugation to all coefficients of the LTI model sys. This function accepts LTI models in transfer function (TF), zero/pole/gain (ZPK), and state space (SS) formats.

## **Examples**

If sys is the transfer function

```
(2+i)/(s+i)
```

then conj(sys) produces the transfer function

```
(2-i)/(s-i)
```

This operation is useful for manipulating partial fraction expansions.

### See Also

append | ss | tf | zpk

Introduced before R2006a

### connect

Block diagram interconnections of dynamic systems

## **Syntax**

```
sysc = connect(sys1,...,sysN,inputs,outputs)
sysc = connect(sys1,...,sysN,inputs,outputs,APs)
sysc = connect(blksys,connections,inputs,outputs)
sysc = connect(____,opts)
```

## **Description**

sysc = connect(sys1,...,sysN,inputs,outputs) connects the block diagram elements sys1,...,sysN based on signal names. The block diagram elements sys1,...,sysN are dynamic system models. These models can include summing junctions that you create using sumblk. The connect command interconnects the block diagram elements by matching the input and output signals that you specify in the InputName and OutputName properties of sys1,...,sysN. The aggregate model sysc is a dynamic system model having inputs and outputs specified by inputs and outputs respectively.

sysc = connect(sys1,...,sysN,inputs,outputs,APs) inserts an AnalysisPoint at every signal location specified in APs. Use analysis points to mark locations of interest which are internal signals in the aggregate model. For instance, a location at which you want to extract a loop transfer function or measure the stability margins is a location of interest.

sysc = connect(blksys,connections,inputs,outputs) uses index-based interconnection to build sysc out of an aggregate, unconnected model blksys. The matrix connections specifies how the outputs and inputs of blksys interconnect. For index-based interconnections, inputs and outputs are index vectors that specify which inputs and outputs of blksys are the external inputs and outputs of sysc. This syntax can be convenient when you do not want to assign names to all inputs and outputs of all models to connect. However, in general, it is easier to keep track of named signals.

sysc = connect( \_\_\_\_, opts) builds the interconnected model using additional
options. You can use opts with the input arguments of any of the previous syntaxes.

## **Input Arguments**

#### sys1,...,sysN

Dynamic system models that correspond to the elements of your block diagram. For example, the elements of your block diagram can include one or more tf or ss models that represent plant dynamics. Block diagram elements can also include a pid or tunablePID model representing a controller. You can also include one or more summing junction that you create using sumblk. Provide multiple arguments sys1,...,sysN to represent all of the block diagram elements and summing junctions.

#### inputs

For name-based interconnection, a character vector or cell array of character vectors that specify the inputs of the aggregate model sysc. The inputs in inputs must correspond to entries in the InputName or OutputName property of one or more of the block diagram elements sys1,...,sysN.

#### outputs

For name-based interconnection, a character vector or cell array of character vectors that specify the outputs of the aggregate model sysc. The outputs in outputs must correspond to entries in the OutputName property of one or more of the block diagram elements sys1,...,sysN.

#### **APs**

Locations (internal signals) of interest in the aggregate model, specified as a character vector or cell array of character vectors, such as 'X' or {'AP1', 'AP2'}. The resulting model contains an analysis point at each such location. (See AnalysisPoint). Each location in APs must correspond to an entry in the InputName or OutputName property of one or more of the block diagram elements sys1,...,sysN.

#### blksys

Unconnected aggregate model. To obtain blksys, use append to join dynamic system models of the elements of your block diagram. For example, if your block diagram contains dynamic system models C, G, and S, create blksys with the following command:

```
blksys = append(C,G,S)
```

#### connections

Matrix that specifies the connections and summing junctions of the block diagram. Each row of connections specifies one connection or summing junction in terms of the input vector u and output vector y of the unconnected aggregate model blksys. For example, the row:

```
[3 2 0 0]
```

specifies that y(2) connects into u(3). The row

```
[7 2 -15 6]
```

indicates that y(2) - y(15) + y(6) feeds into u(7).

If you do not specify any connection for a particular input or output, connect omits that input or output from the aggregate model.

#### opts

Additional options for interconnection, specified as an options set that you create with connectOptions.

## **Output Arguments**

#### sysc

Interconnected system, returned as either a state-space model or frequency-response model. The type of model returned depends on the input models. For example:

- Interconnecting numeric LTI models (other than frd models) returns an ss model.
- Interconnecting a numeric LTI model with a Control Design Block returns a generalized LTI model. For instance, interconnecting a tf model with a tunablePID Control Design Block returns a genss.
- Interconnecting any model with frequency-response data model returns a frequency response data model.

By default, connect automatically discards states that do not contribute to the I/O transfer function from the specified inputs to the specified outputs of the interconnected

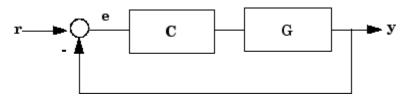
model. To retain the unconnected states, set the Simplify option of connectOptions to false. For example:

```
opt = connectOptions('Simplify', false);
sysc = connect(sys1,sys2,sys3,'r','y',opt);
```

## **Examples**

### **SISO Feedback Loop**

Create an aggregate model of the following block diagram from r to y.



Create C and G, and name the inputs and outputs.

```
C = pid(2,1);
C.u = 'e';
C.y = 'u';
G = zpk([],[-1,-1],1);
G.u = 'u';
G.y = 'y';
```

The notations C.u and C.y are shorthand expressions equivalent to C.InputName and C.OutputName, respectively. For example, entering C.u = 'e' is equivalent to entering C.InputName = 'e'. The command sets the InputName property of C to the value 'e'.

Create the summing junction.

```
Sum = sumblk('e = r - v');
```

Combine C, G, and the summing junction to create the aggregate model from r to y.

```
T = connect(G,C,Sum,'r','y');
```

connect automatically joins inputs and outputs with matching names.

#### MIMO Feedback Loop

Create the control system of the previous example where G and C are both 2-input, 2-output models.

```
C = [pid(2,1),0;0,pid(5,6)];
C.InputName = 'e';
C.OutputName = 'u';
G = ss(-1,[1,2],[1;-1],0);
G.InputName = 'u';
G.OutputName = 'y';
```

When you specify single names for vector-valued signals, the software automatically performs vector expansion of the signal names. For example, examine the names of the inputs to C.

#### C.InputName

```
ans = 'e(1)'
'e(2)'
```

Create a 2-input, 2-output summing junction.

```
Sum = sumblk('e = r-y', 2);
```

sumblk also performs vector expansion of the signal names.

Interconnect the models to obtain the closed-loop system.

```
T = connect(G,C,Sum,'r','y');
```

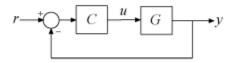
The block diagram elements G, C, and Sum are all 2-input, 2-output models. Therefore, connect performs the same vector expansion. connect selects all entries of the two-input signals 'r' and 'y' as inputs and outputs to T, respectively. For example, examine the input names of T.

#### T.InputName

```
ans = 'r(1)'
'r(2)'
```

### Feedback Loop With Analysis Point Inserted by connect

Create a model of the following block diagram from r to y. Insert an analysis point at an internal location, u.



Create C and G, and name the inputs and outputs.

```
C = pid(2,1);
C.InputName = 'e';
C.OutputName = 'u';
G = zpk([],[-1,-1],1);
G.InputName = 'u';
G.OutputName = 'y';
```

Create the summing junction.

```
Sum = sumblk('e = r - y');
```

Combine C, G, and the summing junction to create the aggregate model, with an analysis point at u.

```
T = connect(G,C,Sum,'r','y','u')
T =
```

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 3 states, and AnalysisPoints\_: Analysis point, 1 channels, 1 occurrences.

```
Type "ss(T)" to see the current value, "get(T)" to see all properties, and "T.Blocks"
```

The resulting T is a genss model. The connect command creates the AnalysisPoint block, AnalysisPoints\_, and inserts it into T. To see the name of the analysis point channel in AnalysisPoints , use getPoints.

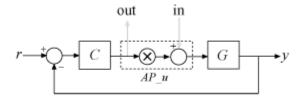
```
getPoints(T)
```

```
ans = 1x1 cell array
{'u'}
```

The analysis point channel is named 'u'. You can use this analysis point to extract system responses. For example, the following commands extract the open-loop transfer at u and the closed-loop response at v to a disturbance injected at u.

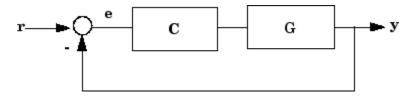
```
L = getLoopTransfer(T, 'u', -1);
Tuy = getIOTransfer(T, 'u', 'y');
```

T is equivalent to the following block diagram, where  $AP\_u$  designates the AnalysisPoint block AnalysisPoints\_ with channel name u.



#### **Index-Based Interconnection**

Create an aggregate model of the following block diagram from r to y using index-based interconnection.



Create C, G, and the unconnected aggregate model blksys.

```
C = pid(2,1);
G = zpk([],[-1,-1],1);
blksys = append(C,G);
```

The inputs u(1), u(2) of blksys correspond to the inputs of C and G, respectively. The outputs w(1), w(2) of blksys correspond to the outputs of C and G, respectively.

Create the matrix connections, which specifies which outputs of blksys connect to which inputs of blksys.

```
connections = [2 1; 1 -2];
```

The first row indicates that w(1) connects to u(2); in other words, that the output of C connects to the input of C. The second row indicates that -w(2) connects to u(1); in other words, that the negative of the output of C connects to the input of C.

Create the connected aggregate model from r to y.

```
T = connect(blksys,connections,1,2)
```

The last two arguments specify the external inputs and outputs in terms of the indices of blksys. The argument 1 specifies that the external input connects to u(1). The last argument, 2, specifies that the external output connects from w(2).

#### See Also

AnalysisPoint | append | connectOptions | feedback | lft | parallel | series | sumblk

#### **Topics**

"Multi-Loop Control System"

"MIMO Control System"

"MIMO Feedback Loop"

"Mark Analysis Points in Closed-Loop Models"

#### Introduced before R2006a

## **connectOptions**

Options for the connect command

## **Syntax**

```
opt = connectOptions
opt = connectOptions(Name, Value)
```

## **Description**

opt = connectOptions returns the default options for connect.

opt = connectOptions(Name, Value) returns an options set with the options
specified by one or more Name, Value pair arguments.

## **Examples**

#### Retain Unconnected States in Model Interconnection

Use connectOptions to cause the connect command to retain unconnected states in an interconnected model.

Suppose you have dynamic system models sys1, sys2, and sys3. Combine these dynamic system models to build an interconnected model with input 'r' and output 'y'. Set the option to retain states in the model that do not contribute to the dynamics in the path from 'r' or 'y'.

```
opt = connectOptions('Simplify', false);
sysc = connect(sys1,sys2,sys3,'r','y',opt);
```

## **Input Arguments**

### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: 'Simplify', false

### Simplify — Automatic elimination of unconnected states

true (default) | false

Automatic elimination of unconnected states, specified as either true or false.

- true connect eliminates all states that do not contribute to the I/O transfer function from the specified inputs to the specified outputs of the interconnected system.
- false connect retains unconnected states. This option can be useful, for example, when you want to compute the interconnected system response from known initial state values of the components.

Data Types: logical

## **Output Arguments**

### opt — Options for connect

connectOptions options set

Options for connect, returned as a connectOptions options set. Use opt as the last argument to connect when interconnecting models.

## **See Also**

connect

Introduced in R2013b

## **Control System Designer**

Design single-input, single-output (SISO) controllers

## **Description**

The **Control System Designer** app lets you design single-input, single-output (SISO) controllers for feedback systems modeled in MATLAB or Simulink (requires Simulink Control Design™ software).

Using this app, you can:

- Design controllers using:
  - Interactive Bode, root locus, and Nichols graphical editors for adding, modifying, and removing controller poles, zeros, and gains.
  - Automated PID, LQG, or IMC tuning.
  - Optimization-based tuning (requires Simulink Design Optimization<sup>™</sup> software).
  - Automated loop shaping (requires Robust Control Toolbox software).
- Tune compensators for single-loop or multiloop control architectures.
- Analyze control system designs using time-domain and frequency-domain responses, such as step responses and pole-zero maps.
- Compare response plots for multiple control system designs.
- Design controllers for multimodel control applications.

### Limitations

When using **Control System Designer** in MATLAB Online<sup>m</sup>, the following features are not available:

- Designing of controllers in Simulink
- Variable editor

## Open the Control System Designer App

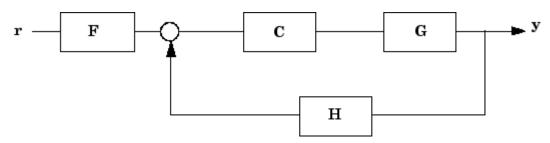
- MATLAB Toolstrip: On the Apps tab, under Control System Design and Analysis, click the app icon.
- MATLAB command prompt: Enter controlSystemDesigner.
- Simulink Toolstrip: On the **Apps** tab, under **Control Systems**, click the app icon.

## **Examples**

- "Control System Designer Tuning Methods"
- "Bode Diagram Design"
- "Root Locus Design"
- "Design Compensator Using Automated Tuning Methods"
- "Design Multiloop Control System"
- "Analyze Designs Using Response Plots"
- "Compare Performance of Multiple Designs"
- "Multimodel Control Design"

### **Programmatic Use**

controlSystemDesigner opens the Control System Designer app using the following
default control architecture:



The architecture consists of the LTI objects:

- G Plant model
- *C* Compensator
- H Sensor model
- F Prefilter

By default, the app configures each of these models as a unit gain.

controlSystemDesigner(plant) initializes the plant, G, to plant. plant can be any SISO LTI model created with ss, tf, zpk or frd, or an array of such models.

controlSystemDesigner(plant,comp) initializes the compensator, C, to the SISO LTI model comp.

controlSystemDesigner(plant,comp,sensor) initializes the sensor model, H, to sensor. sensor can be any SISO LTI model or an array of such models. If you specify both plant and sensor as LTI model arrays, the lengths of the arrays must match.

controlSystemDesigner(plant,comp,sensor,prefilt) initializes the prefilter model, F, to the SISO LTI model prefilt.

controlSystemDesigner(views) opens the app and specifies the initial graphical
editor configuration. views can be any of the following character vectors, or a cell array
of multiple character vectors.

- 'rlocus' Root locus editor
- 'bode' Open-loop Bode Editor
- 'nichols' Open-loop Nichols Editor
- 'filter' Bode Editor for the closed-loop response from prefilter input to the plant output

In addition to opening the specified graphical editors, the app plots the closed-loop, inputoutput step response.

controlSystemDesigner(views,plant,comp,sensor,prefilt) specifies the initial plot configuration and initializes the plant, compensator, sensor, and prefilter using the specified models. If a model is omitted, the app uses the default value.

controlSystemDesigner(initData) opens the app and initializes the system configuration using the initialization data structure initdata. To create initdata, use sisoinit.

controlSystemDesigner(sessionFile) opens the app and loads a previously saved session. sessionFile is the name of a session data file on the MATLAB path. This data includes the current system architecture and plot configuration, and any designs and responses saved in the **Data Browser**.

To save a session, in the **Control System Designer** app, on the **Control System** tab, click Save Session.

### **See Also**

# **Apps Control System Tuner**

# Functions pidTuner|sisoinit

## **Topics**

"Control System Designer Tuning Methods"

"Bode Diagram Design"

"Root Locus Design"

"Design Compensator Using Automated Tuning Methods"

"Design Multiloop Control System"

"Analyze Designs Using Response Plots"

"Compare Performance of Multiple Designs"

"Multimodel Control Design"

### Introduced in R2015a

## **Control System Tuner**

Tune fixed-structure control systems

## **Description**

The **Control System Tuner** app tunes control systems modeled in MATLAB or Simulink (requires Simulink Control Design software). This app lets you tune any control system architecture to meet your design goals. You can tune multiple fixed-order, fixed-structure control elements distributed over one or more feedback loops.

**Control System Tuner** automatically tunes the controller parameters to satisfy the must-have requirements (design constraints) and to best meet the remaining requirements (objectives). The library of tuning goals lets you capture your design requirements in a form suitable for fast automated tuning. Available tuning goals include standard control objectives such as reference tracking, disturbance rejection, loop shapes, closed-loop damping, and stability margins.

### **Limitations**

When using **Control System Tuner** in MATLAB Online, the following features are not available:

- · Tuning of Simulink models
- Tuning reports
- Variable editor

## **Open the Control System Tuner App**

- MATLAB Toolstrip: On the Apps tab, under Control System Design and Analysis, click the app icon.
- MATLAB command prompt: Enter controlSystemTuner.
- Simulink Toolstrip: On the Apps tab, under Control Systems, click the app icon.

## **Examples**

- "Setup for Tuning Control System Modeled in MATLAB"
- "Specify Control Architecture in Control System Tuner"
- "Tune a Control System Using Control System Tuner"

### **Programmatic Use**

controlSystemTuner opens the Control System Tuner app. When invoked without input arguments, Control System Tuner opens for tuning the default single-loop feedback control system architecture. You can then edit the components of this default architecture as described in "Specify Control Architecture in Control System Tuner".

controlSystemTuner(CL) opens the app for tuning the control architecture specified in the genss model CL. If your control architecture does not match Control System Tuner's predefined control architecture, use this syntax with a genss model that has tunable components representing your controller elements. See "Specify Control Architecture in Control System Tuner".

controlSystemTuner(mdl) opens the app for tuning blocks in a Simulink model. mdl is the name of a Simulink model saved in the current working directory or on the MATLAB path. (Requires Simulink Control Design software.)

controlSystemTuner(ST) opens the app for tuning a Simulink model associated with an slTuner interface, ST. Control System Tuner takes information such as analysis points and operating points from ST. (Requires Simulink Control Design software.)

controlSystemTuner(sessionfile) opens the app and loads a previously saved session.

When you use Control System Tuner, you can click Save Session to save session data to disk such as tuning goals you have created, response I/Os you have defined, operating points, and stored designs. sessionfile is the name of a session data file saved in the current working directory or on the MATLAB path.

## **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

#### **Functions**

genss | slTuner | systune

### **Topics**

"Setup for Tuning Control System Modeled in MATLAB"
"Specify Control Architecture in Control System Tuner"
"Tune a Control System Using Control System Tuner"

#### Introduced in R2016a

## **Convert Model Rate**

Convert models between continuous time and discrete time and resample models in the Live Editor

## **Description**

**Convert Model Rate** lets you interactively convert an LTI model between continuous time and discrete time. You can also use it to resample a discrete-time model. The task automatically generates MATLAB code for your live script.

To get started with the **Convert Model Rate** task, select the model you want to convert. You can also specify the target sample time, conversion method, and other parameters. The task generates the converted model in the MATLAB workspace, and can generate a response plot to let you monitor the match between the original and converted models as you experiment with conversion parameters.

### **Related Functions**

**Convert Model Rate** generates code using the following functions.

- c2d
- d2c
- d2d

## **Description (collapsed portion)**

### **Related Functions**

**Convert Model Rate** generates code using the following functions.

- c2d
- d2c
- d2d

## Open the Task

To add the **Convert Model Rate** task to a live script in the MATLAB Editor:

- On the **Live Editor** tab, select **Task > Convert Model Rate**.
- In a code block in your script, type a relevant keyword, such as convert, rate, or c2d. Select Convert Model Rate from the suggested command completions.

## **Examples**

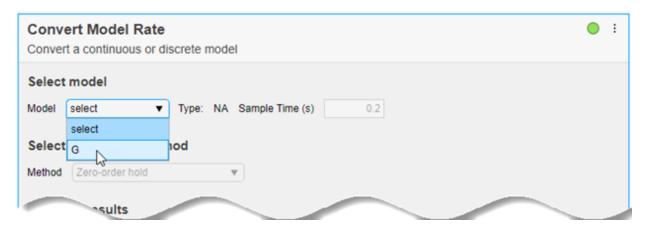
#### Discretize Model in the Live Editor

Use the **Convert Model Rate** task in the Live Editor to interactively convert a model from continuous time to discrete time. Experiment with different methods, options, and response plots. The task automatically generates code reflecting your selections. Open this example to see a preconfigured script containing the **Convert Model Rate** task.

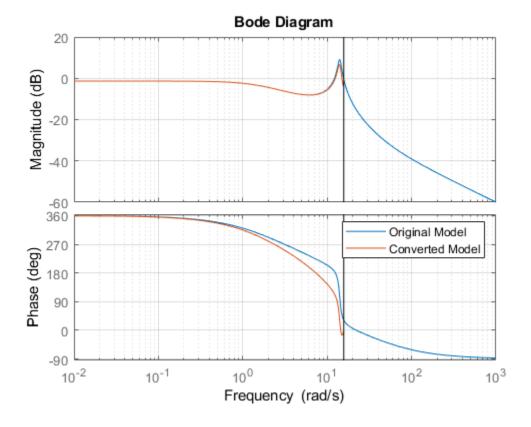
Create a continuous-time transfer-function model.

```
G = tf([1 -50 300], [1 3 200 350]);
```

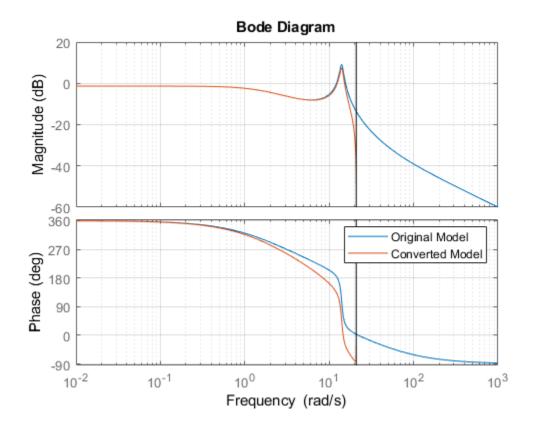
To discretize this model, open the **Convert Model Rate** task in the Live Editor. On the **Live Editor** tab, select **Task > Convert Model Rate**. In the task, select **G** as the model to convert.



The task automatically discretizes the model using the default sample time, 0.2 s, and the default conversion method, Zero-order hold. It also creates a Bode plot, allowing you to compare the responses of the original and converted models.

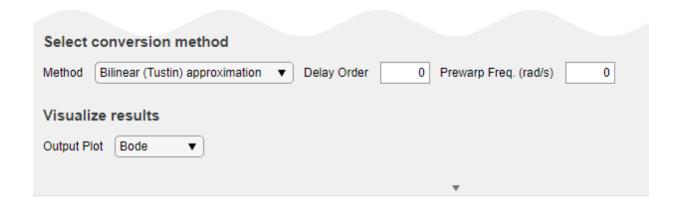


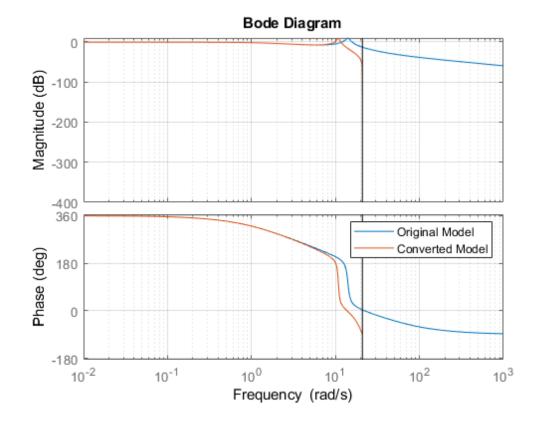
The vertical line on the plot shows the Nyquist frequency associated with the default sample time. Suppose you want to use a sample time of 0.15 seconds. Change the sample time by entering the new value in the **Sample Time** field. The response plot automatically updates to reflect the new sample time.



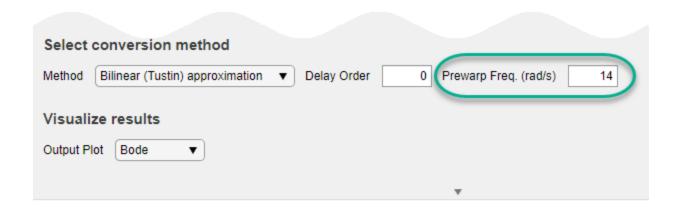
If the precise dynamics of the resonance are important for your application, you can improve the frequency-domain match using a different conversion method. In the task, try experimenting with different methods and observe their effect on the response plot.

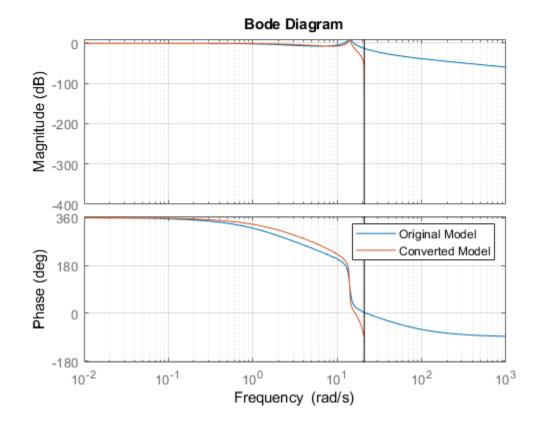
The Tustin method can yield a better match in the frequency domain than the default zero-order hold method. (See "Continuous-Discrete Conversion Methods".) In **Select Conversion Method**, select Bilinear (Tustin) approximation. Initially, the resulting frequency-domain match is poorer than with the zero-order hold method.





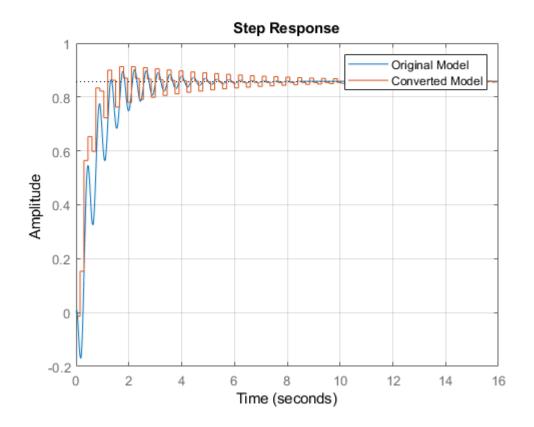
You can improve the match using a *prewarp frequency*. This option forces the discrete-time response to match at the frequency you specify. The resonance of G peaks at about 14 rad/s. Enter that value for the prewarp frequency. The match does improve around the resonance. However, the resonance is very close to the Nyquist frequency for the sample time of  $0.15 \, \text{s}$ , which limits how close the match can be.



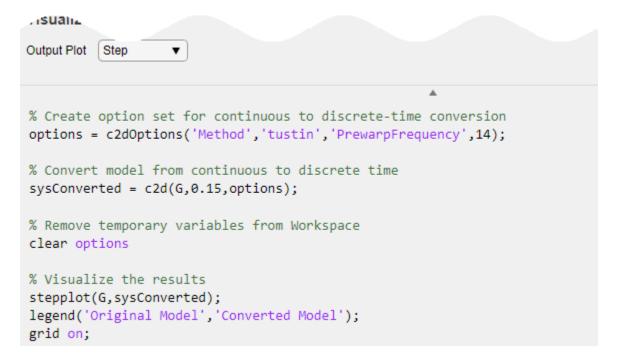


The **Convert Model Rate** task can generate other types of response plots. For instance, to compare the time-domain responses of the original and converted model, in **Output Plot**, select step or impulse.

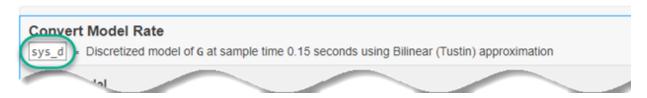




The task generates code in your live script. The generated code reflects the parameters and options you select, and includes code to generate the response plot you specify. To see the generated code, click at the bottom of the task parameter area. The task expands to display the generated code.

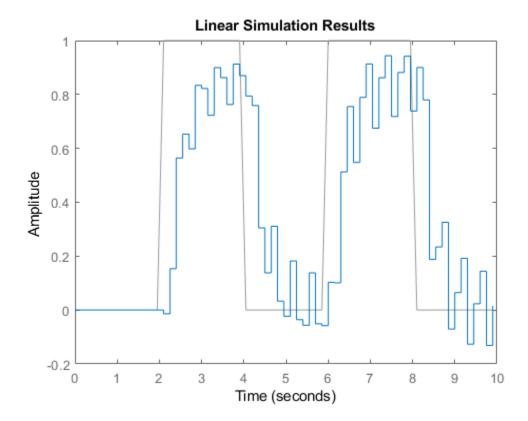


By default, the generated code uses sysConverted as the name of the output variable. The converted model in the MATLAB® workspace using this name. To specify a different output variable name, enter a new name in the summary line at the top of the task. For instance, change the name to sys\_d.



The task updates the generated code to reflect the new variable name, and the new converted model sys\_d appears in the MATLAB workspace. You can use the model for

further analysis or control design as you would any other model object. For instance, simulate the converted system's response to a square-wave input. Use the sample time you specified in the task.



## **Parameters**

Model — Model to convert

LTI model

Select an LTI model. The list contains all suitable continuous-time or discrete-time dynamic system models in the MATLAB workspace, including:

- Numeric LTI models such as tf, ss, or zpk models.
- Identified LTI models such as idss and idtf. (Using identified models requires a System Identification Toolbox license.)

You can convert SISO models or MIMO models, with or without time delays, although some conversion methods are only available for SISO models (see the **Method** parameter description). You cannot use **Convert Model Rate** to convert generalized LTI models such as genss or uss, frequency-response data models such as frd, or process models (idproc).

### Sample Time — Target sample time

0.2 (default) | 0 | positive scalar

Specify the sample time of the converted model in units of 1/TimeUnit, where TimeUnit is the TimeUnit property of the input model.

- To discretize a continuous-time model or resample a discrete-time model, enter the target sample time.
- To convert a discrete-time model to continuous time, enter 0.

#### Method — Rate conversion method

Zero-order hold (default) | First-order hold | Bilinear (Tustin) approximation | ...

Choose a rate conversion method. Available methods are:

- Zero-order hold
- First-order hold
- Impulse-invariant discretization (continuous-to-discrete conversion of SISO models only)
- Bilinear (Tustin) approximation
- Zero-pole matching method
- Least-squares method (continuous-to-discrete conversion of SISO models only)

For information about choosing a conversion method, see "Continuous-Discrete Conversion Methods".

### Delay Order — Approximate order for estimating time delay

0 (default) | positive integer

When you convert the rate of a model that has a time delay, the Bilinear (Tustin) approximation or Zero-pole matching method methods round the time delay to the nearest integer multiple of the sample time. This rounding can degrade the accuracy of dynamics, especially near the Nyquist frequency.

Set **Delay Order** to a nonzero integer to make **Convert Model Rate** approximate the fractional portion of the delay using a Thiran filter, instead of rounding. Use the smallest value that yields sufficiently accurate rate-converted dynamics for your application. For more information about Thiran filters, see thiran.

# Prewarp Freq. (rad/s) — Prewarp frequency for bilinear (Tustin) method 0 (default) | positive scalar

When you discretize a continuous-time model, if your system has important dynamics at a particular frequency that you want the rate conversion to preserve, you can use the Bilinear (Tustin) approximation method with frequency prewarping. This method ensures a match between the original and converted responses at the prewarp frequency you specify. See "Continuous-Discrete Conversion Methods".

### **Output Plot — Type of response plot**

Bode (default) | Step | Impulse | ...

**Convert Model Rate** automatically generates a response plot to help you check that the dynamics important to your application are preserved. Specify one of the following response plot types to compare the responses of the original and converted models while you experiment with conversion parameters.

- Bode
- Step
- Impulse
- Pole-Zero

The code generated by the task includes code to create the selected response plot. To omit the response plot, select None.

### See Also

c2d | d2c | d2d

**Topics**"Continuous-Discrete Conversion Methods"

Introduced in R2019b

### correct

Correct state and state estimation error covariance using extended or unscented Kalman filter, or particle filter and measurements

## **Syntax**

```
[CorrectedState, CorrectedStateCovariance] = correct(obj,y)
[CorrectedState, CorrectedStateCovariance] = correct(obj,y,
Um1,...,Umn)
```

## **Description**

The correct command updates the state and state estimation error covariance of an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object using measured system outputs. To implement extended or unscented Kalman filter, or particle filter, use the correct and predict commands together. If the current output measurement exists, you can use correct and predict. If the measurement is missing, you can only use predict. For information about the order in which to use the commands, see "Using predict and correct Commands" on page 2-191.

[CorrectedState,CorrectedStateCovariance] = correct(obj,y) corrects the state estimate and state estimation error covariance of an extended or unscented Kalman filter, or particle filter object obj using the measured output y.

You create obj using the extendedKalmanFilter, unscentedKalmanFilter or particleFilter commands. You specify the state transition function and measurement function of your nonlinear system in obj. You also specify whether the process and measurement noise terms are additive or nonadditive in these functions. The State property of the object stores the latest estimated state value. Assume that at time step k, obj.State is  $\widehat{x}[k|k-1]$ . This value is the state estimate for time k, estimated using measured outputs until time k-1. When you use the correct command with measured system output y[k], the software returns the corrected state estimate  $\widehat{x}[k|k]$  in the CorrectedState output. Where  $\widehat{x}[k|k]$  is the state estimate at time k, estimated using measured outputs until time k. The command returns the state estimation error covariance of  $\widehat{x}[k|k]$  in the CorrectedStateCovariance output. The software also

updates the State and StateCovariance properties of obj with these corrected values.

Use this syntax if the measurement function h that you specified in obj.MeasurementFcn has one of the following forms:

- y(k) = h(x(k)) for additive measurement noise.
- y(k) = h(x(k), v(k)) for nonadditive measurement noise.

Where y(k), x(k), and v(k) are the measured output, states, and measurement noise of the system at time step k. The only inputs to h are the states and measurement noise.

[CorrectedState, CorrectedStateCovariance] = correct(obj,y, Um1,...,Umn) specifies additional input arguments, if the measurement function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if the measurement function h has one of the following forms:

- y(k) = h(x(k), Um1, ..., Umn) for additive measurement noise.
- y(k) = h(x(k), v(k), Um1, ..., Umn) for nonadditive measurement noise.

correct command passes these inputs to the measurement function to calculate the estimated outputs.

## **Examples**

### **Estimate States Online Using Extended Kalman Filter**

Estimate the states of a van der Pol oscillator using an extended Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an extended Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with the nonlinearity parameter mu equal to 1. The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as [1;0]. This is the guess for the state value at initial time k, based on knowledge of system outputs until time k-1,  $\hat{\chi}[k \mid k-1]$ .

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
```

Load the measured output data y from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp data.mat file.

```
load vdp_data.mat y
```

Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the extended Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct  $\hat{\chi}[k|k-1]$  using measurements at time k to get  $\hat{\chi}[k|k]$ . Then, you predict the state value at the next time step  $\hat{\chi}[k+1|k]$  using  $\hat{\chi}[k|k]$ , the state estimate at time step k that is estimated using measurements until time k.

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual, ResidualCovariance] = residual(obj,y(k));
    [CorrectedState, CorrectedStateCovariance] = correct(obj,y(k));
    [PredictedState, PredictedStateCovariance] = predict(obj);

    residBuf(k,:) = Residual;
    xcorBuf(k,:) = CorrectedState';
    xpredBuf(k,:) = PredictedState';
end
```

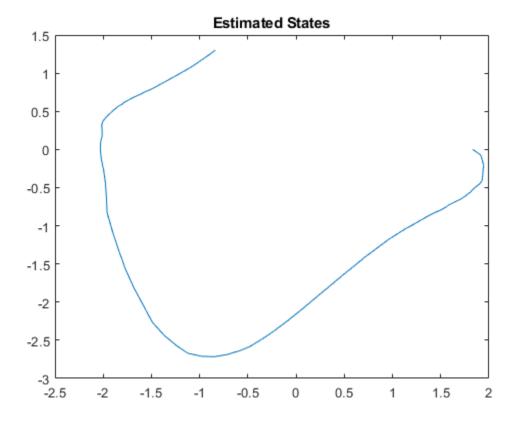
When you use the correct command, obj.State and obj.StateCovariance are updated with the corrected state and state estimation error covariance values for time

step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj.State and obj.StateCovariance are updated with the predicted values for time step k+1, PredictedState and PredictedStateCovariance. When you use the residual command, you do not modify any obj properties.

In this example, you used correct before predict because the initial state value was  $\hat{x}[k \mid k-1]$ , a guess for the state value at initial time k based on system outputs until time k-1. If your initial state value is  $\hat{x}[k-1 \mid k-1]$ , the value at previous time k-1 based on measurements until k-1, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 2-191.

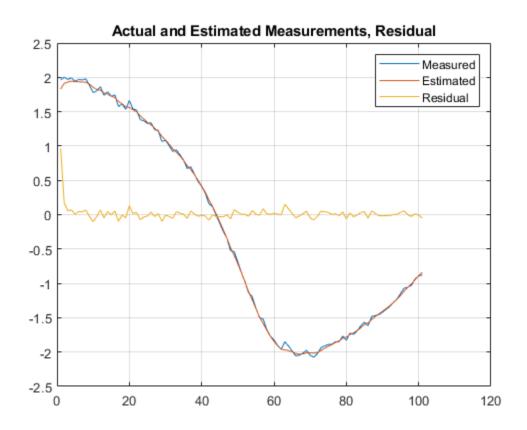
Plot the estimated states, using the postcorrection values.

```
plot(xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')
```



Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.

```
M = [y,xcorBuf(:,1),residBuf];
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')
```



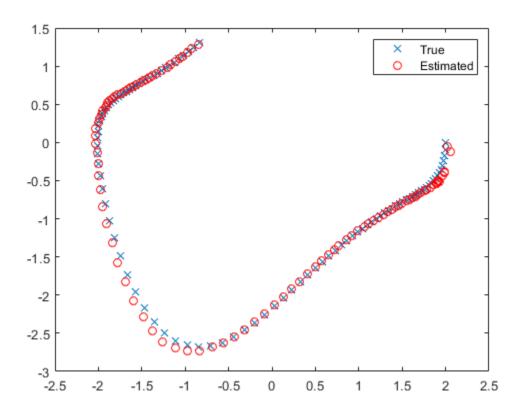
The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

### **Estimate States Online using Particle Filter**

Load the van der Pol ODE data, and specify the sample time.

vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time dt = 0.05.

```
addpath(fullfile(matlabroot, 'examples', 'control', 'main')) % add example data
load ('vdp0DEdata.mat','xTrue','dt')
tSpan = 0:dt:5;
Get the measurements. For this example, a sensor measures the first state with a
Gaussian noise with standard deviation 0.04.
sartR = 0.04:
yMeas = xTrue(:,1) + sqrtR*randn(numel(tSpan),1);
Create a particle filter, and set the state transition and measurement likelihood functions.
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.
initialize(myPF, 1000, [2;0], eye(2));
Pick the mean state estimation and systematic resampling methods.
myPF.StateEstimationMethod = 'mean';
myPF.ResamplingMethod = 'systematic';
Estimate the states using the correct and predict commands, and store the estimated
states.
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
Plot the results, and compare the estimated and true states.
figure(1)
plot(xTrue(:,1),xTrue(:,2),'x',xEst(:,1),xEst(:,2),'ro')
legend('True','Estimated')
```



rmpath(fullfile(matlabroot, 'examples', 'control', 'main')) % remove example data

### Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input u whose state x and measurement y evolve according to the following state transition and measurement equations:

$$x[k] = \sqrt{x[k-1] + u[k-1]} + w[k-1]$$
$$v[k] = x[k] + 2 * u[k] + v[k]^{2}$$

The process noise w of the system is additive while the measurement noise v is nonadditive.

Create the state transition function and measurement function for the system. Specify the functions with an additional input u.

```
f = @(x,u)(sqrt(x+u));

h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.

```
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
```

Specify the measurement noise covariance.

```
obj.MeasurementNoise = 0.01;
```

You can now estimate the state of the system using the predict and correct commands. You pass the values of u to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement y[k]=0.8 and input u[k]=0.2 at time step k.

```
correct(obj, 0.8, 0.2)
```

Predict the state at the next time step, given u[k]=0.2.

```
predict(obj,0.2)
```

Retrieve the error, or residual, between the prediction and the measurement.

```
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);
```

## **Input Arguments**

### obj — Extended or unscented Kalman filter, or particle filter object

extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter
object

Extended or unscented Kalman filter, or particle filter object for online state estimation, created using one of the following commands:

- extendedKalmanFilter Uses the extended Kalman filter algorithm.
- unscentedKalmanFilter Uses the unscented Kalman filter algorithm.
- particleFilter Uses the particle filter algorithm.

### y — Measured system output

vector

Measured system output at the current time step, specified as an N-element vector, where N is the number of measurements.

# Um1, . . . , Umn — Additional input arguments to measurement function input arguments of any type

Additional input arguments to the measurement function of the system, specified as input arguments of any type. The measurement function, h, is specified in the MeasurementFcn or MeasurementLikelihoodFcn property of obj. If the function requires input arguments in addition to the state and measurement noise values, you specify these inputs in the correct command syntax. correct command passes these inputs to the measurement or the measurement likelihood function to calculate estimated outputs. You can specify multiple arguments.

For example, suppose that your measurement or measurement likelihood function calculates the estimated system output y using system inputs u and current time k, in addition to the state x:

$$y(k) = h(x(k), u(k), k)$$

Then when you perform online state estimation at time step k, specify these additional inputs in the correct command syntax:

[CorrectedState, CorrectedStateCovariance] = correct(obj,y,u(k),k);

## **Output Arguments**

#### CorrectedState — Corrected state estimate

vector

Corrected state estimate, returned as a vector of size M, where M is the number of states of the system. If you specify the initial states of obj as a column vector then M is returned as a column vector, otherwise M is returned as a row vector.

For information about how to specify the initial states of the object, see the extendedKalmanFilter, unscentedKalmanFilter and particleFilter reference pages.

# **CorrectedStateCovariance** — **Corrected state estimation error covariance** matrix

Corrected state estimation error covariance, returned as an M-by-M matrix, where M is the number of states of the system.

### **More About**

## Using predict and correct Commands

After you have created an extended or unscented Kalman filter, or particle filter object, obj, to implement the estimation algorithms, use the correct and predict commands together.

At time step k, correct command returns the corrected value of states and state estimation error covariance using measured system outputs y[k] at the same time step. If your measurement function has additional input arguments  $U_m$ , you specify these as inputs to the correct command. The command passes these values to the measurement function.

[CorrectedState,CorrectedCovariance] = correct(obj,y,Um)

The correct command updates the State and StateCovariance properties of the object with the estimated values, CorrectedState and CorrectedCovariance.

The predict command returns the prediction of state and state estimation error covariance at the next time step. If your state transition function has additional input

arguments  $U_s$ , you specify these as inputs to the predict command. The command passes these values to the state transition function.

[PredictedState, PredictedCovariance] = predict(obj,Us)

The predict command updates the State and StateCovariance properties of the object with the predicted values, PredictedState and PredictedCovariance.

If the current output measurement exists at a given time step, you can use correct and predict. If the measurement is missing, you can only use predict. For details about how these commands implement the algorithms, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

The order in which you implement the commands depends on the availability of measured data y,  $U_s$ , and  $U_m$  for your system:

• correct then predict — Assume that at time step k, the value of obj. State is  $\widehat{x}[k|k-1]$ . This value is the state of the system at time k, estimated using measured outputs until time k-1. You also have the measured output y[k] and inputs U<sub>s</sub>[k] and U<sub>m</sub>[k] at the same time step.

Then you first execute the correct command with measured system data y[k] and additional inputs  $U_m[k]$ . The command updates the value of obj. State to be  $\widehat{x}[k|k]$ , the state estimate for time k, estimated using measured outputs up to time k. When you then execute the predict command with input  $U_s[k]$ , obj. State now stores  $\widehat{x}[k+1|k]$ . The algorithm uses this state value as an input to the correct command in the next time step.

• predict then correct — Assume that at time step k, the value of obj.State is  $\widehat{x}[k-1|k-1]$ . You also have the measured output y[k] and input  $U_m[k]$  at the same time step but you have  $U_s[k-1]$  from the previous time step.

Then you first execute the predict command with input  $U_s[k-1]$ . The command updates the value of obj.State to  $\widehat{x}[k|k-1]$ . When you then execute the correct command with input arguments y[k] and  $U_m[k]$ , obj.State is updated with  $\widehat{x}[k|k]$ . The algorithm uses this state value as an input to the predict command in the next time step.

Thus, while in both cases the state estimate for time k,  $\widehat{x}[k|k]$  is the same, if at time k you do not have access to the current state transition function inputs  $U_s[k]$ , and instead have  $U_s[k-1]$ , then use predict first and then correct.

For an example of estimating states using the predict and correct commands, see "Estimate States Online Using Extended Kalman Filter" on page 2-182 or "Estimate States Online using Particle Filter" on page 2-992.

### See Also

clone | extendedKalmanFilter | initialize | particleFilter | predict |
residual | unscentedKalmanFilter

### **Topics**

- "Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
- "Generate Code for Online State Estimation in MATLAB"
- "Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

### Introduced in R2016b

### covar

Output and state covariance of system driven by white noise

## **Syntax**

```
P = covar(sys,W)
[P,Q] = covar(sys,W)
```

## **Description**

**covar** calculates the stationary covariance of the output *y* of an LTI model **sys** driven by Gaussian white noise inputs *w*. This function handles both continuous- and discrete-time cases.

P = covar(sys, W) returns the steady-state output response covariance

$$P = E(yy^T)$$

given the noise intensity

$$E(w(t)w(\tau)^T) = W\delta(t - \tau)$$
 (continuous time)  
 $E(w[k]w[l]^T) = W\delta_{kl}$  (discrete time)

[P,Q] = covar(sys,W) also returns the steady-state state covariance

$$Q = E(xx^T)$$

when sys is a state-space model (otherwise Q is set to  $[\ ]$ ).

When applied to an N-dimensional LTI array sys, covar returns multidimensional arrays P, Q such that

P(:,:,i1,...iN) and Q(:,:,i1,...iN) are the covariance matrices for the model sys(:,:,i1,...iN).

## **Examples**

Compute the output response covariance of the discrete SISO system

$$H(z) = \frac{2z+1}{z^2+0.2z+0.5}, T_S = 0.1$$

due to Gaussian white noise of intensity W = 5. Type

sys = 
$$tf([2 1],[1 0.2 0.5],0.1);$$
  
p =  $covar(sys,5)$ 

These commands produce the following result.

You can compare this output of covar to simulation results.

```
randn('seed',0)
w = sqrt(5)*randn(1,1000); % 1000 samples
% Simulate response to w with LSIM:
y = lsim(sys,w);
% Compute covariance of y values
psim = sum(y .* y)/length(w);
This yields
psim =
    32.6269
```

The two covariance values p and psim do not agree perfectly due to the finite simulation horizon.

# **Algorithms**

Transfer functions and zero-pole-gain models are first converted to state space with ss.

For continuous-time state-space models

$$\dot{x} = Ax + Bw$$

$$y = Cx + Dw$$

the steady-state state covariance Q is obtained by solving the Lyapunov equation

$$AQ + QA^T + BWB^T = 0.$$

In discrete time, the state covariance Q solves the discrete Lyapunov equation

$$AQA^T - Q + BWB^T = 0.$$

In both continuous and discrete time, the output response covariance is given by  $P = CQC^T + DWD^T$ . For unstable systems, P and Q are infinite. For continuous-time systems with nonzero feedthrough, covar returns Inf for the output covariance P.

#### References

[1] Bryson, A.E. and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publishing, 1975, pp. 458-459.

#### See Also

dlyap | lyap

Introduced before R2006a

# ctranspose, '

Conjugate dynamic system model

# **Syntax**

```
H = ctranspose(G)
```

# **Description**

H = ctranspose(G) computes the conjugate of the dynamic system model or static model G. The ctranpose command is equivalent to the 'operator.

If G is a dynamic system model poles P and zeros Z, then the conjugate system H = ctranspose(G) = G' has the following properties.

- In continuous time, the poles of H are -P and the zeros of H are -Z. The frequency response of H is the Hermitian transpose of the frequency response of G:  $H(j\omega) = G(j\omega)^{H}.$
- In discrete time with sample time T<sub>s</sub>, the poles of H are 1/P and the zeros of H are 1/Z.
   The frequency response of H is the Hermitian transpose of the frequency response of G:

```
H(z) = G(z)^{H},
where z = e^{i\omega T_s}.
```

If G is a static model, then H is the complex-conjugate transpose of the matrix.

# **Examples**

#### **Conjugate of Dynamic System Model**

Compute the conjugate of a one-input, two-output transfer function model.

```
s = tf('s');

G1 = (s + 1)/(s^2 + 2*s + 1);
```

Continuous-time transfer function.

The conjugate has two outputs and one input, and takes s to -s.

```
H = ctranspose(G)
```

Continuous-time transfer function.

Using the 'operator yields the same result as ctranspose.

```
From input 2 to output:
-1
--
s
```

Continuous-time transfer function.

#### **Hermitian Transpose of Frequency Response**

Obtain the frequency response and the Hermitian transpose of the frequency response of a transfer function at one frequency.

```
G = [tf(1,[1 0]);tf([1 0],[1 1])];
w = 3;
resp = freqresp(G,w)

resp = 2×1 complex
    0.0000 - 0.3333i
    0.9000 + 0.3000i
```

The Hermitian transpose of the frequency response is the frequency response of the conjugate of G.

```
respH = freqresp(G',w)
respH = 1×2 complex
0.0000 + 0.3333i  0.9000 - 0.3000i
```

# **Input Arguments**

#### G — Input system

dynamic system model | static model

Input system, specified as a dynamic system model or static model such as a tf, zpk, or ss model. G can be a generalized model provided that its control design blocks are scalar and nondynamic, such as a scalar ureal parameter.

# **Output Arguments**

#### **H** — Conjugate model

dynamic system model | static model

Conjugate model of G, returned as a dynamic system model or static model of the same type as G.

# **See Also**

inv

#### **Topics**

"Catalog of Model Interconnections" "Types of Model Objects"

Introduced before R2006a

#### ctrb

Controllability matrix

## **Syntax**

```
Co = ctrb(A,B)
Co = ctrb(sys)
```

# **Description**

Co = ctrb(A,B) returns the controllability matrix:

$$Co = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

where A is an n-by-n matrix, B is an n-by-m matrix, and Co has n rows and nm columns.

Co = ctrb(sys) calculates the controllability matrix of the state-space LTI object sys. This syntax is equivalent to:

```
Co = ctrb(sys.A, sys.B);
```

The system is controllable if **Co** has full rank *n*.

# **Examples**

#### **Check System Controllability**

Define A and B matrices.

$$A = \begin{bmatrix} 1 & 1; \\ & 4 & -2 \end{bmatrix};$$
$$B = \begin{bmatrix} 1 & -1; \\ & 1 & -1 \end{bmatrix};$$

Compute controllability matrix.

```
Co = ctrb(A,B);
```

Determine the number of uncontrollable states.

```
unco = length(A) - rank(Co)
unco = 1
```

The uncontrollable state indicates that Co does not have full rank 2. Therefore the system is not controllable.

#### Limitations

Estimating the rank of the controllability matrix is ill-conditioned; that is, it is very sensitive to roundoff errors and errors in the data. An indication of this can be seen from this simple example.

$$A = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ \delta \end{bmatrix}$$

This pair is controllable if  $\delta \neq 0$  but if  $\delta < \sqrt{eps}$ , where eps is the relative machine precision. ctrb(A,B) returns

$$[B \ AB] = \begin{bmatrix} 1 & 1 \\ \delta & \delta \end{bmatrix}$$

which is not full rank. For cases like these, it is better to determine the controllability of a system using ctrbf.

#### **See Also**

ctrbf|obsv

Introduced before R2006a

#### ctrbf

Compute controllability staircase form

## **Syntax**

[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C) ctrbf(A,B,C,tol)

# **Description**

If the controllability matrix of (A, B) has rank  $r \le n$ , where n is the size of A, then there exists a similarity transformation such that

$$\bar{A} = TAT^T$$
,  $\bar{B} = TB$ ,  $\bar{C} = CT^T$ 

where *T* is unitary, and the transformed system has a *staircase* form, in which the uncontrollable modes, if there are any, are in the upper left corner.

$$\bar{A} = \begin{bmatrix} A_{uc} & 0 \\ A_{21} & A_c \end{bmatrix}, \ \bar{B} = \begin{bmatrix} 0 \\ B_c \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C_{nc}C_c \end{bmatrix}$$

where  $(A_c, B_c)$  is controllable, all eigenvalues of  $A_{uc}$  are uncontrollable, and  $C_c(sI - A_c)^{-1}B_c = C(sI - A)^{-1}B$ .

[Abar, Bbar, Cbar, T, k] = ctrbf(A,B,C) decomposes the state-space system represented by A, B, and C into the controllability staircase form, Abar, Bbar, and Cbar, described above. T is the similarity transformation matrix and k is a vector of length n, where n is the order of the system represented by A. Each entry of k represents the number of controllable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in k indicates how many iterations were necessary to calculate T, and sum(k) is the number of states in  $A_c$ , the controllable portion of Abar.

ctrbf(A,B,C,tol) uses the tolerance tol when calculating the controllable/uncontrollable subspaces. When the tolerance is not specified, it defaults to 10\*n\*norm(A,1)\*eps.

# **Examples**

Compute the controllability staircase form for

and locate the uncontrollable mode.

```
[Abar,Bbar,Cbar,T,k]=ctrbf(A,B,C)
Abar =
   -3.0000
                    0
   -3.0000
               2.0000
Bbar =
    0.0000
               0.0000
    1.4142
              -1.4142
Cbar =
   -0.7071
               0.7071
    0.7071
               0.7071
T =
   -0.7071
               0.7071
    0.7071
               0.7071
k =
     1
           0
```

The decomposed system Abar shows an uncontrollable mode located at -3 and a controllable mode located at 2.

# **Algorithms**

ctrbf implements the Staircase Algorithm of [1].

### **References**

[1] Rosenbrock, M.M., State-Space and Multivariable Theory, John Wiley, 1970.

#### See Also

ctrb|minreal

**Introduced before R2006a** 

# ctrlpref

Set Control System Toolbox preferences

## **Syntax**

ctrlpref

# **Description**

ctrlpref opens a Graphical User Interface (GUI) which allows you to change the Control System Toolbox preferences. Preferences set in this GUI affect future plots only (existing plots are not altered).

Your preferences are stored to disk (in a system-dependent location) and will be automatically reloaded in future MATLAB sessions using the Control System Toolbox software.

#### See Also

**Control System Designer | Linear System Analyzer** 

Introduced in R2006a

#### d2c

Convert model from discrete to continuous time

## **Syntax**

```
sysc = d2c(sysd)
sysc = d2c(sysd,method)
sysc = d2c(sysd,opts)
[sysc,G] = d2c(____)
```

# **Description**

sysc = d2c(sysd) converts a the discrete-time dynamic system model sysd to a continuous-time model using zero-order hold on the inputs.

```
sysc = d2c(sysd, method) specifies the conversion method.
```

sysc = d2c(sysd, opts) specifies conversion options for the discretization.

[sysc,G] =  $d2c(\underline{\phantom{a}}$ ), where sysd is a state-space model, returns a matrix G that maps the states xd[k] of the discrete-time state-space model to the states xc(t) of sysc.

## **Examples**

#### **Convert Discrete-Time Transfer Function to Continuous Time**

Create the following discrete-time transfer function:

$$H(z) = \frac{z - 1}{z^2 + z + 0.3}$$

$$H = tf([1 -1], [1 1 0.3], 0.1);$$

The sample time of the model is  $T_s = 0.1s$ .

Derive a continuous-time, zero-order-hold equivalent model.

Discretize the resulting model, Hc, with the default zero-order hold method and sample time 0.1s to return the original discrete model, H.

Use the Tustin approximation method to convert H to a continuous time model.

```
Hc2 = d2c(H, 'tustin');
```

Discretize the resulting model, Hc2, to get back the original discrete-time model, H.

```
c2d(Hc2,0.1, 'tustin');
```

#### Convert Identified Discrete-Time Transfer Function to Continuous Time

Estimate a discrete-time transfer function model, and convert it to a continuous-time model.

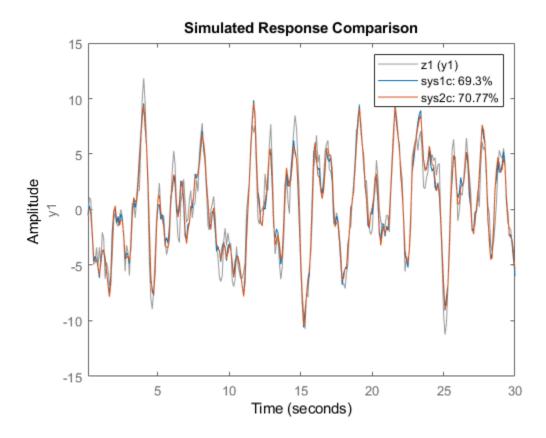
```
load iddata1
sys1d = tfest(z1,2,'Ts',0.1);
sys1c = d2c(sys1d,'zoh');
```

Estimate a continuous-time transfer function model.

```
sys2c = tfest(z1,2);
```

Compare the response of sys1c and the directly estimated continuous-time model, sys2c.

compare(z1,sys1c,sys2c)



The two systems are almost identical.

#### Regenerate Covariance Information After Converting to Continuous-Time Model

Convert an identified discrete-time transfer function model to continuous-time.

```
load iddata1
sysd = tfest(z1,2,'Ts',0.1);
sysc = d2c(sysd,'zoh');
```

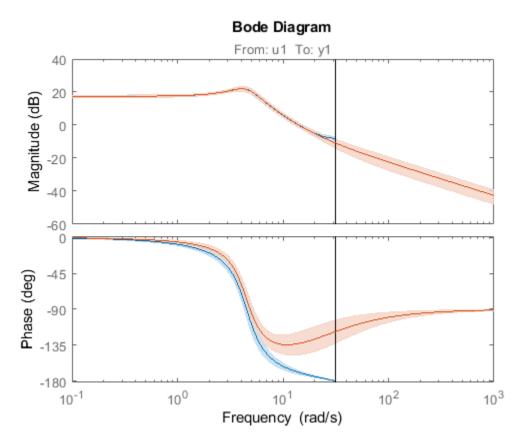
sys1c has no covariance information. The d2c operation leads to loss of covariance data of identified models.

Regenerate the covariance information using a zero iteration update with the same estimation command and estimation data.

```
opt = tfestOptions;
opt.SearchOptions.MaxIterations = 0;
sys1c = tfest(z1,sysc,opt);
```

Analyze the effect on frequency-response uncertainty.

```
h = bodeplot(sysd,sys1c);
showConfidence(h,3)
```



The uncertainties of sys1c and sysd are comparable up to the Nyquist frequency. However, sys1c exhibits large uncertainty in the frequency range for which the estimation data does not provide any information.

If you do not have access to the estimation data, use the translatecov command which is a Gauss-approximation formula based translation of covariance across model type conversion operations.

## **Input Arguments**

#### sysd — Discrete-time dynamic system

dynamic system model

Discrete-time model, specified as a dynamic system model such as tf, ss, or zpk.

You cannot directly use an idgrey model whose FunctionType is 'd' with d2c. Convert the model into idss form first.

#### method — Discrete-to-continuous time conversion method

```
'zoh' (default) | 'foh' | 'tustin' | 'matched'
```

Discrete-to-continuous time conversion method, specified as one of the following values:

- 'zoh' Zero-order hold on the inputs. Assumes that the control inputs are piecewise constant over the sampling period.
- 'foh' Linear interpolation of the inputs (modified first-order hold). Assumes that the control inputs are piecewise linear over the sampling period.
- 'tustin' Bilinear (Tustin) approximation to the derivative. To specify this method with frequency prewarping (formerly known as the 'prewarp' method), use the PrewarpFrequency option of d2cOptions.
- 'matched ' Zero-pole matching method (for SISO systems only). See [1].

For information about the algorithms for each d2c conversion method, see "Continuous-Discrete Conversion Methods".

#### opts — Discrete-to-continuous time conversion options

d2c0ptions object

Discrete-to-continuous time conversion options, created using d2cOptions. For example, specify the prewarp frequency or the conversion method as an option.

## **Output Arguments**

#### sysc — Continuous-time model

dynamic system model

Continuous-time model, returned as a dynamic system model of the same type as the input system sysd.

When sysd is an identified (IDLTI) model, sysc:

- Includes both the measured and noise components of sysd. If the noise variance is  $\lambda$  in sysd, then the continuous-time model sysc has an indicated level of noise spectral density equal to  $Ts*\lambda$ .
- Does not include the estimated parameter covariance of sysd. If you want to translate the covariance while converting the model, use translatecov.

# **G** — Mapping of discrete-time states of state-space model to continuous-time states

matrix

Mapping of the states xd[k] of the state-space model sysd to the states xc(t) of sysc, returned as a matrix. The mapping of the states is as follows:

$$x_C(kT_s) = G \begin{bmatrix} x_d[k] \\ u[k] \end{bmatrix}.$$

Given an initial condition x0 for sysd and an initial input u0 = u[0], the corresponding initial condition for sysc (assuming u[k] = 0 for k < 0 is:

$$x_c(0) = G \begin{bmatrix} x_0 \\ u_0 \end{bmatrix}.$$

#### References

- [1] Franklin, G.F., Powell, D.J., and Workman, M.L., *Digital Control of Dynamic Systems* (3rd Edition), Prentice Hall, 1997.
- [2] Kollár, I., G.F. Franklin, and R. Pintelon, "On the Equivalence of z-domain and s-domain Models in System Identification," *Proceedings of the IEEE Instrumentation and Measurement Technology Conference*, Brussels, Belgium, June, 1996, Vol. 1, pp. 14-19.

#### See Also

Convert Model Rate | c2d | d2cOptions | logm | translatecov

### **Topics**

"Dynamic System Models"
"Convert Discrete-Time System to Continuous Time"
"Continuous-Discrete Conversion Methods"

#### **Introduced before R2006a**

# d2cOptions

Create option set for discrete- to continuous-time conversions

# **Syntax**

```
opts = d2cOptions
opts = d2cOptions(Name, Value)
```

# **Description**

opts = d2cOptions returns the default options for d2c.

opts = d2cOptions(Name, Value) creates an option set with the options specified by
one or more Name, Value pair arguments.

## **Input Arguments**

#### **Name-Value Pair Arguments**

#### method

Discretization method, specified as one of the following values:

'zoh'	Zero-order hold, where d2c assumes that the control inputs are piecewise constant over the sample time Ts.
'foh'	Linear interpolation of the inputs (modified first-order hold). Assumes that the control inputs are piecewise linear over the sampling period.
'tustin'	Bilinear (Tustin) approximation. By default, d2c converts with no prewarp. To include prewarp, use the PrewarpFrequency option.
'matched'	Zero-pole matching method. (See [1] on page 2-217, p. 224.)

For information about the algorithms for each d2c conversion method, see "Continuous-Discrete Conversion Methods".

Default: 'zoh'

#### PrewarpFrequency

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the discrete-time system. Specify the prewarp frequency as a positive scalar value. A value of 0 corresponds to the 'tustin' method without prewarp.

Default: 0

## **Output Arguments**

opts - Option set for d2c

d2c0ptions option set

Option set for d2c, returned as an d2cOptions option set.

## **Examples**

#### **Specify Model Discretization Method**

Consider the following discrete-time transfer function.

$$H(z) = \frac{z+1}{z^2+z+1}$$

Create the discrete-time transfer function with a sample time of 0.1 seconds.

$$Hd = tf([1 1],[1 1 1],0.1);$$

Specify the discretization method as bilinear (Tustin) approximation and the prewarp frequency as 20 rad/seconds.

```
opts = d2cOptions('Method','tustin','PrewarpFrequency',20);
```

Convert the discrete-time model to continuous-time using the specified discretization method.

```
Hc = d2c(Hd, opts);
```

You can use the discretization option set opts to discretize additional models using the same options.

#### References

[1] Franklin, G.F., Powell, D.J., and Workman, M.L., *Digital Control of Dynamic Systems* (3rd Edition), Prentice Hall, 1997.

#### See Also

d2c

Introduced in R2010a

### d2d

Resample discrete-time model

## **Syntax**

```
sys1 = d2d(sys, Ts)
sys1 = d2d(sys, Ts, 'method')
sys1 = d2d(sys, Ts, opts)
```

# **Description**

sys1 = d2d(sys, Ts) resamples the discrete-time dynamic system model sys to produce an equivalent discrete-time model sys1 with the new sample time Ts (in seconds), using zero-order hold on the inputs.

sys1 = d2d(sys, Ts, 'method') uses the specified resampling method 'method':

- 'zoh' Zero-order hold on the inputs
- 'tustin' Bilinear (Tustin) approximation

For information about the algorithms for each d2d conversion method, see "Continuous-Discrete Conversion Methods".

sys1 = d2d(sys, Ts, opts) resamples sys using the option set with d2d0ptions.

# **Examples**

#### **Resample a Discrete-Time Model**

Create the following zero-pole-gain-model with sample time 0.1 seconds.

$$H(z) = \frac{z - 0.7}{z - 0.5}$$

```
H = zpk(0.7,0.5,1,0.1);
Resample the model at 0.05 s.

H2 = d2d(H,0.05)

H2 =
    (z-0.8243)
    (z-0.7071)

Sample time: 0.05 seconds
Discrete-time zero/pole/gain model.

Resample H2 at 0.1 seconds to obtain the original model H.

H3 = d2d(H2,0.1)

H3 =
    (z-0.7)
    (z-0.5)

Sample time: 0.1 seconds
Discrete-time zero/pole/gain model.
```

#### Resample an Identified Discrete-Time Model

Suppose that you estimate a discrete-time output-error polynomial model with sample time commensurate with the estimation data (0.1 seconds). However, your deployment application requires a faster sampling frequency (0.01 seconds). You can use d2d to resample your estimated model.

Load the estimation data.

```
load iddata1 z1
z1.Ts

ans = 0.1000
```

z1 is an iddata object containing the estimation input-output data with sample time 0.1 seconds.

Estimate an output-error polynomial model of order [2 2 1].

```
sys = oe(z1,[2 2 1]);
sys.Ts
ans = 0.1000
```

Resample the model at sample time 0.01 seconds.

```
sys2 = d2d(sys,0.01);
sys2.Ts
ans = 0.0100
```

d2d resamples the model using the zero-order hold method.

## **Tips**

- Use the syntax sys1 = d2d(sys,Ts,'method') to resample sys using the default options for 'method'. To specify tustin resampling with a frequency prewarp, use the syntax sys1 = d2d(sys,Ts,opts). For more information, see d2dOptions.
- When sys is an identified (IDLTI) model, sys1 does not include the estimated
  parameter covariance of sys. If you want to translate the covariance while converting
  the model, use translatecov.

#### **See Also**

 $\verb|c2d|| | d2c|| | d2d0ptions|| | translatecov|| | upsample||$ 

Introduced before R2006a

# d2dOptions

Create option set for discrete-time resampling

## **Syntax**

```
opts = d2dOptions
opts = d2dOptions('OptionName', OptionValue)
```

## **Description**

opts = d2d0ptions returns the default options for d2d.

opts = d2dOptions('OptionName', OptionValue) accepts one or more commaseparated name-value pairs that specify options for the d2d command. Specify OptionName inside single quotes.

This table summarizes the options that the d2d command supports.

# **Input Arguments**

#### **Name-Value Pair Arguments**

#### Method

Discretization method, specified as one of the following values:

'zoh' Zero-order hold, where d2d assumes that the control inputs are

piecewise constant over the sample time Ts.

'tustin' Bilinear (Tustin) approximation. By default, d2d resamples with no

prewarp. To include prewarp, use the PrewarpFrequency option.

For information about the algorithms for each d2d conversion method, see "Continuous-Discrete Conversion Methods".

Default: 'zoh'

#### PrewarpFrequency

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the resampled system. Takes positive scalar values. The prewarp frequency must be smaller than the Nyquist frequency before and after resampling. A value of 0 corresponds to the standard 'tustin' method without prewarp.

**Default:** 0

## **Examples**

#### Specify Method for Resampling a Discrete-Time Model

Create the following discrete-time transfer function with sample time 0.1 seconds.

$$H(z) = \frac{z+1}{z^2+z+1}$$

$$h1 = tf([1 1],[1 1 1],0.1);$$

Specify the discretization method as bilinear Tustin method with a prewarping frequency of 20 rad/seconds.

```
opts = d2d0ptions('Method','tustin','PrewarpFrequency',20);
```

Resample the discrete-time model using the specified options.

$$h2 = d2d(h1, 0.05, opts);$$

You can use the option set opts to resample additional models using the same options.

#### See Also

d2d

#### Introduced in R2010a

# damp

Natural frequency and damping ratio

## **Syntax**

```
damp(sys)
[wn,zeta] = damp(sys)
[wn,zeta,p] = damp(sys)
```

# **Description**

damp(sys) displays the damping ratio, natural frequency, and time constant of the poles of the linear model sys. For a discrete-time model, the table also includes the magnitude of each pole. The poles are sorted in increasing order of frequency values.

[wn,zeta] = damp(sys) returns the natural frequencies wn, and damping ratios zeta
of the poles of sys.

[wn, zeta, p] = damp(sys) also returns the poles p of sys.

# **Examples**

#### Display Natural Frequency, Damping Ratio, and Poles of Continuous-Time System

For this example, consider the following continuous-time transfer function:

$$sys(s) = \frac{2s^2 + 5s + 1}{s^3 + 2s - 3}.$$

Create the continuous-time transfer function.

```
sys = tf([2,5,1],[1,0,2,-3]);
```

Display the natural frequencies, damping ratios, time constants, and poles of sys. damp(sys)

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
1.00e+00	-1.00e+00	1.00e+00	-1.00e+00
-5.00e-01 + 1.66e+00i	2.89e-01	1.73e+00	2.00e+00
-5.00e-01 - 1.66e+00i	2.89e-01	1.73e+00	2.00e+00

The poles of sys contain an unstable pole and a pair of complex conjugates that lie int he left-half of the s-plane. The corresponding damping ratio for the unstable pole is -1, which is called a driving force instead of a damping force since it increases the oscillations of the system, driving the system to instability.

#### Display Natural Frequency, Damping Ratio, and Poles of Discrete-Time System

For this example, consider the following discrete-time transfer function with a sample time of 0.01 seconds:

$$sys(z) = \frac{5z^2 + 3z + 1}{z^3 + 6z^2 + 4z + 4}.$$

Create the discrete-time transfer function.

Display information about the poles of sys using the damp command.

damp(sys)

Pole	Magnitude	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-3.02e-01 + 8.06e-01i	8.61e-01	7.74e-02	1.93e+02	6.68e-02
-3.02e-01 - 8.06e-01i	8.61e-01	7.74e-02	1.93e+02	6.68e-02
-5.40e+00	5.40e+00	-4.73e-01	3.57e+02	-5.93e-03

The Magnitude column displays the discrete-time pole magnitudes. The Damping, Frequency, and Time Constant columns display values calculated using the equivalent continuous-time poles.

#### Natural Frequency and Damping Ratio of Zero-Pole-Gain Model

For this example, create a discrete-time zero-pole-gain model with two outputs and one input. Use sample time of 0.1 seconds.

Compute the natural frequency and damping ratio of the zero-pole-gain model  ${\tt sys.}$ 

```
[wn,zeta] = damp(sys)

wn = 3 \times 1

12.0397

14.7114

14.7114
```

Discrete-time zero/pole/gain model.

```
zeta = 3×1
1.0000
-0.0034
-0.0034
```

Each entry in wn and zeta corresponds to combined number of I/Os in sys. zeta is ordered in increasing order of natural frequency values in wn.

#### Compute Natural Frequency, Damping Ratio and Poles of a State-Space Model

For this example, compute the natural frequencies, damping ratio and poles of the following state-space model:

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Create the state-space model using the state-space matrices.

```
A = [-2 -1;1 -2];
B = [1 1;2 -1];
C = [1 0];
D = [0 1];
sys = ss(A,B,C,D);
```

Use damp to compute the natural frequencies, damping ratio and poles of sys.

```
[wn, zeta, p] = damp(sys)
wn = 2 \times 1
2.2361
2.2361
2.2361
2.2361
2.2361
```

```
p = 2×1 complex

-2.0000 + 1.0000i

-2.0000 - 1.0000i
```

The poles of sys are complex conjugates lying in the left half of the s-plane. The corresponding damping ratio is less than 1. Hence, sys is an underdamped system.

## **Input Arguments**

#### sys — Linear dynamic system

dynamic system model

Linear dynamic system, specified as a SISO, or MIMO dynamic system model. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

damp assumes

- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.

# **Output Arguments**

#### wn — Natural frequency of each pole

vector

Natural frequency of each pole of sys, returned as a vector sorted in ascending order of frequency values. Frequencies are expressed in units of the reciprocal of the TimeUnit property of sys.

If sys is a discrete-time model with specified sample time, wn contains the natural frequencies of the equivalent continuous-time poles. If the sample time is not specified, then damp assumes a sample time value of 1 and calculates wn accordingly. For more information, see "Algorithms" on page 2-229.

#### zeta — Damping ratio of each pole

vector

Damping ratios of each pole, returned as a vector sorted in the same order as wn.

If sys is a discrete-time model with specified sample time, zeta contains the damping ratios of the equivalent continuous-time poles. If the sample time is not specified, then damp assumes a sample time value of 1 and calculates zeta accordingly. For more information, see "Algorithms" on page 2-229.

#### p — Poles of the dynamic system model

vector

Poles of the dynamic system model, returned as a vector sorted in the same order as wn. p is the same as the output of pole(sys), except for the order. For more information on poles, see pole.

# **Algorithms**

damp computes the natural frequency, time constant, and damping ratio of the system poles as defined in the following table:

	Continuous Time	Discrete Time with Sample Time Ts	
Pole Location	S	Z	
Equivalent Continuous- Time Pole	Not applicable	$s = \frac{\ln(z)}{T_s}$	
Natural Frequency	$\omega_n =  s $	$\omega_n =  s  = \left  \frac{\ln(z)}{T_s} \right $	
Damping Ratio	$\zeta = -\cos(\angle s)$	$\zeta = -\cos(\angle s) = -\cos(\angle lt)$	n(z)
Time Constant	$\tau = \frac{1}{\omega_n \zeta}$	$\tau = \frac{1}{\omega_n \zeta}$	

If the sample time is not specified, then damp assumes a sample time value of 1 and calculates zeta accordingly.

# **See Also**

dsort | eig | esort | pole | pzmap | zero

**Introduced before R2006a** 

#### dare

(Not recommended) Solve discrete-time algebraic Riccati equations (DAREs)

**Note** dare is not recommended. Use idare instead. For more information, see "Compatibility Considerations".

## **Syntax**

```
[X,L,G] = dare(A,B,Q,R)
[X,L,G] = dare(A,B,Q,R,S,E)
[X,L,G,report] = dare(A,B,Q,...)
[X1,X2,L,report] = dare(A,B,Q,..., 'factor')
```

## Description

[X,L,G] = dare(A,B,Q,R) computes the unique stabilizing solution X of the discrete-time algebraic Riccati equation

$$A^{T}XA - X - A^{T}XB(B^{T}XB + R)^{-1}B^{T}XA + Q = 0$$

The dare function also returns the gain matrix,  $G = (B^T X B + R)^{-1} B^T X A$ , and the vector L of closed loop eigenvalues, where

```
L=eig(A-B*G,E)
```

[X,L,G] = dare(A,B,Q,R,S,E) solves the more general discrete-time algebraic Riccati equation,

$$A^{T}XA - E^{T}XE - (A^{T}XB + S)(B^{T}XB + R)^{-1}(B^{T}XA + S^{T}) + Q = 0$$

or, equivalently, if R is nonsingular,

$$E^{T}XE = F^{T}XF - F^{T}XB(B^{T}XB + R)^{-1}B^{T}XF + Q - SR^{-1}S^{T}$$

where  $F = A - BR^{-1}S^{T}$ . When omitted, R, S, and E are set to the default values R=I, S=0, and E=I.

The dare function returns the corresponding gain matrix  $G = (B^T X B + R)^{-1} (B^T X A + S^T)$ 

and a vector L of closed-loop eigenvalues, where

$$L= eig(A-B*G,E)$$

[X,L,G,report] = dare(A,B,Q,...) returns a diagnosis report with value:

- -1 when the associated symplectic pencil has eigenvalues on or very near the unit circle
- -2 when there is no finite stabilizing solution X
- · The Frobenius norm if X exists and is finite

[X1,X2,L,report] = dare(A,B,Q,...,'factor') returns two matrices, X1 and X2, and a diagonal scaling matrix D such that X = D\*(X2/X1)\*D. The vector L contains the closed-loop eigenvalues. All outputs are empty when the associated Symplectic matrix has eigenvalues on the unit circle.

#### **Limitations**

The (A, B) pair must be stabilizable (that is, all eigenvalues of A outside the unit disk must be controllable). In addition, the associated symplectic pencil must have no eigenvalue on the unit circle. Sufficient conditions for this to hold are (Q, A) detectable when S = 0 and R > 0, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

## **Algorithms**

dare implements the algorithms described in [1]. It uses the QZ algorithm to deflate the extended symplectic pencil and compute its stable invariant subspace.

## **Compatibility Considerations**

#### dare not recommended

Not recommended starting in R2019a

Starting in R2019a, use the idare command to solve discrete-time Riccati equations. This approach has improved accuracy through better scaling and the computation of K is more accurate when R is ill-conditioned relative to dare. Furthermore, idare includes an optional info structure to gather the implicit solution data of the Riccati equation.

The following table shows some typical uses of dare and how to update your code to use idare instead.

Not Recommended	Recommended
[X,L,G] = dare(A,B,Q,R,S,E)	[X,K,L] = idare(A,B,Q,R,S,E) computes the stabilizing solution X, the state-feedback gain K and the closed-loop eigenvalues L of the discrete-time algebraic Riccati equation. For more information, see idare.
<pre>[X,L,G,report] = dare(A,B,Q,R,S,E)</pre>	[X,K,L,info] = idare(A,B,Q,R,S,E) computes the stabilizing solution X, the state-feedback gain K, the closed-loop eigenvalues L of the discrete-time algebraic Riccati equation. The info structure contains the implicit solution data. For more information, see idare.

There are no plans to remove dare at this time.

#### References

[1] Arnold, W.F., III and A.J. Laub, "Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations," *Proc. IEEE*, 72 (1984), pp. 1746-1754.

# **See Also**

idare

# db2mag

Convert decibels (dB) to magnitude

# **Syntax**

y = db2mag(ydb)

# **Description**

y = db2mag(ydb) returns the corresponding magnitude y for a given decibel (dB) value ydb. The relationship between magnitude and decibels is  $ydb = 20 * log_{10}(y)$ .

#### **See Also**

mag2db

Introduced in R2008a

## dcgain

Low-frequency (DC) gain of LTI system

#### **Syntax**

k = dcgain(sys)

# **Description**

k = dcgain(sys) computes the DC gain k of the LTI model sys.

#### **Continuous Time**

The continuous-time DC gain is the transfer function value at the frequency s = 0. For state-space models with matrices (A, B, C, D), this value is

$$K = D - CA^{-1}B$$

#### **Discrete Time**

The discrete-time DC gain is the transfer function value at z = 1. For state-space models with matrices (A, B, C, D), this value is

$$K = D + C(I - A)^{-1}B$$

# **Examples**

#### Compute the DC Gain of a MIMO Transfer Function

Create the following 2-input 2-output continuous-time transfer function.

$$H(s) = \begin{bmatrix} 1 & \frac{s-1}{s^2 + s + 3} \\ \frac{1}{s+1} & \frac{s+2}{s-3} \end{bmatrix}$$

$$H = [1 tf([1 -1],[1 1 3]) ; tf(1,[1 1]) tf([1 2],[1 -3])];$$

Compute the DC gain of the transfer function. For continuous-time models, the DC gain is the transfer function value at the frequency s = 0.

The DC gain for each input-output pair is returned. K(i,j) is the DC gain from input j to output i.

#### **Compute DC Gain of Identified Model**

Load the estimation data.

```
load iddatal zl
```

z1 is an iddata object containing the input-output estimation data.

Estimate a process model from the data. Specify that the model has one pole and a time delay term.

```
Tp1 = 0.25655
        Td = 0.068
Parameterization:
    'P1D'
   Number of free coefficients: 3
   Use "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using PROCEST on time domain data "z1".
Fit to estimation data: 44.85%
FPE: 6.02, MSE: 5.901
Compute the DC gain of the model.
K = dcgain(sys)
K = 9.0754
This DC gain value is stored in the Kp property of sys.
sys.Kp
ans = 9.0754
```

# **Tips**

The DC gain is infinite for systems with integrators.

#### See Also

evalfr | norm

# delay2z

Replace delays of discrete-time TF, SS, or ZPK models by poles at z=0, or replace delays of FRD models by phase shift

Note delay2z has been removed. Use absorbDelay instead.

## delayss

Create state-space models with delayed inputs, outputs, and states

## **Syntax**

```
sys=delayss(A,B,C,D,delayterms)
sys=delayss(A,B,C,D,ts,delayterms)
```

## **Description**

sys=delayss(A,B,C,D,delayterms) constructs a continuous-time state-space model of the form:

$$\frac{dx}{dt} = Ax(t) + Bu(t) + \sum_{j=1}^{N} (Ajx(t-tj) + Bju(t-tj))$$
$$y(t) = Cx(t) + Du(t) + \sum_{j=1}^{N} (Cjx(t-tj) + Dju(t-tj))$$

where  $t_j$ , j=1,...,N are time delays expressed in seconds. delayterms is a struct array with fields delay, a, b, c, d where the fields of delayterms(j) contain the values of tj, Aj, Bj, Cj, and Dj, respectively. The resulting model sys is a state-space (SS) model with internal delays.

sys=delayss(A,B,C,D,ts,delayterms)constructs the discrete-time counterpart:

$$x[k+1] = Ax[k] + Bu[k] + \sum_{j=1}^{N} \{Ajx[k-nj] + Bju[k-nj]\}$$
$$y[k] = Cx[k] + Du[k] + \sum_{j=1}^{N} \{Cjx[k-nj] + Dju[k-nj]\}$$

where Nj, j=1,...,N are time delays expressed as integer multiples of the sample time ts.

## **Examples**

To create the model:

```
\frac{dx}{dt} = x(t) - x(t - 1.2) + 2u(t - 0.5)
    y(t) = x(t - 0.5) + u(t)
type
DelayT(1) = struct('delay',0.5,'a',0,'b',2,'c',1,'d',0);
DelayT(2) = struct('delay',1.2,'a',-1,'b',0,'c',0,'d',0);
sys = delayss(1,0,0,1,DelayT)
a =
       x1
   x1
b =
       u1
        2
   x1
c =
       x1
        1
   у1
d =
       u1
        1
   у1
(values computed with all internal delays set to zero)
Internal delays: 0.5 0.5 1.2
Continuous-time model.
```

#### See Also

getdelaymodel | ss

Introduced in R2007a

## dlqr

Linear-quadratic (LQ) state-feedback regulator for discrete-time state-space system

## **Syntax**

$$[K,S,e] = dlgr(A,B,Q,R,N)$$

## **Description**

[K,S,e] = dlqr(A,B,Q,R,N) calculates the optimal gain matrix K such that the state-feedback law

$$u[n] = -Kx[n]$$

minimizes the quadratic cost function

$$J(u) = \sum_{n=1}^{\infty} (x[n]^{T} Q x[n] + u[n]^{T} R u[n] + 2x[n]^{T} N u[n])$$

for the discrete-time state-space mode

$$x[n+1] = Ax[n] + Bu[n]$$

The default value N=0 is assumed when N is omitted.

In addition to the state-feedback gain K, dlqr returns the infinite horizon solution S of the associated discrete-time Riccati equation

$$A^{T}SA - S - (A^{T}SB + N)(B^{T}SB + R)^{-1}(B^{T}SA + N^{T}) + Q = 0$$

and the closed-loop eigenvalues e = eig(A-B\*K). Note that K is derived from S by

$$K = (B^T S B + R)^{-1} (B^T S A + N^T)$$

#### Limitations

The problem data must satisfy:

- The pair (A, B) is stabilizable.
- R > 0 and  $Q NR^{-1}N^T \ge 0$
- $(Q NR^{-1}N^{T}, A BR^{-1}N^{T})$  has no unobservable mode on the unit circle.

#### See Also

dare|lqgreg|lqr|lqrd|lqry

# dlyap

Solve discrete-time Lyapunov equations

## **Syntax**

```
X = dlyap(A,Q)
X = dlyap(A,B,C)
X = dlyap(A,Q,[],E)
```

## **Description**

X = dlyap(A,Q) solves the discrete-time Lyapunov equation  $AXA^T - X + Q = 0$ ,

where A and Q are n-by-n matrices.

The solution X is symmetric when Q is symmetric, and positive definite when Q is positive definite and A has all its eigenvalues inside the unit disk.

```
X = dlyap(A,B,C) solves the Sylvester equation AXB - X + C = 0,
```

where A, B, and C must have compatible dimensions but need not be square.

X = dlyap(A,Q,[],E) solves the generalized discrete-time Lyapunov equation  $AXA^T - EXE^T + Q = 0$ ,

where Q is a symmetric matrix. The empty square brackets, [], are mandatory. If you place any values inside them, the function will error out.

# **Diagnostics**

The discrete-time Lyapunov equation has a (unique) solution if the eigenvalues  $\alpha_1$ ,  $\alpha_2$ , ...,  $\alpha_N$  of A satisfy  $\alpha_i \alpha_j \neq 1$  for all (i, j).

If this condition is violated, dlyap produces the error message

Solution does not exist or is not unique.

## **Algorithms**

dlyap uses SLICOT routines SB03MD and SG03AD for Lyapunov equations and SB04QD (SLICOT) for Sylvester equations.

#### References

- [1] Barraud, A.Y., "A numerical algorithm to solve A XA X = Q," *IEEE Trans. Auto. Contr.*, AC-22, pp. 883-885, 1977.
- [2] Bartels, R.H. and G.W. Stewart, "Solution of the Matrix Equation AX + XB = C," *Comm. of the ACM*, Vol. 15, No. 9, 1972.
- [3] Hammarling, S.J., "Numerical solution of the stable, non-negative definite Lyapunov equation," *IMA J. Num. Anal.*, Vol. 2, pp. 303-325, 1982.
- [4] Higham, N.J., "FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation," *A.C.M. Trans. Math. Soft.*, Vol. 14, No. 4, pp. 381-396, 1988.
- [5] Penzl, T., "Numerical solution of generalized Lyapunov equations," *Advances in Comp. Math.*, Vol. 8, pp. 33-48, 1998.
- [6] Golub, G.H., Nash, S. and Van Loan, C.F. "A Hessenberg-Schur method for the problem AX + XB = C," *IEEE Trans. Auto. Contr.*, AC-24, pp. 909-913, 1979.
- [7] Sima, V. C, "Algorithms for Linear-quadratic Optimization," Marcel Dekker, Inc., New York, 1996.

#### See Also

covar|lyap

# dlyapchol

Square-root solver for discrete-time Lyapunov equations

## **Syntax**

```
R = dlyapchol(A,B)
X = dlyapchol(A,B,E)
```

## **Description**

R = dlyapchol(A,B) computes a Cholesky factorization X = R'\*R of the solution X to the Lyapunov matrix equation:

```
A*X*A'-X+B*B'=0
```

All eigenvalues of A matrix must lie in the open unit disk for R to exist.

X = dlyapchol(A,B,E) computes a Cholesky factorization X = R'\*R of X solving the Sylvester equation

```
A*X*A' - E*X*E' + B*B' = 0
```

All generalized eigenvalues of (A,E) must lie in the open unit disk for R to exist.

## **Algorithms**

dlyapchol uses SLICOT routines SB03OD and SG03BD.

#### References

[1] Bartels, R.H. and G.W. Stewart, "Solution of the Matrix Equation AX + XB = C," *Comm. of the ACM*, Vol. 15, No. 9, 1972.

- [2] Hammarling, S.J., "Numerical solution of the stable, non-negative definite Lyapunov equation," *IMA J. Num. Anal.*, Vol. 2, pp. 303-325, 1982.
- [3] Penzl, T., "Numerical solution of generalized Lyapunov equations," *Advances in Comp. Math.*, Vol. 8, pp. 33-48, 1998.

#### **See Also**

dlyap | lyapchol

#### drss

Generate random discrete test model

## **Syntax**

```
sys = drss(n)
drss(n,p)
drss(n,p,m)
drss(n,p,m,s1,...sn)
```

## **Description**

sys = drss(n) generates an n-th order model with one input and one output, and returns the model in the state-space object sys. The poles of sys are random and stable with the possible exception of poles at z = 1 (integrators).

drss(n,p) generates an n-th order model with one input and p outputs.

drss(n,p,m) generates an n-th order model with p outputs and m inputs.

drss(n,p,m,s1,...sn) generates a s1-by-sn array of n-th order models with m inputs and p outputs.

In all cases, the discrete-time state-space model or array returned by drss has an unspecified sample time. To generate transfer function or zero-pole-gain systems, convert sys using tf or zpk.

## **Examples**

Generate a discrete LTI system with three states, four outputs, and two inputs.

```
sys = drss(3,4,2)

a = x1 	 x2 	 x3
```

Sample time: unspecified Discrete-time model.

### **See Also**

 $\verb"rss|tf|zpk"$ 

#### dsort

Sort discrete-time poles by magnitude

## **Syntax**

```
dsort
[s,ndx] = dsort(p)
```

## **Description**

dsort sorts the discrete-time poles contained in the vector p in descending order by magnitude. Unstable poles appear first.

When called with one lefthand argument, dsort returns the sorted poles in s.

[s,ndx] = dsort(p) also returns the vector ndx containing the indices used in the sort.

## **Examples**

Sort the following discrete poles.

```
p =
    -0.2410 + 0.5573i
    -0.2410 - 0.5573i
    0.1503
    -0.0972
    -0.2590

s = dsort(p)

s =
    -0.2410 + 0.5573i
    -0.2410 - 0.5573i
    -0.2590
```

0.1503

-0.0972

# **Limitations**

The poles in the vector p must appear in complex conjugate pairs.

#### **See Also**

eig|esort|pole|pzmap|sort|zero

#### dss

Create descriptor state-space models

## **Syntax**

```
sys = dss(A,B,C,D,E)
sys = dss(A,B,C,D,E,Ts)
sys = dss(A,B,C,D,E,ltisys)
```

## **Description**

sys = dss(A,B,C,D,E) creates the continuous-time descriptor state-space model

$$E\frac{dx}{dt} = Ax + Bu$$
$$v = Cx + Du$$

The output sys is an SS model storing the model data (see "State-Space Models"). Note that ss produces the same type of object. If the matrix D = 0, you can simply set d to the scalar 0 (zero).

sys = dss(A,B,C,D,E,Ts) creates the discrete-time descriptor model

$$Ex[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time Ts (in seconds).

sys = dss(A,B,C,D,E,ltisys) creates a descriptor model with properties inherited
from the LTI model ltisys (including the sample time).

Any of the previous syntaxes can be followed by property name/property value pairs

```
'Property', Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or some notes on the model history. See set and the example below for details.

# **Examples**

The command

```
sys = dss(1,2,3,4,5,'inputdelay',0.1,'inputname','voltage',... 'notes','Just an example'); creates the model 5\dot{x}=x+2u
```

with a 0.1 second input delay. The input is labeled 'voltage', and a note is attached to tell you that this is just an example.

#### See Also

dssdata | get | set | ss

y = 3x + 4u

#### dssdata

Extract descriptor state-space data

## **Syntax**

```
[A,B,C,D,E] = dssdata(sys)
[A,B,C,D,E,Ts] = dssdata(sys)
```

## **Description**

[A,B,C,D,E] = dssdata(sys) returns the values of the A, B, C, D, and E matrices for the descriptor state-space model sys (see dss). dssdata equals ssdata for regular state-space models (i.e., when E=I).

If sys has internal delays, A, B, C, D are obtained by first setting all internal delays to zero (creating a zero-order Padé approximation). For some systems, setting delays to zero creates singular algebraic loops, which result in either improper or ill-defined, zero-delay approximations. For these systems, dssdata cannot display the matrices and returns an error. This error does not imply a problem with the model sys itself.

```
[A,B,C,D,E,Ts] = dssdata(sys) also returns the sample time Ts.
```

You can access other properties of sys using get or direct structure-like referencing (e.g., sys.Ts).

For arrays of SS models with variable order, use the syntax

```
[A,B,C,D,E] = dssdata(sys,'cell')
```

to extract the state-space matrices of each model as separate cells in the cell arrays A, B, C, D, and E.

#### See Also

dss | get | getdelaymodel | ssdata

#### esort

Sort continuous-time poles by real part

## **Syntax**

```
s = esort(p)
[s,ndx] = esort(p)
```

## **Description**

esort sorts the continuous-time poles contained in the vector **p** by real part. Unstable eigenvalues appear first and the remaining poles are ordered by decreasing real parts.

When called with one left-hand argument, s = esort(p) returns the sorted eigenvalues in s.

[s,ndx] = esort(p) returns the additional argument ndx, a vector containing the indices used in the sort.

## **Examples**

Sort the following continuous eigenvalues.

```
p = 
    -0.2410+ 0.5573i 
    -0.2410- 0.5573i 
    0.1503 
    -0.0972 
    -0.2590 
esort(p) 
ans = 
    0.1503 
    -0.0972
```

```
-0.2410+ 0.5573i
```

#### **Limitations**

The eigenvalues in the vector  $\boldsymbol{p}$  must appear in complex conjugate pairs.

#### See Also

dsort | eig | pole | pzmap | sort | zero

<sup>-0.2410- 0.5573</sup>i

<sup>-0.2590</sup> 

#### estim

Form state estimator given estimator gain

## **Syntax**

```
est = estim(sys,L)
est = estim(sys,L,sensors,known)
```

## **Description**

est = estim(sys,L) produces a state/output estimator est given the plant state-space model sys and the estimator gain L. All inputs w of sys are assumed stochastic (process and/or measurement noise), and all outputs y are measured. The estimator est is returned in state-space form (SS object).

For a continuous-time plant sys with equations

$$\dot{x} = Ax + Bw$$
$$y = Cx + Dw$$

estim uses the following equations to generate a plant output estimate  $\widehat{y}$  and a state estimate  $\widehat{x}$ , which are estimates of y(t)=C and x(t), respectively:

$$\dot{\widehat{x}} = A\widehat{x} + L(y - C\widehat{x})$$
$$\begin{vmatrix} \widehat{y} \\ \widehat{x} \end{vmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} \widehat{x}$$

For a discrete-time plant sys with the following equations:

$$x[n+1] = Ax[n] + Bw[n]$$
$$y[n] = Cx[n] + Dw[n]$$

estim uses estimator equations similar to those for continuous-time to generate a plant output estimate y[n|n-1] and a state estimate x[n|n-1], which are estimates of y[n] and x[n], respectively. These estimates are based on past measurements up to y[n-1].

est = estim(sys, L, sensors, known) handles more general plants sys with both known (deterministic) inputs u and stochastic inputs w, and both measured outputs y and nonmeasured outputs z.

$$\dot{x} = Ax + B_1 w + B_2 u$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix} w + \begin{bmatrix} D_{12} \\ D_{22} \end{bmatrix} u$$

The index vectors sensors and known specify which outputs of sys are measured (y), and which inputs of sys are known (u). The resulting estimator est, found using the following equations, uses both u and v to produce the output and state estimates.

$$\begin{split} \dot{\widehat{x}} &= A\widehat{x} + B_2 u + L(y - C_2 \widehat{x} - D_{22} u) \\ \left[ \begin{matrix} \widehat{y} \\ \widehat{x} \end{matrix} \right] &= \begin{bmatrix} C_2 \\ I \end{matrix} \widehat{x} + \begin{bmatrix} D_{22} \\ 0 \end{bmatrix} u \\ u \text{ (known)} & & & \\ y \text{ (sensors)} & & & \\ & & & \\ \end{matrix} \end{split}$$

## **Examples**

Consider a state-space model sys with seven outputs and four inputs. Suppose you designed a Kalman gain matrix L using outputs 4, 7, and 1 of the plant as sensor measurements and inputs 1, 4, and 3 of the plant as known (deterministic) inputs. You can then form the Kalman estimator by

```
sensors = [4,7,1];
known = [1,4,3];
est = estim(sys,L,sensors,known)
```

See the function kalman for direct Kalman estimator design.

# **Tips**

You can use the functions place (pole placement) or kalman (Kalman filtering) to design an adequate estimator gain L. Note that the estimator poles (eigenvalues of A-LC) should be faster than the plant dynamics (eigenvalues of A) to ensure accurate estimation.

#### See Also

kalman | kalmd | lqgreg | place | predict | reg | ss | ssest

#### evalfr

Evaluate frequency response at given frequency

## **Syntax**

$$frsp = evalfr(sys, f)$$

## **Description**

frsp = evalfr(sys, f) evaluates the transfer function of the TF, SS, or ZPK model sys at the complex number f. For state-space models with data (A, B, C, D), the result is

$$H(f) = D + C(fI - A)^{-1}B$$

evalfr is a simplified version of freqresp meant for quick evaluation of the response at a single point. Use freqresp to compute the frequency response over a set of frequencies.

## **Examples**

#### **Evaluate Discrete-Time Transfer Function**

Create the following discrete-time transfer function.

$$H(z) = \frac{z-1}{z^2+z+1}$$

$$H = tf([1 -1],[1 1 1],-1);$$

Evaluate the transfer function at z = 1+j.

#### **Evaluate Frequency Response of Identified Model at Given Frequency**

Create the following continuous-time transfer function model:

$$H(s) = \frac{1}{s^2 + 2s + 1}$$

```
sys = idtf(1,[1 2 1]);
```

Evaluate the transfer function at frequency 0.1 rad/second.

```
w = 0.1;
s = j*w;
evalfr(sys,s)
ans = 0.9705 - 0.1961i
```

Alternatively, use the freqresp command.

```
freqresp(sys,w)
ans = 0.9705 - 0.1961i
```

### **Limitations**

The response is not finite when f is a pole of sys.

#### **See Also**

bode | freqresp | sigma

### evalGoal

Evaluate tuning goals for tuned control system

#### **Syntax**

```
[Hspec,fval] = evalGoal(Req,T)
```

## **Description**

[Hspec, fval] = evalGoal(Req, T) returns the normalized value fval of a tuning goal evaluated for a tuned control system T. The evalGoal command also returns the transfer function Hspec used to compute this value.

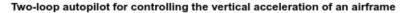
## **Examples**

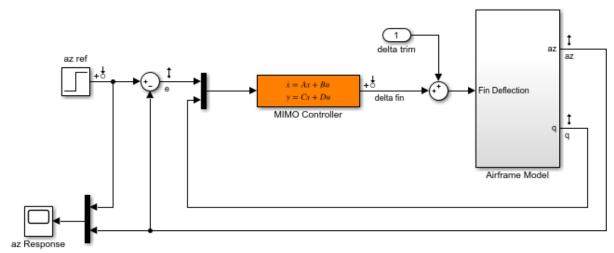
#### **Evaluate Requirements for Tuned System**

Tune a control system with systune, and evaluate the tuning goals with evalGoal.

Open the Simulink® model rct\_airframe2.

```
open_system('rct_airframe2')
```





Create tracking, roll-off, stability margin, and disturbance rejection requirements for tuning the control system.

```
Req1 = TuningGoal.Tracking('az ref','az',1);
Req2 = TuningGoal.Gain('delta fin','delta fin',tf(25,[1 0]));
Req3 = TuningGoal.Margins('delta fin',7,45);
MaxGain = frd([2 200 200],[0.02 2 200]);
Req4 = TuningGoal.Gain('delta fin','az',MaxGain);
```

Create an slTuner interface, and tune the model using these tuning goals.

```
ST0 = slTuner('rct_airframe2', 'MIMO Controller');
rng default
[ST1,fSoft] = systune(ST0,[Req1,Req2,Req3,Req4]);
Final: Soft = 1.13, Hard = -Inf, Iterations = 96
```

ST1 is a tuned version of the slTuner interface to the control system. ST1 contains the tuned values of the tunable parameters of the MIMO controller in the model.

Evaluate the margin goal for the tuned system.

```
[hspec,fval] = evalGoal(Req3,ST1);
fval
```

```
fval = 0.5140
```

The normalized value of the tuning goal is less than 1, indicating that the tuned system satisfies the margin requirement. For more information about how the normalized value of this tuning goal is calculated, see the TuningGoal.Margins reference page.

Evaluate the tracking goal for the tuned system.

```
[hspec,fval] = evalGoal(Req1,ST1);
fval

fval =
    1.1327
```

The tracking requirement is nearly met, but the value exceeds 1, indicating a small violation. To further assess the violation, you can use viewGoal to visualize the requirement against the corresponding response of the tuned system.

## **Input Arguments**

#### Req — Tuning goal to evaluate

TuningGoal object | vector of TuningGoal objects

Tuning goal to evaluate, specified as a TuningGoal object or vector of TuningGoal objects. For a list of all TuningGoal objects, see "Tuning Goals".

#### T — Tuned control system

generalized state-space model | slTuner interface object

Tuned control system, specified as a generalized state-space (genss) model or an slTuner interface to a Simulink model. T is typically the result of using the tuning goal to tune control system parameters with systume.

```
Example: [T,fSoft,gHard] = systume(T0,SoftReq,HardReq), where T0 is a tunable genss model
```

Example: [T,fSoft,gHard] = systume(ST0,SoftReq,HardReq), where ST0 is a slTuner interface object

## **Output Arguments**

#### Hspec — transfer function associated with tuning goal

state-space model

Transfer function associated with the tuning goal, returned as a state-space (ss) model. evalGoal uses Hspec to compute the evaluated tuning goal, fval.

For example, suppose Req is a TuningGoal.Gain goal that limits the gain H(s) between some specified input and output to the gain profile w(s). In that case, Hspec is given by:

$$Hspec(s) = \frac{1}{w(s)}H(s).$$

fval is the peak gain of Hspec. If H(s) satisfies the tuning goal, fval  $\leq 1$ .

For more information about the transfer function associated with the tuning goal, see the reference page for each tuning goal.

#### **fval** — **Normalized value of tuning requirement**

positive scalar

Normalized value of tuning requirement, returned as a positive scalar. The normalized value is a measure of how closely the requirement is met in the tuned system. The tuning requirement is satisfied if fval < 1. For information about how each type of TuningGoal requirement is converted into a normalized value, see the reference page for each tuning goal.

#### **Tips**

 For MIMO feedback loops, the LoopShape, MinLoopGain, MaxLoopGain, Margins, Sensitivity, and Rejection goals are sensitive to the relative scaling of each SISO loop. systune tries to balance the overall loop-transfer matrix while enforcing such goals. The optimal loop scaling is stored in the tuned closed-loop model or slTuner interface T returned by systune. For consistency, evalGoal(R,T) applies this same scaling when evaluating the tuning goals. To omit this scaling, use evalGoal(R,clearTuningInfo(T)).

Modifying T might compromise the validity of the stored scaling. Therefore, if you make significant modifications to T, retuning is recommended to update the scaling data.

### See Also

genss|systune|systune (for slTuner)|viewGoal

## **Topics**

"Visualize Tuning Goals"
"Tuning Goals"

Introduced in R2012b

## evalSpec

(Not recommended) Evaluate tuning goals for tuned control system

**Note** evalSpec is not recommended. Use evalGoal instead.

## **Syntax**

```
[Hspec,fval] = evalSpec(Req,T)
[Hspec,fval] = evalSpec(Req,T,[])
```

## **Description**

[Hspec, fval] = evalSpec(Req, T) returns the normalized value fval of a tuning goal evaluated for a tuned control system T. The evalSpec command also returns the transfer function Hspec used to compute this value.

[Hspec, fval] = evalSpec(Req,T,[]) disregards scaling information stored with the tuned control system T when evaluating the tuning goal. For more information, see "Tips" on page 2-271.

## **Examples**

#### **Evaluate Requirements for Tuned System**

Tune a control system with systune, and evaluate the tuning goals with evalSpec.

Open the Simulink  ${\tt @}$  model rct\_airframe 2.

```
open_system('rct_airframe2')
```

Create tracking, roll-off, stability margin, and disturbance rejection requirements for tuning the control system.

```
Req1 = TuningGoal.Tracking('az ref','az',1);
Req2 = TuningGoal.Gain('delta fin','delta fin',tf(25,[1 0]));
Req3 = TuningGoal.Margins('delta fin',7,45);
MaxGain = frd([2 200 200],[0.02 2 200]);
Req4 = TuningGoal.Gain('delta fin','az',MaxGain);
```

Create an slTuner interface, and tune the model using these tuning goals.

```
ST0 = slTuner('rct_airframe2', 'MIMO Controller');
rng default
[ST1,fSoft] = systune(ST0,[Req1,Req2,Req3,Req4]);
Final: Soft = 1.13, Hard = -Inf, Iterations = 68
```

ST1 is a tuned version of the slTuner interface to the control system. ST1 contains the tuned values of the tunable parameters of the MIMO controller in the model.

Evaluate the margin goal for the tuned system.

```
[hspec,fval] = evalSpec(Req3,ST1);
fval

fval =
    0.5140
```

The normalized value of the tuning goal is less than 1, indicating that the tuned system satisfies the margin requirement. For more information about how the normalized value of this tuning goal is calculated, see the TuningGoal.Margins reference page.

Evaluate the tracking goal for the tuned system.

```
[hspec,fval] = evalSpec(Req1,ST1);
fval

fval =
    1.1327
```

The tracking requirement is nearly met, but the value exceeds 1, indicating a small violation. To further assess the violation, you can use viewSpec to visualize the requirement against the corresponding response of the tuned system.

## **Input Arguments**

#### Req — Tuning goal to evaluate

TuningGoal object | vector of TuningGoal objects

Tuning goal to evaluate, specified as a TuningGoal object or vector of TuningGoal objects. For a list of all TuningGoal objects, see "Tuning Goals".

#### T — Tuned control system

generalized state-space model | slTuner interface object

Tuned control system, specified as a generalized state-space (genss) model or an slTuner interface to a Simulink model. T is typically the result of using the tuning goal to tune control system parameters with systune.

Example: [T,fSoft,gHard] = systume(T0,SoftReq,HardReq), where T0 is a tunable genss model

Example: [T,fSoft,gHard] = systume(ST0,SoftReq,HardReq), where ST0 is a slTuner interface object

## **Output Arguments**

#### Hspec — transfer function associated with tuning goal

state-space model

Transfer function associated with the tuning goal, returned as a state-space (ss) model. evalSpec uses Hspec to compute the evaluated tuning goal, fval.

For example, suppose Req is a TuningGoal.Gain goal that limits the gain H(s) between some specified input and output to the gain profile w(s). In that case, Hspec is given by:

$$Hspec(s) = \frac{1}{w(s)}H(s)$$
.

fval is the peak gain of Hspec. If H(s) satisfies the tuning goal, fval  $\leq 1$ .

For more information about the transfer function associated with the tuning goal, see the reference page for each tuning goal.

#### fval — Normalized value of tuning requirement

positive scalar

Normalized value of tuning requirement, returned as a positive scalar. The normalized value is a measure of how closely the requirement is met in the tuned system. The tuning requirement is satisfied if fval < 1. For information about how each type of TuningGoal requirement is converted into a normalized value, see the reference page for each tuning goal.

## **Tips**

• For MIMO feedback loops, the LoopShape, MinLoopGain, MaxLoopGain, Margins, Sensitivity, and Rejection goals are sensitive to the relative scaling of each SISO loop. systune tries to balance the overall loop-transfer matrix while enforcing such goals. The optimal loop scaling is stored in the tuned closed-loop model CL returned by systune. For consistency, evalSpec(R,CL) applies this same scaling when evaluating the tuning goals. To omit this scaling, use evalSpec(R,CL,[]).

Modifying CL might compromise the validity of the stored scaling. Therefore, if you make significant modifications to CL, retuning is recommended to update the scaling data.

## **Compatibility Considerations**

### evalSpec is not recommended

Not recommended starting in R2017b

Beginning in R2017b, evalSpec is not recommended. Use evalGoal instead.

#### See Also

evalGoal|genss|systune|systune (for slTuner)|viewGoal

**Topics**"Visualize Tuning Goals"
"Tuning Goals"

Introduced in R2012b

## evalSurf

Evaluate gain surfaces at specific design points

## **Syntax**

```
GV = evalSurf(GS,X)
GV = evalSurf(GS,X1,...,XM)
GV = evalSurf( ,gridflag)
```

## **Description**

 $\mathsf{GV} = \mathsf{evalSurf}(\mathsf{GS},\mathsf{X})$  evaluates a gain surface at the list of points specified in the array  $\mathsf{X}$ . A point is a combination of scheduling-variable values. Thus  $\mathsf{X}$  is an N-by-M array, where N is the number of points at which to evaluate the gain, and M is the number of scheduling variables in  $\mathsf{GS}$ .

GV = evalSurf(GS, X1, ..., XM) evaluates the gain surface over the rectangular grid generated by the vectors X1, ..., XM. Each vector contains values for one scheduling variable of GS.

```
GV = evalSurf(____, gridflag) specifies the layout of GV.
```

## **Examples**

#### **Evaluate 1-D Gain Surface at Specified Values**

Create a gain surface with one scheduling variable and evaluate the gain at a list of scheduling-variable values.

When you create a gain surface using tunableSurface, you specify design points at which the gain coefficients are tuned. These points are the typically the scheduling-variable values at which you have sampled or linearized the plant. However, you might want to implement the gain surface as a lookup table with breakpoints that are different

from the specified design points. In this example, you create a gain surface with a set of design points and then evaluate the surface using a different set of scheduling variable values.

Create a scalar gain that varies as a quadratic function of one scheduling variable, t. Suppose that you have linearized your plant every five seconds from t = 0 to t = 40.

```
t = 0:5:40;
domain = struct('t',t);
shapefcn = @(x) [x,x^2];
GS = tunableSurface('GS',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS, [12.1, 4.2, 2]);
```

Evaluate the gain surface at a different set of time values.

```
tvals = [0,4,11,18,25,32,39,42]; % eight values
GV = evalSurf(GS,tvals)

GV = 8×1

    9.9000
    10.0200
    10.6150
    11.7000
    13.2750
    15.3400
    17.8950
    19.1400
```

GV is an 8-by-1 array. You can use tvals and GV to implement the variable gain as a lookup table.

#### **Evaluate Gain Surface on Grid of Values**

Evaluate a gain surface with two scheduling variables over a grid of values of those variables.

When you create a gain surface using tunableSurface, you specify design points at which the gain coefficients are tuned. These points are the typically the scheduling-variable values at which you have sampled or linearized the plant. However, you might want to implement the gain surface as a lookup table with breakpoints that are different from the specified design points. In this example, you create a gain surface with a set of design points and then evaluate the surface using a different set of scheduling-variable values.

Create a scalar-valued gain surface that is a bilinear function of two independent variables,  $\alpha$  and V.

```
[alpha,V] = ndgrid(0:1.5:15,300:30:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
GS = tunableSurface('GS',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS, [100, 28, 40, 10]);
```

Evaluate the gain at selected values of  $\alpha$  and V.

```
alpha_vec = [7:1:13]; % N1 = 7 points
V_vec = [400:25:625]; % N2 = 10 points
GV = evalSurf(GS,alpha vec,V vec);
```

The breakpoints at which you evaluate the gain surface need not fall within the range specified by domain. However, if you attempt to evaluate the gain too far outside the range used for tuning, the software issues a warning.

The breakpoints also need not be regularly spaced. evalSurf evaluates the gain surface over the grid formed by ndgrid(alpha\_vec,V\_vec). Examine the dimensions of the resulting array.

```
size(GV) ans = 1 \times 2 7 10
```

By default, the grid dimensions N1-by-N2 are first in the array, followed by the gain dimensions. GS is scalar-valued gain, so the dimensions of GV are [7,10,1,1], or equivalently [7,10].

The value in each location of GV is the gain evaluated at the corresponding (alpha\_vec, V\_vec) pair in the grid. For example, GV(2,3) is the gain evaluated at (alpha\_vec(2), V\_vec(3)) or (8,450).

#### **Evaluate Array-Valued Gain Surface**

Evaluate an array-valued gain surface with two scheduling variables over a grid of values of those variables.

Create a vector-valued gain that has two scheduling variables.

```
[alpha,V] = ndgrid(0:1.5:15,300:30:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
GS = tunableSurface('GS',ones(2,2),domain,shapefcn);
```

Setting the initial constant coefficient to ones(2,2) causes tunableSurface to generate a 2-by-2 gain matrix. Each entry in that matrix is an independently tunable gain surface that is a bilinear function of two scheduling variables. In other words, the gain surface is given by:

$$GS = K_0 + K_1 \alpha + K_2 V + K_3 \alpha V,$$

where each of the coefficients  $K_0, ..., K_3$  is itself a 2-by-2 matrix.

Typically, you would tune the coefficients of those gain surfaces as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
K0 = 10*rand(2);
K1 = 10*rand(2);
K2 = 10*rand(2);
K3 = 10*rand(2);
```

The tunableSurface object stores array-valued coefficients by concatenating them into a 2-by-8 array (see the tunableSurface reference page). Therefore, concatenate these values of  $K_0, ..., K_3$  to change the coefficients of GS.

```
GS = setData(GS, [K0 K1 K2 K3]);
```

Now evaluate the gain surface at selected values of the scheduling variables.

```
alpha_vec = [7:1:13]; % N1 = 7 points
V_vec = [400:25:625]; % N2 = 10 points
GV = evalSurf(GS,alpha vec,V vec,'gridlast');
```

The 'gridlast' orders the array GV such that the dimensions of the grid of gain values, 7-by-10, are last. The dimensions of the gain array itself, 2-by-2, are first.

```
size(GV) ans = 1 \times 4
2 2 7 10
```

## **Input Arguments**

#### **GS** — **Gain** surface

tunableSurface object

Gain surface to evaluate, specified as a tunableSurface object. GS can have any number of scheduling variables, and can be scalar-valued or array-valued.

#### X — Points

array

Points at which to evaluate the gain surface, specified as an array. A point is a combination of scheduling-variable values. X has dimensions N-by-M, where M is the number of scheduling variables in GS and N is the number of points at which to evaluate GS. Thus, X is a list of scheduling-variable-value combinations at which to evaluate the gain. For example, suppose GS has two scheduling variables, a and b, and you want to evaluate GS at 10 (a,b) pairs. In that case, X is a 10-by-2 array that lists the (a,b). The points in X need not match the design points in GS.SamplingGrid.

## X1,...,XM — Scheduling-variable values arrays

Scheduling-variable values at which to evaluate the gain surface, specified as M arrays, where M is the number of scheduling variables in GS. For example, if GS has two

scheduling variables, a and b, then X1 and X2 are vectors of a and b values, respectively. The gain surface is evaluated over the grid ndgrid(X1,X2). The values in that grid need not match the design points in GS.SamplingGrid.

#### gridflag — Layout of output array

'gridfirst' (default) | 'gridlast'

Layout of output array, specified as either 'gridfirst' or 'gridlast'.

- 'gridfirst' GV is of size [N1,...,NM,Ny,Nu] with the grid dimensions first and the gain dimensions last. This layout is the natural format for a scalar gain, where Ny = Nu = 1.
- 'gridlast' GV is of size [Ny, Nu, N1, ..., NM] with the gain dimensions first. This format is more readable for matrix-valued gains.

## **Output Arguments**

#### **GV** — Gain values

array

Gain values, returned as an array. GV contains the gain evaluated at the points (scheduling-variable values) specified by X or X1,...,XM. The size of GV depends on the number of scheduling variables in GS, the I/O dimensions of the gain defined by GS, and the value of gridflag.

If you compute the gain at a list of N points specified in an array X, then the size of GV is [N,Ny,Nu]. Here, [Ny,Nu] are the I/O dimensions of the gain. For example, suppose GS is a scalar gain surface with two scheduling variables, A and A is a 10-by-2 array containing 10 A0 pairs. Then GV1 is a column vector of ten values.

If you compute the gain over a grid specified by vectors  $X1, \ldots, XM$ , then the dimensions of GV depend on the value of gridflag.

• gridflag = 'gridfirst' (default) — The size of GV is [N1,...,NM,Ny,Nu]. Each Ni is the length of Xi, the number of values of the i-th scheduling variable. For example, suppose GS is a scalar gain surface with two scheduling variables, a and b, and X1 and X2 are vectors of 4 a values and 5 b values, respectively. Then, the size of GV is [4,5,1,1] or equivalently, [4,5]. Or, if GS is a three-output, two-input vector-valued gain, then the size of GV is [4,5,3,2].

• gridflag = 'gridlast' — The size of GV is [Ny,Nu,N1,...,NM]. For example, suppose GS is a scalar gain surface with two scheduling variables, a and b, and X1 and X2 are vectors of 4 a values and 5 b values, respectively. Then, the size of GV is [1,1,4,5]. Or, if GS is a three-output, two-input vector-valued gain, then the size of GV is [3,2,4,5].

## **Tips**

• Use evalSurf to turn tuned gain surfaces into lookup tables. Set X1,..., XM to the desired table breakpoints and use GV as table data. The table breakpoints do not need to match the design points used for tuning GS.

### See Also

getData | setData | tunableSurface | viewSurf

Introduced in R2015b

## exp

Create pure continuous-time delays

## **Syntax**

```
d = exp(tau,s)
```

## **Description**

d = exp(tau, s) creates pure continuous-time delays. The transfer function of a pure delay tau is:

$$d(s) = exp(-tau*s)$$

You can specify this transfer function using exp.

```
s = zpk('s')
d = exp(-tau*s)
```

More generally, given a 2D array M,

$$s = zpk('s')$$
  
 $D = exp(-M*s)$ 

creates an array  $\boldsymbol{D}$  of pure delays where

$$D(i,j) = \exp(-M(i,j)s).$$

All entries of M should be non negative for causality.

## **See Also**

tf|zpk

Introduced in R2006a

## extendedKalmanFilter

Create extended Kalman filter object for online state estimation

## **Syntax**

```
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn,
InitialState)
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn,
InitialState, Name, Value)

obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn)
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn,
Name, Value)
obj = extendedKalmanFilter(Name, Value)
```

## **Description**

obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn, InitialState) creates an extended Kalman filter object for online state estimation of a discrete-time nonlinear system. StateTransitionFcn is a function that calculates the state of the system at time k, given the state vector at time k-1. MeasurementFcn is a function that calculates the output measurement of the system at time k, given the state at time k. InitialState specifies the initial value of the state estimates.

After creating the object, use the correct and predict commands to update state estimates and state estimation error covariance values using a first-order discrete-time extended Kalman filter algorithm and real-time data.

obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn, InitialState, Name, Value) specifies additional attributes of the extended Kalman filter object using one or more Name, Value pair arguments.

obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn) creates an extended Kalman filter object using the specified state transition and measurement functions. Before using the predict and correct commands, specify the initial state values using dot notation. For example, for a two-state system with initial state values [1;0], specify obj.State = [1;0].

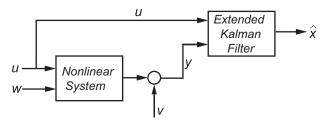
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn, Name, Value) specifies additional attributes of the extended Kalman filter object using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the initial state values using Name, Value pair arguments or dot notation.

obj = extendedKalmanFilter(Name, Value) creates an extended Kalman filter object with properties specified using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the state transition function, measurement function, and initial state values using Name, Value pair arguments or dot notation.

## **Object Description**

extendedKalmanFilter creates an object for online state estimation of a discrete-time nonlinear system using the first-order discrete-time extended Kalman filter algorithm.

Consider a plant with states x, input u, output y, process noise w, and measurement noise v. Assume that you can represent the plant as a nonlinear system.



The algorithm computes the state estimates  $\widehat{x}$  of the nonlinear system using state transition and measurement functions specified by you. The software lets you specify the noise in these functions as additive or nonadditive:

• **Additive Noise Terms** — The state transition and measurements equations have the following form:

$$x[k] = f(x[k-1], u_s[k-1]) + w[k-1]$$
  
$$y[k] = h(x[k], u_m[k]) + v[k]$$

Here f is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function h relates x to the measurements y at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional input arguments that are denoted by  $u_s$  and  $u_m$  in the equations. For example, the additional arguments could be time step k or the inputs u to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, x(k) is linearly related to the process noise w(k-1), and y(k) is linearly related to the measurement noise v(k).

• Nonadditive Noise Terms — The software also supports more complex state transition and measurement functions where the state x[k] and measurement y[k] are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$x[k] = f(x[k-1], w[k-1], u_s[k-1])$$
  
$$y[k] = h(x[k], v[k], u_m[k])$$

When you perform online state estimation, you first create the nonlinear state transition function f and measurement function h. You then construct the <code>extendedKalmanFilter</code> object using these nonlinear functions, and specify whether the noise terms are additive or nonadditive. You can also specify the Jacobians of the state transition and measurement functions. If you do not specify them, the software numerically computes the Jacobians.

After you create the object, you use the predict command to predict state estimate at the next time step, and correct to correct state estimates using the algorithm and real-time data. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You can use the following commands with extendedKalmanFilter objects:

Command	Description
	Correct the state and state estimation error covariance at time step $k$ using measured data at time step $k$ .

Command	Description
predict	Predict the state and state estimation error covariance at time the next time step.
residual	Return the difference between the actual and predicted measurements.
clone	Create another object with the same object property values.
	Do not create additional objects using syntax obj2 = obj. Any changes made to the properties of the new object created in this way (obj2) also change the properties of the original object (obj).

For extendedKalmanFilter object properties, see "Properties" on page 2-290.

## **Examples**

#### **Create Extended Kalman Filter Object for Online State Estimation**

To define an extended Kalman filter object for estimating the states of your system, you first write and save the state transition function and measurement function for the system.

In this example, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with nonlinearity parameter, mu, equal to 1. The oscillator has two states.

Specify an initial guess for the two states. You specify the guess as an M-element row or column vector, where M is the number of states.

```
initialStateGuess = [1;0];
```

Create the extended Kalman filter object. Use function handles to provide the state transition and measurement functions to the object.

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,initialStateGuess);
```

The object has a default structure where the process and measurement noise are additive.

To estimate the states and state estimation error covariance from the constructed object, use the correct and predict commands and real-time data.

## Specify Process and Measurement Noise Covariances in Extended Kalman Filter Object

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions are written for additive process and measurement noise terms. Specify the initial state values for the two states as [2;0].

Since the system has two states and the process noise is additive, the process noise is a 2-element vector and the process noise covariance is a 2-by-2 matrix. Assume there is no cross-correlation between process noise terms, and both the terms have the same variance 0.01. You can specify the process noise covariance as a scalar. The software uses the scalar value to create a 2-by-2 diagonal matrix with 0.01 on the diagonals.

Specify the process noise covariance during object construction.

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0],...
    'ProcessNoise',0.01);
```

Alternatively, you can specify noise covariances after object construction using dot notation. For example, specify the measurement noise covariance as 0.2.

```
obj.MeasurementNoise = 0.2;
```

Since the system has only one output, the measurement noise is a 1-element vector and the MeasurementNoise property denotes the variance of the measurement noise.

#### **Specify Jacobians for State and Measurement Functions**

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement

functions, vdpStateFcn.m and vdpMeasurementFcn.m. Specify the initial state values for the two states as [2;0].

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0]);
```

The extended Kalman filter algorithm uses Jacobians of the state transition and measurement functions for state estimation. You write and save the Jacobian functions and provide them as function handles to the object. In this example, use the previously written and saved functions vdpStateJacobianFcn.m and vdpMeasurementJacobianFcn.m.

```
obj.StateTransitionJacobianFcn = @vdpStateJacobianFcn;
obj.MeasurementJacobianFcn = @vdpMeasurementJacobianFcn;
```

Note that if you do not specify the Jacobians of the functions, the software numerically computes the Jacobians. This numerical computation may result in increased processing time and numerical inaccuracy of the state estimation.

#### Specify Nonadditive Measurement Noise in Extended Kalman Filter Object

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Assume that the process noise terms in the state transition function are additive. That is, there is a linear relation between the state and process noise. Also assume that the measurement noise terms are nonadditive. That is, there is a nonlinear relation between the measurement and measurement noise.

```
obj = extendedKalmanFilter('HasAdditiveMeasurementNoise',false);
```

Specify the state transition function and measurement functions. Use the previously written and saved functions, vdpStateFcn.m and vdpMeasurementNonAdditiveNoiseFcn.m.

The state transition function is written assuming the process noise is additive. The measurement function is written assuming the measurement noise is nonadditive.

```
obj.StateTransitionFcn = @vdpStateFcn;
obj.MeasurementFcn = @vdpMeasurementNonAdditiveNoiseFcn;
```

Specify the initial state values for the two states as [2;0].

```
obj.State = [2;0];
```

You can now use the correct and predict commands to estimate the state and state estimation error covariance values from the constructed object.

#### **Specify State Transition and Measurement Functions with Additional Inputs**

Consider a nonlinear system with input u whose state x and measurement y evolve according to the following state transition and measurement equations:

$$x[k] = \sqrt{x[k-1] + u[k-1]} + w[k-1]$$
$$v[k] = x[k] + 2 * u[k] + v[k]^{2}$$

The process noise w of the system is additive while the measurement noise v is nonadditive.

Create the state transition function and measurement function for the system. Specify the functions with an additional input u.

```
f = @(x,u)(sqrt(x+u));

h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.

```
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
```

Specify the measurement noise covariance.

```
obj.MeasurementNoise = 0.01;
```

You can now estimate the state of the system using the predict and correct commands. You pass the values of u to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement y[k]=0.8 and input u[k]=0.2 at time step k.

```
correct(obj, 0.8, 0.2)
```

Predict the state at the next time step, given u[k]=0.2.

```
predict(obj,0.2)
```

Retrieve the error, or *residual*, between the prediction and the measurement.

```
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);
```

## **Input Arguments**

#### StateTransitionFcn — State transition function

function handle

State transition function f, specified as a function handle. The function calculates the Ns-element state vector of the system at time step k, given the state vector at time step k-1. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system, and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

HasAdditiveProcessNoise is true — The process noise w is additive, and the state
transition function specifies how the states evolve as a function of state values at the
previous time step:

```
x(k) = f(x(k-1), Us1, \dots, Usn)
```

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.

 HasAdditiveProcessNoise is false — The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$x(k) = f(x(k-1), w(k-1), Us1, ..., Usn)$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

#### MeasurementFcn — Measurement function

function handle

Measurement function h, specified as a function handle. The function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. N is the number of measurements of the system. You write and save the measurement function, and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn. as gvdpMeasurementFcn. You can also specify MeasurementFcn. as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

 HasAdditiveMeasurementNoise is true — The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:

$$y(k) = h(x(k),Um1,...,Umn)$$

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command, which in turn passes them to the measurement function.

• HasAdditiveMeasurementNoise is false — The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

$$y(k) = h(x(k), v(k), Um1, ..., Umn)$$

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement

function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

#### InitialState — Initial state estimate value

vector

Initial state estimate value, specified as an Ns-element vector, where Ns is the number of states in the system. Specify the initial state values based on your knowledge of the system.

The specified value is stored in the State property of the object. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned.

If you want a filter with single-precision floating-point variables, specify InitialState as a single-precision vector variable. For example, for a two-state system with state transition and measurement functions vdpStateFcn.m and vdpMeasurementFcn.m, create the extended Kalman filter object with initial state estimates [1;2] as follows:

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,single([1;2]))
Data Types: double | single
```

#### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify properties on page 2-290 of extendedKalmanFilter object during object creation. For example, to create an extended Kalman filter object and specify the process noise covariance as 0.01:

```
obj = extended Kalman Filter (State Transition Fcn, Measurement Fcn, Initial State, 'Process Noise', 0.01); \\
```

## **Properties**

extendedKalmanFilter object properties are of three types:

 Tunable properties that you can specify multiple times, either during object construction using Name, Value arguments, or any time afterward during state estimation. After object creation, use dot notation to modify the tunable properties.

```
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn, InitialState);
obj.ProcessNoise = 0.01;
```

The tunable properties are State, StateCovariance, ProcessNoise, and MeasurementNoise.

- Nontunable properties that you can specify once, either during object construction or afterward using dot notion. Specify these properties before state estimation using correct and predict. The StateTransitionFcn, MeasurementFcn, StateTransitionJacobianFcn, and MeasurementJacobianFcn properties belong to this category.
- Nontunable properties that you must specify during object construction. The HasAdditiveProcessNoise and HasAdditiveMeasurementNoise properties belong to this category.

# HasAdditiveMeasurementNoise — Measurement noise characteristics true (default) | false

Measurement noise characteristics, specified as one of the following values:

 true — Measurement noise v is additive. The measurement function h that is specified in MeasurementFcn has the following form:

```
y(k) = h(x(k),Um1,...,Umn)
```

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function.

• false — Measurement noise is nonadditive. The measurement function specifies how the output measurement evolves as a function of the state *and* measurement noise:

```
y(k) = h(x(k), v(k), Um1, ..., Umn)
```

HasAdditiveMeasurementNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

#### HasAdditiveProcessNoise — Process noise characteristics

true (default) | false

Process noise characteristics, specified as one of the following values:

• true — Process noise w is additive. The state transition function f specified in StateTransitionFcn has the following form:

$$x(k) = f(x(k-1), Us1, \dots, Usn)$$

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function.

• false — Process noise is nonadditive. The state transition function specifies how the states evolve as a function of the state *and* process noise at the previous time step:

$$x(k) = f(x(k-1), w(k-1), Us1, ..., Usn)$$

HasAdditiveProcessNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

#### MeasurementFcn — Measurement function

function handle

Measurement function h, specified as a function handle. The function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. N is the number of measurements of the system. You write and save the measurement function and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn. as gvdpMeasurementFcn. You can also specify MeasurementFcn. as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

 HasAdditiveMeasurementNoise is true — The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:

$$y(k) = h(x(k), Um1, ..., Umn)$$

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these

additional arguments to the correct command which in turn passes them to the measurement function.

• HasAdditiveMeasurementNoise is false — The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

```
y(k) = h(x(k), v(k), Um1, ..., Umn)
```

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

MeasurementFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

## MeasurementJacobianFcn — Jacobian of measurement function [] (default) | function handle

[] (default) | fulletion handle

Jacobian of measurement function h, specified as one of the following:

- [] The Jacobian is numerically computed at every call to the correct command. This may increase processing time and numerical inaccuracy of the state estimation.
- function handle You write and save the Jacobian function and specify the handle to the function. For example, if vdpMeasurementJacobianFcn.m is the Jacobian function, specify MeasurementJacobianFcn as @vdpMeasurementJacobianFcn.

The function calculates the partial derivatives of the measurement function with respect to the states and measurement noise. The number of inputs to the Jacobian function must equal the number of inputs to the measurement function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true The function calculates the partial derivatives of the measurement function with respect to the states  $(\partial h/\partial x)$ . The output is as an N-by-Ns Jacobian matrix, where N is the number of measurements of the system and Ns is the number of states.
- HasAdditiveMeasurementNoise is false The function also returns a second output that is the partial derivative of the measurement function with respect to

the measurement noise terms  $(\partial h/\partial v)$ . The second output is returned as an *N*-by-*V* Jacobian matrix, where *V* is the number of measurement noise terms.

To see an example of a Jacobian function for additive measurement noise, type edit vdpMeasurementJacobianFcn at the command line.

MeasurementJacobianFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

#### MeasurementNoise — Measurement noise covariance

1 (default) | scalar | matrix

Measurement noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true Specify the covariance as a scalar or an *N*-by-*N* matrix, where *N* is the number of measurements of the system. Specify a scalar if there is no cross-correlation between measurement noise terms and all the terms have the same variance. The software uses the scalar value to create an *N*-by-*N* diagonal matrix.
- HasAdditiveMeasurementNoise is false Specify the covariance as a V-by-V matrix, where V is the number of measurement noise terms. MeasurementNoise must be specified before using correct. After you specify MeasurementNoise as a matrix for the first time, to then change MeasurementNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the measurement noise terms and all the terms have the same variance. The software extends the scalar to a V-by-V diagonal matrix with the scalar on the diagonals.

MeasurementNoise is a tunable property. You can change it using dot notation.

#### ProcessNoise — Process noise covariance

1 (default) | scalar | matrix

Process noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveProcessNoise property:

• HasAdditiveProcessNoise is true — Specify the covariance as a scalar or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. The software uses the scalar value to create an *Ns*-by-*Ns* diagonal matrix.

• HasAdditiveProcessNoise is false — Specify the covariance as a W-by-W matrix, where W is the number of process noise terms. ProcessNoise must be specified before using predict. After you specify ProcessNoise as a matrix for the first time, to then change ProcessNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the process noise terms and all the terms have the same variance. The software extends the scalar to a W-by-W diagonal matrix.

ProcessNoise is a tunable property. You can change it using dot notation.

#### State — State of nonlinear system

[] (default) | vector

State of the nonlinear system, specified as a vector of size *Ns*, where *Ns* is the number of states of the system.

When you use the predict command, State is updated with the predicted value at time step k using the state value at time step k-1. When you use the correct command, State is updated with the estimated value at time step k using measured data at time step k.

The initial value of State is the value you specify in the InitialState input argument during object creation. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned. If you want a filter with single-precision floating-point variables, you must specify State as a single-precision variable during object construction using the InitialState input argument.

State is a tunable property. You can change it using dot notation.

#### **StateCovariance** — **State estimation error covariance**

1 (default) | scalar | matrix

State estimation error covariance, specified as a scalar or an Ns-by-Ns matrix, where Ns is the number of states of the system. If you specify a scalar, the software uses the scalar value to create an Ns-by-Ns diagonal matrix.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in the InitialState input argument.

When you use the predict command, StateCovariance is updated with the predicted value at time step k using the state value at time step k-1. When you use the correct

command, StateCovariance is updated with the estimated value at time step k using measured data at time step k.

StateCovariance is a tunable property. You can change it using dot notation after using the correct or predict commands.

#### StateTransitionFcn — State transition function

function handle

State transition function f, specified as a function handle. The function calculates the Ns-element state vector of the system at time step k, given the state vector at time step k-1. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

• HasAdditiveProcessNoise is true — The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at previous time step:

$$x(k) = f(x(k-1), Us1, \dots, Usn)$$

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.

 HasAdditiveProcessNoise is false — The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$x(k) = f(x(k-1),w(k-1),Us1,...,Usn)$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

StateTransitionFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## **StateTransitionJacobianFcn** — **Jacobian of state transition function** [] (default) | function handle

Jacobian of state transition function f, specified as one of the following:

- [] The Jacobian is numerically computed at every call to the predict command. This may increase processing time and numerical inaccuracy of the state estimation.
- function handle You write and save the Jacobian function and specify the handle to the function. For example, if vdpStateJacobianFcn.m is the Jacobian function, specify StateTransitionJacobianFcn as @vdpStateJacobianFcn.

The function calculates the partial derivatives of the state transition function with respect to the states and process noise. The number of inputs to the Jacobian function must equal the number of inputs of the state transition function and must be specified in the same order in both functions. The number of outputs of the function depends on the HasAdditiveProcessNoise property:

- HasAdditiveProcessNoise is true The function calculates the partial derivative of the state transition function with respect to the states  $(\partial f/\partial x)$ . The output is an *Ns*-by-*Ns* Jacobian matrix, where *Ns* is the number of states.
- HasAdditiveProcessNoise is false The function must also return a second output that is the partial derivative of the state transition function with respect to the process noise terms  $(\partial f/\partial w)$ . The second output is returned as an *Ns*-by-*W* Jacobian matrix, where *W* is the number of process noise terms.

The extended Kalman filter algorithm uses the Jacobian to compute the state estimation error covariance.

To see an example of a Jacobian function for additive process noise, type edit vdpStateJacobianFcn at the command line.

StateTransitionJacobianFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## **Output Arguments**

#### obj — Extended Kalman filter object for online state estimation

extendedKalmanFilter object

Extended Kalman filter object for online state estimation, returned as an extendedKalmanFilter object. This object is created using the specified properties on page 2-290. Use the correct and predict commands to estimate the state and state estimation error covariance using the extended Kalman filter algorithm.

When you use predict, obj.State and obj.StateCovariance are updated with the predicted value at time step k using the state value at time step k-1. When you use correct, obj.State and obj.StateCovariance are updated with the estimated values at time step k using measured data at time step k.

## **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using MATLAB® Coder™.

Usage notes and limitations:

For more information, see "Generate Code for Online State Estimation in MATLAB".

Supports MATLAB Function block: No

### See Also

#### **Functions**

clone | correct | kalman | kalmd | predict | residual | unscentedKalmanFilter

#### **Blocks**

Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

#### **Topics**

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"

#### Introduced in R2016b

<sup>&</sup>quot;Generate Code for Online State Estimation in MATLAB"

<sup>&</sup>quot;Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

<sup>&</sup>quot;Validate Online State Estimation at the Command Line"

<sup>&</sup>quot;Troubleshoot Online State Estimation"

## fcat

Concatenate FRD models along frequency dimension

## **Syntax**

```
sys = fcat(sys1, sys2,...)
```

## **Description**

sys = fcat(sys1, sys2, ...) takes two or more frd models and merges their frequency responses into a single frd model sys. The resulting frequency vector is sorted by increasing frequency. The frequency vectors of sys1, sys2,... should not intersect. If the frequency vectors do intersect, use fdel to remove intersecting data from one or more of the models.

### See Also

fdel|frd|fselect|interp

Introduced in R2006a

## fdel

Delete specified data from frequency response data (FRD) models

## **Syntax**

```
sysout = fdel(sys, freq)
```

## **Description**

sysout = fdel(sys, freq) removes from the frd model sys the data nearest to the frequency values specified in the vector freq.

## **Input Arguments**

#### sys

frd model.

#### freq

Vector of frequency values.

## **Output Arguments**

#### sysout

frd model containing the data remaining in sys after removing the frequency points closest to the entries of freq.

## **Examples**

#### **Delete Specified Data from Frequency Response Data Model**

Create a frequency response data (FRD) model at specified frequencies from a transfer function model.

```
w = logspace(0,1,10);
sys = frd(tf([1],[1 1]),w)
sys =
    Frequency(rad/s)
                           Response
         1.0000
                      0.5000 - 0.5000i
         1.2915
                       0.3748 - 0.4841i
         1.6681
                       0.2644 - 0.4410i
         2.1544
                       0.1773 - 0.3819i
         2.7826
                       0.1144 - 0.3183i
         3.5938
                       0.0719 - 0.2583i
         4.6416
                       0.0444 - 0.2059i
         5.9948
                       0.0271 - 0.1623i
         7.7426
                       0.0164 - 0.1270i
        10.0000
                       0.0099 - 0.0990i
```

Continuous-time frequency response.

freq = [2, 3.5, 6];

w is a logarithmically-spaced grid of 10 frequency points between 1 and 10 rad/second.

Remove the data nearest 2, 3.5, and 6 rad/s from sys.

```
sys2 = fdel(sys, freq)
sys2 =
    Frequency(rad/s)
                            Response
                        0.5000 - 0.5000i
          1.0000
          1.2915
                        0.3748 - 0.4841i
                        0.2644 - 0.4410i
          1.6681
          2.7826
                        0.1144 - 0.3183i
          4.6416
                        0.0444 - 0.2059i
          7.7426
                        0.0164 - 0.1270i
         10.0000
                        0.0099 - 0.0990i
```

Continuous-time frequency response.

Note that you do not have to specify the exact frequency of the data to remove. The fdel command removes the data corresponding to frequencies that are nearest to the specified frequencies.

# **Tips**

- Use fdel to remove unwanted data (for example, outlier points) at specified frequencies.
- Use fdel to remove data at intersecting frequencies from frd models before merging them with fcat. fcat produces an error when you attempt to merge frd models that have intersecting frequency data.
- To remove data from an frd model within a range of frequencies, use fselect.

## See Also

fcat | frd | fselect

Introduced in R2010a

## feedback

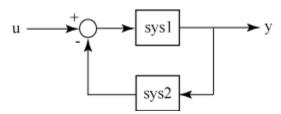
Feedback connection of multiple models

# **Syntax**

```
sys = feedback(sys1,sys2)
sys = feedback(sys1,sys2,feedin,feedout)
sys = feedback(sys1,sys2,'name')
sys = feedback(____,sign)
```

# **Description**

sys = feedback(sys1,sys2) returns a model object sys for the negative feedback
interconnection of model objects sys1,sys2.



From the figure, the closed-loop model sys has u as input vector and y as output vector. Both models, sys1 and sys2, must either be continuous or discrete with identical sample times.

sys = feedback(sys1,sys2,feedin,feedout) computes a closed-loop model sys
using the input and output connections specified using feedin and feedout. Use this
syntax when you want to connect only a subset of the available I/Os of MIMO systems.

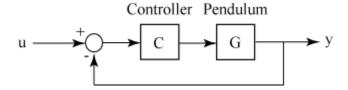
sys = feedback(sys1,sys2, 'name') computes a closed-loop model sys with the feedback connections specified by the respective I/O names of MIMO models sys1 and sys2. Use the 'name' flag only when all the required I/Os in the set of MIMO systems are properly named.

 $sys = feedback(\_\_, sign)$  returns a model object sys for a feedback loop with the type of feedback specified by sign. By default, feedback assumes negative feedback and is equivalent to feedback(sys1, sys2, -1). To compute the closed-loop system with positive feedback, use sign = +1.

# **Examples**

#### Plant and Controller with Unit Feedback

pendulumModelAndController.mat contains a SISO inverted pendulum transfer function model G and its associated PID controller C.



Load the inverted pendulum and controller model to the workspace.

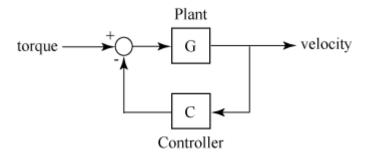
```
load('pendulumModelAndController','G','C');
size(G)
Transfer function with 1 outputs and 1 inputs.
size(C)
PID controller with 1 output and 1 input.
```

Use feedback to create the negative feedback loop with G and C.

sys is the resultant closed loop continuous-time transfer function obtained using negative feedback. feedback converts the PID controller model C to a transfer function before connecting it to the continuous-time transfer function model G. For more information, see "Rules That Determine Model Type".

#### Plant with Controller in Negative Feedback Path

For this example, consider two transfer functions that describe a plant G and controller C respectively.



$$G(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3} \qquad C(s) = \frac{5(s+2)}{s+10}$$

Create the plant and controller transfer functions.

```
G = tf([2 5 1],[1 2 3],'inputname',"torque",'outputname',"velocity");
C = tf([5,10],[1,10]);
```

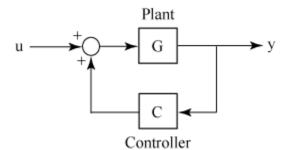
Use feedback to create the negative feedback loop using G and C.

Continuous-time transfer function.

sys is the resultant closed loop transfer function obtained using negative feedback with *torque* as the input and *velocity* as the output.

#### Positive Feedback Loop with Plant and Controller

For this example, consider two transfer functions that describe a plant G and controller C respectively.



$$G(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$
  $C(s) = \frac{5(s+2)}{s+10}$ 

Create the plant and controller transfer functions.

```
G = tf([2 5 1],[1 2 3],'inputname',"torque",'outputname',"velocity");
C = tf([5,10],[1,10]);
```

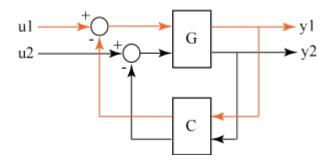
Use feedback to create the positive feedback loop using G and C.

Continuous-time transfer function.

sys is the resultant closed loop transfer function obtained using positive feedback with *torque* as the input and *velocity* as the output.

#### **Negative Feedback Loop with MIMO Systems**

Based on the figure below, consider connecting two MIMO transfer functions with two inputs and two outputs in a negative feedback loop.



For this example, create two random continuous state-space models using rss.

```
G = rss(4,2,2);
C = rss(2,2,2);
size(G)

State-space model with 2 outputs, 2 inputs, and 4 states.
size(C)

State-space model with 2 outputs, 2 inputs, and 2 states.
```

Use feedback to connect the two state-space models in a negative feedback loop according to the above figure.

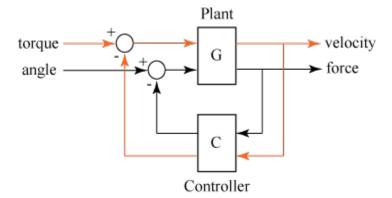
```
sys = feedback(G,C,-1);
size(sys)
State-space model with 2 outputs, 2 inputs, and 6 states.
```

The resulting state-space model sys is a 2 input, 2 output model with 6 states. The negative feedback loop is completed such that,

- The first output of G is connected to the first input of C
- The second output of G is connected to the second input of C

#### Feedback Loop Based on I/O Names

mimoPlantAndController.mat contains a 2 input, 2 output transfer function plant model G and a 2 input, 2 output transfer function controller model C to be connected as follows:



First, load the plant and controller models to the workspace.

```
load('mimoPlantAndController.mat','G','C');
size(G)
Transfer function with 2 outputs and 2 inputs.
size(C)
Transfer function with 2 outputs and 2 inputs.
```

By default, feedback would connect the first output of G to the first input of C and the second output of G to the second input of C. In order to connect the plant and controller according to the figure, name the respective I/Os of the two systems to ensure the correct connections.

#### G.InputName

```
ans = 2x1 cell array
{'torque'}
{'angle'}
```

#### G.OutputName

```
ans = 2x1 cell array
{'velocity'}
{'force' }
```

#### C.InputName

```
ans = 2x1 cell array
{'force' }
{'velocity'}
```

#### C.OutputName

```
ans = 2x1 cell array
{'angle' }
{'torque'}
```

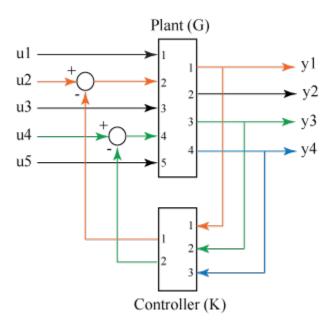
Then use the 'name' flag with the feedback command to make the connections according to the I/O names.

```
sys = feedback(G,C,'name');
```

The resulting closed loop negative feedback transfer function sys has the feedback connections in the required order.

### **Specify Input and Output Connections in a Feedback Loop**

Consider a state-space plant G with five inputs and four outputs and a state-space feedback controller K with three inputs and two outputs. The outputs 1, 3, and 4 of the plant G must be connected the controller K inputs, and the controller outputs to inputs 4 and 2 of the plant.



For this example, generate randomized continuous-time state-space models using rss for both G and K.

```
G = rss(3,4,5);

K = rss(3,2,3);
```

Define the feedout and feedin vectors based on the inputs and outputs to be connected in a feedback loop.

```
feedin = [4 2];
feedout = [1 3 4];
sys = feedback(G,K,feedin,feedout,-1);
size(sys)
State-space model with 4 outputs, 5 inputs, and 6 states.
```

 ${\tt sys}$  is the resultant closed loop state-space model obtained by connecting the specified inputs and outputs of  ${\tt G}$  and  ${\tt K}.$ 

# **Input Arguments**

#### sys1, sys2 — Systems to connect in a feedback loop

dvnamic system models

Systems to connect in a feedback loop, specified as dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models such as tf, zpk, pid, pidstd, or ss models.
- Frequency response models such as frd or genfrd.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

The resulting feedback loop assumes

- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.

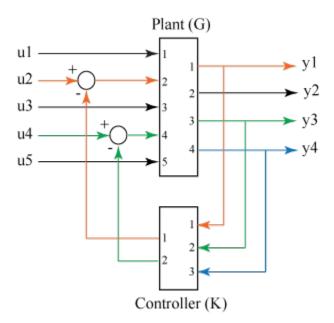
For more information, see dynamic system models.

When sys1 and sys2 are two different model types, feedback uses precedence rules to determine the resulting model sys. For example, when a state-space model and a transfer function is connected in a feedback loop, the resulting system is a state-space model based on the precedence rules. For more information, see "Rules That Determine Model Type".

### feedin — Subset of inputs to be used

vector

Subset of inputs to be used, specified as a vector.



From the figure, feedin contains indices of the input vector of MIMO plant P and specifies which subset of inputs *u* are involved in the feedback loop. The resulting model sys has the same inputs as G, with their orders preserved.

For an example, see "Specify Input and Output Connections in a Feedback Loop" on page 2-310.

### feedout — Subset of outputs to be used

vector

Subset of outputs to be used, specified as a vector.

feedout specifies which outputs of MIMO plant G are used for feedback. The resulting model sys has the same outputs as G, with their orders preserved.

For an example, see "Specify Input and Output Connections in a Feedback Loop" on page 2-310.

### sign — Type of feedback

-1 (default) | +1

Type of feedback, specified as -1 for negative feedback or +1 for positive feedback. feedback assumes negative feedback by default.

# **Output Arguments**

#### sys - Closed-loop system

dynamic system model

Closed-loop system, returned as a SISO, or MIMO dynamic system model. sys can be one of the following depending on the precedence rules:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, ss, pid, or pidstd models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

When sys1 and sys2 are two different model types, feedback uses precedence rules to determine the resulting model sys. For example, when a state-space model and a transfer function is connected in a feedback loop, the resulting system is a state-space model based on the precedence rules outlined in "Rules That Determine Model Type".

## Limitations

- The feedback connection must be free of algebraic loops. For instance, if  $D_1$  and  $D_2$  are the feedthrough matrices of sys1 and sys2, this condition is equivalent to:
  - $I + D_1D_2$  nonsingular when using negative feedback
  - $I D_1D_2$  nonsingular when using positive feedback

## **Tips**

For complicated feedback structures, use append and connect.

### See Also

append | connect | lft | parallel | series | sumblk

## **Topics**

"Rules That Determine Model Type"
"MIMO Feedback Loop"
"Using FEEDBACK to Close Feedback Loops"

### **Introduced before R2006a**

## filt

Specify discrete transfer functions in DSP format

# **Syntax**

```
sys = filt(num,den)
sys = filt(num,den,Ts)
sys = filt(M)
```

# **Description**

In digital signal processing (DSP), it is customary to write transfer functions as rational expressions in  $z^{-1}$  and to order the numerator and denominator terms in *ascending* powers of  $z^{-1}$ . For example:

$$H(z^{-1}) = \frac{2 + z^{-1}}{1 + 0.4z^{-1} + 2z^{-2}}$$

The function filt is provided to facilitate the specification of transfer functions in DSP format.

sys = filt(num, den) creates a discrete-time transfer function sys with numerator(s) num and denominator(s) den. The sample time is left unspecified (sys.Ts = -1) and the output sys is a TF object.

sys = filt(num, den, Ts) further specifies the sample time Ts (in seconds).

sys = filt(M) specifies a static filter with gain matrix M.

Any of the previous syntaxes can be followed by property name/property value pairs of the form

'Property', Value

Each pair specifies a particular property of the model, for example, the input names or the transfer function variable. For information about the available properties and their values, see the tf reference page.

## **Arguments**

For SISO transfer functions, num and den are row vectors containing the numerator and denominator coefficients ordered in ascending powers of  $z^{-1}$ . For example, den = [1 0.4 2] represents the polynomial  $1 + 0.4z^{-1} + 2z^{-2}$ .

MIMO transfer functions are regarded as arrays of SISO transfer functions (one per I/O channel), each of which is characterized by its numerator and denominator. The input arguments num and den are then cell arrays of row vectors such that:

- num and den have as many rows as outputs and as many columns as inputs.
- Their (i, j) entries num{i,j} and den{i,j} specify the numerator and denominator of the transfer function from input j to output i.

If all SISO entries have the same denominator, you can also set **den** to the row vector representation of this common denominator.

# **Examples**

# **Tips**

filt behaves as tf with the Variable property set to  $'z^-1'$ . See tf entry below for details.

## See Also

ss|tf|zpk

**Introduced before R2006a** 

## fnorm

Pointwise peak gain of FRD model

# **Syntax**

```
fnrm = fnorm(sys)
fnrm = fnorm(sys,ntype)
```

# **Description**

fnrm = fnorm(sys) computes the pointwise 2-norm of the frequency response
contained in the FRD model sys, that is, the peak gain at each frequency point. The
output fnrm is an FRD object containing the peak gain across frequencies.

fnrm = fnorm(sys,ntype) computes the frequency response gains using the matrix
norm specified by ntype. See norm for valid matrix norms and corresponding NTYPE
values.

### See Also

abs | norm

Introduced in R2006a

## **fourierBasis**

Fourier basis functions for tunable gain surface

# **Syntax**

```
shapefcn = fourierBasis(N)
shapefcn = fourierBasis(N,nvars)
shapefcn = fourierBasis( ,varnames)
```

# **Description**

You use basis function expansions to parameterize gain surfaces for tuning gain-scheduled controllers. fourierBasis generates periodic Fourier series expansions for parameterizing gain surfaces that depend periodically on the scheduling variables, such as a gain that varies with angular position. Use the output of fourierBasis to create tunable gain surfaces with tunableSurface.

shapefon = fourierBasis(N) generates a function that evaluates the first N harmonics of  $e^{i\pi x}$ :

```
F(x) = \left[\cos(\pi x), \sin(\pi x), \cos(2\pi x), \sin(2\pi x), ..., \cos(N\pi x), \sin(N\pi x)\right].
```

F is the function represented by shapefon. The term of F are the first 2\*N basis functions in the Fourier series expansion of a periodically varying gain, K(x), with K(-1) = K(1). That expansion is given by:

$$K(x) = \frac{a_0}{2} + \sum_{k} \{a_k \cos(k\pi x) + b_k \sin(k\pi x)\}.$$

shapefcn = fourierBasis(N,nvars) generates an nvars-dimensional Fourier basis for periodic functions on the region  $[-1,1]^{nvars}$ . This basis is the outer product of nvars Fourier bases with N harmonics along each dimension. The resulting function shapefcn takes nvars input arguments and returns a vector with  $(2*N+1)^{nvars-1}$  -1 entries.

To specify basis functions of multiple scheduling variables where the expansions are different for each variable, use ndBasis.

shapefcn = fourierBasis(\_\_\_\_, varnames) specifies variable names. Use this
syntax with any of the previous syntaxes to name the variables in shapefcn. Using
variable names improves readability of the tunableSurface object display and of any
MATLAB code you generate using codegen.

# **Examples**

#### Fourier Basis Functions of One Scheduling Variable

Create basis functions for a gain that varies as a periodic function of one scheduling variable.

```
shapefcn = fourierBasis(2);
```

shapefcn is a handle to a function of one variable that returns an array of four values corresponding to the first two harmonics of a periodic function on x = [-1,1]:

```
F(x) = [\cos(\pi x), \sin(\pi x), \cos(2\pi x), \sin(2\pi x)].
```

Use shapefcn as an input argument to tunableSurface to define a gain surface of the form:

```
K(x) = K_0 + K_1 \cos(\pi x) + K_2 \sin(\pi x) + K_3 \cos(2\pi x) + K_4 \sin(2\pi x).
```

The variable x is a normalized version of the scheduling variable for your tunable surface. Because the basis functions created by fourierBasis act on normalized variables, your gain-scheduled system must use design points whose endpoint values delineate exactly one period. For example, suppose you use the following design points:

```
alpha = [-7,-4,-1,2,5];
domain = struct('alpha',alpha);
K = tunableSurface('K',0,domain,shapefcn);
```

In normalizing the domain, the software assumes that the gain surface, K, is periodic in alpha such that K(-7) = K(5).

#### **Fourier Basis Functions in Higher Dimensions**

Create a two-dimensional Fourier basis for periodic functions of x and y on the domain  $[-1,1]^N$ . The basis functions should go up to the third harmonic in both the x and y dimensions.

```
F2D = fourierBasis(3,2);
```

This function is the outer product of two vectors:

```
x = fourierBasis(3);
y = fourierBasis(3);
```

Equivalently, you can obtain the outer product using ndBasis.

```
F = fourierBasis(3);
F2D = ndBasis(F,F);
```

The values in the vector returned by F include cross-terms such as  $\sin(\pi x)\cos(\pi y)$  and  $\sin(3\pi x)\cos(2\pi y)$ .

# **Input Arguments**

#### N — Number of harmonics of Fourier expansion

positive integer

Number of harmonics of Fourier expansion, specified as a positive integer.

#### nvars — Number of variables

```
1 (default) | positive integer
```

Number of scheduling variables, specified as a positive integer.

#### varnames — Variable names

```
{'x1', 'x2', ...} (default) | character vector | cell array of character vectors
```

Variable names in the generated function shapefon, specified as a:

- · Character vector, for monovariable basis functions.
- Cell array of character vectors, for multivariable basis functions.

If you do not specify varnames, then the variables in shapefon are named  $\{'x1', 'x2', \ldots\}$ .

Example: {'alpha','V'}

# **Output Arguments**

#### shapefcn — Fourier expansion

function handle

Fourier expansion, specified as a function handle. shapefcn takes as input arguments the number of variables specified by nvars. It returns a vector of polynomials in those variables, defined on the interval [-1,1] for each input variable. When you use shapefcn to create a gain surface, tunableSurface automatically generates tunable coefficients for each polynomial term in the vector.

# **Tips**

• If the gain surface K is periodic in the scheduling variable x with period P, make sure that the corresponding entry in K.Normalization.InputScaling is set to P/2 to ensure consistency with the fourierBasis period, P = 2. When using the default normalization, the x values in K.SamplingGrid must span exactly one period, [a, a +P], to satisfy this requirement. See the Normalization property of tunableSurface for more details.

### See Also

ndBasis | polyBasis | tunableSurface

Introduced in R2015b

## frd

Create frequency-response data model, convert to frequency-response data model

# **Syntax**

```
sys = frd(response,frequency)
sys = frd(response,frequency,Ts)
sys = frd
sysfrd = frd(sys,frequency)
sysfrd = frd(sys,frequency,units)
```

# **Description**

sys = frd(response, frequency) creates a frequency-response data (frd) model object sys from the frequency response data stored in the multidimensional array response. The vector frequency represents the underlying frequencies for the frequency response data. See Data Format for the Argument Response in FRD Models for a list of response data formats.

sys = frd(response, frequency, Ts) creates a discrete-time frd model object sys with scalar sample time Ts. Set Ts = -1 to create a discrete-time frd model object without specifying the sample time.

```
sys = frd creates an empty frd model object.
```

The input argument list for any of these syntaxes can be followed by property name/property value pairs of the form

```
'PropertyName',PropertyValue
```

You can use these extra arguments to set the various properties the model. For more information about available properties of frd models, see "Properties" on page 2-325.

To force an FRD model sys to inherit all of its generic LTI properties from any existing LTI model refsys, use the syntax

```
sys = frd(response, frequency, ltisys)
```

sysfrd = frd(sys,frequency) converts a dynamic system model sys to frequency
response data form. The frequency response is computed at the frequencies provided by
the vector frequency, in rad/TimeUnit, where TimeUnit is the time units of the input
dynamic system, specified in the TimeUnit property of sys.

sysfrd = frd(sys,frequency,units) converts a dynamic system model to an frd model and interprets frequencies in the frequency vector to have the units specified by units. For a list of values that units can take, see the FrequencyUnit property in "Properties" on page 2-325.

# **Arguments**

When you specify a SISO or MIMO FRD model, or an array of FRD models, the input argument frequency is always a vector of length Nf, where Nf is the number of frequency data points in the FRD. The specification of the input argument response is summarized in the following table.

#### **Data Format for the Argument Response in FRD Models**

Model Form	Response Data Format
SISO model	Vector of length Nf for which response(i) is the frequency response at the frequency frequency(i)
MIMO model with Ny outputs and Nu inputs	Ny-by-Nu-by-Nf multidimensional array for which response(i,j,k) specifies the frequency response from input j to output i at frequency frequency(k)
S1-byby-Sn array of models with Ny outputs and Nu inputs	Multidimensional array of size [Ny Nu S1 $\dots$ Sn] for which response (i,j,k,:) specifies the array of frequency response data from input j to output i at frequency frequency(k)

# **Properties**

frd objects have the following properties:

### **Frequency**

Frequency points of the frequency response data. Specify Frequency values in the units specified by the FrequencyUnit property.

#### FrequencyUnit

Frequency units of the model.

FrequencyUnit specifies the units of the frequency vector in the Frequency property. Set FrequencyUnit to one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

The units 'rad/TimeUnit' and 'cycles/TimeUnit' are relative to the time units specified in the TimeUnit property.

Changing this property changes the overall system behavior. Use chgFreqUnit to convert between frequency units without modifying system behavior.

Default: 'rad/TimeUnit'

#### ResponseData

Frequency response data.

The 'ResponseData' property stores the frequency response data as a 3-D array of complex numbers. For SISO systems, 'ResponseData' is a vector of frequency response values at the frequency points specified in the 'Frequency' property. For MIMO systems with Nu inputs and Ny outputs, 'ResponseData' is an array of size [Ny Nu Nw], where Nw is the number of frequency points.

### **IODelay**

Transport delays. IODelay is a numeric array specifying a separate transport delay for each input/output pair.

For continuous-time systems, specify transport delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify transport delays in integer multiples of the sample time, Ts.

For a MIMO system with Ny outputs and Nu inputs, set IODelay to a Ny-by-Nu array. Each entry of this array is a numerical value that represents the transport delay for the corresponding input/output pair. You can also set IODelay to a scalar value to apply the same delay to all input/output pairs.

**Default:** 0 for all input/output pairs

#### **InputDelay**

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

#### **Default:** 0

#### **OutputDelay**

Output delays. OutputDelay is a numeric vector specifying a time delay for each output channel. For continuous-time systems, specify output delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time Ts. For example, OutputDelay = 3 means a delay of three sampling periods.

For a system with Ny outputs, set OutputDelay to an Ny-by-1 vector, where each entry is a numerical value representing the output delay for the corresponding output channel. You can also set OutputDelay to a scalar value to apply the same delay to all channels.

**Default:** 0 for all output channels

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit

specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- · 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'vears'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### **InputName**

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** '' for all input channels

#### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### InputGroup

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

#### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all output channels

### **OutputUnit**

Output channel units, specified as one of the following:

- Character vector For single-output models. For example, 'seconds'.
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

**Default:** ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as

a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

### **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

#### Default: []

#### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

#### М

```
M(:,:,2,1) [zeta=0.35, w=5] =

25

s^2 + 3.5 s + 25
```

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

#### **Create Frequency-Response Model**

Create a SISO FRD model from a frequency vector and response data:

```
% generate a frequency vector and response data
freq = logspace(1,2);
resp = .05*(freq).*exp(i*2*freq);
% Create a FRD model
sys = frd(resp,freq);
```

### See Also

chgFreqUnit | chgTimeUnit | frdata | idfrd | set | ss | tf | zpk

### **Topics**

- "Create Frequency-Response Model from Data"
- "MIMO Frequency Response Data Models"
- "What Are Model Objects?"
- "Frequency Response Data (FRD) Models"

#### Introduced before R2006a

## frdata

Access data for frequency response data (FRD) object

# **Syntax**

```
[response,freq] = frdata(sys)
[response,freq,covresp] = frdata(sys)
[response,freq,Ts,covresp] = frdata(sys,'v')
[response,freq,Ts] = frdata(sys)
```

# **Description**

[response,freq] = frdata(sys) returns the response data and frequency samples
of the FRD model sys. For an FRD model with Ny outputs and Nu inputs at Nf
frequencies:

- response is an Ny-by-Nu-by-Nf multidimensional array where the (i,j) entry specifies the response from input j to output i.
- freq is a column vector of length Nf that contains the frequency samples of the FRD model.

See the frd reference page for more information on the data format for FRD response data.

[response,freq,covresp] = frdata(sys) also returns the covariance covresp of the response data resp for idfrd model sys. (Using idfrd models requires System Identification Toolbox software.) The covariance covresp is a 5D-array where covH(i,j,k,:,:) contains the 2-by-2 covariance matrix of the response resp(i,j,k). The (1,1) element is the variance of the real part, the (2,2) element the variance of the imaginary part and the (1,2) and (2,1) elements the covariance between the real and imaginary parts.

```
For SISO FRD models, the syntax
[response,freq] = frdata(sys,'v')
```

forces frdata to return the response data as a column vector rather than a 3-dimensional array (see example below). Similarly

[response, freq, Ts, covresp] = frdata(sys, 'v') for an IDFRD model sys returns covresp as a 3-dimensional rather than a 5-dimensional array.

```
[response, freq, Ts] = frdata(sys) also returns the sample time Ts.
```

Other properties of sys can be accessed with get or by direct structure-like referencing (e.g., sys.Frequency).

# **Arguments**

The input argument sys to frdata must be an FRD model.

# **Examples**

### **Extract Data from Frequency Response Data Model**

Create a frequency response data model by computing the response of a transfer function on a grid of frequencies.

```
H = tf([-1.2,-2.4,-1.5],[1,20,9.1]);
w = logspace(-2,3,101);
sys = frd(H,w);
```

sys is a SISO frequency response data (frd) model containing the frequency response at 101 frequencies.

Extract the frequency response data from sys.

```
[response,freq] = frdata(sys);
```

response is a 1-by-1-by-101 array. response(1,1,k) is the complex frequency response at the frequency freq(k).

# **See Also**

frd | freqresp | get | set

**Introduced before R2006a** 

# freqresp

Frequency response over grid

# **Syntax**

```
[H,wout] = freqresp(sys)
H = freqresp(sys,w)
H = freqresp(sys,w,units)
[H,wout,covH] = freqresp(idsys,...)
```

# **Description**

[H,wout] = freqresp(sys) returns the frequency response on page 2-341 of the dynamic system model sys at frequencies wout. The freqresp command automatically determines the frequencies based on the dynamics of sys.

H = freqresp(sys, w) returns the frequency response on page 2-341 on the real frequency grid specified by the vector w.

H = freqresp(sys,w,units) explicitly specifies the frequency units of w with units.

[H,wout,covH] = freqresp(idsys,...) also returns the covariance covH of the frequency response of the identified model idsys.

# **Input Arguments**

#### sys

Any dynamic system model or model array.

W

Vector of real frequencies at which to evaluate the frequency response. Specify frequencies in units of rad/TimeUnit, where TimeUnit is the time units specified in the TimeUnit property of sys.

#### units

Units of the frequencies in the input frequency vector w, specified as one of the following values:

- 'rad/TimeUnit' radians per the time unit specified in the TimeUnit property of sys
- 'cycles/TimeUnit' cycles per the time unit specified in the TimeUnit property of sys
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

Default: 'rad/TimeUnit'

#### idsys

Any identified model.

# **Output Arguments**

#### Н

Array containing the frequency response values.

If sys is an individual dynamic system model having Ny outputs and Nu inputs, H is a 3D array with dimensions Ny-by-Nu-by-Nw, where Nw is the number of frequency points. Thus, H(:,:,k) is the response at the frequency w(k) or wout(k).

If sys is a model array of size [Ny Nu S1 . . . Sn], H is an array with dimensions Ny-by-Nu-by-Nu-by-S1-by-...-by-Sn] array.

If sys is a frequency response data model (such as frd, genfrd, or idfrd), freqresp(sys,w) evaluates to NaN for values of w falling outside the frequency interval defined by sys.frequency. The freqresp command can interpolate between

frequencies in sys.frequency. However, freqresp cannot extrapolate beyond the frequency interval defined by sys.frequency.

#### wout

Vector of frequencies corresponding to the frequency response values in H. If you omit w from the inputs to freqresp, the command automatically determines the frequencies of wout based on the system dynamics. If you specify w, then wout = w

#### covH

Covariance of the response H. The covariance is a 5D array where covH(i,j,k,:,:) contains the 2-by-2 covariance matrix of the response from the ith input to the jth output at frequency w(k). The (1,1) element of this 2-by-2 matrix is the variance of the real part of the response. The (2,2) element is the variance of the imaginary part. The (1,2) and (2,1) elements are the covariance between the real and imaginary parts of the response.

## **Examples**

### **Compute Frequency Response of System**

Create the following 2-input, 2-output system:

$$sys = \begin{bmatrix} 0 & \frac{1}{s+1} \\ \frac{s-1}{s+2} & 1 \end{bmatrix}$$

```
sys11 = 0;
sys22 = 1;
sys12 = tf(1,[1 1]);
sys21 = tf([1 -1],[1 2]);
sys = [sys11,sys12;sys21,sys22];
```

Compute the frequency response of the system.

```
[H,wout] = freqresp(sys);
```

H is a 2-by-2-by-45 array. Each entry H(:,:,k) in H is a 2-by-2 matrix giving the complex frequency response of all input-output pairs of sys at the corresponding frequency

wout(k). The 45 frequencies in wout are automatically selected based on the dynamics of sys.

### **Compute Frequency Response on Specified Frequency Grid**

Create the following 2-input, 2-output system:

$$sys = \begin{bmatrix} 0 & \frac{1}{s+1} \\ \frac{s-1}{s+2} & 1 \end{bmatrix}$$

```
sys11 = 0;
sys22 = 1;
sys12 = tf(1,[1 1]);
sys21 = tf([1 -1],[1 2]);
sys = [sys11,sys12;sys21,sys22];
```

Create a logarithmically-spaced grid of 200 frequency points between 10 and 100 radians per second.

```
w = logspace(1,2,200);
```

Compute the frequency response of the system on the specified frequency grid.

```
H = freqresp(sys,w);
```

H is a 2-by-2-by-200 array. Each entry H(:,:,k) in H is a 2-by-2 matrix giving the complex frequency response of all input-output pairs of sys at the corresponding frequency w(k).

### **Compute Frequency Response and Associated Covariance**

Compute the frequency response and associated covariance for an identified process model at its peak response frequency.

Load estimation data z1.

```
load iddatal zl
```

Estimate a SISO process model using the data.

```
model = procest(z1, 'P2UZ');
```

Compute the frequency at which the model achieves the peak frequency response gain. To get a more accurate result, specify a tolerance value of 1e-6.

```
[gpeak, fpeak] = getPeakGain(model, 1e-6);
```

Compute the frequency response and associated covariance for model at its peak response frequency.

```
[H,wout,covH] = freqresp(model,fpeak);
```

H is the response value at fpeak frequency, and wout is the same as fpeak.

covH is a 5-dimensional array that contains the covariance matrix of the response from the input to the output at frequency fpeak. Here covH(1,1,1,1,1) is the variance of the real part of the response, and covH(1,1,1,2,2) is the variance of the imaginary part. The covH(1,1,1,1,2) and covH(1,1,1,2,1) elements are the covariance between the real and imaginary parts of the response.

## **More About**

### **Frequency Response**

In continuous time, the frequency response at a frequency  $\omega$  is the transfer function value at  $s = j\omega$ . For state-space models, this value is given by

$$H(i\omega) = D + C(i\omega I - A)^{-1}B$$

In discrete time, the frequency response is the transfer function evaluated at points on the unit circle that correspond to the real frequencies. freqresp maps the real frequencies w(1),..., w(N) to points on the unit circle using the transformation  $z=e^{j\omega T_s}$ .  $T_s$  is the sample time. The function returns the values of the transfer function at the resulting z values. For models with unspecified sample time, freqresp uses  $T_s=1$ .

# **Algorithms**

For transfer functions or zero-pole-gain models, freqresp evaluates the numerator(s) and denominator(s) at the specified frequency points. For continuous-time state-space models (A, B, C, D), the frequency response is

$$D + C(j\omega - A)^{-1}B$$
,  $\omega = \omega_1, ..., \omega_N$ 

For efficiency, A is reduced to upper Hessenberg form and the linear equation  $(j\omega - A)X = B$  is solved at each frequency point, taking advantage of the Hessenberg structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

## **Alternatives**

Use evalfr to evaluate the frequency response at individual frequencies or small numbers of frequencies. freqresp is optimized for medium-to-large vectors of frequencies.

## References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 407-408.

## **See Also**

bode | evalfr | interp | nichols | nyquist | sigma | spectrum

Introduced before R2006a

# freqsep

Slow-fast decomposition

## **Syntax**

```
[Gs,Gf] = freqsep(G,fcut)
[Gs,Gf] = freqsep(G,fcut,options)
```

# **Description**

[Gs,Gf] = freqsep(G,fcut) decomposes a linear dynamic system into slow and fast components around the specified cutoff frequency. The decomposition is such that G = Gs + Gf.

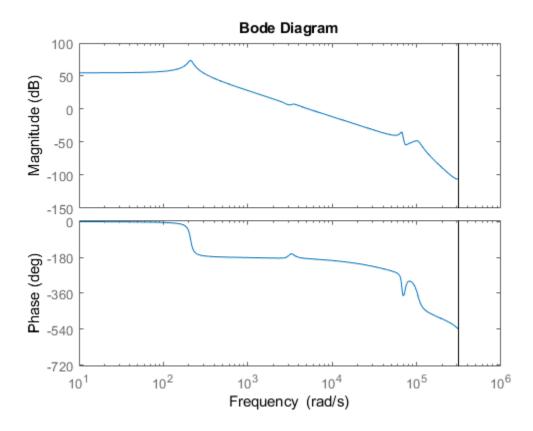
[Gs,Gf] = freqsep(G,fcut,options) specifies additional options for the decomposition.

# **Examples**

### **Decompose Model into Fast and Slow Dynamics**

Load a dynamic system model.

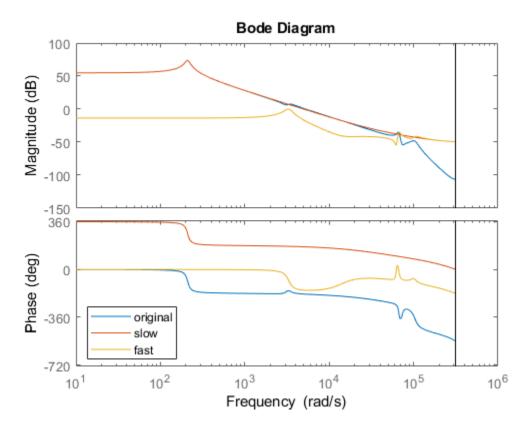
```
load numdemo Pd
bode(Pd)
```



Pd has four complex poles and one real pole. The Bode plot shows a resonance around 210 rad/s and a higher-frequency resonance below 10,000 rad/s.

Decompose this model around 1000 rad/s to separate these two resonances.

```
[Gs,Gf] = freqsep(Pd,10^3);
bode(Pd,Gs,Gf)
legend('original','slow','fast','Location','Southwest')
```



The Bode plot shows that the slow component, Gs, contains only the lower-frequency resonance. This component also matches the DC gain of the original model. The fast component, Gf, contains the higher-frequency resonances and matches the response of the original model at high frequencies. The sum of the two components Gs+Gf yields the original model.

## **Separate Nearby Modes by Adjusting Tolerance**

Decompose a model into slow and fast components between poles that are closely spaced.

The following system includes a real pole and a complex pair of poles that are all close to s = -2.

```
G = zpk(-.5, [-1.9999 -2+1e-4i -2-1e-4i], 10);
```

Try to decompose the model about 2 rad/s, so that the slow component contains the real pole and the fast component contains the complex pair.

```
[Gs,Gf] = freqsep(G,2);
```

Warning: One or more fast modes could not be separated from the slow modes. To force se

These poles are too close together for freqsep to separate. Increase the relative tolerance to allow the separation.

```
options = freqsepOptions('RelTol',1e-4);
[Gs,Gf] = freqsep(G,2,options);
```

Now freqsep successfully separates the dynamics about 2 rad/s.

```
slowpole = pole(Gs)
slowpole = -1.9999
fastpole = pole(Gf)
fastpole = 2×1 complex
    -2.0000 + 0.0001i
    -2.0000 - 0.0001i
```

## **Input Arguments**

### **G** — Dynamic system to decompose

numeric LTI model

Dynamic system to decompose, specified as a numeric LTI model, such as a ss or tf model.

### fcut — Cutoff frequency

positive scalar

Cutoff frequency for fast-slow decomposition, specified as a positive scalar. The output Gs contains all poles with natural frequency less than fcut. The output Gf contains all poles with natural frequency greater than or equal to fcut.

### options — Options for decomposition

freqsepOptions options set

Options for the decomposition, specified as an options set you create with freqsepOptions. Available options include absolute and relative tolerance for accuracy of the decomposed systems.

## **Output Arguments**

### Gs — Slow dynamics

numeric LTI model

Slow dynamics of the decomposed system, returned as a numeric LTI model of the same type as G. Gs contains all poles of G with natural frequency less than fcut, and is such that G = Gs + Gf.

### **Gf** — **Fast dynamics**

numeric LTI model

Fast dynamics of the decomposed system, returned as a numeric LTI model of the same type as G. Gf contains all poles of G with natural frequency greater than or equal to fcut, and is such that G = Gs + Gf.

# **Alternative Functionality**

## App

**Model Reducer** 

## **Live Editor Task**

Reduce Model Order

# **See Also**

Model Reducer | Reduce Model Order | freqsepOptions

## **Topics**

"Model Reduction Basics"

"Mode-Selection Model Reduction"

### Introduced in R2014a

# freqsepOptions

Options for slow-fast decomposition

## **Syntax**

```
opt = freqsep0ptions
opt = freqsep0ptions(Name, Value)
```

# **Description**

```
opt = freqsepOptions returns the default options for freqsep.
```

opt = freqsepOptions(Name, Value) returns an options set with the options
specified by one or more Name, Value pair arguments.

# **Examples**

## **Separate Nearby Modes by Adjusting Tolerance**

Decompose a model into slow and fast components between poles that are closely spaced.

The following system includes a real pole and a complex pair of poles that are all close to s = -2.

```
G = zpk(-.5,[-1.9999 -2+1e-4i -2-1e-4i],10);
```

Try to decompose the model about 2 rad/s, so that the slow component contains the real pole and the fast component contains the complex pair.

```
[Gs,Gf] = freqsep(G,2);
```

Warning: One or more fast modes could not be separated from the slow modes. To force so

These poles are too close together for freqsep to separate. Increase the relative tolerance to allow the separation.

## **Input Arguments**

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

```
Example: 'AbsTol', 1e-4
```

### AbsTol — Absolute tolerance for decomposition

0 (default) | nonnegative scalar

Absolute tolerance for slow-fast decomposition, specified as a nonnegative scalar value. freqresp ensures that the frequency responses of the original system, G, and the sum of the decomposed systems Gs+Gf, differ by no more than AbsTol + RelTol\*abs(G). Increase AbsTol to help separate nearby modes, at the expense of the accuracy of the decomposition.

## **RelTol** — **Relative tolerance for decomposition**

```
1e-8 (default) | nonnegative scalar
```

Relative tolerance for slow-fast decomposition, specified as a nonnegative scalar value. freqresp ensures that the frequency responses of the original system, G, and the sum of

the decomposed systems Gs+Gf, differ by no more than AbsTol + RelTol\*abs(G). Increase RelTol to help separate nearby modes, at the expense of the accuracy of the decomposition.

# **Output Arguments**

### opt — Options for freqsep

freqsepOptions options set

Options for freqsep, returned as a freqsepOptions options set. Use opt as the last argument to freqsep when computing slow-fast decomposition.

## See Also

freqsep

Introduced in R2014a

## **fselect**

Select frequency points or range in FRD model

# **Syntax**

```
subsys = fselect(sys,fmin,fmax)
subsys = fselect(sys,index)
```

# **Description**

subsys = fselect(sys,fmin,fmax) takes an FRD model sys and selects the portion
of the frequency response between the frequencies fmin and fmax. The selected range
[fmin,fmax] should be expressed in the FRD model units. For an IDFRD model
(requires System Identification Toolbox software), the SpectrumData, CovarianceData
and NoiseCovariance values, if non-empty, are also selected in the chosen range.

subsys = fselect(sys,index) selects the frequency points specified by the vector of
indices index. The resulting frequency grid is

```
sys.Frequency(index)
```

## See Also

fcat | fdel | frd | interp

Introduced before R2006a

## gcare

(Not recommended) Generalized solver for continuous-time algebraic Riccati equation

**Note** gcare is not recommended. Use icare instead. For more information, see "Compatibility Considerations".

## **Syntax**

```
[X,L,report] = gcare(H,J,ns)
[X1,X2,D,L] = gcare(H,...,'factor')
```

## **Description**

[X,L,report] = gcare(H,J,ns) computes the unique stabilizing solution X of the continuous-time algebraic Riccati equation associated with a Hamiltonian pencil of the form

$$H - tJ = \begin{bmatrix} A & F & S1 \\ G & -A' & -S2 \\ S2' & S1' & R \end{bmatrix} - \begin{bmatrix} E & 0 & 0 \\ 0 & E' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The optional input ns is the row size of the A matrix. Default values for J and ns correspond to E = I and R = [ ].

Optionally, gcare returns the vector L of closed-loop eigenvalues and a diagnosis report with value:

- -1 if the Hamiltonian pencil has jw-axis eigenvalues
- -2 if there is no finite stabilizing solution X
- 0 if a finite stabilizing solution X exists

This syntax does not issue any error message when X fails to exist.

[X1,X2,D,L] = gcare(H,..., 'factor') returns two matrices X1, X2 and a diagonal scaling matrix D such that X = D\*(X2/X1)\*D. The vector L contains the closed-loop

eigenvalues. All outputs are empty when the associated Hamiltonian matrix has eigenvalues on the imaginary axis.

# **Compatibility Considerations**

## gcare is not recommended

Not recommended starting in R2019a

Starting in R2019a, use the icare command to solve continuous-time Riccati equations. This approach has improved accuracy through better scaling and the computation of K is more accurate when R is ill-conditioned relative to gcare. Furthermore, icare includes an optional info structure to gather the implicit solution data of the Riccati equation.

The following table shows some typical uses of gcare and how to update your code to use icare instead.

Not Recommended	Recommended
<pre>[X,L] = gcare(H,J,NS)</pre>	[X,K,L] = icare(A,B,Q,R,S,E,G) computes the stabilizing solution X, the state-feedback gain K and the closed-loop eigenvalues L of the continuous-time algebraic Riccati equation. For more information, see icare.
<pre>[X,L,report] = gcare(H,J,NS)</pre>	[X,K,L,info] = icare(A,B,Q,R,S,E,G) computes the stabilizing solution X, the state-feedback gain K, the closed-loop eigenvalues L of the continuous-time algebraic Riccati equation. The info structure contains the implicit solution data. For more information, see icare.

There are no plans to remove gcare at this time.

## See Also

icare

### **Introduced before R2006a**

# gdare

(Not recommended) Generalized solver for discrete-time algebraic Riccati equation

**Note** gdare is not recommended. Use idare instead. For more information, see "Compatibility Considerations".

## **Syntax**

```
[X,L,report] = gdare(H,J,ns)
[X1,X2,D,L] = gdare(H,J,NS,'factor')
```

## **Description**

[X,L,report] = gdare(H,J,ns) computes the unique stabilizing solution X of the discrete-time algebraic Riccati equation associated with a Symplectic pencil of the form

$$H - tJ = \begin{bmatrix} A & F & B \\ -Q & E' & -S \\ S' & 0 & R \end{bmatrix} - \begin{bmatrix} E & 0 & 0 \\ 0 & A' & 0 \\ 0 & B' & 0 \end{bmatrix}$$

The third input ns is the row size of the *A* matrix.

Optionally, gdare returns the vector L of closed-loop eigenvalues and a diagnosis report with value:

- -1 if the Symplectic pencil has eigenvalues on the unit circle
- -2 if there is no finite stabilizing solution X
- ullet 0 if a finite stabilizing solution X exists

This syntax does not issue any error message when X fails to exist.

[X1,X2,D,L] = gdare(H,J,NS,'factor') returns two matrices X1, X2 and a diagonal scaling matrix D such that X = D\*(X2/X1)\*D. The vector L contains the closed-loop eigenvalues. All outputs are empty when the Symplectic pencil has eigenvalues on the unit circle.

# **Compatibility Considerations**

## gdare not recommended

Not recommended starting in R2019a

Starting in R2019a, use the idare command to solve discrete-time Riccati equations. This approach has improved accuracy through better scaling and the computation of K is more accurate when R is ill-conditioned relative to gdare. Furthermore, idare includes an optional info structure to gather the implicit solution data of the Riccati equation.

The following table shows some typical uses of gdare and how to update your code to use idare instead.

Not Recommended	Recommended
[X,L] = gdare(H,J,NS)	[X,K,L] = idare(A,B,Q,R,S,E) computes the stabilizing solution X, the state-feedback gain K and the closed-loop eigenvalues L of the discrete-time algebraic Riccati equation. For more information, see idare.
<pre>[X,L,report] = gdare(H,J,NS)</pre>	[X,K,L,info] = idare(A,B,Q,R,S,E) computes the stabilizing solution X, the state-feedback gain K, the closed-loop eigenvalues L of the discrete-time algebraic Riccati equation. The info structure contains the implicit solution data. For more information, see idare.

There are no plans to remove gdare at this time.

## **See Also**

idare

Introduced before R2006a

# genfrd

Generalized frequency response data (FRD) model

# **Description**

Generalized FRD (genfrd) models arise when you combine numeric FRD models with models containing tunable components (Control Design Blocks). genfrd models keep track of how the tunable blocks interact with the tunable components. For more information about Control Design Blocks, see "Generalized Models".

## Construction

To construct a genfrd model, use series, parallel, lft, or connect, or the arithmetic operators +, -, \*, /, and  $^$ , to combine a numeric FRD model with control design blocks.

You can also convert any numeric LTI model or control design block sys to genfrd form.

frdsys = genfrd(sys, freqs, frequnits) converts any static model or dynamic system sys to a generalized FRD model. If sys is not an frd model object, genfrd computes the frequency response of each frequency point in the vector freqs. The frequencies freqs are in the units specified by the optional argument frequnits. If frequnits is omitted, the units of freqs are 'rad/TimeUnit'.

frdsys = genfrd(sys, freqs, frequnits, timeunits) further specifies the time
units for converting sys to genfrd form.

For more information about time and frequency units of **genfrd** models, see "Properties" on page 2-360.

## **Input Arguments**

#### SYS

A static model or dynamic system model object.

### freqs

Vector of frequency points. Express frequencies in the unit specified in frequnits.

### frequnits

Frequency units of the genfrd model, specified as one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

Default: 'rad/TimeUnit'

#### **timeunits**

Time units of the genfrd model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Default: 'seconds'

## **Properties**

#### **Blocks**

Structure containing the control design blocks included in the generalized LTI model or generalized matrix. The field names of Blocks are the Name property of each control design block.

You can change some attributes of these control design blocks using dot notation. For example, if the generalized LTI model or generalized matrix M contains a realp tunable parameter a, you can change the current value of a using:

```
M.Blocks.a.Value = -1;
```

### **Frequency**

Frequency points of the frequency response data. Specify Frequency values in the units specified by the FrequencyUnit property.

### FrequencyUnit

Frequency units of the model.

FrequencyUnit specifies the units of the frequency vector in the Frequency property. Set FrequencyUnit to one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

The units 'rad/TimeUnit' and 'cycles/TimeUnit' are relative to the time units specified in the TimeUnit property.

Changing this property changes the overall system behavior. Use chgFreqUnit to convert between frequency units without modifying system behavior.

### Default: 'rad/TimeUnit'

### **InputDelay**

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

#### **Default:** 0

### **OutputDelay**

Output delays. OutputDelay is a numeric vector specifying a time delay for each output channel. For continuous-time systems, specify output delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time Ts. For example, OutputDelay = 3 means a delay of three sampling periods.

For a system with Ny outputs, set OutputDelay to an Ny-by-1 vector, where each entry is a numerical value representing the output delay for the corresponding output channel. You can also set OutputDelay to a scalar value to apply the same delay to all channels.

**Default:** 0 for all output channels

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- ullet Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- · Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

### OutputUnit

Output channel units, specified as one of the following:

- $\bullet \quad \text{Character vector} \text{For single-output models. For example, 'seconds'}.$
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

## **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

Default: Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ' '

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

## **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

### Default: []

### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

```
М
```

. . .

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Tips**

 You can manipulate genfrd models as ordinary frd models. Frequency-domain analysis commands such as bode evaluate the model by replacing each tunable parameter with its current value.

## See Also

chgFreqUnit | frd | genss | getValue

## **Topics**

"Models with Tunable Coefficients"

"Generalized Models"

Introduced in R2011a

# genmat

Generalized matrix with tunable parameters

# **Description**

Generalized matrices (genmat) are matrices that depend on tunable parameters (see realp). You can use generalized matrices for parameter studies. You can also use generalized matrices for building generalized LTI models (see genss) that represent control systems having a mixture of fixed and tunable components.

## Construction

Generalized matrices arise when you combine numeric values with static blocks such as realp objects. You create such combinations using any of the arithmetic operators +, -, \*, /, and  $^$ . For example, if a and b are tunable parameters, the expression M = a + b is represented as a generalized matrix.

The internal data structure of the genmat object M keeps track of how M depends on the parameters a and b. The Blocks property of M lists the parameters a and b.

M = genmat(A) converts the numeric array or tunable parameter A into a genmat object.

## **Input Arguments**

#### Α

Static control design block, such as a realp object.

If A is a numeric array, M is a generalized matrix of the same dimensions as A, with no tunable parameters.

If A is a static control design block, M is a generalized matrix whose Blocks property lists A as the only block.

## **Properties**

#### **Blocks**

Structure containing the control design blocks included in the generalized LTI model or generalized matrix. The field names of Blocks are the Name property of each control design block.

You can change some attributes of these control design blocks using dot notation. For example, if the generalized LTI model or generalized matrix M contains a realp tunable parameter a, you can change the current value of a using:

```
M.Blocks.a.Value = -1;
```

### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

M
M(:,:,1,1) [zeta=0.3, w=5] =

25

5^2 + 3 s + 25

M(:,:,2,1) [zeta=0.35, w=5] =

25

5^2 + 3.5 s + 25

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

### Default: []

#### Name

System name, specified as a character vector. For example, 'mat\_1'. When you convert a static control design block such as tunableSurface to a generalized matrix using genmat(blk), the Name property of the block is preserved.

Default: ''

# **Examples**

#### **Generalized Matrix With Two Tunable Parameters**

This example shows how to use algebraic combinations of tunable parameters to create the generalized matrix:

$$M = \begin{bmatrix} 1 & a+b \\ 0 & ab \end{bmatrix},$$

where *a* and *b* are tunable parameters with initial values -1 and 3, respectively.

**1** Create the tunable parameters using realp.

```
a = realp('a',-1);
b = realp('b',3);
```

2 Define the generalized matrix using algebraic expressions of a and b.

```
M = [1 a+b;0 a*b]
```

M is a generalized matrix whose Blocks property contains a and b. The initial value of M is  $M = [1 \ 2; 0 \ -3]$ , from the initial values of a and b.

**3** (Optional) Change the initial value of the parameter **a**.

```
M.Blocks.a.Value = -3;
```

**4** (Optional) Use **double** to display the new value of M.

```
double(M)
```

The new value of M is  $M = [1 \ 0; 0 \ -9]$ .

## See Also

genss | getValue | realp

## **Topics**

"Models with Tunable Coefficients"

#### Introduced in R2011a

<sup>&</sup>quot;Dynamic System Models"

# gensig

Generate test input signals for lsim

## **Syntax**

```
[u,t] = gensig(type,tau)
[u,t] = gensig(type,tau,Tf,Ts)
```

# **Description**

[u,t] = gensig(type,tau) generates a scalar signal u of class type and with period tau (in seconds). The following types of signals are available.

'sin'	Sine wave.
'square'	Square wave.
'pulse'	Periodic pulse.

gensig returns a vector t of time samples and the vector u of signal values at these samples. All generated signals have unit amplitude.

[u,t] = gensig(type,tau,Tf,Ts) also specifies the time duration Tf of the signal and the spacing Ts between the time samples t.

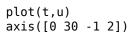
You can feed the outputs u and t directly to lsim and simulate the response of a single-input linear system to the specified signal. Since t is uniquely determined by Tf and Ts, you can also generate inputs for multi-input systems by repeated calls to gensig.

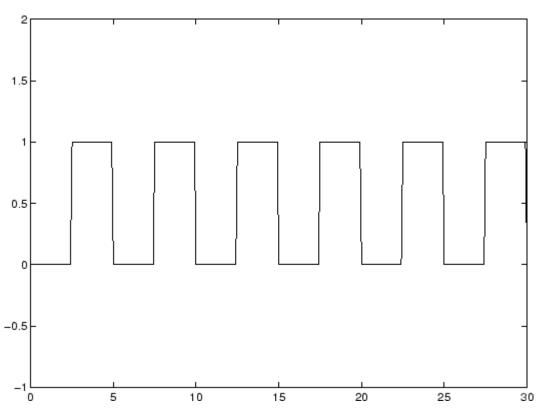
# **Examples**

Generate a square wave with period 5 seconds, duration 30 seconds, and sampling every 0.1 second.

```
[u,t] = gensig('square',5,30,0.1)
```

Plot the resulting signal.





# **See Also**

lsim

**Introduced before R2006a** 

## genss

Generalized state-space model

# **Description**

Generalized state-space (genss) models are state-space models that include tunable parameters or components. genss models arise when you combine numeric LTI models with models containing tunable components (control design blocks). For more information about numeric LTI models and control design blocks, see "Models with Tunable Coefficients".

You can use generalized state-space models to represent control systems having a mixture of fixed and tunable components. Use generalized state-space models for control design tasks such as parameter studies and parameter tuning with commands such as systune and looptune.

## Construction

To construct a genss model:

- Use series, parallel, lft, or connect, or the arithmetic operators +, -, \*, /, \, and ^, to combine numeric LTI models with control design blocks.
- Use tf or ss with one or more input arguments that is a generalized matrix (genmat) instead of a numeric array
- Convert any numeric LTI model, control design block, or slTuner interface (requires Simulink Control Design), for example, sys, to genss form using:

```
gensys = genss(sys)
```

When sys is an slTuner interface, gensys contains all the tunable blocks and analysis points specified in this interface. To compute a tunable model of a particular I/O transfer function, call getIOTransfer(gensys,in,out). Here, in and out are the analysis points of interest. (Use getPoints(sys) to get the full list of analysis points.) Similarly, to compute a tunable model of a particular open-loop transfer function, use getLoopTransfer(gensys,loc). Here, loc is the analysis point of interest.

## **Properties**

#### **Blocks**

Structure containing the control design blocks included in the generalized LTI model or generalized matrix. The field names of Blocks are the Name property of each control design block.

You can change some attributes of these control design blocks using dot notation. For example, if the generalized LTI model or generalized matrix M contains a realp tunable parameter a, you can change the current value of a using:

```
M.Blocks.a.Value = -1;
```

#### A,B,C,D

Dependency of state-space matrices on tunable and uncertain parameters, stored as a generalized matrix (genmat), uncertain matrix (umat), or double array.

These properties model the dependency of the state-space matrices on static control design blocks, realp, ureal, ucomplex, or ucomplexm. Dynamic control design blocks such as tunableGain or tunableSS set to their current values, and internal delays are set to zero.

When the corresponding state-space matrix does not depend on any static control design blocks, these properties evaluate to double matrices.

For an example, see "Dependence of State-Space Matrices on Parameters" on page 2-388.

#### Ε

E matrix, stored as a double matrix when the generalized state-space equations are implicit. The value E = [] means that the generalized state-space equations are explicit. For more information about implicit state-space models, see "State-Space Models".

#### **StateName**

State names, stored as one of the following:

- $\bullet \quad \hbox{Character vector} \hbox{For first-order models, for example, '} \\ \hbox{velocity'}.$
- Cell array of character vectors For models with two or more states, for example, {'position';'velocity'}.

• '' — For unnamed states.

You can assign state names to a genss model only when all its control design blocks are static. Otherwise, specify the state names for the component models before interconnecting them to create the genss model. When you do so, the genss model tracks the assigned state names. For an example, see "Track State Names in Generalized State-Space Model" on page 2-387.

Default: ' ' for all states

#### StateUnit

State unit labels, stored as one of the following:

- Character vector For first-order models, for example, 'm/s'.
- Cell array of character vectors For models with two or more states, for example, {'m'; 'm/s'}.
- '' For unnamed states.

StateUnit labels the units of each state for convenience, and has no effect on system behavior.

You can assign state units to a genss model only when all its control design blocks are static. Otherwise, specify the state units for the component models before interconnecting them to create the genss model. When you do so, the genss model tracks the assigned state units. For an example, see "Track State Names in Generalized State-Space Model" on page 2-387.

**Default:** ' ' for all states

### **InternalDelay**

Vector storing internal delays.

Internal delays arise, for example, when closing feedback loops on systems with delays, or when connecting delayed systems in series or parallel. For more information about internal delays, see "Closing Feedback Loops with Time Delays".

For continuous-time models, internal delays are expressed in the time unit specified by the TimeUnit property of the model. For discrete-time models, internal delays are expressed as integer multiples of the sample time Ts. For example, InternalDelay = 3 means a delay of three sampling periods.

You can modify the values of internal delays. However, the number of entries in sys.InternalDelay cannot change, because it is a structural property of the model.

### **InputDelay**

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

#### Default: 0

### **OutputDelay**

Output delays. OutputDelay is a numeric vector specifying a time delay for each output channel. For continuous-time systems, specify output delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time Ts. For example, OutputDelay = 3 means a delay of three sampling periods.

For a system with Ny outputs, set OutputDelay to an Ny-by-1 vector, where each entry is a numerical value representing the output delay for the corresponding output channel. You can also set OutputDelay to a scalar value to apply the same delay to all channels.

**Default:** 0 for all output channels

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- · Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: '' for all output channels

### OutputUnit

Output channel units, specified as one of the following:

- $\bullet \quad \hbox{Character vector} \hbox{For single-output models. For example, 'seconds'}.$
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ' '

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

### **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

### Default: []

### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

```
М
```

. . .

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

#### **Tunable Low-Pass Filter**

In this example, you will create a low-pass filter with one tunable parameter *a*:

$$F = \frac{a}{s+a}$$

Since the numerator and denominator coefficients of a tunableTF block are independent, you cannot use tunableTF to represent F. Instead, construct F using the tunable real parameter object realp.

Create a real tunable parameter with an initial value of 10.

Use tf to create the tunable low-pass filter F.

```
numerator = a;
denominator = [1,a];
F = tf(numerator,denominator)

F =

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 1 states, and a: Scalar parameter, 2 occurrences.
```

Type "ss(F)" to see the current value, "get(F)" to see all properties, and "F.Blocks"

F is a genss object which has the tunable parameter a in its Blocks property. You can connect F with other tunable or numeric models to create more complex control system models. For an example, see "Control System with Tunable Components".

### **Create State-Space Model with Both Fixed and Tunable Parameters**

This example shows how to create a state-space genss model having both fixed and tunable parameters.

$$A = \begin{bmatrix} 1 & a + b \\ 0 & ab \end{bmatrix}, \quad B = \begin{bmatrix} -3.0 \\ 1.5 \end{bmatrix}, \quad C = [0.3 \ 0], \quad D = 0,$$

where a and b are tunable parameters, whose initial values are -1 and 3, respectively.

Create the tunable parameters using realp.

```
a = realp('a',-1);
b = realp('b',3);
```

Define a generalized matrix using algebraic expressions of a and b.

```
A = [1 a+b;0 a*b];
```

A is a generalized matrix whose Blocks property contains a and b. The initial value of A is  $[1\ 2;0\ -3]$ , from the initial values of a and b.

Create the fixed-value state-space matrices.

```
B = [-3.0;1.5];
C = [0.3 0];
D = 0;
```

Use ss to create the state-space model.

```
sys = ss(A,B,C,D)
sys =

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 2 states, and
a: Scalar parameter, 2 occurrences.
b: Scalar parameter, 2 occurrences.
```

Type "ss(sys)" to see the current value, "get(sys)" to see all properties, and "sys.Blo

sys is a generalized LTI model (genss) with tunable parameters a and b. Confirm that the A property of sys is stored as a generalized matrix.

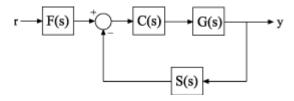
```
ans =
Generalized matrix with 2 rows, 2 columns, and the following blocks:
    a: Scalar parameter, 2 occurrences.
    b: Scalar parameter, 2 occurrences.
```

Type "double(ans)" to see the current value, "get(ans)" to see all properties, and "and

### **Control System Model With Both Numeric and Tunable Components**

This example shows how to create a tunable model of a control system that has both fixed plant and sensor dynamics and tunable control components.

Consider the control system of the following illustration.



Suppose that the plant response is  $G(s) = 1/(s+1)^2$ , and that the model of the sensor dynamics is S(s) = 5/(s+4). The controller C is a tunable PID controller, and the prefilter F = a/(s+a) is a low-pass filter with one tunable parameter, a.

Create models representing the plant and sensor dynamics. Because the plant and sensor dynamics are fixed, represent them using numeric LTI models.

```
G = zpk([],[-1,-1],1);

S = tf(5,[1 4]);
```

To model the tunable components, use Control Design Blocks. Create a tunable representation of the controller C.

```
C = tunablePID('C', 'PID');
```

C is a tunablePID object, which is a Control Design Block with a predefined proportional-integral-derivative (PID) structure.

Create a model of the filter F = a/(s + a) with one tunable parameter.

```
a = realp('a',10);
F = tf(a,[1 a]);
```

a is a realp (real tunable parameter) object with initial value 10. Using a as a coefficient in tf creates the tunable genss model object F.

Interconnect the models to construct a model of the complete closed-loop response from r to y.

T is a genss model object. In contrast to an aggregate model formed by connecting only numeric LTI models, T keeps track of the tunable elements of the control system. The tunable elements are stored in the Blocks property of the genss model object. Examine the tunable elements of T.

#### T.Blocks

```
ans = struct with fields:
    C: [1x1 tunablePID]
```

```
a: [1x1 realp]
```

When you create a genss model of a control system that has tunable components, you can use tuning commands such as systune to tune the free parameters to meet design requirements you specify.

### Track State Names in Generalized State-Space Model

Create a genss model with labeled state names. To do so, label the states of the component LTI models before connecting them. For instance, connect a two-state fixed-coefficient plant model and a one-state tunable controller.

```
A = [-1 -1; 1 0];
B = [1; 0];
C = [0 1];
D = 0;
G = ss(A,B,C,D);
G.StateName = {'Pstate1', 'Pstate2'};
C = tunableSS('C',1,1,1);
L = G*C;
```

The genss model L preserves the state names of the components that created it. Because you did not assign state names to the tunable component C, the software automatically does so. Examine the state names of L to confirm them.

#### L.StateName

```
ans = 3x1 cell array
     {'Pstate1'}
     {'Pstate2'}
     {'C.x1' }
```

The automatic assignment of state names to control design blocks allows you to trace which states in the generalized model are contributed by tunable components.

State names are also preserved when you convert a genss model to a fixed-coefficient state-space model. To confirm, convert L to ss form.

```
Lfixed = ss(L);
Lfixed.StateName

ans = 3x1 cell array
     {'Pstate1'}
     {'Pstate2'}
     {'C.x1'}
```

State unit labels, stored in the StateUnit property of the genss model, behave similarly.

### **Dependence of State-Space Matrices on Parameters**

Create a generalized model with a tunable parameter, and examine the dependence of the A matrix on that parameter. To do so, examine the A property of the generalized model.

```
G = tf(1,[1 10]);
k = realp('k',1);
F = tf(k,[1 k]);
L1 = G*F;
L1.A

ans =

Generalized matrix with 2 rows, 2 columns, and the following blocks:
    k: Scalar parameter, 2 occurrences.
```

The A property is a generalized matrix that preserves the dependence on the real tunable parameter k. The state-space matrix properties A, B, C, and D only preserve dependencies on static parameters. When the genss model has dynamic control design blocks, these are set to their current value for evaluating the state-space matrix properties. For example, examine the A matrix property of a genss model with a tunable PI block.

Type "double(ans)" to see the current value, "get(ans)" to see all properties, and "ans

```
C = tunablePID('C','PI');
L2 = G*C;
L2.A
ans = 2×2
-10.0000 0.0010
```

0 0

Here, the A matrix is stored as a double matrix, whose value is the A matrix of the current value of L2.

```
L2cur = ss(L2);
L2cur.A
ans = 2 \times 2
-10.0000 0.0010
```

Additionally, extracting state-space matrices using ssdata sets all control design blocks to their current or nominal values, including static blocks. Thus, the following operations all return the current value of the A matrix of L1.

```
[A,B,C,D] = ssdata(L1);
Α
A = 2 \times 2
   - 10
            1
           - 1
double(L1.A)
ans = 2 \times 2
   - 10
            1
            -1
L1cur = ss(L1);
L1cur.A
ans = 2 \times 2
   - 10
            1
      0
            - 1
```

# **Tips**

 You can manipulate genss models as ordinary ss models. Analysis commands such as bode and step evaluate the model by replacing each tunable parameter with its current value.

## See Also

connect | feedback | genfrd | genmat | getValue | realp | ss | tf | tunablePID

## **Topics**

"Models with Tunable Coefficients"

"Dynamic System Models"

"Control Design Blocks"

Introduced in R2011a

# get

Access model property values

# **Syntax**

```
Value = get(sys,'PropertyName')
Struct = get(sys)
```

# **Description**

Value = get(sys,'PropertyName') returns the current value of the property PropertyName of the model object sys. 'PropertyName' can be the full property name (for example, 'UserData') or any unambiguous case-insensitive abbreviation (for example, 'user'). See reference pages for the individual model object types for a list of properties available for that model.

Struct = get(sys) converts the TF, SS, or ZPK object sys into a standard MATLAB structure with the property names as field names and the property values as field values.

Without left-side argument,

```
get(sys)
```

displays all properties of sys and their values.

# **Examples**

### **Display Model Property Values**

Create the following discrete-time SISO transfer function model:

$$H(z) = \frac{1}{z+2}$$

Specify the sample time as 0.1 seconds and input channel name as Voltage.

```
h = tf(1,[1 2],0.1,'InputName','Voltage')
h =
   From input "Voltage" to output:
     1
     ....
z + 2

Sample time: 0.1 seconds
Discrete-time transfer function.
```

Display all the properties of the transfer function.

get(h)

```
Numerator: {[0 1]}
Denominator: {[1 2]}
   Variable: 'z'
    IODelay: 0
  InputDelay: 0
 OutputDelay: 0
         Ts: 0.1000
   TimeUnit: 'seconds'
  InputName: {'Voltage'}
  InputUnit: {''}
  InputGroup: [1x1 struct]
  OutputName: {''}
  OutputUnit: {''}
 OutputGroup: [1x1 struct]
       Notes: [0x1 string]
   UserData: []
       Name: ''
SamplingGrid: [1x1 struct]
```

Display the numerator of the transfer function.

```
num = get(h,'Numerator')
num = 1x1 cell array
{1x2 double}
```

The numerator data is stored as a cell array, thus the Numerator property is a cell array containing the row vector  $[0 \ 1]$ .

```
\begin{array}{l} \text{num\{1\}} \\ \text{ans} = 1 \times 2 \\ \\ 0 \end{array}
```

Display the sample time Ts of the transfer function.

```
get(h, 'Ts')
ans = 0.1000
```

Alternatively, use dot notation to access the property value.

```
h.Ts
ans = 0.1000
```

# **Tips**

sys.user

```
An alternative to the syntax
```

```
Value = get(sys,'PropertyName')
is the structure-like referencing
Value = sys.PropertyName
For example,
sys.Ts
sys.A
```

return the values of the sample time, A matrix, and UserData property of the (state-space) model sys.

## **See Also**

frdata|idssdata|polydata|set|ssdata|tfdata|zpkdata

**Introduced before R2006a** 

# getBlockValue

Current value of Control Design Block in Generalized Model

# **Syntax**

```
val = getBlockValue(M,blockname)
[val1,val2,...] = getBlockValue(M,blockname1,blockname2,...)
S = getBlockValue(M)
```

# **Description**

val = getBlockValue(M, blockname) returns the current value of the Control Design
Block blockname in the Generalized Model M. (For uncertain blocks, the "current value"
is the nominal value of the block.)

[val1,val2,...] = getBlockValue(M,blockname1,blockname2,...) returns the values of the specified Control Design Blocks.

S = getBlockValue(M) returns the values of all Control Design Blocks of the generalized model in a structure. This syntax lets you transfer the block values from one generalized model to another model that uses the same Control Design Blocks, as follows:

```
S = getBlockValue(M1);
setBlockValue(M2,S);
```

## **Input Arguments**

#### М

Generalized LTI (genss) model or generalized matrix (genmat).

#### blockname

Name of the Control Design Block in the model M whose current value is evaluated.

To get a list of the Control Design Blocks in M, enter M.Blocks.

# **Output Arguments**

#### val

Numerical LTI model or numerical value, equal to the current value of the Control Design Block blockname.

#### S

Current values of all Control Design Blocks in M, returned as a structure. The names of the fields in S are the names of the blocks in M. The values of the fields are numerical LTI models or numerical values equal to the current values of the corresponding Control Design Blocks.

# **Examples**

### **Get Current Values of Single Blocks**

Create a tunable genss model, and evaluate the current value of the Control Design Blocks of the model.

Typically, you use getBlockValue to retrieve the tuned values of control design blocks after tuning the genss model using a tuning command such as systume. For this example, create the model and retrieve the initial block values.

Cval is a numeric pid controller object.

```
aval = getBlockValue(T,'a')
aval =
    10
```

aval is a numeric scalar, because a is a real scalar parameter.

### **Get All Current Values as Structure**

Using the genss model of the previous example, get the current values of all blocks in the model.

## See Also

getValue | setBlockValue | showBlockValue

## **Topics**

Generalized Model Control Design Block

#### Introduced in R2011b

# getCompSensitivity

Complementary sensitivity function from generalized model of control system

# **Syntax**

T = getCompSensitivity(CL,location)
T = getSensitivity(CL,location,opening)

# **Description**

T = getCompSensitivity(CL,location) returns the complementary sensitivity on page 2-402 measured at the specified location for a generalized model of a control system.

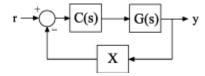
T = getSensitivity(CL,location,opening) specifies additional loop openings for the complementary sensitivity function calculation. Use an opening, for example, to calculate the complementary sensitivity function of an inner loop, with the outer loop open.

If opening and location list the same point, the software opens the loop after adding the disturbance signal at the point.

# **Examples**

### **Complementary Sensitivity Function at a Location**

Compute the complementary sensitivity at the plant output, X, of the control system of the following illustration.



Create a model of the system by specifying and connecting a numeric LTI plant model G, a tunable controller C, and the AnalysisPoint block X. Use the AnalysisPoint block to mark the location where you assess the complementary sensitivity, which in this example is the plant output.

```
G = tf([1],[1 5]);
C = tunablePID('C','p');
C.Kp.Value = 3;
X = AnalysisPoint('X');
CL = feedback(G*C,X);
```

 ${\sf CL}$  is a genss model that represents the closed-loop response of the control system from r to y. Examine the Control Design Blocks of the model.

#### CL.Blocks

```
ans = struct with fields:
    C: [1x1 tunablePID]
    X: [1x1 AnalysisPoint]
```

The model's blocks include the AnalysisPoint block, X, that identifies the analysis-point location.

Calculate the complementary sensitivity, T, at X.

```
T = getCompSensitivity(CL,'X')
T =
Generalized continuous-time state-space model with 1 outputs, 1 inputs, 1 states, and C: Parametric PID controller, 1 occurrences.
   X: Analysis point, 1 channels, 1 occurrences.
Type "ss(T)" to see the current value, "get(T)" to see all properties, and "T.Blocks" of the current value, "get(T)" to see all properties.

Type "ss(T)" to see the current value, "get(T)" to see all properties, and "T.Blocks" of the current value, "get(T)" to see all properties.
```

getCompSensitivity preserves the Control Design Blocks of CL, and returns a genss model. To get a numeric model, you can convert T to transfer-function form, using the current value of the tunable block.

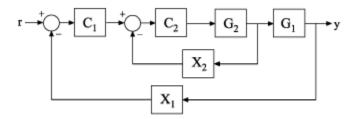
```
Tnum = tf(T)
Tnum =
  From input "X" to output "X":
```

```
-3
-----
s + 8
```

Continuous-time transfer function.

# Specify Additional Loop Opening for Complementary Sensitivity Function Calculation

In the multiloop system of the following illustration, calculate the inner-loop sensitivity at the output of G2, with the outer loop open.



Create a model of the system by specifying and connecting the numeric plant models, tunable controllers, and AnalysisPoint blocks. G1 and G2 are plant models, C1 and C2 are tunable controllers, and X1 and X2 are AnalysisPoint blocks that mark potential loop-opening locations.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
CL = feedback(G1*feedback(G2*C2,X2)*C1,X1);
```

Calculate the complementary sensitivity, *T*, at X2, with the outer loop open at X1. Specifying X1 as the third input argument tells getCompSensitivity to open the loop at that location.

# **Input Arguments**

### CL — Model of control system

generalized state-space model

Model of a control system, specified as a generalized state-space model (genss).

Locations at which you can perform sensitivity analysis or open loops are marked by AnalysisPoint blocks in CL. Use getPoints(CL) to get the list of such locations.

#### location — Location

character vector | cell array of character vectors

Location at which you calculate the complementary sensitivity function on page 2-402, specified as a character vector or cell array of character vectors. To extract the complementary sensitivity function at multiple locations, use a cell array of character vectors.

Each specified location must match an analysis point in CL. Analysis points are marked using AnalysisPoint blocks. To get the list of available analysis points in CL, use getPoints(CL).

```
Example: u' or \{u', y'\}
```

### opening — Additional loop opening

character vector | cell array of character vectors

Additional loop opening used to calculate the complementary sensitivity function on page 2-402, specified as a character vector or cell array of character vectors. To open the loop at multiple locations, use a cell array of character vectors.

Each specified opening must match an analysis point in CL. Analysis points are marked using AnalysisPoint blocks. To get the list of available analysis points in CL, use getPoints(CL).

Use an opening, for example, to calculate the complementary sensitivity function of an inner loop, with the outer loop open.

If opening and location list the same point, the software opens the loop after adding the disturbance signal at the point.

```
Example: 'y_outer' or {'y_outer', 'y_outer2'}
```

# **Output Arguments**

### T — Complementary sensitivity function

generalized state-space model

Complementary sensitivity function on page 2-402 of the control system, T, measured at location, returned as a generalized state-space model (genss).

- If location specifies a single analysis point, then T is a SISO genss model.
- If location is a vector signal, or specifies multiple analysis points, then T is a MIMO genss model.

### **More About**

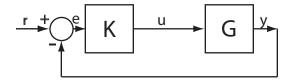
### **Complementary Sensitivity**

The complementary sensitivity function, T, at a point is the closed-loop transfer function around the feedback loop measured at the specified location. It is related to the open-loop transfer function, L, and the sensitivity function, S, at the same point as follows:

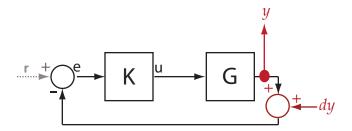
$$T = \frac{L}{1 - L} = S - 1.$$

Use getLoopTransfer and getSensitivity to compute L and S.

Consider the following model:



The complementary sensitivity, *T*, at y is defined as the transfer function from dy to y.



Observe that, in contrast to the sensitivity function, the disturbance, dy, is added *after* the measurement, y.

$$y = -GK(y + dy)$$

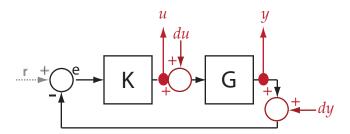
$$\rightarrow y = -GKy - GKdy$$

$$\rightarrow (I + GK)y = -GKdy$$

$$\rightarrow y = \prod_{T} (I + GK)^{-1}GKdy.$$

Here, I is an identity matrix of the same size as GK. The complementary sensitivity transfer function at y is equal to -1 times the closed-loop transfer function from r to y.

Complementary sensitivity at multiple locations, for example, u and y, is defined as the MIMO transfer function from the disturbances to measurements:



$$T = \begin{bmatrix} T_{du \to u} & T_{dy \to u} \\ T_{du \to y} & T_{dy \to y} \end{bmatrix}.$$

## See Also

AnalysisPoint | genss | getCompSensitivity | getIOTransfer |
getLoopTransfer | getPoints | getSensitivity | getValue | systume

Introduced in R2014a

# getComponents

Extract SISO control components from a 2-DOF PID controller

## **Syntax**

```
[C,X] = getComponents(C2,looptype)
```

# **Description**

[C,X] = getComponents(C2,looptype) decomposes the 2-DOF PID controller C2 into two SISO control components. One of the control components, C, is a 1-DOF PID controller. The other, X, is a SISO dynamic system. When C and X are connected in the loop structure specified by looptype, the resulting closed-loop system is equivalent to the 2-DOF control loop.

For more information about 2-DOF PID control architectures, see "Two-Degree-of-Freedom PID Controllers".

# **Examples**

### **Extract SISO Components from 2-DOF PID Controller**

Decompose a 2-DOF PID controller into SISO control components, using each of the feedforward, feedback, and filter configurations.

To start, obtain a 2-DOF PID controller. For this example, create a plant model and tune a 2-DOF PID controller for it.

```
G = tf(1,[1 0.5 0.1]);
C2 = pidtune(G,'pidf2',1.5)
C2 =
```

C2 is a pid2 controller object, with two inputs and one output. Decompose C2 into SISO control components using the feedforward configuration.

As the display shows, this command returns the SISO PID controller Cff as a pid object. The feedforward compensator X is returned as a zpk object.

Decompose C2 using the feedback configuration. In this case as well, Cfb is a pid controller object, and the feedback compensator X is a zpk model.

```
[Cfb,Xfb] = getComponents(C2, 'feedback');
```

Decompose C2 using the filter configuration. Again, the components are a SISO pid controller and a zpk model representing the prefilter.

```
[Cfr,Xfr] = getComponents(C2, 'filter');
```

# **Input Arguments**

### C2 - 2-DOF PID controller

pid2 object | pidstd2 object

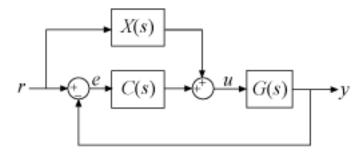
2-DOF PID controller to decompose, specified as a pid2 or pidstd2 controller object.

### looptype — Loop structure

'feedforward' (default) | 'feedback' | 'filter'

Loop structure for decomposing the 2-DOF controller, specified as 'feedforward', 'feedback', or 'filter'. These correspond to the following control decompositions and architectures:

• 'feedforward' — C is a conventional SISO PID controller that takes the error signal as its input. X is a feedforward controller, as shown:



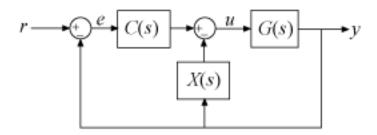
If C2 is a continuous-time, parallel-form controller, then the components are given by:

$$\begin{split} C(s) &= K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}, \\ X(s) &= (b-1) K_p + \frac{(c-1) K_d s}{T_f s + 1} \,. \end{split}$$

The following command constructs the closed-loop system from r to y for the feedforward configuration.

$$T = G*(C+X)*feedback(1,G*C);$$

• 'feedback' — C is a conventional SISO PID controller that takes the error signal as its input. X is a feedback controller from y to u, as shown:



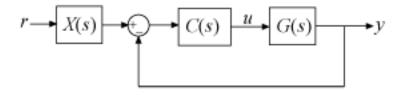
If C2 is a continuous-time, parallel-form controller, then the components are given by:

$$\begin{split} C(s) &= bK_p + \frac{K_i}{s} + \frac{cK_ds}{T_fs+1},\\ X(s) &= (1-b)K_p + \frac{(1-c)K_ds}{T_fs+1}\,. \end{split}$$

The following command constructs the closed-loop system from r to y for the feedback configuration.

$$T = G*C*feedback(1,G*(C+X));$$

'filter' — X is a prefilter on the reference signal. C is a conventional SISO PID controller that takes as its input the difference between the filtered reference and the output, as shown:



If C2 is a continuous-time, parallel-form controller, then the components are given by:

$$\begin{split} C(s) &= K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}, \\ X(s) &= \frac{\left(bK_p T_f + cK_d\right) s^2 + \left(bK_p + K_i T_f\right) s + K_i}{\left(K_p T_f + K_d\right) s^2 + \left(K_p + K_i T_f\right) s + K_i} \,. \end{split}$$

The following command constructs the closed-loop system from r to y for the filter configuration.

T = X\*feedback(G\*C,1);

The formulas shown above pertain to continuous-time, parallel-form controllers. Standard-form controllers and controllers in discrete time can be decomposed into analogous configurations. The <code>getComponents</code> command works on all 2-DOF PID controller objects.

# **Output Arguments**

#### C — SISO PID controller

pid object | pidstd object

SISO PID controller, returned as a pid or pidstd controller object. The form of C corresponds to the form of the input controller C2. For example, if C2 is a standard-form pidstd2 controller, then C is a pidstd object.

The precise functional form of C depends on the loop structure you specify with the looptype argument, as described in "Input Arguments" on page 2-407.

### X — SISO control component

zpk model

SISO control component, specified as a zero-pole-gain (zpk) model. The precise functional form of X depends on the loop structure you specify with the looptype argument, as described in "Input Arguments" on page 2-407.

### See Also

make1D0F | make2D0F | pid2 | pidstd2

## **Topics**

"Decompose a 2-DOF PID Controller into SISO Components" "Two-Degree-of-Freedom PID Controllers"

### Introduced in R2015b

## getData

Get current values of tunable-surface coefficients

## **Syntax**

```
Kco = getData(K)
KcoJ = getData(K,J)
```

## **Description**

Kco = getData(K) extracts the current value of a tunable surface. K is a
tunableSurface object that represents the parametric gain surface:

$$K(n(\sigma)) = \gamma [K_0 + K_1 F_1(n(\sigma)) + ... + K_M F_M(n(\sigma))],$$

#### where:

- $\sigma$  is a vector of scheduling variables.
- $n(\sigma)$  is a normalization function (see the Normalization property of tunableSurface).
- $\gamma$  is a scaling factor (see the Normalization property of tunableSurface).
- $F_1,...,F_M$  are basis functions.
- $K_0,...,K_M$  are tunable coefficients.

getData evaluates K at the current values of the coefficients  $K_0,...,K_M$ .

 $\mathsf{KcoJ} = \mathsf{getData}(\mathsf{K}, \mathsf{J})$  extracts the current value of the coefficient of the Jth basis function  $F_J$ . Use  $\mathsf{J} = 0$  to get the constant coefficient  $K_0$ .

## **Input Arguments**

#### K — Gain surface

tunableSurface object

Gain surface, specified as a tunableSurface object,

#### J - Index of basis function

nonnegative integer

Index of basis function, specified as a nonnegative integer. To extract the constant coefficient  $K_0$ , use J = 0.

## **Output Arguments**

#### Kco - Current coefficient values

array

Current coefficient values of the tunable surface, returned as an array.

If the tunable surface K is a scalar-valued gain, then the length of K is (M+1), where M is the number of basis functions in the parameterization. For example, if K represents the tunable gain surface:

$$K(\alpha, V) = K_0 + K_1 \alpha + K_2 V + K_3 \alpha V,$$

then Kco is the 1-by-4 vector  $[K_0,K_1,K_2,K_3]$ .

For array-valued gains, each coefficient expands to the I/O dimensions of the gain. These expanded coefficients are concatenated horizontally in Kco. (See tunableSurface.) For example, for a two-input, two-output gain surface, Kco has dimensions [2,2(M+1)].

### **KcoJ** — Coefficient of Jth basis function

scalar | array

Coefficient of the *J*th basis function in the tunable surface parameterization, returned as a scalar or an array.

If the tunable surface K is a scalar-valued gain, then KcoJ is a scalar. If K is an array-valued gain, then KcoJ is an array that matches the I/O dimensions of the gain.

### See Also

evalSurf|setData|tunableSurface|viewSurf

### Introduced in R2015b

# getDelayModel

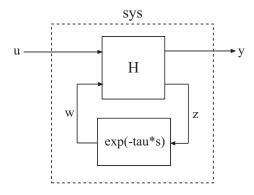
State-space representation of internal delays

## **Syntax**

```
[H,tau] = getDelayModel(sys)
[A,B1,B2,C1,C2,D11,D12,D21,D22,E,tau] = getDelayModel(sys)
```

## **Description**

[H,tau] = getDelayModel(sys) decomposes a state-space model sys with internal delays into a delay-free state-space model, H, and a vector of internal delays, tau. The relationship among sys, H, and tau is shown in the following diagram.



[A,B1,B2,C1,C2,D11,D12,D21,D22,E,tau] = getDelayModel(sys) returns the set of state-space matrices and internal delay vector, tau, that explicitly describe the state-space model sys. These state-space matrices are defined by the state-space equations:

Continuous-time sys:

$$E\frac{dx(t)}{dt} = Ax(t) + B_1 u(t) + B_2 w(t)$$

$$y(t) = C_1 x(t) + D_{11} u(t) + D_{12} w(t)$$

$$z(t) = C_2 x(t) + D_{21} u(t) + D_{22} w(t)$$

$$w(t) = z(t - \tau)$$

Discrete-time sys:

$$Ex[k + 1] = Ax[k] + B_1u[k] + B_2w[k]$$

$$y[k] = C_1x[k] + D_{11}u[k] + D_{12}w[k]$$

$$z[k] = C_2x[k] + D_{21}u[k] + D_{22}w[k]$$

$$w[k] = z[k - \tau]$$

## **Input Arguments**

sys

Any state-space (ss) model.

## **Output Arguments**

Н

Delay-free state-space model (ss). H results from decomposing sys into a delay-free component and a component exp(-tau\*s) that represents all internal delays.

If sys has no internal delays, H is equal to sys.

#### tau

Vector of internal delays of sys, expressed in the time units of sys. The vector tau results from decomposing sys into a delay-free state-space model H and a component exp(-tau\*s) that represents all internal delays.

If sys has no internal delays, tau is empty.

#### A,B1,B2,C1,C2,D11,D12,D21,D22,E

Set of state-space matrices that, with the internal delay vector tau, explicitly describe the state-space model sys.

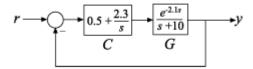
For explicit state-space models (E = I, or sys.e = []), the output E = [].

If sys has no internal delays, the outputs B2, C2, D12, D21, and D22 are all empty ([]).

## **Examples**

#### **Get Delay-Free State-Space Model and Internal Delay**

Decompose the following closed-loop system with internal delay into a delay-free component and a component representing the internal delay.



Create the closed-loop model sys from r to y.

```
G = tf(1,[1 10],'InputDelay',2.1);
C = pid(0.5,2.3);
sys = feedback(C*G,1);
```

sys is a state-space (ss) model with an internal delay that arises from closing the feedback loop on a plant with an input delay.

Decompose sys into a delay-free state-space model and the value of the internal delay.

```
[H,tau] = getDelayModel(sys);
```

Confirm that the internal delay matches the original input delay on the plant.

tau

tau = 2.1000

### **See Also**

setDelayModel

# **Topics**

"Internal Delays"

Introduced in R2006a

# getGainCrossover

Crossover frequencies for specified gain

### **Syntax**

wc = getGainCrossover(sys,gain)

## **Description**

wc = getGainCrossover(sys,gain) returns the vector wc of frequencies at which
the frequency response of the dynamic system model, sys, has principal gain of gain.
For SISO systems, the principal gain is the frequency response. For MIMO models, the
principal gain is the largest singular value of sys.

## **Examples**

### **Unity Gain Crossover**

Find the 0dB crossover frequencies of a single-loop control system with plant given by:

$$G(s) = \frac{1}{\left(s+1\right)^3},$$

and PI controller given by:

$$C(s) = 1.14 + \frac{0.454}{s}.$$

$$G = \text{zpk}([],[-1,-1,-1],1);$$

$$C = \text{pid}(1.14,0.454);$$

$$\text{sys} = G*C;$$

$$\text{wc} = \text{getGainCrossover(sys,1)}$$

$$\text{wc} = 0.5214$$

The 0 dB crossover frequencies are the frequencies at which the open-loop response sys = G\*C has unity gain. Because this system only crosses unity gain once, getGainCrossover returns a single value.

#### **Notch Filter Stopband**

Find the 20 dB stopband of

$$sys = \frac{s^2 + 0.05s + 100}{s^2 + 5s + 100}.$$

sys is a notch filter centered at 10 rad/s.

The db2mag command converts the gain value of -20 dB to absolute units. The getGainCrossover command returns the two frequencies that define the stopband.

## **Input Arguments**

### sys — Input dynamic system

dynamic system model

Input dynamic system, specified as any SISO or MIMO dynamic system model.

### gain — Input gain

positive real scalar

Input gain in absolute units, specified as a positive real scalar.

- If sys is a SISO model, the gain is the frequency response magnitude of sys.
- If sys is a MIMO model, gain means the largest singular value of sys.

## **Output Arguments**

#### wc - Crossover frequencies

column vector

Crossover frequencies, returned as a column vector. This vector lists the frequencies at which the gain or largest singular value of sys is gain.

## **Algorithms**

getGainCrossover computes gain crossover frequencies using structure-preserving eigensolvers from the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

### **See Also**

bandwidth|bode|freqresp|getPeakGain|sigma

### **Topics**

"Dynamic System Models"

Introduced in R2012a

# getGoal

Evaluate variable tuning goal at specified design point

# **Syntax**

```
TG = getGoal(VG,'index',k)
TG = getGoal(VG,'index',k1,k2,...)
TG = getGoal(VG,'value',x1,x2,...)
```

## **Description**

When tuning controllers for multiple operating conditions, varyingGoal lets you adjust the tuning objectives as a function of the design point. Use getGoal to evaluate a variable goal at a particular design point or for particular values of the sampling variables.

TG = getGoal(VG, 'index',k) returns the effective tuning goal at the kth design point. The absolute index k is relative to the arrays of parameter values in the Parameters property of the varying goal VG. If you have specified VG.SamplingGrid, then k is relative to the grid of design points in that property.

TG = getGoal(VG, 'index', k1, k2, ...) returns the effective tuning goal at the design point with coordinates (k1, k2, ...). These coordinates are indices into the multidimensional arrays in VG.Parameters and VG.SamplingGrid. This syntax is useful when your design grid includes multiple scheduling variables.

TG = getGoal(VG, 'value', x1, x2, ...) returns the effective tuning goal at the design point whose scheduling-variable values are (x1, x2, ...). Use this syntax only if you have specified design points in VG.SamplingGrid. For example, if VG.SamplingGrid specifies a grid of design points (a,b), then TG = getGoal(VG, 'value', -1,3) returns the tuning goal at the design point (a,b) = (-1,3). If (x1,x2,...) does not match any point in VG.SamplingGrid, then getGoal returns the nearest point, in a relative sense.

## **Examples**

#### Get Goal at Design Point from Varying Tuning Goal

Suppose you use the following 5-by-5 grid of design points to tune your controller.

```
[alpha,V] = ndgrid(linspace(0,20,5),linspace(700,1300,5));
```

Create a variable tuning goal that specifies gain and phase margins at a signal named 'u' that vary across a grid of design points.

```
[GM,PM] = ndgrid(linspace(7,20,5),linspace(45,70,5));
FH = @(gm,pm) TuningGoal.Margins('u',gm,pm);
VG = varyingGoal(FH,GM,PM);
```

Use the grid to specify the SamplingGrid property of VG.

```
VG.SamplingGrid = struct('alpha',alpha,'V',V);
```

Evaluate this variable design goal at (alpha, V) = (5, 1150). This point is the second alpha value and the fourth V value, so you can index into the varyingGoal using (k1, k2) = (2, 4).

```
TGi = getGoal(VG, 'index', 2, 4);
```

Because you have the specific (alpha, V) values at which you want the tuning goal, you can use those values instead of indexing.

```
TGv = getGoal(VG,'value',5,1150)
TGv =
  Margins with properties:
    GainMargin: 10.2500
    PhaseMargin: 63.7500
    ScalingOrder: 0
        Focus: [0 Inf]
    Location: {'u'}
        Models: 17
        Openings: {0x1 cell}
        Name: ''
```

## **Input Arguments**

#### VG — Varying tuning goal

varyingGoal object

Varying tuning goal, specified as a varyingGoal object. VG captures the variation of a tuning goal over a grid of design points for gain-scheduled tuning (see tunableSurface).

#### k — Index into design-point grid

integer

Index into design point grid, specified as an integer. You can provide one integer index, k, or multiple indices  $k1, k2, \ldots$ 

If you provide a single index, k, then getGoal treats k as a linear index into the parameter arrays of VG.Parameters or the structures of VG.SamplingGrid that specify the design points.

- If VG varies over a 1-D sampling grid (one scheduling variable), then TG = getGoal (VG, 'index', k) returns the tuning goal for the kth entry in VG.Parameters.
- If VG varies over two or more scheduling-variables, then TG = getGoal(VG, 'index', k) returns the kth entry in that grid, determined by linear indexing. (See "Array Indexing" (MATLAB).)

If you provide multiple indices,  $k1, k2, \ldots$ , then getGoal treats them as indices into the multidimensional arrays of VG.Parameters or VG.SamplingGrid.

### x — Variable value at design point

scalar

Variable value at design point, specified as a scalar. Use inputs  $x1, x2, \ldots$ , to get the tuning goal for a particular set of scheduling-variable values. Provide as many values as you have scheduling variables in your system. For example, if the operating conditions are described by two scheduling variables (a,b), then use (x1,x2) to specify the (a,b) value at which you want to extract the tuning goal.

# **Output Arguments**

### TG — Tuning goal at design point

TuningGoal object | []

Tuning goal at the specified design point, returned as a TuningGoal object. If any of the tuning goal parameters is NaN at the specified design point, then TG = []. (See varyingGoal).

### See Also

tunableSurface | varyingGoal

Introduced in R2017b

# getIOTransfer

Closed-loop transfer function from generalized model of control system

# **Syntax**

```
H = getIOTransfer(T,in,out)
H = getIOTransfer(T,in,out,openings)
```

## **Description**

H = getIOTransfer(T,in,out) returns the transfer function from specified inputs to specified outputs of a control system, computed from a closed-loop generalized model of the control system.

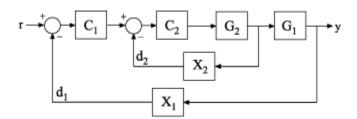
H = getIOTransfer(T,in,out,openings) returns the transfer function calculated with one or more loops open.

## **Examples**

### **Closed-Loop Responses of Control System Model**

Analyze responses of a control system by using getIOTransfer to compute responses between various inputs and outputs of a closed-loop model of the system.

Consider the following control system.



Create a genss model of the system by specifying and connecting the numeric plant models G1 and G2, the tunable controllers C1 and C2, and the AnalysisPoint blocks X1 and X2 that mark potential loop-opening or signal injection sites.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
T = feedback(G1*feedback(G2*C2,X2)*C1,X1);
T.InputName = 'r';
T.OutputName = 'y';
```

If you tuned the free parameters of this model (for example, using the tuning command systune), you might want to analyze the tuned system performance by examining various system responses.

For example, examine the response at the output, y, to a disturbance injected at the point  $d_1$ .

```
H1 = getIOTransfer(T, 'X1', 'y');
```

H1 represents the closed-loop response of the control system to a disturbance injected at the implicit input associated with the AnalysisPoint block X1, which is the location of  $d_1$ :



H1 is a genss model that includes the tunable blocks of T. If you have tuned the free parameters of T, H1 allows you to validate the disturbance response of your tuned system. For example, you can use analysis commands such as bodeplot or stepplot to examine the responses of H1. You can also use getValue to obtain the current value of H1, in which all the tunable blocks are evaluated to their current numeric values.

Similarly, examine the response at the output to a disturbance injected at the point  $d_2$ .

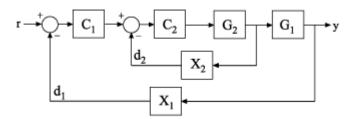
```
H2 = getIOTransfer(T, 'X2', 'y');
```

You can also generate a two-input, one-output model representing the response of the control system to simultaneous disturbances at both  $d_1$  and  $d_2$ . To do so, provide getIOTransfer with a cell array that specifies the multiple input locations.

```
H = getIOTransfer(T,{'X1','X2'},'y');
```

#### Responses with Some Loops Open and Others Closed

Compute the response from r to y of the following cascaded control system, with the inner loop open, and the outer loop closed.



Create a genss model of the system by specifying and connecting the numeric plant models G1 and G2, the tunable controllers C1 and C2, and the AnalysisPoint blocks X1 and X2 that mark potential loop-opening or signal injection sites.

```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
T = feedback(G1*feedback(G2*C2,X2)*C1,X1);
T.InputName = 'r';
T.OutputName = 'y';
```

If you tuned the free parameters of this model (for example, using the tuning command systune), you might want to analyze the tuned system performance by examining various system responses.

For example, compute the response of the system with the inner loop open, and the outer loop closed.

```
H = getIOTransfer(T, 'r', 'y', 'X2');
```

By default, the loops are closed at the analysis points X1 and X2. Specifying 'X2' for the openings argument causes getIOTransfer to open the loop at X2 for the purposes of computing the requested transfer from r to y. The switch at X1 remains closed for this computation.

## **Input Arguments**

#### T — Model of control system

generalized state-space model

Model of a control system, specified as a generalized state-space model (genss).

#### in — Input to extracted transfer function

character vector | cell array of character vectors

Input to extracted transfer function, specified as a character vector or cell array of character vectors. To extract a multiple-input transfer function from the control system, use a cell array of character vectors. Each specified input must match either:

- An input of the control system model T; that is, a channel name from T. Input Name.
- An analysis point in T, corresponding to a channel of an AnalysisPoint block in T.
   To get the list of available analysis points in T, use getPoints(T).

When you specify an analysis point as an input in, getIOTransfer uses the input implicitly associated with the AnalysisPoint channel, arranged as follows.



This input signal models a disturbance entering at the output of the switch.

If an analysis point has the same name as an input of T, then getIOTransfer uses the input of T.

```
Example: {'r','X1'}
```

#### out — Output of extracted transfer function

character vector | cell array of character vectors

Output of extracted transfer function, specified as a character vector or cell array of character vectors. To extract a multiple-output transfer function from the control system, use a cell array of character vectors. Each specified output must match either:

- An output of the control system model T; that is, a channel name from T.OutputName.
- An analysis point in T, corresponding to a channel of an AnalysisPoint block in T. To get the list of available analysis points in T, use getPoints(T).

When you specify an analysis point as an output out, getIOTransfer uses the output implicitly associated with the AnalysisPoint channel, arranged as follows.



If an analysis point has the same name as an output of  $\mathsf{T}$ , then  $\mathsf{getIOTransfer}$  uses the output of  $\mathsf{T}$ .

Example: {'y','X2'}

### openings — Locations for opening feedback loops

character vector | cell array of character vectors

Locations for opening feedback loops for computation of the response from in to out, specified as a character vector or cell array of character vectors that identify analysis points in T. Analysis points are marked by AnalysisPoint blocks in T. To get the list of available analysis points in T, use getPoints(T).

Use openings when you want to compute the response from in to out with some loops in the control system open. For example, in a cascaded loop configuration, you can calculate the response from the system input to the system output with the inner loop open.

## **Output Arguments**

#### H — Closed-loop transfer function

generalized state-space model

Closed-loop transfer function of the control system T from in to out, returned as a generalized state-space model (genss).

- If both in and out specify a single signal, then T is a SISO genss model.
- If in or out specifies multiple signals, then T is a MIMO genss model.

## **Tips**

- You can use getIOTransfer to extract various subsystem responses, given a generalized model of the overall control system. This is useful for validating responses of a control system that you tune with tuning commands such as systume.
  - For example, in addition to evaluating the overall response of a tuned control system from inputs to outputs, you can use <code>getIOTransfer</code> to extract the transfer function from a disturbance input to a system output. Evaluate the responses of that transfer function (such as with <code>step</code> or <code>bode</code>) to confirm that the tuned system meets your disturbance rejection requirements.
- getIOTransfer is the genss equivalent to the Simulink Control Design getIOTransfer command, which works with the slTuner and slLinearizer interfaces. Use the Simulink Control Design command when your control system is modeled in Simulink.

### See Also

AnalysisPoint | genss | getIOTransfer | getLoopTransfer | getPoints | systume

Introduced in R2012b

# getLFTModel

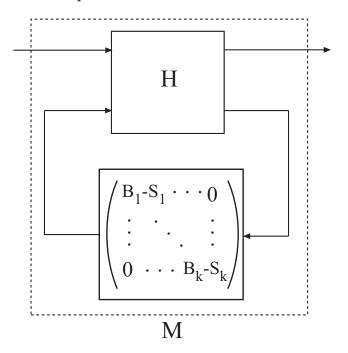
Decompose generalized LTI model

# **Syntax**

[H,B,S] = getLFTModel(M)

# **Description**

[H,B,S] = getLFTModel(M) extracts the components H, B, and S that make up the Generalized matrix or Generalized LTI model M. The model M decomposes into H, B, and S. These components are related to M as shown in the following illustration.



The cell array B contains the Control Design Blocks of M. The component H is a numeric matrix, ss model, or frd model that describes the fixed portion of M and the interconnections between the blocks of B. The matrix S = blkdiag(S1, ..., Sk) contains numerical offsets that ensure that the interconnection is well-defined when the current (nominal) value of M is finite.

You can recombine H, B, and S into M using lft, as follows:

```
M = lft(H,blkdiag(B{:})-S);
```

## **Input Arguments**

М

Generalized LTI model (genss or genfrd) or Generalized matrix (genmat).

## **Output Arguments**

Н

Matrix, ss model, or frd model describing the numeric portion of M and how it the numeric portion is connected to the Control Design Blocks of M.

В

Cell array of Control Design Blocks (for example, realp or tunableSS) of M.

S

Matrix of offset values. The software might introduce offsets when you build a Generalized model to ensure that H is finite when the current (nominal) value of M is finite.

## **Tips**

 getLFTModel gives you access to the internal representation of Generalized LTI models and Generalized Matrices. For more information about this representation, see "Internal Structure of Generalized Models".

### See Also

genfrd | genmat | genss | getValue | lft | nblocks

### **Topics**

- "Generalized Matrices"
- "Generalized and Uncertain LTI Models"
- "Models with Tunable Coefficients"
- "Internal Structure of Generalized Models"

#### Introduced in R2011a

# getLoopTransfer

Open-loop transfer function of control system

## **Syntax**

```
L = getLoopTransfer(T,Locations)
L = getLoopTransfer(T,Locations,sign)
L = getLoopTransfer(T,Locations,sign,openings)
```

# **Description**

L = getLoopTransfer(T,Locations) returns the point-to-point open-loop transfer function of a control system measured at specified analysis points. The point-to-point open-loop transfer function is the open-loop response obtained by injecting signals at the specified locations and measuring the return signals at the same locations.

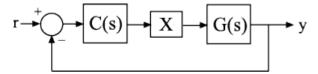
L = getLoopTransfer(T, Locations, sign) specifies the feedback sign for calculating the open-loop response. The relationship between the closed-loop response T and the open-loop response L is T = feedback(L, 1, sign).

L = getLoopTransfer(T,Locations,sign,openings) specifies additional loopopening locations to open for computing the open-loop response at Locations.

## **Examples**

### **Open-Loop Transfer Function at Analysis Point**

Compute the open-loop response of the following control system model at an analysis point specified by an AnalysisPoint block, X.



Create a model of the system by specifying and connecting a numeric LTI plant model, G, a tunable controller, C, and the AnalysisPoint, X.

```
G = tf([1 2],[1 0.2 10]);
C = tunablePID('C','pi');
X = AnalysisPoint('X');
T = feedback(G*X*C,1);
```

T is a genss model that represents the closed-loop response of the control system from r to y. The model contains AnalysisPoint block X, which identifies the potential loop-opening location.

Calculate the open-loop point-to-point loop transfer at location X.

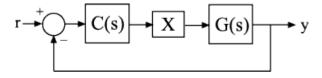
```
L = getLoopTransfer(T, 'X');
```

This command computes the positive-feedback transfer function you would obtain by opening the loop at X, injecting a signal into G, and measuring the resulting response at the output of C. By default, getLoopTransfer computes the positive feedback transfer function. In this example, the positive feedback transfer function is L(s) = -G(s)C(s)

The output L is a genss model that includes the tunable block C. You can use getValue to obtain the current value of L, in which all the tunable blocks of L are evaluated to their current numeric value.

### Negative-Feedback Open-Loop Transfer Function

Compute the negative-feedback open-loop transfer of the following control system model at an analysis point specified by an AnalysisPoint block, X.



Create a model of the system by specifying and connecting a numeric LTI plant model G, a tunable controller C, and the AnalysisPoint block X.

```
G = tf([1 2],[1 0.2 10]);
C = tunablePID('C','pi');
```

```
X = AnalysisPoint('X');
T = feedback(G*X*C,1);
```

T is a genss model that represents the closed-loop response of the control system from r to y. The model contains the AnalysisPoint block X that identifies the potential loop-opening location.

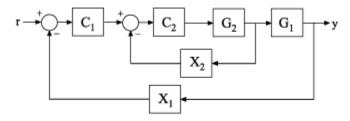
Calculate the open-loop point-to-point loop transfer at the location X.

```
L = getLoopTransfer(T, 'X', -1);
```

This command computes the open-loop transfer function from the input of G to the output of C, assuming that the loop is closed with negative feedback. That is, the relationships between L and T is given by T = feedback(L, 1). In this example, the positive feedback transfer function is L(s) = G(s)C(s)

#### **Transfer Function with Additional Loop Openings**

Compute the open-loop response of the inner loop of the following cascaded control system, with the outer loop open.



Create a model of the system by specifying and connecting the numeric plant models G1 and G2, the tunable controllers C1, and the AnalysisPoint blocks X1 and X2 that mark potential loop-opening locations.

```
G1 = tf(10,[1 10]);

G2 = tf([1 2],[1 0.2 10]);

C1 = tunablePID('C','pi');

C2 = tunableGain('G',1);

X1 = AnalysisPoint('X1');
```

```
X2 = AnalysisPoint('X2');
T = feedback(G1*feedback(G2*C2,X2)*C1,X1);
```

Compute the negative-feedback open-loop response of the inner loop, at the location X2, with the outer loop opened at X1.

```
L = getLoopTransfer(T, 'X2', -1, 'X1');
```

By default, the loop is closed at the analysis-point location marked by the AnalysisPoint block X1. Specifying 'X1' for the openings argument causes getLoopTransfer to open the loop at X1 for the purposes of computing the requested loop transfer at X2. In this example, the negative-feedback open-loop response  $L(s) = G_2(s)C_2(s)$ .

## **Input Arguments**

#### T — Model of control system

generalized state-space model

Model of a control system, specified as a Generalized State-Space (genss) Model. Locations at which you can open loops and perform open-loop analysis are marked by AnalysisPoint blocks in T.

### Locations — Analysis-point locations

character vector | cell array of character vectors

Analysis-point locations in the control system model at which to compute the open-loop point-to-point response, specified as a character vector or a cell array of character vectors that identify analysis-point locations in T.

Analysis-point locations are marked by AnalysisPoint blocks in T. An AnalysisPoint block can have single or multiple channels. The Location property of an AnalysisPoint block gives names to these feedback channels.

The name of any channel in an AnalysisPoint block in T is a valid entry for the Locations argument to getLoopTransfer. To get a full list of available analysis points in T, use getPoints(T).

getLoopTransfer computes the open-loop response you would obtain by injecting a signal at the implicit input associated with an AnalysisPoint channel, and measuring the response at the implicit output associated with the channel. These implicit inputs and outputs are arranged as follows.



L is the open-loop transfer function from in to out.

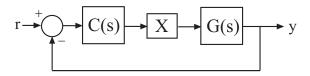
#### sign - Feedback sign

+1 (default) | -1

Feedback sign, specified as +1 or -1 The feedback sign determines the sign of the open-loop transfer function.

- +1 Compute the positive-feedback loop transfer. In this case, the relationship between the closed-loop response T and the open-loop response L is T = feedback(L,1,+1).
- -1 Compute the negative-feedback loop transfer. In this case, the relationship between the closed-loop response T and the open-loop response L is T = feedback(L,1).

Choose a feedback sign that is consistent with the conventions of the analysis you intend to perform with the loop transfer function. For example, consider the following system, where T is the closed-loop transfer function from r to y.



To compute the stability margins of this system with the margin command, which assumes negative feedback, you need to use the negative-feedback open-loop response. Therefore, you can use L = getLoopTransfer(T, 'X', -1) to obtain the negative-feedback transfer function L = GC.

### openings — Additional locations for opening feedback loops

character vector | cell array of character vectors

Additional locations for opening feedback loops for computation of the open-loop response, specified as character vector or cell array of character vectors that identify analysis-point locations in T. Analysis-point locations are marked by AnalysisPoint blocks in T. Any channel name contained in the Location property of an AnalysisPoint block in T is a valid entry for openings.

Use openings when you want to compute the open-loop response at one analysis-point location with other loops also open at other locations. For example, in a cascaded loop configuration, you can calculate the inner loop open-loop response with the outer loop also open. Use getPoints(T) to get a full list of available analysis-point locations in T.

### **Output Arguments**

#### L — Point-to-point open-loop response

generalized state-space model

Point-to-point open-loop response of the control system T measured at the analysis points specified by Locations, returned as a generalized state-space (genss) model.

- If Locations specifies a single analysis point, then L is a SISO genss model. In this case, L represents the response obtained by opening the loop at Locations, injecting signals and measuring the return signals at the same location.
- If Locations is a vector signal, or specifies multiple analysis points, then L is a MIMO genss model. In this case, L represents the open-loop MIMO response obtained by opening loops at all locations listed in Locations, injecting signals and measuring the return signals at those locations.

### **Tips**

- You can use getLoopTransfer to extract open-loop responses given a generalized
  model of the overall control system. This is useful, for example, for validating openloop responses of a control system that you tune with the a tuning command such as
  systune.
- getLoopTransfer is the genss equivalent to the Simulink Control Design command getLoopTransfer, which works with the slTuner and slLinearizer interfaces. Use the Simulink Control Design command when your control system is modeled in Simulink.

## **See Also**

 $\label{lem:analysisPoint} A \textit{nalysisPoint} \mid \textit{genss} \mid \textit{getIOTransfer} \mid \textit{getLoopTransfer} \mid \textit{getPoints} \mid \textit{systune} \\$ 

**Introduced in R2012b** 

# getoptions

Return @PlotOptions handle or plot options property

## **Syntax**

```
p = getoptions(h)
p = getoptions(h,propertyname)
```

# **Description**

p = getoptions(h) returns the plot options handle associated with plot handle h. p
contains all the settable options for a given response plot.

p = getoptions(h,propertyname) returns the specified options property,
propertyname, for the plot with handle h. You can use this to interrogate a plot handle.
For example,

```
p = getoptions(h,'Grid')
```

returns 'on' if a grid is visible, and 'off' when it is not.

For a list of the properties and values available for each plot type, see "Properties and Values Reference".

### See Also

setoptions

Introduced before R2006a

# getPassiveIndex

Compute passivity index of linear system

## **Syntax**

```
R = getPassiveIndex(G)
nu = getPassiveIndex(G,'input')
rho = getPassiveIndex(G,'output')
tau = getPassiveIndex(G,'io')
DX = getPassiveIndex(G,dQ)

index = getPassiveIndex(____,tol)
index = getPassiveIndex(____,tol,fband)
[index,FI] = getPassiveIndex(____)
[index,FI,Qout,dQout] = getPassiveIndex(____)
```

## **Description**

getPassiveIndex computes various measures of the excess or shortage of passivity for a given system.

A linear system G(s) is passive if all its I/O trajectories (u(t),y(t)) satisfy:

$$\int_0^T y(t)^T u(t)dt > 0,$$

for all T > 0. Equivalently, a system is passive if its frequency response is positive real, such that for all  $\omega > 0$ ,

$$G(i\omega) + G(i\omega)^H > 0$$

(or the discrete-time equivalent).

R = getPassiveIndex(G) computes the relative passivity index. G is passive when R is less than one. R measures the relative excess (R < 1) or shortage (R > 1) of passivity.

For more information about the notion of passivity indices, see "About Passivity and Passivity Indices".

nu = getPassiveIndex(G, 'input') computes the input passivity index. The system is input strictly passive when nu > 0. nu is also called the input feedforward passivity (IFP) index. The value of nu is the minimum feedforward action such that the resulting system is passive.

For more information about the notion of passivity indices, see "About Passivity and Passivity Indices".

 ${\sf rho} = {\sf getPassiveIndex}({\sf G,'output'})$  computes the output passivity index. The system is output strictly passive when  ${\sf rho} > 0$ .  ${\sf rho}$  is also called the output feedback passivity (OFP) index. The value of  ${\sf rho}$  is the minimum feedback action such that the resulting system is passive.

For more information about the notion of passivity indices, see "About Passivity and Passivity Indices".

tau = getPassiveIndex(G, 'io') computes the combined I/O passivity index. The system is very strictly passive when tau > 0.

For more information about the notion of passivity indices, see "About Passivity and Passivity Indices".

DX = getPassiveIndex(G,dQ) computes the directional passivity index in the direction specified by the matrix dQ.

index = getPassiveIndex(\_\_\_\_, tol) computes the passivity index with relative
accuracy specified by tol. Use this syntax with any of the previous combinations of input
arguments. index is the corresponding passivity index R, nu, rho, tau, or DX.

index = getPassiveIndex( \_\_\_\_, tol, fband) computes passivity indices restricted
to a specific frequency interval.

[index,FI] = getPassiveIndex(\_\_\_\_) also returns the frequency at which the returned index value is achieved.

[index,FI,Qout,dQout] = getPassiveIndex(\_\_\_\_) also returns the sector matrix Qout for passivity and the directional index matrix dQout.

## **Examples**

#### Relative, Input, and Output Passivity Indices

Compute passivity indices for the following dynamic system:

$$G(s) = \frac{s^2 + s + 5s + 0.1}{s^3 + 2s^2 + 3s + 4}$$

$$G = tf([1,1,5,.1],[1,2,3,4]);$$

Compute the relative passivity index.

```
R = getPassiveIndex(G)
```

```
R = 0.9512
```

The system is passive, but with a relatively small excess of passivity.

Compute the input and output passivity indices.

```
nu = getPassiveIndex(G,'input')
nu = 0.0250
rho = getPassiveIndex(G,'output')
rho = 0.2583
```

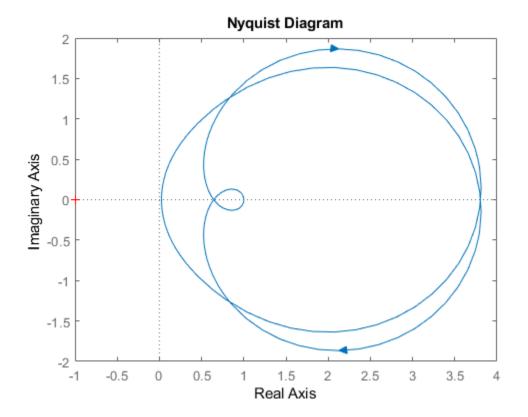
This system is both input strictly passive and output strictly passive.

Compute the combined I/O passivity index.

```
tau = getPassiveIndex(G,'io')
tau = 0.0250
```

The system is very strictly passive as well. A system that is very strictly passive is also strictly positive real. Examining the Nyquist plot confirms this, showing that the frequency response lies entirely within the right half-plane.

```
nyquistplot(G)
```



The relatively small tau value is reflected in how close the frequency response comes to the imaginary axis.

## **Input Arguments**

### **G** — Model to analyze

dynamic system model | model array

Model to analyze for passivity, specified as a dynamic system model such as a tf, ss, or genss model. G can be MIMO, if the number of inputs equals the number of outputs. G

can be continuous or discrete. If G is a generalized model with tunable or uncertain blocks, getPassiveIndex evaluates passivity of the current, nominal value of G.

If G is a model array, then getPassiveIndex returns the passivity index as an array of the same size, where:

```
index(k) = getPassivityIndex(G(:,:,k),___)
```

Here, index is any of R, nu, rho, tau, or DX, depending on which input arguments you use.

#### d0 — Custom direction

matrix

Custom direction in which to compute passivity, specified as a symmetric square matrix that is 2\*ny on a side, where ny is the number of outputs of G.

The rho, nu, and tau indices each correspond to a particular direction in the y/u space of the system, with a corresponding dQ value. (See dQout for these values.) Use this argument to specify your own value for this direction.

#### tol — Relative accuracy

0.01 (default) | positive real value

Relative accuracy for the calculated passivity index. By default, the tolerance is 1%, meaning that the returned passivity index is within 1% of the actual passivity index.

### fband — Frequency interval

1-by-2 array

Frequency interval for determining passivity index, specified as an array of the form [fmin,fmax]. When you provide fband, then getPassiveIndex restricts the frequency-domain computation of the passivity index to that frequency range. For example, the relative passivity index R is the peak gain of the bilinear-transformed system  $(I-G)(I+G)^-1$  (for minimum-phase (I+G)). When you provide fband, then R is the peak gain within the frequency band.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the dynamic system model G.

## **Output Arguments**

### R — Relative passivity index

scalar | array

Relative passivity index, returned as a scalar, or an array if G is an array.

The system G is passive when R is less than one.

- R < 1 indicates a relative excess of passivity.</li>
- R > 1 indicates a relative shortage of passivity.

When I + G is minimum phase, R is the peak gain of the bilinear-transformed system (I - G)(I + G)^-1.

For more information about the notion of passivity indices, see "About Passivity and Passivity Indices".

### nu - Input passivity index

scalar | array

Input passivity index, returned as a scalar, or an array if G is an array. nu is defined as the largest value of  $\nu$  for which:

$$\int_0^T y(t)^T u(t) dt > \nu \int_0^T u(t)^T u(t) dt,$$

for all T > 0. Equivalently, nu is the largest  $\nu$  for which:

$$G(j\omega) + G(j\omega)^H > 2\nu I$$

(or the discrete-time equivalent). The system is input strictly passive when nu > 0. nu is also called the input feedforward passivity (IFP) index. The value of nu is the minimum feedforward action such that the resulting system is passive.

### rho — Output passivity index

scalar | array

Output passivity index, returned as a scalar, or an array if G is an array. rho is defined as the largest value of  $\rho$  for which:

$$\int_0^T y(t)^T u(t)dt > \rho \int_0^T y(t)^T y(t)dt,$$

for all T > 0. The system is output strictly passive when rho > 0. rho is also called the output feedback passivity (OFP) index. The value of rho is the minimum feedback action such that the resulting system is passive.

### tau - Combined I/O passivity index

scalar | array

Combined I/O passivity index, returned as a scalar, or an array if G is an array. tau is defined as the largest value of  $\tau$  for which:

$$\int_0^T y(t)^T u(t) dt > \tau \int_0^T \left( u(t)^T u(t) + y(t)^T y(t) \right) dt,$$

for all T > 0. The system is very strictly passive when tau > 0.

### DX — Directional passivity index

scalar | array

Directional passivity index in the direction specified by dQ, returned as a scalar, or an array if G is an array. The directional passivity index is the largest value of D for which:

$$\int_{0}^{T} y(t)^{T} u(t) dt > D \int_{0}^{T} \left[ \begin{pmatrix} y(t) \\ u(t) \end{pmatrix}^{T} dQ \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} \right] dt,$$

for all T > 0. The rho, nu, and tau indices correspond to particular choices of dQ (see the output argument dQout). To compute DX, the software uses the custom dQ value you supply, dQ.

### FI — Frequency at which index is achieved

nonnegative scalar | array

Frequency at which the returned passivity index is achieved, returned as a nonnegative scalar, or an array if G is an array. In general, the passivity indices vary with frequency (see passiveplot). For each index type, the returned value is the largest value over all frequencies. FI is the frequency at which this value occurs, returned in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of G.

### **Qout** — Sector geometry

matrix

Sector geometry used for computing the passivity index, returned as a matrix. For passivity indices, **Qout** is given by:

```
Qout = [zeros(ny), -1/2*eye(ny); -1/2*eye(ny), zeros(ny)];
where ny is the number of outputs of G. For example, for a SISO G,
Qout = [ 0,   -0.5;
        -0.5, 0     ];
```

For more information about sector geometry, see getSectorIndex.

### dQout — Direction

matrix

Direction in which passivity is computed, returned as a square matrix that is 2\*ny on a side, where ny is the number of outputs of G. The value returned for dQout depends on what kind of passivity index you calculate:

• nu — For the input passivity index, dQout is given by:

```
dQout = [zeros(ny),zeros(ny);zeros(ny),eye(ny)];
```

For instance, for a SISO system, dQout = [0,0;0,1].

• rho — For the output passivity index, dQout is given by:

```
dQout = [eye(ny),zeros(ny);zeros(ny),zeros(ny)];
```

For instance, for a SISO system, dQout = [1,0;0,0].

• tau — For the combined I/O passivity index, dQout is given by:

```
dQout = eye(2*ny);
```

For instance, for a SISO system, dQout = [1,0;0,1].

- DX dQout is the custom value you provide in the dQ input argument.
- R The relative passivity index does not involve a direction, so in this case the function returns dQout = [].

For more information about directional indices, see  ${\tt getSectorIndex}$ .

### See Also

getSectorCrossover | getSectorIndex | isPassive | nyquist | passiveplot |
sectorplot

# **Topics**

"Passivity Indices"
"About Passivity and Passivity Indices"

### Introduced in R2016a

# getPeakGain

Peak gain of dynamic system frequency response

## **Syntax**

```
gpeak = getPeakGain(sys)
gpeak = getPeakGain(sys,tol)
gpeak = getPeakGain(sys,tol,fband)
[gpeak,fpeak] = getPeakGain( )
```

## **Description**

gpeak = getPeakGain(sys) returns the peak input/output gain in absolute units of the
dynamic system model, sys.

- If sys is a SISO model, then the peak gain is the largest value of the frequency response magnitude.
- If sys is a MIMO model, then the peak gain is the largest value of the frequency response 2-norm (the largest singular value across frequency) of sys. This quantity is also called the  $L_{\infty}$  norm of sys, and coincides with the  $H_{\infty}$  norm for stable systems (see norm).
- If sys is a model that has tunable or uncertain parameters, getPeakGain evaluates the peak gain at the current or nominal value of sys.
- If sys is a model array, getPeakGain returns an array of the same size as sys, where gpeak(k) = getPeakGain(sys(:,:,k)).

gpeak = getPeakGain(sys,tol) returns the peak gain of sys with relative accuracy
tol.

gpeak = getPeakGain(sys,tol,fband) returns the peak gain in the frequency
interval fband.

[gpeak, fpeak] = getPeakGain(\_\_\_\_) also returns the frequency fpeak at which the gain achieves the peak value gpeak, and can include any of the input arguments in previous syntaxes.

## **Examples**

#### **Peak Gain of Transfer Function**

Compute the peak gain of the resonance in the following transfer function:

$$sys = \frac{90}{s^2 + 1.5s + 90}.$$

$$sys = tf(90,[1,1.5,90]);$$

$$gpeak = getPeakGain(sys)$$

$$qpeak = 6.3444$$

The getPeakGain command returns the peak gain in absolute units.

### **Peak Gain with Specified Accuracy**

Compute the peak gain of the resonance in the transfer function with a relative accuracy of 0.01%.

```
sys = \frac{90}{s^2 + 1.5s + 90}.
sys = tf(90,[1,1.5,90]);
gpeak = getPeakGain(sys,0.0001)
qpeak = 6.3444
```

The second argument specifies a relative accuracy of 0.0001. The getPeakGain command returns a value that is within 0.0001 (0.01%) of the true peak gain of the transfer function. By default, the relative accuracy is 0.01 (1%).

### **Peak Gain Within Specified Band**

Compute the peak gain of the higher-frequency resonance in the transfer function

$$sys = \left(\frac{1}{s^2 + 0.2s + 1}\right)\left(\frac{100}{s^2 + s + 100}\right).$$

sys is the product of resonances at 1 rad/s and 10 rad/s.

```
sys = tf(1,[1,.2,1])*tf(100,[1,1,100]);
fband = [8,12];
gpeak = getPeakGain(sys,0.01,fband);
```

The fband argument causes getPeakGain to return the local peak gain between 8 and 12 rad/s.

#### Frequency of Peak Gain

Identify which of the two resonances has higher gain in the transfer function

$$sys = \left(\frac{1}{s^2 + 0.2s + 1}\right)\left(\frac{100}{s^2 + s + 100}\right).$$

sys is the product of resonances at 1 rad/s and 10 rad/s.

```
sys = tf(1,[1,.2,1])*tf(100,[1,1,100]);
[gpeak,fpeak] = getPeakGain(sys)

gpeak = 5.0747

fpeak = 0.9902
```

fpeak is the frequency corresponding to the peak gain gpeak. The peak at 1 rad/s is the overall peak gain of sys.

## **Input Arguments**

### sys — Input dynamic system

dynamic system model | model array

Input dynamic system, specified as any dynamic system model or model array. sys can be SISO or MIMO.

#### tol — Relative accuracy

0.01 (default) | positive real scalar

Relative accuracy of the peak gain, specified as a positive real scalar value. getPeakGain calculates gpeak such that the fractional difference between gpeak and the true peak gain of sys is no greater than tol. The default value is 0.01, meaning that gpeak is within 1% of the true peak gain.

### fband — Frequency interval

[0, Inf] (default) | 1-by-2 vector of positive real values

Frequency interval in which to calculate the peak gain, specified as a 1-by-2 vector of positive real values. Specify fband as a row vector of the form [fmin,fmax].

## **Output Arguments**

#### gpeak — Peak gain of dynamic system

scalar | array

Peak gain of the dynamic system model or model array sys, returned as a scalar value or an array.

- If sys is a single model, then gpeak is a scalar value.
- If sys is a model array, then gpeak is an array of the same size as sys, where gpeak(k) = getPeakGain(sys(:,:,k)).

### fpeak — Frequency of peak gain

nonnegative real scalar | array of nonnegative real values

Frequency at which the gain achieves the peak value <code>gpeak</code>, returned as a nonnegative real scalar value or an array of nonnegative real values. The frequency is expressed in units of rad/TimeUnit, relative to the TimeUnit property of <code>sys</code>.

- If sys is a single model, then fpeak is a scalar.
- If sys is a model array, then fpeak is an array of the same size as sys, where fpeak(k) is the peak gain frequency of sys(:,:,k).

# **Algorithms**

getPeakGain uses the algorithm of [1]. All eigenvalue computations are performed using structure-preserving algorithms from the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

### References

[1] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the  $H_{\infty}$ -Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287-293.

## See Also

bode | freqresp | getGainCrossover | hinfnorm | norm | sigma

### **Topics**

"Dynamic System Models"

Introduced in R2012a

# getPIDLoopResponse

Closed-loop and open-loop responses of systems with PID controllers

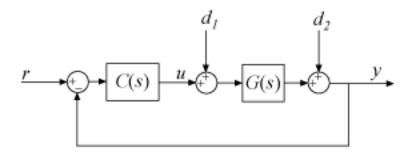
## **Syntax**

response = getPIDLoopResponse(C,G,looptype)

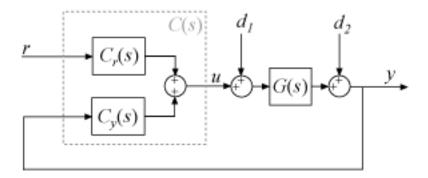
# **Description**

response = getPIDLoopResponse(C,G,looptype) returns a response of the control loop formed by the PID controller C and the plant G. The function returns the closed-loop, open-loop, controller action, or disturbance response that you specify with the looptype argument. The function assumes the following control architecture.

• When C is a pid or pidstd controller object (1-DOF controller):



When C is a pid2 or pidstd2 controller object (2-DOF controller):

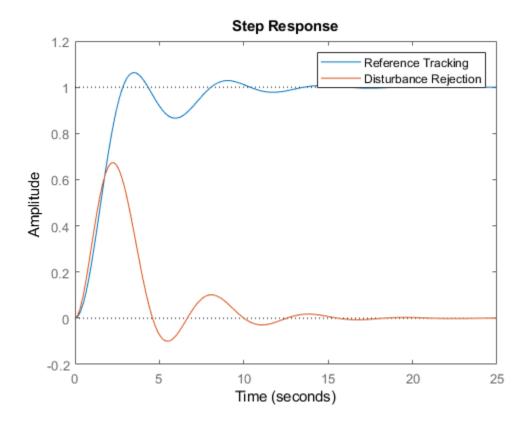


## **Examples**

### **Validate Controller Performance by Examining Closed-Loop Responses**

Design a PI controller for a SISO plant and examine its performance in reference tracking and disturbance rejection. For reference tracking, use the "closed-loop" response. For rejection of a load disturbance, use "input-disturbance".

```
G = tf(1,[1 1 1]);
C = pidtune(G,'PI');
Tref = getPIDLoopResponse(C,G,"closed-loop");
Tdist = getPIDLoopResponse(C,G,"input-disturbance");
step(Tref,Tdist)
legend("Reference Tracking","Disturbance Rejection")
```

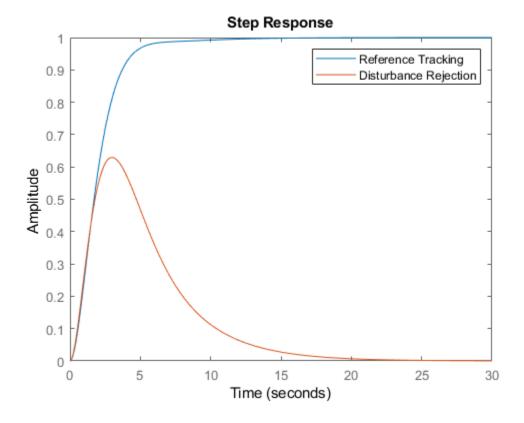


Validate the tuned controller by comparing the extracted responses to your design requirements for settling time and overshoot.

## Responses of System with 2-DOF PID Controller

Design a two-degree-of-freedom (2-DOF) PID controller for a plant and examine its performance in reference tracking and disturbance rejection. For reference tracking, use the "closed-loop" response. For rejection of a load disturbance, use "input-disturbance".

```
G = tf(1,[1 0.5 0.1]);
w0 = 1.5;
C = pidtune(G,'PID2',w0);
Tref = getPIDLoopResponse(C,G,"closed-loop");
Tdist = getPIDLoopResponse(C,G,"input-disturbance");
step(Tref,Tdist)
legend("Reference Tracking","Disturbance Rejection")
```



# **Input Arguments**

### **C** — **PID** controller

PID controller object

PID controller, specified as a PID controller object (pid, pidstd, pid2, or pidstd2).

#### **G** — Plant

dynamic system model

Plant, specified as a SISO dynamic system model, such as a tf, ss, zpk, or frd model object. If G is a model with tunable or uncertain elements (such as a genss or uss model), then the function uses the current or nominal value of the model.

### looptype — Loop response to return

string | character vector

Loop response to return, specified as a string or character vector. The available loop responses are given in the following table.

Response	1-DOF Controller	2-DOF Controller	Description
"open-loop"	GC	$-GC_y$	Response of the open- loop controller-plant system. Use for frequency-domain design. Use when your design specifications include robustness criteria such as open-loop gain margin and phase margin.
"closed-loop"	$\frac{GC}{1+GC} \text{ (from } r \text{ to } y\text{)}$	$\frac{GC_r}{1 - GC_y} \text{ (from } r \text{ to } y\text{)}$	Closed-loop system response to a step change in setpoint. Use when your design specifications include setpoint tracking.

Response	1-DOF Controller	2-DOF Controller	Description
"controller- effort"	$\frac{C}{1+GC} \text{ (from } r \text{ to } u\text{)}$	$\frac{C_r}{1 - GC_y} \text{ (from } r \text{ to } u\text{)}$	Closed-loop controller output response to a step change in setpoint. Use when your design is limited by practical constraints, such as controller saturation.
"input- disturbance"	$\frac{G}{1+GC} \text{ (from } d_1 \text{ to } y)$	$\frac{G}{1 - GC_y} \text{ (from } d_1 \text{ to } y\text{)}$	Closed-loop system response to load disturbance (a step disturbance at the plant input). Use when your design specifications include input disturbance rejection.
"output- disturbance"	$\frac{1}{1+GC} (\text{from } d_2 \text{ to } y)$	$\frac{1}{1 - GC_y} \text{ (from } d_2 \text{ to } y\text{)}$	Closed-loop system response to a step disturbance at plant output. Use when you want to analyze sensitivity to modeling errors.

# **Output Arguments**

### response — Selected loop response

ss model | frd model

Selected loop response, returned as a state-space (ss) or frequency-response data (frd) model. If G is an frd model, then response is also an frd model with the same frequencies as G. Otherwise, response is an ss model.

## See Also

pid|pid2|pidstd|pidstd2|pidtune

### Introduced in R2019a

# getPoints

Get list of analysis points in generalized model of control system

## **Syntax**

points = getPoints(T)

# **Description**

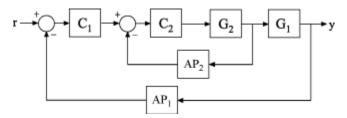
points = getPoints(T) returns the names of all analysis-point locations in a generalized state-space model of a control system. Use this function to query the list of available analysis points in the model for control system analysis or tuning. You can refer to the analysis-point locations by name to create design goals control system tuning or to compute open-loop and closed-loop responses using analysis commands such as getLoopTransfer and getIOTransfer.

## **Examples**

### **Analysis-Point Locations in Control System Model**

Build a closed-loop model of a cascaded feedback loop system, and get a list of analysis point locations in the model.

Create a model of the following cascaded feedback loop.  $C_1$  and  $C_2$  are tunable controllers.  $AP_1$  and  $AP_2$  are points of interest for analysis, which you mark with AnalysisPoint blocks.



```
G1 = tf(10,[1 10]);
G2 = tf([1 2],[1 0.2 10]);
C1 = tunablePID('C','pi');
C2 = tunableGain('G',1);
AP1 = AnalysisPoint('AP1');
AP2 = AnalysisPoint('AP2');
T = feedback(G1*feedback(G2*C2,AP2)*C1,AP1);
```

T is a genss model whose Control Design Blocks include the tunable controllers and the switches AP1 and AP2.

Get a list of the loop-opening sites in T.

getPoints returns a cell array listing loop-opening sites in the model.

For more complicated closed-loop models, you can use getPoints to keep track of a larger number of analysis points.

## **Input Arguments**

### T — Model of control system

generalized state-space model

Model of a control system, specified as a generalized state-space (genss) model. Locations in the model at which you can calculate system responses or specify design goals for tuning are marked by AnalysisPoint blocks in T.

# **Output Arguments**

### points — Analysis-point locations

cell array of character vectors

Analysis-point locations in the control system model, returned as a cell array of character vectors. This output is obtained by concatenating the Location properties of all AnalysisPoint blocks in the control system model.

## See Also

AnalysisPoint | genss | getIOTransfer | getLoopTransfer

## **Topics**

"Generalized Models"

Introduced in R2014b

# getSectorCrossover

Crossover frequencies for sector bound

# **Syntax**

wc = getSectorCrossover(H,Q)

## **Description**

wc = getSectorCrossover(H,Q) returns the frequencies at which the following matrix  $M(\omega)$  is singular:

$$M(\omega) = H(j\omega)^H Q H(j\omega).$$

When a frequency-domain sector plot exists, these frequencies are the frequencies at which the relative sector index (R-index) for H and Q equals 1. See "About Sector Bounds and Sector Indices" for details.

## **Examples**

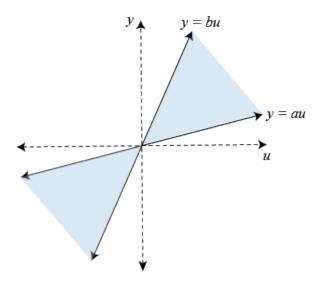
### **Find Sector Crossover Frequency**

Find the crossover frequencies for the dynamic system G(s) = (s + 2)/(s + 1) and the sector defined by:

$$S = \left\{ (y, u) \colon au^2 < uy < bu^2 \right\},$$

for various values of a and b.

In U/Y space, this sector is the shaded region of the following diagram (for a, b > 0).



The Q matrix for this sector is given by:

$$Q = \begin{bmatrix} 1 & -(a+b)/2 \\ -(a+b)/2 & ab \end{bmatrix}; \quad a = 0.1, \ b = 10.$$

getSectorCrossover finds the frequencies at which  $H(s)^H QH(s)$  is singular, for H(s) = [G(s); I]. For instance, find these frequencies for the sector defined by Q with a = 0.1 and b = 10.

```
G = tf([1 2],[1 1]);
H = [G;1];

a = 0.1;
b = 10;
Q = [1 -(a+b)/2 ; -(a+b)/2 a*b];
w = getSectorCrossover(H,Q)
w =
    0x1 empty double column vector
```

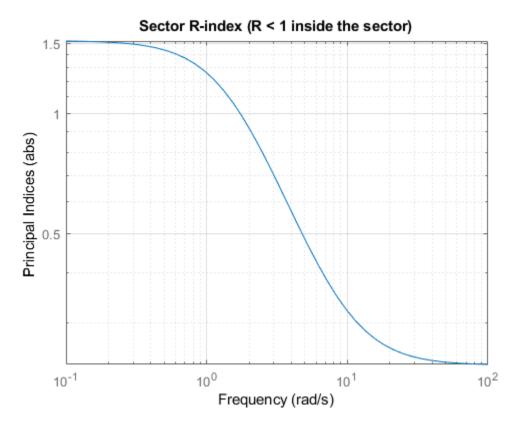
The empty result means that there are no such frequencies.

Now find the frequencies at which  $H^HQH$  is singular for a narrower sector, with a=0.5 and b=1.5.

```
a2 = 0.5;
b2 = 1.5;
Q2 = [1 -(a2+b2)/2 ; -(a2+b2)/2 a2*b2];
w2 = getSectorCrossover(H,Q2)
w2 = 1.7321
```

Here the resulting frequency is where the R-index for H and Q2 is equal to 1, as shown in the sector plot.

```
sectorplot(H,Q2)
```



Thus, when a sector plot exists for a system H and sector Q, getSectorCrossover finds the frequencies at which the R-index is 1.

## **Input Arguments**

## **H** — Model to analyze

dynamic system model

Model to analyze against sector bounds, specified as a dynamic system model such as a tf, ss, or genss model. H can be continuous or discrete. If H is a generalized model with

tunable or uncertain blocks, getSectorCrossover analyzes the current, nominal value of H.

To get the frequencies at which the I/O trajectories (u,y) of a linear system G lie in a particular sector, use H = [G;I], where I = eyes(nu), and nu is the number of inputs of G.

#### Q — Sector geometry

matrix | LTI model

Sector geometry, specified as:

- A matrix, for constant sector geometry. Q is a symmetric square matrix that is ny on a side, where ny is the number of outputs of H.
- An LTI model, for frequency-dependent sector geometry. **Q** satisfies Q(s)' = Q(-s). In other words, Q(s) evaluates to a Hermitian matrix at each frequency.

The matrix **Q** must be indefinite to describe a well-defined conic sector. An indefinite matrix has both positive and negative eigenvalues.

For more information, see "About Sector Bounds and Sector Indices".

## **Output Arguments**

### wc — Sector crossover frequencies

vector | []

Sector crossover frequencies, returned as a vector. The frequencies are expressed in rad/ TimeUnit, relative to the TimeUnit property of H. If the trajectories of H never cross the boundary, wc = [].

### See Also

getGainCrossover|getSectorIndex|sectorplot

### **Topics**

"About Sector Bounds and Sector Indices"

### Introduced in R2016a

# getSectorIndex

Compute conic-sector index of linear system

## **Syntax**

```
RX = getSectorIndex(H,Q)
RX = getSectorIndex(H,Q,tol)
RX = getSectorIndex(H,Q,tol,fband)
[RX,FX] = getSectorIndex(___)
[RX,FX,W1,W2,Z] = getSectorIndex(___)
DX = getSectorIndex(H,Q,dQ)
DX = getSectorIndex(H,Q,dQ,tol)
```

# **Description**

RX = getSectorIndex(H,Q) computes the relative index RX for the linear system H and the conic sector specified by Q. When RX < 1, all output trajectories y(t) = Hu(t) lie in the sector defined by:

$$\int_0^T y(t)^T Q y(t) dt < 0,$$

for all  $T \ge 0$ .

getSectorIndex can also check whether all I/O trajectories  $\{u(t),y(t)\}$  of a linear system G lie in the sector defined by:

$$\int_{0}^{T} {y(t) \choose u(t)}^{T} Q {y(t) \choose u(t)} dt < 0,$$

for all  $T \ge 0$ . To do so, use getSectorIndex with H = [G;I], where I = eyes(nu), and nu is the number of inputs of G.

For more information about sector bounds and the relative index, see "About Sector Bounds and Sector Indices".

RX = getSectorIndex(H,Q,tol) computes the index with relative accuracy specified by tol.

RX = getSectorIndex(H,Q,tol,fband) computes the passivity index by restricting the inequalities that define the index to a specified frequency interval. This syntax is available only when Q has as many negative eigenvalues as there are inputs in H.

 $[RX,FX] = getSectorIndex( ____ )$  also returns the frequency at which the index value RX is achieved. FX is set to NaN when the number of negative eigenvalues in Q differs from the number of inputs in H. You can use this syntax with any of the previous combinations of input arguments.

[RX,FX,W1,W2,Z] = getSectorIndex( $\underline{\phantom{a}}$ ) also returns the decomposition of Q into its positive and negative parts, as well as the spectral factor Z when Q is dynamic. When Q is a matrix (constant sector bounds), Z = 1. You can use this syntax with any of the previous combinations of input arguments.

DX = getSectorIndex(H,Q,dQ) computes the index in the direction specified by the matrix dQ. If DX > 0, then the output trajectories of H fit in the conic sector specified by Q. For more information about the directional index, see "About Sector Bounds and Sector Indices".

The directional index is not available if H is a frequency-response data (frd) model.

DX = getSectorIndex(H,Q,dQ,tol) computes the index with relative accuracy specified by tol.

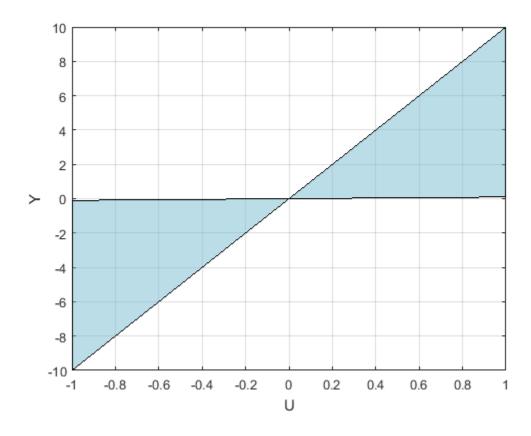
## **Examples**

#### Check Sector Bounds

Test whether, on average, the I/O trajectories of G(s) = (s + 2)/(s + 1) belong within the sector defined by:

$$S = \left\{ (y, u) : 0 . 1u^2 < uy < 10u^2 \right\}.$$

In U/Y space, this sector is the shaded region of the following diagram.



The Q matrix corresponding to this sector is given by:

$$Q = \begin{bmatrix} 1 & -(a+b)/2 \\ -(a+b)/2 & ab \end{bmatrix}; \quad a = 0.1, \ b = 10.$$

A trajectory y(t) = Gu(t) lies within the sector S when for all T > 0,

$$0.1 \int_{0}^{T} u(t)^{2} < \int_{0}^{T} u(t)y(t)dt < 10 \int_{0}^{T} u(t)^{2}dt.$$

To check whether trajectories of G satisfy the sector bound, represented by Q, compute the R-index for H = [G;1].

```
G = tf([1 2],[1 1]);

a = 0.1; b = 10;

Q = [1 -(a+b)/2 ; -(a+b)/2 a*b];

R = getSectorIndex([G;1],Q)

R = 0.4074
```

This resulting R is less than 1, indicating that the trajectories fit within the sector. The value of R tells you how much tightly the trajectories fit in the sector. This value, R = 0.41, means that the trajectories would fit in a narrower sector with a base 1/0.41 = 2.4 times smaller.

## **Input Arguments**

#### **H** — Model to analyze

dynamic system model | model array

Model to analyze against sector bounds, specified as a dynamic system model such as a tf, ss, or genss model. H can be continuous or discrete. If H is a generalized model with tunable or uncertain blocks, getSectorIndex analyzes the current, nominal value of H.

To analyze whether all I/O trajectories  $\{u(t),y(t)\}$  of a linear system G lie in a particular sector, use H = [G;I].

If H is a model array, then getSectorIndex returns the passivity index as an array of the same size, where:

```
index(k) = getSectorIndex(H(:,:,k),___)
```

Here, index is either RX, or DX, depending on which input arguments you use.

### Q — Sector geometry

matrix | LTI model

Sector geometry, specified as:

• A matrix, for constant sector geometry. Q is a symmetric square matrix that is ny on a side, where ny is the number of outputs of H.

• An LTI model, for frequency-dependent sector geometry. Q satisfies Q(s)' = Q(-s). In other words, Q(s) evaluates to a Hermitian matrix at each frequency.

The matrix **Q** must be indefinite to describe a well-defined conic sector. An indefinite matrix has both positive and negative eigenvalues.

For more information, see "About Sector Bounds and Sector Indices".

#### tol — Relative accuracy

0.01 (default) | positive real value

Relative accuracy for the calculated sector index. By default, the tolerance is 1%, meaning that the returned index is within 1% of the actual index.

### fband — Frequency interval

1-by-2 array

Frequency interval for calculating the sector index, specified as an array of the form [fmin,fmax]. When you provide fband, getSectorIndex restricts to the specified frequency interval the inequalities that define the index. Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the dynamic system model H.

#### d0 - Direction

matrix

Direction in which to compute directional sector index, specified as a nonnegative definite matrix. The matrix dQ is a symmetric square matrix that is ny on a side, where ny is the number of outputs of H.

## **Output Arguments**

#### RX — Relative sector index

scalar | array

Relative index of the system H for the sector specified by Q, returned as a scalar value, or an array if H is an array. If RX < 1, then the output trajectories of H fit inside the cone of Q.

The value of RX provides a measure of how tightly the output trajectories of H fit inside the cone. Let the following be an orthogonal decomposition of the symmetric matrix Q into its positive and negative parts.

$$Q = W_1 W_1^T - W_2 W_2^T, \quad W_1^T W_2 = 0.$$

(Such a decomposition is readily obtained from the Schur decomposition of Q.) Then, RX is the smallest R that satisfies:

$$\int_{0}^{T} y(t)^{T} \left( W_{1} W_{1}^{T} - R^{2} W_{2} W_{2}^{T} \right) y(t) dt < 0,$$

for all  $T \ge 0$ . Varying R is equivalent to adjusting the slant angle of the cone specified by Q until the cone fits tightly around the output trajectories of H. The cone base-to-height ratio is proportional to R.

For more information about interpretations of the relative index, see "About Sector Bounds and Sector Indices".

### FX — Frequency at which index is achieved

nonnegative scalar | array

Frequency at which the index RX is achieved, returned as a nonnegative scalar, or an array if H is an array. In general, the index varies with frequency (see sectorplot). The returned value is the largest value over all frequencies. FX is the frequency at which this value occurs, returned in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of H.

### W1, W2 — Positive and negative factors of Q

matrices

Positive and negative factors of Q, returned as matrices. For a constant Q, W1 and W2 satisfy:

$$Q = W_1 W_1^T - W_2 W_2^T, \quad W_1^T W_2 = 0.$$

#### Z — Bistable model

state-space model | 1

Bistable model in the factorization of Q, returned as:

- If Q is a constant matrix, Z = 1.
- If Q is frequency-dependent, then Z is a state-space (ss) model such that:

$$Q(j\omega) = Z(j\omega)^H (W_1 W_1^T - W_2 W_2^T) Z(j\omega).$$

### DX — Directional sector index

scalar | array

Directional sector index of the system H for the sector specified by Q in the direction dQ, returned as a scalar value, or an array if H is an array. The directional index is the largest  $\tau$  which satisfies:

$$\int_0^T y(t)^T (Q + \tau dQ) \, y(t) dt < 0,$$

for all  $T \ge 0$ .

## See Also

getPassiveIndex | getPeakGain | getSectorCrossover | nyquist | sectorplot

## **Topics**

"About Sector Bounds and Sector Indices"

#### Introduced in R2016a

# getSensitivity

Sensitivity function from generalized model of control system

## **Syntax**

S = getSensitivity(T,location)
S = getSensitivity(T,location,opening)

## **Description**

S = getSensitivity(T,location) returns the sensitivity function on page 2-483 at the specified location for a generalized model of a control system.

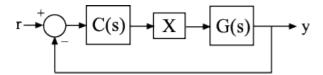
S = getSensitivity(T,location,opening) specifies additional loop openings for the sensitivity function calculation. Use an opening, for example, to calculate the sensitivity function of an inner loop, with the outer loop open.

If opening and location list the same point, the software opens the loop after measuring the signal at the point.

## **Examples**

### **Sensitivity Function at a Location**

Compute the sensitivity at the plant input, marked by the analysis point X.



Create a model of the system by specifying and connecting a numeric LTI plant model G, a tunable controller C, and the AnalysisPoint block X. Use the AnalysisPoint block to mark the location where you assess the sensitivity (plant input in this example).

```
G = tf([1],[1 5]);
C = tunablePID('C','p');
C.Kp.Value = 3;
X = AnalysisPoint('X');
T = feedback(G*X*C,1);
```

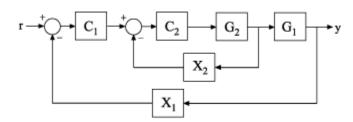
T is a genss model that represents the closed-loop response of the control system from r to y. The model contains the AnalysisPoint block, X, that identifies the analysis point.

Calculate the sensitivity, S, at X.

```
S = getSensitivity(T,'X');
tf(S)
ans =
From input "X" to output "X":
    s + 5
    ----
    s + 8
Continuous-time transfer function.
```

### **Specify Additional Loop Opening for Sensitivity Function Calculation**

Calculate the inner-loop sensitivity at the output of G2, with the outer loop open.



Create a model of the system by specifying and connecting the numeric plant models, tunable controllers, and AnalysisPoint blocks. G1 and G2 are plant models, C1 and C2 are tunable controllers, and X1 and X2 are AnalysisPoint blocks that mark potential loop-opening locations.

## **Input Arguments**

### T — Model of control system

generalized state-space model

Model of a control system, specified as a generalized state-space model (genss).

Locations at which you can perform sensitivity analysis or open loops are marked by AnalysisPoint blocks in T. Use getPoints(T) to get the list of such locations.

#### location — Location

character vector | cell array of character vectors

Location at which you calculate the sensitivity function on page 2-483, specified as a character vector or cell array of character vectors. To extract the sensitivity function at multiple locations, use a cell array of character vectors.

Each specified location must match an analysis point in T. Analysis points are marked using AnalysisPoint blocks. To get the list of available analysis points in T, use getPoints(T).

```
Example: 'u' or {'u', 'y'}
```

### opening - Additional loop opening

character vector | cell array of character vectors

Additional loop opening used to calculate the sensitivity function on page 2-479, specified as a character vector or cell array of character vectors. To open the loop at multiple locations, use a cell array of character vectors.

Each specified opening must match an analysis point in T. Analysis points are marked using AnalysisPoint blocks. To get the list of available analysis points in T, use getPoints(T).

Use an opening, for example, to calculate the sensitivity function of an inner loop, with the outer loop open.

If opening and location list the same point, the software opens the loop after measuring the signal at the point.

```
Example: 'y_outer' or {'y_outer', 'y_outer2'}
```

## **Output Arguments**

### S — Sensitivity function

generalized state-space model

Sensitivity function on page 2-483 of the control system, T, measured at location, returned as a generalized state-space model (genss).

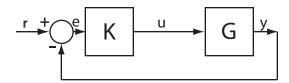
- If location specifies a single analysis point, then S is a SISO genss model.
- If location is a vector signal, or specifies multiple analysis points, then S is a MIMO genss model.

#### **More About**

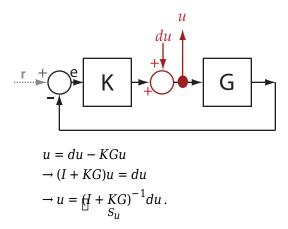
#### **Sensitivity Function**

The sensitivity function, also referred to simply as sensitivity, measures how sensitive a signal is to an added disturbance. Feedback reduces the sensitivity in the frequency band where the open-loop gain is greater than 1.

Consider the following model:

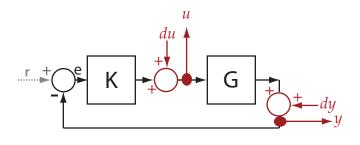


The sensitivity,  $S_u$ , at u is defined as the transfer function from du to u:



Here, I is an identity matrix of the same size as KG.

Sensitivity at multiple locations, for example, u and y, is defined as the MIMO transfer function from the disturbances to sensitivity measurements:



$$S = \begin{bmatrix} S_{du \to u} & S_{dy \to u} \\ S_{du \to y} & S_{dy \to y} \end{bmatrix}.$$

#### **See Also**

AnalysisPoint | genss | getCompSensitivity | getIOTransfer |
getLoopTransfer | getPoints | getSensitivity | getValue | systume

#### Introduced in R2014a

## getStateEstimate

Extract best state estimate and covariance from particles

## **Syntax**

```
State = getStateEstimate(pf)
[State,StateCovariance] = getStateEstimate(pf)
```

## **Description**

State = getStateEstimate(pf) returns the best state estimate based on the current
set of particles. The estimate is extracted based on the StateEstimationMethod
property from the particleFilter object, pf.

[State, StateCovariance] = getStateEstimate(pf) also returns the covariance of the state estimate. The covariance is a measure of the uncertainty of the state estimate. Not all state estimation methods support covariance output. In this case, getStateEstimate returns StateCovariance as [].

The State and StateCovariance information can directly be accessed as properties of the particle filter object pf, as pf.State and pf.StateCovariance. However, when both these quantities are needed, using the getStateEstimation method with two output arguments is more computationally efficient.

## **Examples**

#### **State Estimation using Particle Filter**

Create a particle filter, and set the state transition and measurement likelihood functions.

myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF, 1000, [2;0], eye(2));
Pick the mean state estimation and systematic resampling methods.
myPF.StateEstimationMethod = 'mean';
myPF.ResamplingMethod = 'systematic';
myPF
myPF =
  particleFilter with properties:
           NumStateVariables: 2
                NumParticles: 1000
          StateTransitionFcn: @vdpParticleFilterStateFcn
    MeasurementLikelihoodFcn: @vdpMeasurementLikelihoodFcn
     IsStateVariableCircular: [0 0]
            ResamplingPolicy: [1x1 particleResamplingPolicy]
            ResamplingMethod: 'systematic'
       StateEstimationMethod: 'mean'
            StateOrientation: 'column'
                    Particles: [2x1000 double]
                      Weights: [1x1000 double]
                        State: 'Use the getStateEstimate function to see the value.'
             StateCovariance: 'Use the getStateEstimate function to see the value.'
Assuming a measurement 2.1, run one predict and correct step.
[PredictedState, PredictedStateCovariance] = predict(myPF);
[CorrectedState, CorrectedStateCovariance] = correct(myPF,2.1);
Get the best state estimate and covariance based on the StateEstimationMethod
property.
[State, StateCovariance] = getStateEstimate(myPF)
State = 2 \times 1
    2.1018
   -0.1413
StateCovariance = 2 \times 2
    0.0175 -0.0096
```

-0.0096 0.5394

## **Input Arguments**

#### pf — Particle filter

particleFilter object

Particle filter, specified as an object. See particleFilter for more information.

## **Output Arguments**

#### State — Best state estimate

[] (default) | vector

Best state estimate, defined as a vector based on the condition of the StateOrientation property:

- If StateOrientation is 'row' then State is a 1-by-NumStateVariables vector
- If StateOrientation is 'column' then State is a NumStateVariables-by-1 vector

#### StateCovariance — Current estimate of state estimation error covariance

NumStateVariables-by-NumStateVariables array (default) | [] | array

Current estimate of state estimation error covariance, defined as an NumStateVariables-by-NumStateVariables array. StateCovariance is calculated based on the StateEstimationMethod. If you specify a state estimation method that does not support covariance, then the function returns StateCovariance as [].

#### See Also

correct | initialize | particleFilter | predict

#### **Topics**

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"

#### Introduced in R2017b

# **getValue**

Current value of Generalized Model

## **Syntax**

```
curval = getValue(M)
curval = getValue(M,blockvalues)
curval = getValue(M,Mref)
```

## **Description**

curval = getValue(M) returns the current value curval of the Generalized LTI model
or Generalized matrix M. The current value is obtained by replacing all Control Design
Blocks in M by their current value. (For uncertain blocks, the "current value" is the
nominal value of the block.)

curval = getValue(M,blockvalues) uses the block values specified in the structure
blockvalues to compute the current value. The field names and values of blockvalues
specify the block names and corresponding values. Blocks of M not specified in
blockvalues are replaced by their current values.

curval = getValue(M,Mref) inherits block values from the generalized model Mref.
This syntax is equivalent to curval = getValue(M,Mref.Blocks). Use this syntax to
evaluate the current value of M using block values computed elsewhere (for example,
tuned values obtained with tuning commands such as systume, looptume, or the Robust
Control Toolbox command hinfstruct).

## **Input Arguments**

М

Generalized LTI model or Generalized matrix.

#### **blockvalues**

Structure specifying blocks of M to replace and the values with which to replace those blocks.

The field names of blockvalues match names of Control Design Blocks of M. Use the field values to specify the replacement values for the corresponding blocks of M. The field values can be numeric values, dynamic system models, or static models. If some field values are Control Design Blocks or Generalized LTI models, the current values of those models are used to compute curval.

#### Mref

Generalized LTI model. If you provide Mref, getValue computes curval using the current values of the blocks in Mref whose names match blocks in M.

## **Output Arguments**

#### curval

Numeric array or Numeric LTI model representing the current value of M.

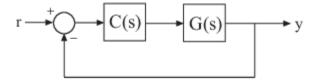
If you do not specify a replacement value for a given Control Design Block of M, getValue uses the current value of that block.

## **Examples**

#### **Evaluate Model for Specified Values of its Blocks**

This example shows how to replace a Control Design Block in a Generalized LTI model with a specified replacement value using getValue.

Consider the following closed-loop system:



The following code creates a genss model of this system with  $G(s) = \frac{(s-1)}{(s+1)^3}$  and a tunable PI controller C.

```
G = zpk(1,[-1,-1,-1],1);
C = tunablePID('C','pi');
Try = feedback(G*C,1)
```

The genss model Try has one Control Design Block, C. The block C is initialized to default values, and the model Try has a current value that depends on the current value of C. Use getValue to evaluate C and Try to examine the current values.

**1** Evaluate C to obtain its current value.

```
Cnow = getValue(C)
```

This command returns a numeric pid object whose coefficients reflect the current values of the tunable parameters in C.

**2** Evaluate Try to obtain its current value.

```
Tnow = getValue(Try)
```

This command returns a numeric model that is equivalent to feedback(G\*Cnow, 1).

#### Access Values of Tuned Models and Blocks

Propagate changes in block values from one model to another using getValue.

This technique is useful for accessing values of models and blocks tuned with tuning commands such as systune, looptune, or hinfstruct. For example, if you have a closed-loop model of your control system T0, with two tunable blocks, C1 and C2, you can tune it using:

```
[T,fSoft] = systune(T0,SoftRegs);
```

You can then access the tuned values of C1 and C2, as well as any closed-loop model H that depends on C1 and C2, using the following:

```
C1t = getValue(C1,T);
C2t = getValue(C2,T);
Ht = getValue(H,T);
```

## **See Also**

genss | hinfstruct | looptune | replaceBlock | systume

Introduced in R2011b

#### gram

Controllability and observability Gramians

## **Syntax**

```
Wc = gram(sys,'c')
Wc = gram(sys,'o')
Wc = gram(____,opt)
```

## **Description**

Wc = gram(sys,'c') calculates the controllability Gramian of the state-space (ss)
model sys.

Wc = gram(sys,'o') calculates the observability Gramian of the ss model sys.

Wc = gram(\_\_\_\_,opt) calculates time-limited or frequency-limited Gramians. opt is an option set that specifies time or frequency intervals for the computation. Create opt using the gramOptions command.

You can use Gramians to study the controllability and observability properties of statespace models and for model reduction [1]. They have better numerical properties than the controllability and observability matrices formed by ctrb and obsv.

Given the continuous-time state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

the controllability Gramian is defined by

$$W_C = \int_0^\infty e^{A\tau} B B^T e^{A^T \tau} d\tau$$

The controllability Gramian is positive definite if and only if (A, B) is controllable.

The observability Gramian is defined by

$$W_o = \int_0^\infty e^{A^T \tau} C^T C e^{A \tau} d\tau$$

The observability Gramian is positive definite if and only if (A, C) is observable.

The discrete-time counterparts of the controllability and observability Gramians are

$$W_{c} = \sum_{k=0}^{\infty} A^{k} B B^{T} (A^{T})^{k}, \ W_{o} = \sum_{k=0}^{\infty} (A^{T})^{k} C^{T} C A^{k}$$

respectively.

Use time-limited or frequency-limited Gramians to examine the controllability or observability of states within particular time or frequency intervals. The definition of these Gramians is as described in [2].

## **Examples**

#### **Compute Frequency-Limited Gramian**

Compute the controllability Gramian of the following state-space model. Focus the computation on the frequency interval with the most energy.

```
sys = ss([-.1 -1;1 0],[1;0],[0 1],0);
```

The model contains a peak at 1 rad/s. Use gramOptions to specify an interval around that frequency.

#### Limitations

The *A* matrix must be stable (all eigenvalues have negative real part in continuous time, and magnitude strictly less than one in discrete time).

## **Algorithms**

The controllability Gramian  $W_c$  is obtained by solving the continuous-time Lyapunov equation

$$AW_C + W_C A^T + BB^T = 0$$

or its discrete-time counterpart

$$AW_CA^T - W_C + BB^T = 0$$

Similarly, the observability Gramian  $W_o$  solves the Lyapunov equation

$$A^T W_o + W_o A + C^T C = 0$$

in continuous time, and the Lyapunov equation

$$A^T W_0 A - W_0 + C^T C = 0$$

in discrete time.

The computation of time-limited and frequency-limited Gramians is as described in [2].

#### References

- [1] Kailath, T., Linear Systems, Prentice-Hall, 1980.
- [2] Gawronski, W. and J.N. Juang. "Model Reduction in Limited Time and Frequency Intervals." International Journal of Systems Science. Vol. 21, Number 2, 1990, pp. 349–376.

## **See Also**

balreal|dlyap|gramOptions|hsvd|lyap

**Introduced before R2006a** 

# gramOptions

Options for the gram command

## **Syntax**

```
opt = gramOptions
opt = gramOptions(Name, Value)
```

## **Description**

opt = gramOptions returns an option set with the default options for gram.

opt = gramOptions(Name, Value) returns an options set with the options specified by
one or more Name, Value pair arguments.

## **Examples**

#### **Compute Frequency-Limited Gramian**

Compute the controllability Gramian of the following state-space model. Focus the computation on the frequency interval with the most energy.

```
sys = ss([-.1 -1;1 0],[1;0],[0 1],0);
```

The model contains a peak at 1 rad/s. Use gramOptions to specify an interval around that frequency.

## **Input Arguments**

#### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: 'FregIntervals', [0.8 1.2]

#### FreqIntervals — Frequency intervals for computing Gramians

[] (default) | two-column matrix

Frequency intervals for computing frequency-limited controllability and observability Gramians, specified as a matrix with two columns. Each row specifies a frequency interval [fmin fmax], where fmin and fmax are nonnegative frequencies, expressed in the frequency unit of the model. For example:

- To restrict the computation to the range between 3 rad/s and 15 rad/s, assuming the frequency unit of the model is rad/s, set FreqIntervals to [3 15].
- To restrict the computation to two frequency intervals, 3-15 rad/s and 40-60 rad/s, use [3 15; 40 60].
- To specify all frequencies below a cutoff frequency fcut, use [0 fcut].
- To specify all frequencies above the cutoff, use [fcut Inf] in continuous time, or [fcut pi/Ts] in discrete time, where Ts is the sample time of the model.

The default value, [], imposes no frequency limitation and is equivalent to [0 Inf] in continuous time or [0 pi/Ts] in discrete time. However, if you specify a TimeIntervals value other than [], then this limit overrides FreqIntervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

#### TimeIntervals — Time intervals for computing Gramians

[] (default) | two-column matrix

Time intervals for computing time-limited controllability and observability Gramians, specified as a matrix with two columns. Each row specifies a time interval [tmin tmax],

where tmin and tmax are nonnegative times, expressed in the time unit of the model. For example:

- To restrict the computation to the range between 3 s and 15 s, assuming the time unit of the model is seconds, set TimeIntervals to [3 15].
- To restrict the computation to two time intervals, 3-15 s and 40-60 s, use [3 15; 40 60].
- To specify all times from zero up to a cutoff time tcut, use [0 tcut]. To specify all times after the cutoff, use [tcut Inf].

The default value, [], imposes no time limitation and is equivalent to [0 Inf]. However, if you specify a FreqIntervals value other than [], then this limit overrides Timeintervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

## **Output Arguments**

#### opt — Options for gram

gramOptions options set

Options for gram, returned as a gramOptions options set. Use opt as the last argument to gram to compute time-limited or frequency-limited Gramians.

#### See Also

gram|hsvd

Introduced in R2016a

# hasdelay

True for linear model with time delays

#### **Syntax**

```
B = hasdelay(sys)
B = hasdelay(sys,'elem')
```

## **Description**

B = hasdelay(sys) returns 1 (true) if the model sys has input delays, output delays, I/O delays, or internal delays, and 0 (false) otherwise. If sys is a model array, then B is true if least one model in sys has delays.

B = hasdelay(sys,'elem') returns a logical array of the same size as the model array sys. The logical array indicates which models in sys have delays.

#### See Also

absorbDelay | totaldelay

Introduced before R2006a

# hasInternalDelay

Determine if model has internal delays

## **Syntax**

```
B = hasInternalDelay(sys)
B = hasInternalDelay(sys,'elem')
```

## **Description**

B = hasInternalDelay(sys) returns 1 (true) if the model sys has internal delays, and 0 (false) otherwise. If sys is a model array, then B is true if least one model in sys has delays.

B = hasInternalDelay(sys, 'elem') checks each model in the model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys have internal delays.

## **Examples**

#### **Check Model for Internal Delays**

Build a dynamic system model of the following closed-loop system.

```
s = tf('s');
G = exp(-2.4*s)/(s-5);
C = pid(5,0.1);
```

sys = feedback(G\*C,1);

Check the model for internal delays.

```
B = hasInternalDelay(sys)
B = logical
1
```

The model, sys, has an internal delay because of the transport delay in the plant G. Therefore, hasInternalDelay returns 1.

## **Input Arguments**

#### sys — Model or array to check

dynamic system model | model array

Model or array to check for internal delays, specified as a dynamic system model or array of dynamic system models.

## **Output Arguments**

#### **B** — Flag indicating presence of internal delays

logical | logical array

Flag indicating presence of internal delays in input model or array, returned as a logical value or logical array.

#### See Also

getDelayModel | hasdelay

Introduced in R2013a

#### hsvd

Hankel singular values of dynamic system

## **Syntax**

```
hsv = hsvd(sys)
hsv = hsvd(sys,opts)
[hsv,baldata] = hsvd(____)
hsvd(____)
```

## **Description**

hsv = hsvd(sys) computes the Hankel singular values hsv of the dynamic system sys. In state coordinates that equalize the input-to-state and state-to-output energy transfers, the Hankel singular values measure the contribution of each state to the input/output behavior. Hankel singular values are to model order what singular values are to matrix rank. In particular, small Hankel singular values signal states that can be discarded to simplify the model (see balred).

For models with unstable poles, hsvd only computes the Hankel singular values of the stable part and entries of hsv corresponding to unstable modes are set to Inf.

hsv = hsvd(sys,opts) computes the Hankel singular values using options that you specify using hsvdOptions. Options include offset and tolerance options for computing the stable-unstable decompositions. The options also allow you to limit the HSV computation to energy contributions within particular time and frequency intervals. See hsvdOptions for details.

[hsv,baldata] = hsvd() returns additional data to speed up model order reduction with balred. You can use this syntax with any of the previous combination	ns of
input arguments.	

hsvd( \_\_\_\_) displays a Hankel singular values plot.

## **Examples**

#### **Compute Hankel Singular Values of System With Near-Unstable Pole**

Create a system with a stable pole very near to 0, and display the Hankel singular values.

```
sys = zpk([1 2],[-1 -2 -3 -10 -1e-7],1);
hsv = hsvd(sys)

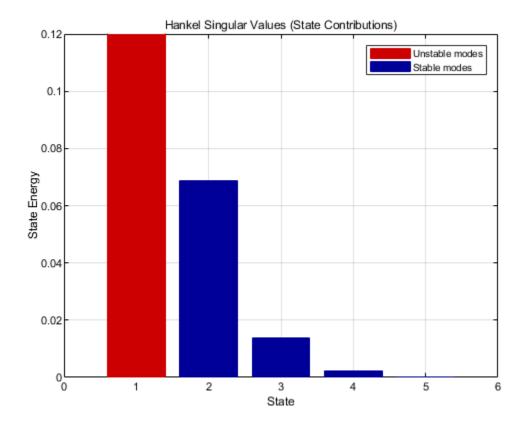
hsv = 5×1
10<sup>5</sup> ×

1.6667
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
```

Notice the dominant Hankel singular value with magnitude  $10^5$ , which is so much larger that the significant digits of the other modes are not displayed. This value is due to the near-unstable mode at  $s = 10^{-7}$ . Use the 'Offset' option to treat this mode as unstable.

The Hankel singular value of modes that are unstable, or treated as unstable, is returned as Inf. Create a Hankel singular-value plot while treating this mode as unstable.

```
hsvd(sys,opts)
```



The unstable mode is shown in red on the plot.

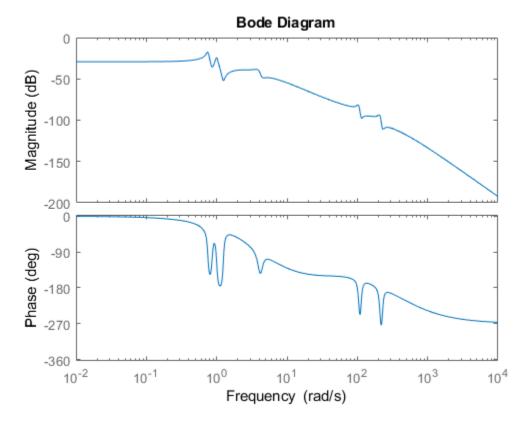
By default, hsvd uses a linear scale. To switch the plot to a log scale, right-click on the plot and select **Y Scale > Log**. For information about programmatically changing properties of HSV plots, see hsvplot.

#### **Frequency-Limited Hankel Singular Values**

Compute the Hankel singular values of a model with low-frequency and high-frequency dynamics. Focus the calculation on the high-frequency modes.

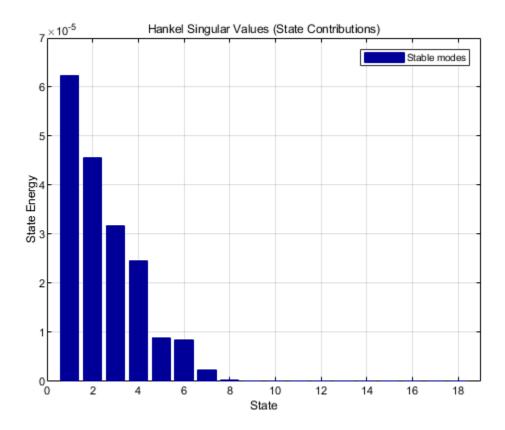
Load the model and examine its frequency response.

load modeselect Gms
bodeplot(Gms)



Gms has two sets of resonances, one at relatively low frequency and the other at relatively high frequency. Compute the Hankel singular values of the high-frequency modes, excluding the energy contributions to the low-frequency dynamics. To do so, use hsvdOptions to specify a frequency interval above 30 rad/s.

```
opts = hsvdOptions('FreqInterval',[30 Inf]);
hsvd(Gms,opts)
```



# **Tips**

To create a Hankel singular-value plot with more flexibility to programmatically customize the plot, use hsvplot.

# **Algorithms**

The AbsTol, RelTol, and Offset options of hsvdOptions are only used for models with unstable or marginally stable dynamics. Because Hankel singular values are only

meaningful for stable dynamics, hsvd must first split such models into the sum of their stable and unstable parts:

$$G = G_s + G_ns$$

This decomposition can be tricky when the model has modes close to the stability boundary (e.g., a pole at s=-1e-10), or clusters of modes on the stability boundary (e.g., double or triple integrators). While hsvd is able to overcome these difficulties in most cases, it sometimes produces unexpected results such as

- **1** Large Hankel singular values for the stable part.
  - This happens when the stable part G\_s contains some poles very close to the stability boundary. To force such modes into the unstable group, increase the 'Offset' option to slightly grow the unstable region.
- **2** Too many modes are labeled "unstable." For example, you see 5 red bars in the HSV plot when your model had only 2 unstable poles.

The stable/unstable decomposition algorithm has built-in accuracy checks that reject decompositions causing a significant loss of accuracy in the frequency response. Such loss of accuracy arises, e.g., when trying to split a cluster of stable and unstable modes near s=0. Because such clusters are numerically equivalent to a multiple pole at s=0, it is actually desirable to treat the whole cluster as unstable. In some cases, however, large relative errors in low-gain frequency bands can trip the accuracy checks and lead to a rejection of valid decompositions. Additional modes are then absorbed into the unstable part  $G_ns$ , unduly increasing its order.

Such issues can be easily corrected by adjusting the AbsTol and RelTol tolerances. By setting AbsTol to a fraction of smallest gain of interest in your model, you tell the algorithm to ignore errors below a certain gain threshold. By increasing RelTol, you tell the algorithm to sacrifice some relative model accuracy in exchange for keeping more modes in the stable part  ${\tt G}$   ${\tt S}$ .

If you use the TimeIntervals or FreqIntervals options of hsvdOptions, then hsvd bases the computation of state energy contributions on time-limited or frequency-limited controllability and observability Gramians. For information about calculating time-limited and frequency-limited Gramians, see gram and [1].

#### References

[1] Gawronski, W. and J.N. Juang. "Model Reduction in Limited Time and Frequency Intervals." *International Journal of Systems Science*. Vol. 21, Number 2, 1990, pp. 349–376.

#### See Also

balreal | balred | hsvdOptions | hsvplot

**Introduced before R2006a** 

# **hsvdOptions**

Create option set for computing Hankel singular values and input/output balancing

## **Syntax**

```
opts = hsvdOptions
opts = hsvdOptions(Name, Value)
```

## **Description**

opts = hsvdOptions returns the default options for the hsvd and balreal commands.

opts = hsvdOptions(Name, Value) returns an options set with the options specified
by one or more Name, Value pair arguments.

## **Input Arguments**

#### Name-Value Pair Arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

#### **FreqIntervals**

Frequency intervals for computing frequency-limited Hankel singular values (hsvd) or balanced realization (balreal), specified as a matrix with two columns. Each row specifies a frequency interval [fmin fmax], where fmin and fmax are nonnegative frequencies, expressed in the frequency unit of the model. For example:

• To restrict the computation to the range between 3 rad/s and 15 rad/s, assuming the frequency unit of the model is rad/s, set FreqIntervals to [3 15].

- To restrict the computation to two frequency intervals, 3-15 rad/s and 40-60 rad/s, use [3 15; 40 60].
- To specify all frequencies below a cutoff frequency fcut, use [0 fcut].
- To specify all frequencies above the cutoff, use [fcut Inf] in continuous time, or [fcut pi/Ts] in discrete time, where Ts is the sample time of the model.

The default value, [], imposes no frequency limitation and is equivalent to [0 Inf] in continuous time or [0 pi/Ts] in discrete time. However, if you specify a TimeIntervals value other than [], then this limit overrides FreqIntervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

#### Default: []

#### **TimeIntervals**

Time intervals for computing time-limited Hankel singular values (hsvd) or balanced realization (balreal), specified as a matrix with two columns. Each row specifies a time interval [tmin tmax], where tmin and tmax are nonnegative times, expressed in the time unit of the model. The software computes state contributions to the system's impulse response in these time intervals only. For example:

- To restrict the computation to the range between 3 s and 15 s, assuming the time unit of the model is seconds, set TimeIntervals to [3 15].
- To restrict the computation to two time intervals, 3-15 s and 40-60 s, use [3 15; 40 60].
- To specify all times from zero up to a cutoff time tcut, use [0 tcut]. To specify all times after the cutoff, use [tcut Inf].

The default value, [], imposes no time limitation and is equivalent to [0 Inf]. However, if you specify a FreqIntervals value other than [], then this limit overrides Timeintervals = []. If you specify both a TimeIntervals value and a FreqIntervals value, then the computation uses the union of these intervals.

#### AbsTol, RelTol

Absolute and relative error tolerance for stable/unstable decomposition. Positive scalar values. For an input model G with unstable poles, hsvd and balreal first extract the stable dynamics by computing the stable/unstable decomposition  $G \to GS + GU$ . The AbsTol and RelTol tolerances control the accuracy of this decomposition by ensuring

that the frequency responses of G and GS + GU differ by no more than AbsTol + RelTol\*abs(G). Increasing these tolerances helps separate nearby stable and unstable modes at the expense of accuracy. See stabsep for more information.

**Default:** AbsTol = 0; RelTol = 1e-8

#### Offset

Offset for the stable/unstable boundary. Positive scalar value. In the stable/unstable decomposition, the stable term includes only poles satisfying:

- Re(s) < -Offset \* max(1, |Im(s)|) (Continuous time)
- |z| < 1 Offset (Discrete time)

Increase the value of Offset to treat poles close to the stability boundary as unstable.

Default: 1e-8

For additional information on the options and how they affect the calculation, see hsvd. The time-limited and frequency-limited state contributions are calculated using the time-limited and frequency-limited controllability and observability Gramians, as described in gram and in [1].

## **Examples**

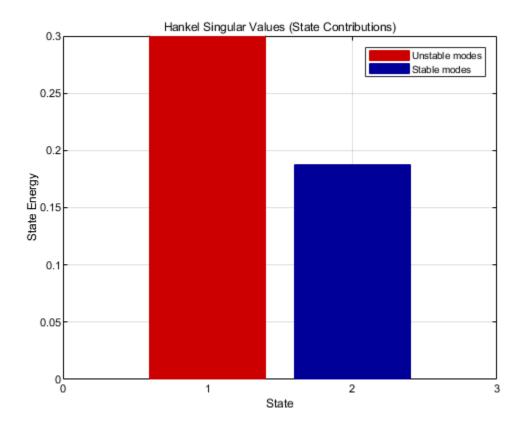
#### Hankel Singular-Value Plot with Near-Unstable Pole

Compute the Hankel singular values of the system given by:

$$sys = \frac{(s+0.5)}{(s+10^{-6})(s+2)}$$

Use the Offset option to force hsvd to exclude the pole at  $s=10^{-6}$  from the stable term of the stable/unstable decomposition.

```
sys = zpk(-.5,[-1e-6 -2],1);
opts = hsvdOptions('Offset',.001);
hsvd(sys,opts)
```



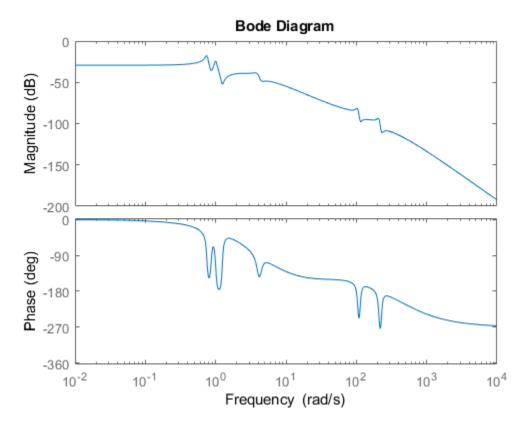
The plot shows that one state is treated as unstable. hsvd computes the energy contributions of the stable states only.

#### **Frequency-Limited Hankel Singular Values**

Compute the Hankel singular values of a model with low-frequency and high-frequency dynamics. Focus the calculation on the high-frequency modes.

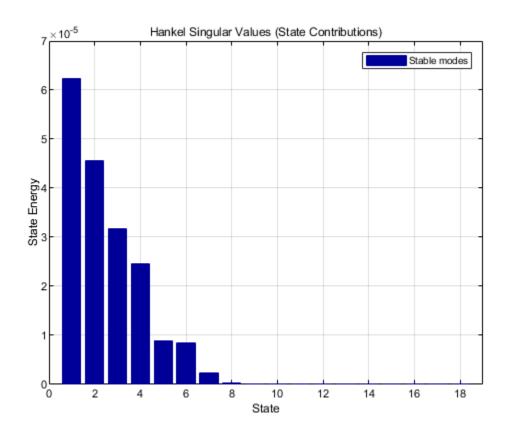
Load the model and examine its frequency response.

load modeselect Gms
bodeplot(Gms)



Gms has two sets of resonances, one at relatively low frequency and the other at relatively high frequency. Compute the Hankel singular values of the high-frequency modes, excluding the energy contributions to the low-frequency dynamics. To do so, use hsvdOptions to specify a frequency interval above 30 rad/s.

```
opts = hsvdOptions('FreqInterval',[30 Inf]);
hsvd(Gms,opts)
```



#### References

[1] Gawronski, W. and J.N. Juang. "Model Reduction in Limited Time and Frequency Intervals." *International Journal of Systems Science*. Vol. 21, Number 2, 1990, pp. 349–376.

## **See Also**

balreal | balred | gram | hsvd

#### Introduced in R2010a

# **hsvoptions**

Plot options for hsvplot

## **Syntax**

P = hsvoptions
P = hsvoptions('cstpref')

## **Description**

P = hsvoptions returns a list of available options for Hankel singular value (HSV) plots with default values set. Use dot notation to change the option values. You can use these options to customize the appearance of a Hankel singular value plot created with hsvplot.

P = hsvoptions('cstpref') initializes the plot options you selected in the Control System Toolbox Preferences Editor dialog box. For more information about the editor, see "Toolbox Preferences Editor".

The Hankel singular-value plot options include:

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid — [off on]	Show or hide the grid
GridColor — [Vector of RGB values in the range [0,1]  color   'none']	Color of the grid lines
XlimMode, YlimMode	Limit modes
Xlim, Ylim	Axes limits
YScale — [linear log]	Scale for Y-axis

Option	Description
<ul><li>FreqIntervals</li><li>TimeIntervals</li><li>AbsTol</li><li>RelTol</li><li>Offset</li></ul>	Options for the Hankel singular value computation. See hsvdOptions for detailed information about these options.

# **Tips**

• Both hsvd and hsvplot generate Hankel singular-value plots. hsvplot is useful when you want to customize properties of your plot such as axis limits, scale, and label styles. Use hsvoptions with hsvplot to define properties for your plot. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

# **Examples**

#### **Set Properties in HSV Plot**

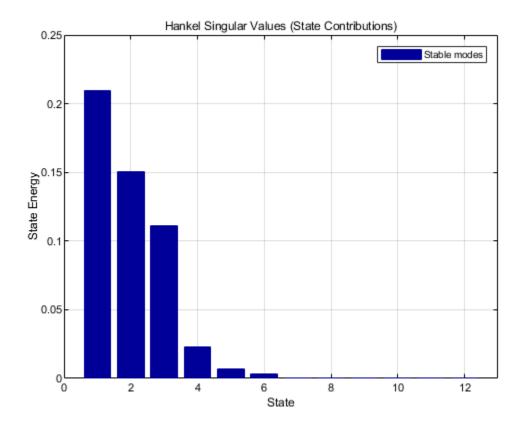
Use hsvplot to create a Hankel singular-value plot with and customized plot properties.

Create an options set for hsvplot that sets the Yscale property and the title font size.

```
P = hsvoptions;
P.YScale = 'linear';
P.Title.FontSize = 14;
```

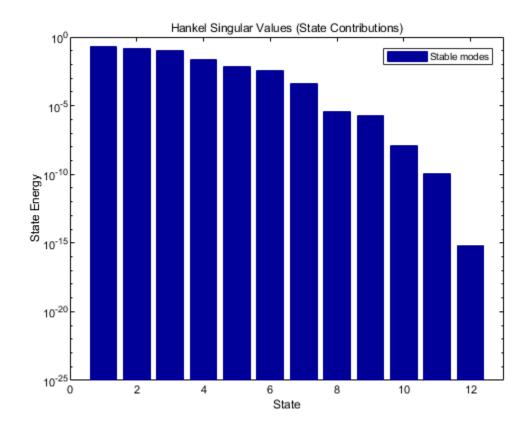
Use the options set to generate an HSV plot. Note the linear y-axis scale in the plot.

```
h = hsvplot(rss(12),P);
```



hsvplot returns a plot handle. You can use the plot handle to change properties of the existing plot. For example, switch to log scale and turn off the grid.

setoptions(h,'Yscale','log','Grid','Off')



**See Also** 

getoptions | hsvd | hsvdOptions | hsvplot | setoptions | stabsep

Introduced in R2008a

# hsvplot

Plot Hankel singular values and return plot handle

### **Syntax**

```
h = hsvplot(sys)
hsvplot(sys)
hsvplot(sys, AbsTol',ATOL,'RelTol',RTOL,'Offset',ALPHA)
hsvplot(AX,sys,...)
```

### **Description**

h = hsvplot(sys) plots the Hankel singular values of an LTI system sys and returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. See hsvoptions for a list of some available plot options.

hsvplot(sys) plots the Hankel singular values of the LTI model sys. See hsvd for details on the meaning and purpose of Hankel singular values. The Hankel singular values for the stable and unstable modes of sys are shown in blue and red, respectively.

hsvplot(sys, AbsTol',ATOL,'RelTol',RTOL,'Offset',ALPHA) specifies additional options for computing the Hankel singular values.

hsvplot(AX, sys,...) attaches the plot to the axes with handle AX.

# **Examples**

### **Set Properties in HSV Plot**

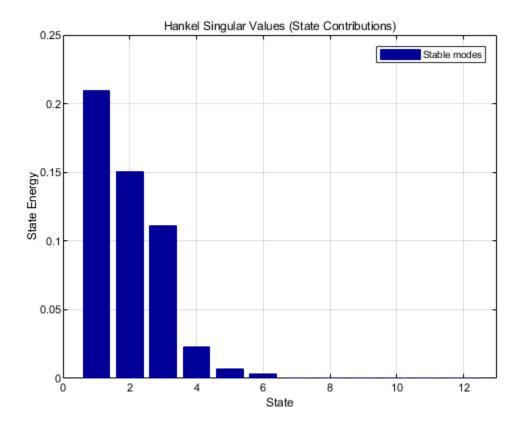
Use hsvplot to create a Hankel singular-value plot with and customized plot properties.

Create an options set for hsvplot that sets the Yscale property and the title font size.

```
P = hsvoptions;
P.YScale = 'linear';
P.Title.FontSize = 14;
```

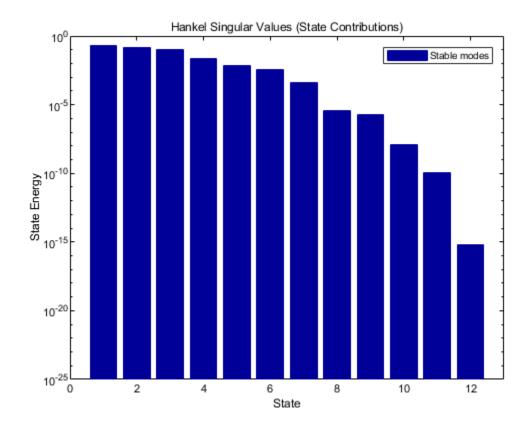
Use the options set to generate an HSV plot. Note the linear y-axis scale in the plot.

```
h = hsvplot(rss(12),P);
```



hsvplot returns a plot handle. You can use the plot handle to change properties of the existing plot. For example, switch to log scale and turn off the grid.

```
setoptions(h,'Yscale','log','Grid','Off')
```



# **Tips**

• Both hsvd and hsvplot generate Hankel singular-value plots. hsvplot is useful when you want to customize properties of your plot such as axis limits, scale, and label styles. Use hsvoptions with hsvplot to define properties for your plot. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

### **See Also**

getoptions | hsvd | hsvoptions | setoptions

**Introduced before R2006a** 

### icare

Implicit solver for continuous-time algebraic Riccati equations

### **Syntax**

```
[X,K,L] = icare(A,B,Q,R,S,E,G)
[X,K,L,info] = icare(____)
[___] = icare(____,'noscaling')
[___] = icare(____,'anti')
```

### **Description**

[X,K,L] = icare(A,B,Q,R,S,E,G) computes the unique stabilizing solution X, state-feedback gain K, and the closed-loop eigenvalues L of the following continuous-time algebraic Riccati equation.

$$A^{T}XE + E^{T}XA + E^{T}XGXE - (E^{T}XB + S)R^{-1}(B^{T}XE + S^{T}) + Q = 0$$

The stabilizing solution X puts all the eigenvalues L in the left half-plane.

Algebraic Riccati equations play a key role in LQR/LQG control, H2- and H-infinity control, Kalman filtering, and spectral or co-prime factorizations.

[X,K,L,info] = icare(\_\_\_\_) also returns a structure info which contains additional information about the solution to the continuous-time algebraic Riccati equation.

[\_\_\_] = icare(\_\_\_,'noscaling') turns off the built-in scaling and sets all entries of the scaling vectors info.Sx and info.Sr to 1. Turning off scaling speeds up computation but can be detrimental to accuracy when A,B,Q,R,S,E,G are poorly scaled.

 $[ \_\_]$  = icare( $\_\_$ , 'anti') computes the anti-stabilizing solution X that puts all eigenvalues L in the right half-plane.

# **Examples**

#### **Solve Continuous-Time Algebraic Riccati Equation**

To solve the algebraic Riccati equation  $A^TX + XA - XBB^TX + CC^T = 0$ , consider the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix} \qquad C = [7 - 8 \ 9].$$

The least effort approach is to use  $G = -BB^T$  and  $Q = C^TC$ , and then find the solution using icare.

```
A = [-1,2,3;4,5,-6;7,-8,9];
B = [5;6;-7];
C = [7, -8, 9];
G = -B*B';
0 = C'*C;
[X1,K1,L1] = icare(A,[],Q,[],[],[],G)
X1 = .3 \times .3
   15.3201 4.2369 17.0090
   4.2369 2.6252
                      4.4123
   17.0090
              4.4123
                      19.0374
K1 =
  0x3 empty double matrix
L1 = 3 \times 1
   -3.2139
  -10.1191
  -76.9693
```

The above approach may lead to numerical inaccuracies when matrices B and C have large entries, since they are squared-up to compute the G and Q matrices. Due to limited numerical range, the computation may be less accurate or even fail. For example, if norm(B) is 1e6, then norm(G) is 1e12, and any eigenvalue within 1e-4 of the imaginary axis may be diagnosed as 'imaginary' due to numerical errors.

For greater numerical accuracy, re-write the algebraic Riccati equation in the following way:

$$\boldsymbol{A}^T\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A} - \left[\boldsymbol{X}\boldsymbol{B}, \boldsymbol{C}^T\right] * \left[\boldsymbol{I}, \boldsymbol{0}; \boldsymbol{0}, \, -\boldsymbol{I}\right] \left[\boldsymbol{B}^T\boldsymbol{X}; \boldsymbol{C}\right] = \boldsymbol{0}.$$

The above equation is the standard form of  $A^TX + XA - (XB + S)R^{-1}(B^TX + S^T) = 0$ ,

where 
$$B = [B, 0]$$
,  $S = [0, C^T]$ , and  $R = [I, 0; 0, -I]$ .

Compute the solution using icare with the above values.

```
n = size(A,1);
m = size(B,2);
p = size(C,1);
R = blkdiag(eye(m), -eye(p));
BB = [B, zeros(n,p)];
S = [zeros(n,m),C'];
[X2,K2,L2,info] = icare(A,BB,0,R,S,[],[])
X2 = 3 \times 3
   15.3201
               4.2369
                        17.0090
               2.6252
    4.2369
                        4.4123
               4.4123
   17.0090
                         19.0374
K2 = 2 \times 3
  -17.0406
              6.0501 -21.7435
   -7.0000
               8.0000
                         -9.0000
L2 = 3 \times 1
   -3.2139
  -10.1191
  -76.9693
info = struct with fields:
        Sx: [3x1 double]
        Sr: [2x1 double]
```

U: [3x3 double]

```
V: [3x3 double]
W: [2x3 double]
Report: 0
```

Here, X2 is the unique stabilizing solution, K2 contains the state-feedback gain and L2 contains the closed-loop eigenvalues.

#### Anti-Stabilizing Solution of Continuous-Time Algebraic Riccati Equation

To find the anti-stabilizing solution of the continuous-time algebraic Riccati equation  $A^TX + XA - XBB^TX + CC^T = 0$ , consider the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix} \qquad C = [7 -8 \ 9].$$

For greater numerical accuracy, re-write the algebraic Riccati equation in the following way:

$$A^TX + XA - \left[XB, C^T\right] * \left[I, 0; 0, -I\right] \left[B^TX; C\right] = 0.$$

The above equation is the standard form of  $A^TX + XA - (XB + S)R^{-1}(B^TX + S^T) = 0$ ,

where 
$$B = [B, 0]$$
,  $S = [0, C^T]$ , and  $R = [I, 0; 0, -I]$ .

Compute the anti-stabilizing solution using the 'anti' option.

```
A = [-1,2,3;4,5,-6;7,-8,9];
B = [5;6;-7];
C = [7,-8,9];
n = size(A,1);
m = size(B,2);
p = size(C,1);
R = blkdiag(eye(m),-eye(p));
BB = [B,zeros(n,p)];
S = [zeros(n,m),C'];
[X,K,L] = icare(A,BB,0,R,S,[],[],'anti')
```

```
X = 3 \times 3
  -18.0978
             10.9090
                         -1.8466
   10.9090
            -6.7195
                        1.4354
              1.4354
   -1.8466
                        -0.9426
K = 2 \times 3
  -12.1085
              4.1803
                       5.9774
               8.0000
   -7.0000
                       -9.0000
L = 3 \times 1
   76.9693
   10.1191
    3.2139
```

Here, X is the unique anti-stabilizing solution, K contains the state-feedback gain and L contains the closed-loop eigenvalues.

### **Input Arguments**

### A,B,Q,R,S,E,G — Input matrices

matrices

Input matrices, specified as matrices.

The matrices Q, R and G must be Hermitian. A square matrix is Hermitian if it is equal to its complex conjugate transpose, that is,  $a_{i,j} = \bar{a}_{j,j}$ .

For more information on Hermitian matrices, see ishermitian.

Matrices R and E must be invertible.

When matrices B, R, S, E and G are omitted or set to [], icare uses the following default values:

• B = 0

- R = I
- S = 0
- E = I
- G = 0

If the inputs Q, R and G are scalar-valued, icare interprets them as multiples of the identity matrix.

# 'noscaling' — Option to turn off built-in scaling 'noscaling'

Option to turn off built-in scaling, specified as 'noscaling'. When you turn off the built-in scaling, icare sets all entries in the scaling vectors info.Sx and info.Sr to 1. Turning off scaling speeds up computation but can be detrimental to accuracy when A,B,Q,R,S,E,G are poorly scaled.

# 'anti' — Option to compute the anti-stabilizing solution 'anti'

Option to compute the anti-stabilizing solution, specified as 'anti'. When you enable this option, icare computes the anti-stabilizing solution X that puts all eigenvalues of (A + G\*X\*E-B\*K, E) in the right half-plane.

The unique stabilizing and anti-stabilizing are both needed to know the complete phase portrait of the Riccati differential equations.

# **Output Arguments**

### X — Unique solution to the Riccati equation

matrix

Unique solution to the continuous-time algebraic Riccati equation, returned as a matrix.

By default, X is the stabilizing solution of the continuous-time algebraic Riccati equation. When the 'anti' option is used, X is the anti-stabilizing solution.

icare returns [] for X when the associated Hamiltonian matrix has eigenvalues on the imaginary axis.

#### K — State-feedback gain

matrix

State-feedback gain, returned as a matrix.

The state-feedback gain K is computed as:

$$K = R^{-1} (B^T X E + S^T).$$

icare returns [] for K when the associated Hamiltonian matrix has eigenvalues on the imaginary axis.

#### L — Closed-loop eigenvalues

matrix

Closed-loop eigenvalues, returned as a matrix.

The closed-loop eigenvalues L is computed as:

$$L = eig(A + GXE - BK, E).$$

icare returns [] for X and K when the associated Hamiltonian matrix has eigenvalues on the imaginary axis. In other words, L is non-empty even when X and K are empty matrices.

### info — Information about the unique solution

structure

Information about the unique solution, returned as a structure with the following fields:

- Sx Vector of values used to scale the states.
- Sr Vector of values used to scale the R matrix.
- U, V and W Vectors of values representing the basis of the stable invariant subspace
  of the associated scaled matrix pencil. For more information, see "Algorithms" on page
  2-532.
- Report A scalar with one of the following values:
  - 0 The unique solution is accurate.
  - 1 The solution accuracy is poor.
  - 2 The solution is not finite.

3 — No solution found since the Hamiltonian spectrum, denoted by [L;-L], has
eigenvalues on the imaginary axis.

### Limitations

- (A-sE,B) must be stabilizable, and E and R must be invertible for a finite stabilizing solution X to exist and be finite. While these conditions are not sufficient in general, they become sufficient when the following conditions are met:

  - $[A BR^{-1}S^T Q SR^{-1}S^T]$  is detectible

# **Algorithms**

#### Basis of the invariant subspace

icare works with the following pencil, and computes a basis [U;V;W] of the invariant subspace associated with the stable or anti-stable finite eigenvalues of this pencil.

$$M - sN = \begin{bmatrix} A & G & B \\ -Q & -A^T & -S \\ S^T & B^T & R \end{bmatrix} - s \begin{bmatrix} E & 0 & 0 \\ 0 & E^T & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The data is automatically scaled to reduce the sensitivity of eigenvalues near the imaginary axis and increase separation between the stable and anti-stable invariant subspaces.

# Relationship between the solution, the state-feedback gain, and the scaling vectors

The solution X and state-feedback gain K are related to the scaling vectors, and U, V, W by the following set of equations:

$$X = D_X V U^{-1} D_X E^{-1},$$
  
$$K = -D_r W U^{-1} D_X.$$

where,

```
D_{\chi} = \operatorname{diag}(S_{\chi}),

D_{r} = \operatorname{diag}(S_{r}).
```

#### Basis of the invariant subspace

icare works with the following pencil, and computes a basis [U;V;W] of the invariant subspace associated with the stable or anti-stable finite eigenvalues of this pencil.

### See Also

h2syn|hinfsyn|idare|ishermitian|kalman|lncf|lqg|lqr|lyap|rncf|spectralfact

#### Introduced in R2019a

### idare

Implicit solver for discrete-time algebraic Riccati equations

### **Syntax**

```
[X,K,L] = idare(A,B,Q,R,S,E)
[X,K,L,info] = idare(____)
[___] = idare(____,'noscaling')
[___] = idare(____,'anti')
```

# **Description**

[X,K,L] = idare(A,B,Q,R,S,E) computes the unique stabilizing solution X, state-feedback gain K, and the closed-loop eigenvalues L of the following discrete-time algebraic Riccati equation.

$$A^{T}XA - E^{T}XE - (A^{T}XB + S)(B^{T}XB + R)^{-1}(A^{T}XB + S)^{T} + Q = 0$$

The stabilizing solution  $\boldsymbol{X}$  puts all the eigenvalues  $\boldsymbol{L}$  inside the unit disk.

Algebraic Riccati equations play a key role in LQR/LQG control, H2- and H-infinity control, Kalman filtering, and spectral or co-prime factorizations.

[X,K,L,info] = idare(\_\_\_\_) also returns a structure info which contains additional information about the solution to the discrete-time algebraic Riccati equation.

[\_\_\_] = idare(\_\_\_, 'noscaling') turns off the built-in scaling and sets all entries of the scaling vectors info.Sx and info.Sr to 1. Turning off scaling speeds up computation but can be detrimental to accuracy when A,B,Q,R,S,E are poorly scaled.

 $[ \_\_] = idare( \_\_, 'anti')$  computes the anti-stabilizing solution X that puts all eigenvalues L outside the unit disk.

# **Examples**

#### **Solve Discrete-Time Algebraic Riccati Equation**

For this example, solve the discrete-time algebraic Riccati equation considering the following set of matrices:

$$A = \begin{bmatrix} -0.9 & -3 \\ 0.7 & 0.1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \qquad R = 0.1.$$

Find the stabilizing solution using idare to solve for the above matrices with default values for S and E.

```
A = [-0.9, -0.3; 0.7, 0.1];
B = [1;1];
Q = [1,0;0,3];
R = 0.1;
[X,K,L,info] = idare(A,B,Q,R,[],[])
X = 2 \times 2
    4.7687
               0.9438
    0.9438
               3.2369
K = 1 \times 2
   -0.2216 -0.1297
L = 2 \times 1
   -0.4460
   -0.0027
info = struct with fields:
         Sx: [2x1 double]
         Sr: 1
          U: [2x2 double]
          V: [2x2 double]
          W: [-0.0232 0.0428]
```

```
Report: 0
```

Here, X is the unique stabilizing solution, K contains the state-feedback gain, L contains the closed-loop eigenvalues, while info contains additional information about the solution.

#### Anti-Stabilizing Solution of Discrete-Time Algebraic Riccati Equation

For this example, solve the discrete-time algebraic Riccati equation considering the following set of matrices:

$$A = \begin{bmatrix} -0.9 & -3 \\ 0.7 & 0.1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \qquad R = 0.1.$$

Find the anti-stabilizing solution using the 'anti' option to solve for the above matrices with default values for S and E.

Here, X is the unique anti-stabilizing solution, K contains the state-feedback gain, and L contains the closed-loop eigenvalues.

### **Input Arguments**

#### A,B,Q,R,S,E — Input matrices

matrices

Input matrices, specified as matrices.

The matrices **Q** and **R** must be Hermitian. A square matrix is Hermitian if it is equal to its complex conjugate transpose, that is,  $a_{i,j} = \bar{a}_{j,i}$ .

For more information on Hermitian matrices, see ishermitian.

Matrix E must be non-singular.

When matrices R, S and E are omitted or set to [], idare uses the following default values:

- $\bullet$  R = I
- S = 0
- E = I

If the inputs Q and R are scalar-valued, idare interprets them as multiples of the identity matrix.

# 'noscaling' — Option to turn off built-in scaling 'noscaling'

Option to turn off built-in scaling, specified as 'noscaling'. When you turn off the built-in scaling, idare sets all entries in the scaling vectors info.Sx and info.Sr to 1. Turning off scaling speeds up computation but can be detrimental to accuracy when A,B,Q,R,S,E are poorly scaled.

# 'anti' — Option to compute the anti-stabilizing solution 'anti'

Option to compute the anti-stabilizing solution, specified as 'anti'. When you enable this option, idare computes the anti-stabilizing solution X that puts all eigenvalues of (A-B\*K,E) outside the unit disk.

The unique stabilizing and anti-stabilizing are both needed to know the complete phase portrait of the Riccati differential equations.

### **Output Arguments**

#### X — Unique solution to the Riccati equation

matrix

Unique solution to the discrete-time algebraic Riccati equation, returned as a matrix.

By default, X is the stabilizing solution of the discrete-time algebraic Riccati equation. When the 'anti' option is used, X is the anti-stabilizing solution.

idare returns [] for X when there is no finite stabilizing solution.

### K - State-feedback gain

matrix

State-feedback gain, returned as a matrix.

The state-feedback gain K is computed as:

$$K = (B^T X B + R)^{-1} (B^T X A + S^T).$$

idare returns [] for K when there is no finite stabilizing solution.

### L — Closed-loop eigenvalues

matrix

Closed-loop eigenvalues, returned as a matrix.

The closed-loop eigenvalues L is computed as:

$$L = eig(A - BK, E).$$

idare returns [] for X and K when there is no finite stabilizing solution. L is non-empty even when X and K are empty matrices.

#### info — Information about the unique solution

structure

Information about the unique solution, returned as a structure with the following fields:

- Sx Vector of values used to scale the states.
- Sr Vector of values used to scale the R matrix.
- U, V and W Vectors of values representing the basis of the stable invariant subspace
  of the associated scaled matrix pencil. For more information, see "Algorithms" on page
  2-532.
- Report A scalar with one of the following values:
  - 0 The unique solution is accurate.
  - 1 The solution accuracy is poor.
  - 2 The solution is not finite.
  - 3 No solution found since the Symplectic spectrum, denoted by [L;1./L], has eigenvalues on the unit circle.

### Limitations

- (A-zE,B) must be stabilizable, E and R must be invertible, and [B;S;R] have full column rank for a finite stabilizing solution X to exist and be finite. While these conditions are not sufficient in general, they become sufficient when the following conditions are met:

  - $[A BR^{-1}S^T O SR^{-1}S^T]$  is detectible

# **Algorithms**

### Basis of the invariant subspace

idare works with the following pencil, and computes a basis [U;V;W] of the invariant subspace associated with the stable or anti-stable finite eigenvalues of this pencil.

$$M - zN = \begin{bmatrix} A & 0 & B \\ -Q & E^T - S \\ S^T & 0 & R \end{bmatrix} - z \begin{bmatrix} E & 0 & 0 \\ 0 & A^T & 0 \\ 0 & -B^T & 0 \end{bmatrix}$$

The data is automatically scaled to reduce the sensitivity of eigenvalues near the unit circle and increase separation between the stable and anti-stable invariant subspaces.

# Relationship between the solution, the state-feedback gain, and the scaling vectors

The solution X and state-feedback gain K are related to the scaling vectors, and U, V, W by the following set of equations:

$$X = D_X V U^{-1} D_X E^{-1},$$
  

$$K = -D_r W U^{-1} D_X,$$

where,

$$D_X = \operatorname{diag}(S_X),$$
  
 $D_T = \operatorname{diag}(S_T).$ 

### See Also

dlyap | h2syn | hinfsyn | icare | ishermitian | kalman | lncf | lqg | lqr | rncf |
spectralfact

#### Introduced in R2019a

# imp2exp

Convert implicit linear relationship to explicit input-output relation

### **Syntax**

```
B = imp2exp(A, yidx, uidx)
```

# **Description**

B = imp2exp(A,yidx,uidx) transforms a linear constraint between variables y and u of the form A(:,[yidx;uidx])\*[y;u] = 0 into an explicit input/output relationship y = B\*u. The vectors yidx and uidx refer to the columns (inputs) of A as referenced by the explicit relationship for B.

# **Examples**

### **Uncertain Matrix Algebraic Constraint**

Consider two uncertain motor/generator constraints among 4 variables  $\begin{bmatrix} V \\ I \\ T \end{bmatrix}$ , namely

$$\begin{bmatrix} 1 - R & 0 - K \\ 0 - K & 1 & 0 \end{bmatrix} * \begin{bmatrix} V \\ I \\ T \\ W \end{bmatrix} = 0.$$
 Find the uncertain 2-by-2 matrix B so that  $\begin{bmatrix} V \\ T \end{bmatrix} = B * \begin{bmatrix} W \\ I \end{bmatrix}.$ 

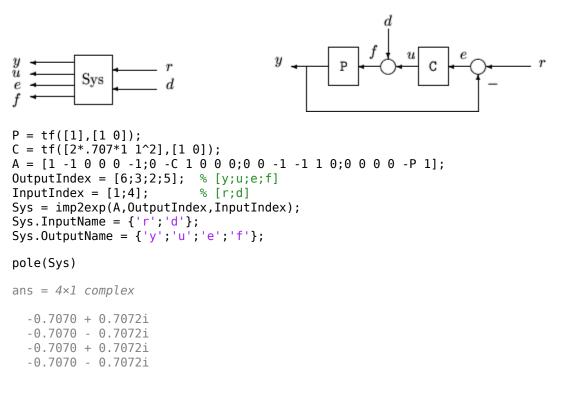
```
R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
A = [1 -R 0 -K;0 -K 1 0];
Yidx = [1 3];
Uidx = [4 2];
B = imp2exp(A,Yidx,Uidx)
```

```
B =
   Uncertain matrix with 2 rows and 2 columns.
The uncertainty consists of the following blocks:
   K: Uncertain real, nominal = 0.002, variability = [-30,30]%, 2 occurrences
   R: Uncertain real, nominal = 1, variability = [-10,40]%, 1 occurrences
```

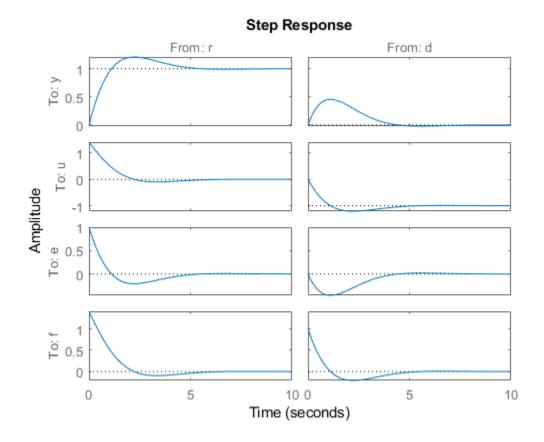
Type "B.NominalValue" to see the nominal value, "get(B)" to see all properties, and "B

#### **Scalar Dynamic System Constraint**

Consider a standard single-loop feedback connection of controller C and an uncertain plant P, described by the equations e = r-y; u = Ce; f = d+u; y = Pf.



stepplot(Sys)



### **Input Arguments**

#### A — Implicit system

static or dynamic input/output model

Implicit system, specified as a static, or dynamic input/output model. A can be:

- A numeric LTI model such as tf, zpk, ss, or frd object
- An uncertain model such as umat, uss, or ufrd object

The number of rows in model A must equal the length of the index vector yidx.

# yidx — Indices to partition implicit model channels into output signals vector

Indices to partition the model channel into output signals, specified as a vector. The index vectors yidx and uidx specify how to partition the input channels of A into y and u signals, respectively.

If [yidx,uidx] does not include all inputs of A, imp2exp excludes the missing y channels from the output model B. In such cases, imp2exp retains only a subset B(I,:) of the outputs/rows of B and does not affect how the explicit model B is computed.

#### uidx — Indices to partition implicit model channels into input signals vector

Indices to partition the model channel into input signals, specified as a vector. The index vectors yidx and uidx specify how to partition the input channels of A into y and u signals, respectively.

# **Output Arguments**

#### B — Explicit system

static or dynamic input/output model

Explicit system, returned as a static, or dynamic input/output model. The output explicit model B is of the same subclass as model A. For instance, if A is specified as an ss model, then imp2exp also returns B as an ss model.

### See Also

connect | inv

Introduced in R2011b

# impulse

Impulse response plot of dynamic system; impulse response data

### **Syntax**

```
impulse(sys)
impulse(sys,Tfinal)
impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,Tfinal)
impulse(sys1,sys2,...,sysN,t)
[y,t] = impulse(sys)
[y,t] = impulse(sys)
[y,t] = impulse(sys,Tfinal)
y = impulse(sys,t)
[y,t,x] = impulse(sys)
[y,t,x,ysd] = impulse(sys)
```

# Description

impulse calculates the unit impulse response of a dynamic system model. For continuous-time dynamic systems, the impulse response is the response to a Dirac input  $\delta(t)$ . For discrete-time systems, the impulse response is the response to a unit area pulse of length Ts and height 1/Ts, where Ts is the sample time of the system. (This pulse approaches  $\delta(t)$  as Ts approaches zero.) For state-space models, impulse assumes initial state values are zero.

impulse(sys) plots the impulse response of the dynamic system model sys. This model can be continuous or discrete, and SISO or MIMO. The impulse response of multi-input systems is the collection of impulse responses for each input channel. The duration of simulation is determined automatically to display the transient behavior of the response.

impulse(sys, Tfinal) simulates the impulse response from t = 0 to the final time t = Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), impulse interprets Tfinal as the number of sampling periods to simulate.

impulse(sys,t) uses the user-supplied time vector t for simulation. Express t in the
system time units, specified in the TimeUnit property of sys. For discrete-time models, t
should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time
models, t should be of the form Ti:dt:Tf, where dt becomes the sample time of a
discrete approximation to the continuous system (see "Algorithms" on page 2-550). The
impulse command always applies the impulse at t=0, regardless of Ti.

To plot the impulse responses of several models sys1,..., sysN on a single figure, use:

```
impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,Tfinal)
impulse(sys1,sys2,...,sysN,t)
```

As with bode or plot, you can specify a particular color, linestyle, and/or marker for each system, for example,

```
impulse(sys1,'y:',sys2,'g--')
```

See "Plotting and Comparing Multiple Systems" and the bode entry in this section for more details.

When invoked with output arguments:

```
[y,t] = impulse(sys)
[y,t] = impulse(sys,Tfinal)
y = impulse(sys,t)
```

impulse returns the output response y and the time vector t used for simulation (if not supplied as an argument to impulse). No plot is drawn on the screen. For single-input systems, y has as many rows as time samples (length of t), and as many columns as outputs. In the multi-input case, the impulse responses of each input channel are stacked up along the third dimension of y. The dimensions of y are then

For state-space models only:

```
[y,t,x] = impulse(sys)
(length of t) \times (number of outputs) \times (number of inputs)
```

and y(:,:,j) gives the response to an impulse disturbance entering the jth input channel. Similarly, the dimensions of x are

(length of t)  $\times$  (number of states)  $\times$  (number of inputs)

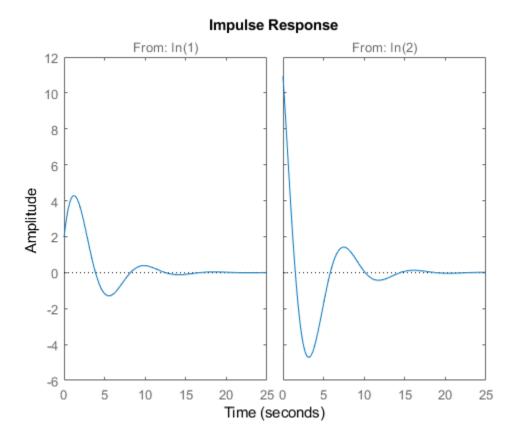
[y,t,x,ysd] = impulse(sys) returns the standard deviation YSD of the response Y of an identified system SYS. YSD is empty if SYS does not contain parameter covariance information.

# **Examples**

### Impulse Response Plot of Second-Order State-Space Model

Plot the impulse response of the second-order state-space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
 
$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
 
$$a = \begin{bmatrix} -0.5572 & -0.7814; 0.7814 & 0 \end{bmatrix};$$
 
$$b = \begin{bmatrix} 1 & -1; 0 & 2 \end{bmatrix};$$
 
$$c = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix};$$
 
$$sys = ss(a,b,c,0);$$
 
$$impulse(sys)$$



The left plot shows the impulse response of the first input channel, and the right plot shows the impulse response of the second input channel.

You can store the impulse response data in MATLAB arrays by

[y,t] = impulse(sys);

Because this system has two inputs, y is a 3-D array with dimensions

size(y)

ans =  $1 \times 3$ 

139 1 2

(the first dimension is the length of t). The impulse response of the first input channel is then accessed by

```
ch1 = y(:,:,1);
size(ch1)
ans = 1 \times 2
```

### Impulse Data from Identified System

Fetch the impulse response and the corresponding  ${\bf 1}$  std uncertainty of an identified linear system .

```
load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
z = iddata(y, u, 0.1, 'Name', 'DC-motor');
set(z, 'InputName', 'Voltage', 'InputUnit', 'V');
set(z, 'OutputName', {'Angular position', 'Angular velocity'});
set(z, 'OutputUnit', {'rad', 'rad/s'});
set(z, 'Tstart', 0, 'TimeUnit', 's');

model = tfest(z,2);
[y,t,~,ysd] = impulse(model,2);
% Plot 3 std uncertainty
subplot(211)
plot(t,y(:,1), t,y(:,1)+3*ysd(:,1),'k:', t,y(:,1)-3*ysd(:,1),'k:')
subplot(212)
plot(t,y(:,2), t,y(:,2)+3*ysd(:,2),'k:', t,y(:,2)-3*ysd(:,2),'k:')
```

### Limitations

The impulse response of a continuous system with nonzero D matrix is infinite at t=0. impulse ignores this discontinuity and returns the lower continuity value Cb at t=0.

### **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

# **Algorithms**

Continuous-time models are first converted to state space. The impulse response of a single-input state-space model

$$\dot{x} = Ax + bu$$
$$y = Cx$$

is equivalent to the following unforced response with initial state b.

$$\dot{x} = Ax, \ x(0) = b$$
$$y = Cx$$

To simulate this response, the system is discretized using zero-order hold on the inputs. The sample time is chosen automatically based on the system dynamics, except when a time vector t = 0:dt:Tf is supplied (dt is then used as sample time).

### See Also

Linear System Analyzer | initial | lsim | step

Introduced before R2006a

# impulseplot

Plot impulse response and return plot handle

### **Syntax**

```
impulseplot(sys)
impulseplot(sys,Tfinal)
impulseplot(sys,t)
impulseplot(sys1,sys2,...,sysN)
impulseplot(sys1,sys2,...,sysN,Tfinal)
impulseplot(sys1,sys2,...,sysN,t)
impulseplot(AX,...)
impulseplot(..., plotoptions)
h = impulseplot(...)
```

# **Description**

impulseplot plots the impulse response of the dynamic system model sys. For multiinput models, independent impulse commands are applied to each input channel. The
time range and number of points are chosen automatically. For continuous systems with
direct feedthrough, the infinite pulse at t=0 is disregarded. impulseplot can also return
the plot handle, h. You can use this handle to customize the plot with the getoptions
and setoptions commands. Type

```
help timeoptions
```

for a list of available plot options.

impulseplot(sys) plots the impulse response of the LTI model without returning the
plot handle.

impulseplot(sys, Tfinal) simulates the impulse response from t=0 to the final time t=Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), impulseplot interprets Tfinal as the number of sampling intervals to simulate.

impulseplot(sys,t) uses the user-supplied time vector t for simulation. Express t in
the system time units, specified in the TimeUnit property of sys. For discrete-time
models, t should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuoustime models, t should be of the form Ti:dt:Tf, where dt becomes the sample time of a
discrete approximation to the continuous system (see impulse). The impulseplot
command always applies the impulse at t=0, regardless of Ti.

To plot the impulse response of multiple LTI models sys1,sys2,... on a single plot, use:

```
impulseplot(sys1,sys2,...,sysN,Tfinal)
impulseplot(sys1,sys2,...,sysN,t)
You can also specify a color, line style, and marker for each system, as in
impulseplot(sys1,'r',sys2,'y--',sys3,'gx')
impulseplot(AX,...) plots into the axes with handle AX.
impulseplot(..., plotoptions) plots the impulse response with the options
specified in plotoptions. Type
help timeoptions
for more detail.
h = impulseplot(...) plots the impulse response and returns the plot handle h.
```

### **Examples**

### **Example 1**

Normalize the impulse response of a third-order system.

```
sys = rss(3);
h = impulseplot(sys);
% Normalize responses
setoptions(h,'Normalize','on');
```

### **Example 2**

Plot the impulse response and the corresponding 1 std "zero interval" of an identified linear system.

```
load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
z = iddata(y, u, 0.1, 'Name', 'DC-motor');
set(z, 'InputName', 'Voltage', 'InputUnit', 'V');
set(z, 'OutputName', {'Angular position', 'Angular velocity'});
set(z, 'OutputUnit', {'rad', 'rad/s'});
set(z, 'Tstart', 0, 'TimeUnit', 's');
model = n4sid(z,4,n4sidOptions('Focus', 'simulation'));
h = impulseplot(model,2);
showConfidence(h);
```

### **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

### See Also

getoptions | impulse | setoptions

Introduced before R2006a

### initial

Initial condition response of state-space model

### **Syntax**

```
initial(sys,x0)
initial(sys,x0,Tfinal)
initial(sys,x0,t)
initial(sys1,sys2,...,sysN,x0)
initial(sys1,sys2,...,sysN,x0,Tfinal)
initial(sys1,sys2,...,sysN,x0,t)
[y,t,x] = initial(sys,x0)
[y,t,x] = initial(sys,x0,Tfinal)
[y,t,x] = initial(sys,x0,t)
```

# **Description**

initial(sys,x0) calculates the unforced response of a state-space (ss) model sys with an initial condition on the states specified by the vector x0:

```
\dot{x} = Ax, \ x(0) = x_0y = Cx
```

This function is applicable to either continuous- or discrete-time models. When invoked without output arguments, initial plots the initial condition response on the screen.

initial(sys,x0,Tfinal) simulates the response from t=0 to the final time t=Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), initial interprets Tfinal as the number of sampling periods to simulate.

initial(sys,x0,t) uses the user-supplied time vector t for simulation. Express t in the system time units, specified in the TimeUnit property of sys. For discrete-time models, t should be of the form 0:Ts:Tf, where Ts is the sample time. For continuous-time models, t should be of the form 0:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see impulse).

To plot the initial condition responses of several LTI models on a single figure, use

When invoked with output arguments,

return the output response y, the time vector t used for simulation, and the state trajectories x. No plot is drawn on the screen. The array y has as many rows as time samples (length of t) and as many columns as outputs. Similarly, x has length(t) rows and as many columns as states.

### **Examples**

#### **Response of State-Space Model to Initial Condition**

Plot the response of the following state-space model:

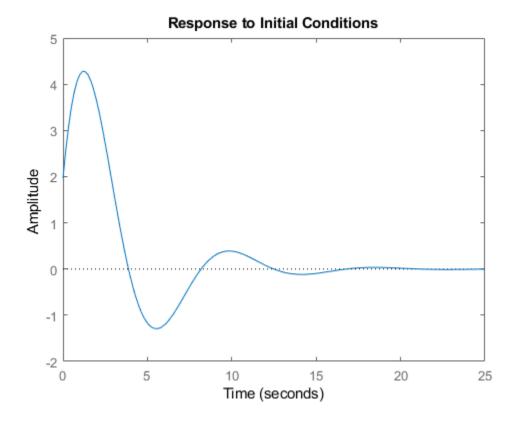
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Take the following initial condition:

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

```
a = [-0.5572, -0.7814; 0.7814, 0];
c = [1.9691  6.4493];
x0 = [1 ; 0];

sys = ss(a,[],c,[]);
initial(sys,x0)
```



# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

### **See Also**

Linear System Analyzer | impulse | lsim | step

**Introduced before R2006a** 

#### initialize

Initialize the state of the particle filter

### **Syntax**

```
initialize(pf,numParticles,mean,covariance)
initialize(pf,numParticles,stateBounds)
initialize(____,Name,Value)
```

# **Description**

initialize(pf,numParticles,mean,covariance) initializes a particle filter object with a specified number of particles. The initial states of the particles in the state space are determined by sampling from the multivariate normal distribution with the specified mean and covariance. The number of state variables (NumStateVariables) is retrieved automatically based on the length of the mean vector.

initialize(pf,numParticles,stateBounds) determines the initial location of numParticles particles by sampling from the multivariate uniform distribution with the given stateBounds.

initialize( \_\_\_\_, Name, Value) initializes the particles with additional options
specified by one or more Name, Value pair arguments.

### **Examples**

#### **Initialize Particle Filter Object for Online State Estimation**

To create a particle filter object for estimating the states of your system, create appropriate state transition function and measurement function for the system.

In this example, the functions vdpParticleFilterStateFcn and vdpMeasurementLikelihoodFcn describe a discrete-approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1.

Create the particle filter object. Use function handles to provide the state transition and measurement likelihood functions to the object.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.
initialize(myPF, 1000, [2;0], eye(2));
myPF
mvPF =
  particleFilter with properties:
           NumStateVariables: 2
                NumParticles: 1000
          StateTransitionFcn: @vdpParticleFilterStateFcn
    MeasurementLikelihoodFcn: @vdpMeasurementLikelihoodFcn
     IsStateVariableCircular: [0 0]
            ResamplingPolicy: [1x1 particleResamplingPolicy]
            ResamplingMethod: 'multinomial'
       StateEstimationMethod: 'mean'
            StateOrientation: 'column'
                   Particles: [2x1000 double]
                     Weights: [1x1000 double]
                       State: 'Use the getStateEstimate function to see the value.'
             StateCovariance: 'Use the getStateEstimate function to see the value.'
```

To estimate the states and state estimation error covariance from the constructed object, use the predict and correct commands.

### **Input Arguments**

#### pf — Particle filter

particleFilter object

Particle filter, specified as a object. See particleFilter for more information.

### numParticles — Number of particles used in the filter

scalar

Number of particles used in the filter, specified as a scalar.

Unless performance is an issue, do not use fewer than 1000 particles. A higher number of particles can improve the estimate but sacrifices performance speed, because the algorithm has to process more particles. Tuning the number of particles is the best way to improve the tracking of your particle filter.

#### mean — Mean of particle distribution

vector

Mean of particle distribution, specified as a vector. The NumStateVariables property of pf is set based on the length of this vector.

#### covariance — Covariance of particle distribution

*N*-by-*N* matrix

Covariance of particle distribution, specified as an N-by-N matrix, where N is the value of NumStateVariables property from pf.

#### stateBounds — Bounds of state variables

*n*-by-2 matrix

Bounds of state variables, specified as an n-by-2 matrix. The NumStateVariables property of pf is set based on the value of n. Each row corresponds to the lower and upper limit of the corresponding state variable. The number of state variables (NumStateVariables) is retrieved automatically based on the number of rows of the stateBounds array.

#### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: ...'StateOrientation','row'

#### Circular Variables — Circular variables

logical vector

Circular variables, the comma-separated pair consisting of CircularVariables and specified as a logical vector. Each state variable that uses circular or angular coordinates is indicated with a 1. The length of the vector is equal to the NumStateVariables property of particleFilter.

#### StateOrientation — Orientation of states

'column' (default) | 'row'

Orientation of states, specified as the comma-separated pair consisting of StateOrientation as one of these values: 'column' or 'row'. If it is 'column', State property and getStateEstimate method of the object pf returns the states as a column vector, and the Particles property has dimensions NumStateVariables-by-NumParticles. If it is 'row', the states have the row orientation and Particles has dimensions NumParticles-by-NumStateVariables.

#### See Also

clone | correct | extendedKalmanFilter | particleFilter | predict |
unscentedKalmanFilter

#### **Topics**

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter" "Generate Code for Online State Estimation in MATLAB"

#### Introduced in R2017b

# initialplot

Plot initial condition response and return plot handle

### **Syntax**

```
initialplot(sys,x0)
initialplot(sys,x0,Tfinal)
initialplot(sys,x0,t)
initialplot(sys1,sys2,...,sysN,x0)
initialplot(sys1,sys2,...,sysN,x0,Tfinal)
initialplot(sys1,sys2,...,sysN,x0,t)
initialplot(AX,...)
initialplot(..., plotoptions)
h = initialplot(...)
```

# Description

initialplot(sys, x0) plots the undriven response of the state-space (ss) model sys with initial condition x0 on the states. This response is characterized by these equations:

```
Continuous time: x = A x, y = C x, x(0) = x0
Discrete time: x[k+1] = A x[k], y[k] = C x[k], x[0] = x0
```

The time range and number of points are chosen automatically. initialplot also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

```
help timeoptions
```

for a list of available plot options.

initialplot(sys,x0,Tfinal) simulates the response from t=0 to the final time t=Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), initialplot interprets Tfinal as the number of sampling periods to simulate.

initialplot(sys,x0,t) uses the user-supplied time vector t for simulation. Express t in the system time units, specified in the TimeUnit property of sys. For discrete-time models, t should be of the form 0:Ts:Tf, where Ts is the sample time. For continuous-time models, t should be of the form 0:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see impulse).

To plot the initial condition responses of several LTI models on a single figure, use

```
initialplot(sys1,sys2,...,sysN,x0)
initialplot(sys1,sys2,...,sysN,x0,Tfinal)
initialplot(sys1,sys2,...,sysN,x0,t)
You can also specify a color, line style, and marker for each system, as in
initialplot(sys1,'r',sys2,'y--',sys3,'gx',x0).
initialplot(AX,...) plots into the axes with handle AX.
initialplot(..., plotoptions) plots the initial condition response with the options specified in plotoptions. Type
help timeoptions
for more detail.
h = initialplot(...) plots the system response and returns the plot handle h.
```

### **Examples**

Plot a third-order system's response to initial conditions and use the plot handle to change the plot's title.

```
sys = rss(3);
h = initialplot(sys,[1,1,1])
p = getoptions(h); % Get options for plot.
p.Title.String = 'My Title'; % Change title in options.
setoptions(h,p); % Apply options to the plot.
```

# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

#### See Also

getoptions | initial | setoptions

Introduced before R2006a

# interp

Interpolate FRD model

### **Syntax**

isys = interp(sys,freqs)

# **Description**

isys = interp(sys,freqs) interpolates the frequency response data contained in the FRD model sys at the frequencies freqs. interp, which is an overloaded version of the MATLAB function interp, uses linear interpolation and returns an FRD model isys containing the interpolated data at the new frequencies freqs. If sys is an IDFRD model (requires System Identification Toolbox software), the noise spectrum, if non-empty, is also interpolated. The response and noise covariance data, if available, are also interpolated.

You should express the frequency values freqs in the same units as sys.frequency. The frequency values must lie between the smallest and largest frequency points in sys (extrapolation is not supported).

#### See Also

frd | freqresp

Introduced before R2006a

#### inv

Invert models

### **Syntax**

inv

### **Description**

inv inverts the input/output relation

$$y = G(s)u$$

to produce the model with the transfer matrix  $H(s) = G(s)^{-1}$ .

$$u = H(s)y$$

This operation is defined only for square systems (same number of inputs and outputs) with an invertible feedthrough matrix D. inv handles both continuous- and discrete-time systems.

### **Examples**

Consider

$$H(s) = \begin{bmatrix} 1 & \frac{1}{s+1} \\ 0 & 1 \end{bmatrix}$$

At the MATLAB prompt, type

$$H = [1 tf(1,[1 1]);0 1]$$
  
 $Hi = inv(H)$ 

to invert it. These commands produce the following result.

```
Transfer function from input 1 to output...
#1: 1
#2:
     0
Transfer function from input 2 to output...
#1:
      s + 1
#2:
     1
```

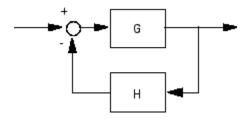
You can verify that

H \* Hi

is the identity transfer function (static gain I).

#### Limitations

Do not use inv to model feedback connections such as



While it seems reasonable to evaluate the corresponding closed-loop transfer function  $(I + GH)^{-1}G$  as

$$inv(1+g*h) * g$$

this typically leads to nonminimal closed-loop models. For example,

```
g = zpk([],1,1)
h = tf([2 1],[1 0])
cloop = inv(1+g*h) * g
```

yields a third-order closed-loop model with an unstable pole-zero cancellation at s = 1.

#### **Introduced before R2006a**

# iopzmap

Plot pole-zero map for I/O pairs of model

### **Syntax**

```
iopzmap(sys)
iopzmap(sys1,sys2,...)
```

# **Description**

iopzmap(sys) computes and plots the poles and zeros of each input/output pair of the dynamic system model sys. The poles are plotted as x's and the zeros are plotted as o's.

iopzmap(sys1,sys2,...) shows the poles and zeros of multiple models sys1,sys2,...
on a single plot. You can specify distinctive colors for each model, as in
iopzmap(sys1,'r',sys2,'y',sys3,'g').

The functions sgrid or zgrid can be used to plot lines of constant damping ratio and natural frequency in the s or z plane.

For model arrays, iopzmap plots the poles and zeros of each model in the array on the same diagram.

### **Examples**

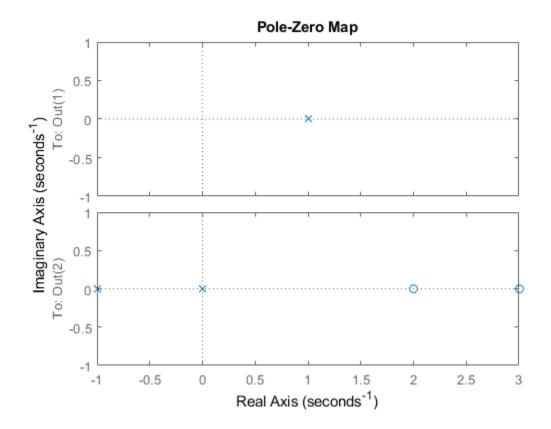
#### Pole-Zero Map for MIMO System

Create a one-input, two-output dynamic system.

```
H = [tf(-5, [1-1]); tf([1-56], [110])];
```

Plot a pole-zero map.

```
iopzmap(H)
```

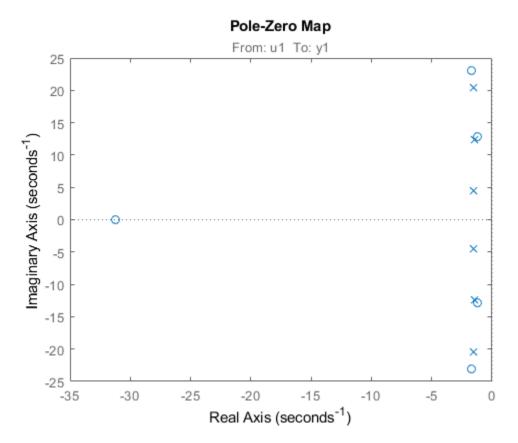


iopzmap generates a separate map for each I/O pair in the system.

#### **Pole-Zero Map of Identified Model**

View the poles and zeros of an over-parameterized state-space model estimated from input-output data. (Requires System Identification Toolbox $^{\text{\tiny TM}}$ ).

```
load iddata1
sys = ssest(z1,6,ssestOptions('focus','simulation'));
iopzmap(sys)
```



The plot shows that there are two pole-zero pairs that almost overlap, which hints are their potential redundancy.

# **Tips**

For additional options for customizing the appearance of the pole-zero plot, use iopzplot.

#### See Also

iopzplot | pole | pzmap | sgrid | zero | zgrid

#### **Introduced before R2006a**

# iopzplot

Plot pole-zero map for I/O pairs and return plot handle

### **Syntax**

```
h = iopzplot(sys)
iopzplot(sys1,sys2,...)
iopzplot(AX,...)
iopzplot(..., plotoptions)
```

# **Description**

h = iopzplot(sys) computes and plots the poles and zeros of each input/output pair of the dynamic system model sys. The poles are plotted as x's and the zeros are plotted as o's. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

```
help proptions
```

for a list of available plot options. For more information on the ways to change properties of your plots, see "Ways to Customize Plots".

iopzplot(sys1,sys2,...) shows the poles and zeros of multiple dynamic system
models sys1,sys2,... on a single plot. You can specify distinctive colors for each
model, as in

```
iopzplot(sys1,'r',sys2,'y',sys3,'g')
```

iopzplot(AX,...) plots into the axes with handle AX.

iopzplot(..., plotoptions) plots the poles and zeros with the options specified in plotoptions. Type

help proptions

for more detail.

The function sgrid or zgrid can be used to plot lines of constant damping ratio and natural frequency in the s or z plane.

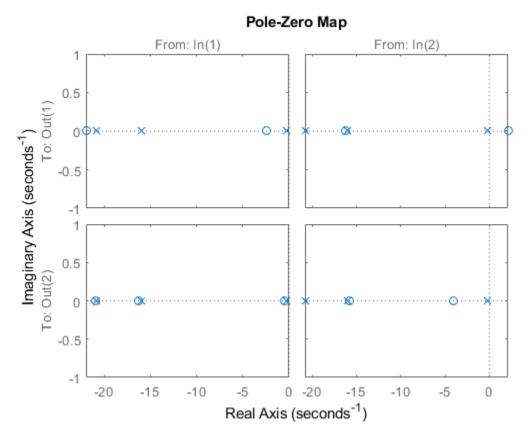
For arrays sys of LTI models, iopzplot plots the poles and zeros of each model in the array on the same diagram.

# **Examples**

#### Change I/O Grouping on Pole/Zero Map

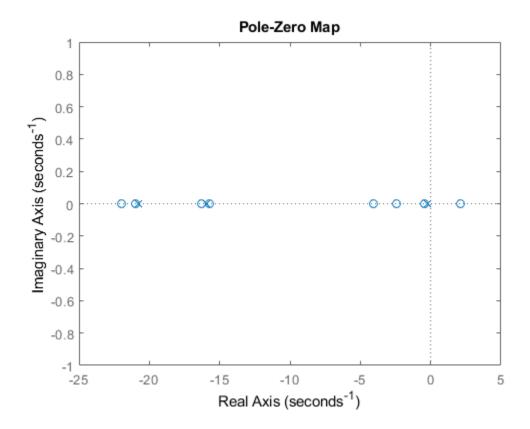
Create a pole/zero map of a two-input, two-output dynamic system.

```
sys = rss(3,2,2);
h = iopzplot(sys);
```



By default, the plot displays the poles and zeros of each I/O pair on its own axis. Use the plot handle to view all I/Os on a single axis.

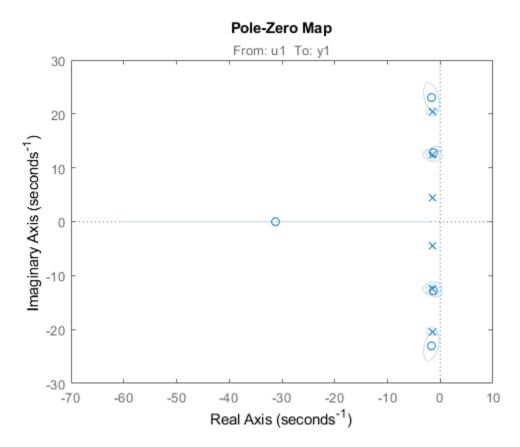
setoptions(h,'IOGrouping','all')



#### **Use Pole-Zero Map to Examine Identified Model**

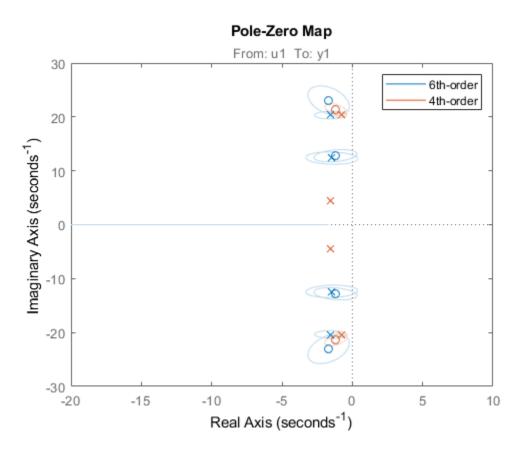
View the poles and zeros of a sixth-order state-space model estimated from input-output data. Use the plot handle to display the confidence intervals of the identified model's pole and zero locations.

```
load iddata1
sys = ssest(z1,6,ssestOptions('focus','simulation'));
h = iopzplot(sys);
showConfidence(h)
```



There is at least one pair of complex-conjugate poles whose locations overlap with those of a complex zero, within the 1- $\sigma$  confidence region. This suggests their redundancy. Hence, a lower (4th) order model might be more robust for the given data.

```
sys2 = ssest(z1,4,ssestOptions('focus','simulation'));
h = iopzplot(sys,sys2);
showConfidence(h)
legend('6th-order','4th-order')
axis([-20, 10 -30 30])
```



The fourth-order model sys2 shows less variability in the pole-zero locations.

#### See Also

getoptions | iopzmap | setoptions

**Introduced before R2006a** 

#### isct

Determine if dynamic system model is in continuous time

### **Syntax**

bool = isct(sys)

### **Description**

bool = isct(sys) returns a logical value of 1 (true) if the dynamic system model sys is a continuous-time model. The function returns a logical value of 0 (false) otherwise.

### **Input Arguments**

#### sys

Dynamic system model or array of such models.

### **Output Arguments**

#### bool

Logical value indicating whether sys is a continuous-time model.

bool = 1 (true) if sys is a continuous-time model (sys.Ts = 0). If sys is a discrete-time model, bool = 0 (false).

For a static gain, both isct and isdt return true unless you explicitly set the sample time to a nonzero value. If you do so, isdt returns true and isct returns false.

For arrays of models, bool is true if the models in the array are continuous.

# **See Also**

isdt|isstable

Introduced in R2007a

#### isdt

Determine if dynamic system model is in discrete time

### **Syntax**

bool = isdt(sys)

### **Description**

bool = isdt(sys) returns a logical value of 1 (true) if the dynamic system model sys is a discrete-time model. The function returns a logical value of  $\theta$  (false) otherwise.

### **Input Arguments**

#### sys

Dynamic system model or array of such models.

### **Output Arguments**

#### bool

Logical value indicating whether sys is a discrete-time model.

bool = 1 (true) if sys is a discrete-time model (sys.Ts  $\neq$  0). If sys is a continuous-time model, bool = 0 (false).

For a static gain, both isct and isdt return true unless you explicitly set the sample time to a nonzero value. If you do so, isdt returns true and isct returns false.

For arrays of models, bool is true if the models in the array are discrete.

# **See Also**

isct|isstable

Introduced in R2007a

# isempty

Determine whether dynamic system model is empty

### **Syntax**

```
isempty(sys)
```

# **Description**

isempty(sys) returns a logical value of 1 (true) if the dynamic system model sys has no input or no output, and a logical value of 0 (false) otherwise. Where sys is a frd model, isempty(sys) returns 1 when the frequency vector is empty. Where sys is a model array, isempty(sys) returns 1 when the array has empty dimensions or when the LTI models in the array are empty.

### **Examples**

#### **Determine Whether Dynamic Model Is Empty**

Create a continuous-time state-space model with 1 input and no outputs. In this example, specify the A and B matrices as 1 and 2, respectively.

```
sys1 = ss(1,2,[],[]);
Determine whether sys1 is empty.
isempty(sys1)
ans = logical
1
```

The isempty command returns 1 because the system does not have any outputs.

Similarly, isempty returns 1 for an empty transfer function.

#### isempty(tf)

```
ans = logical
```

Now create a state-space model with 1 input and 1 output. In this example, specify the A, B, C, and D matrices as 1, 2, 3, and 4, respectively.

```
sys2 = ss(1,2,3,4);
```

Determine whether sys2 is empty.

#### isempty(sys2)

```
ans = logical
```

The command returns  $\theta$  because the system has inputs and outputs.

#### See Also

issiso|size

**Introduced before R2006a** 

#### isfinite

Determine if model has finite coefficients

### **Syntax**

```
B = isfinite(sys)
B = isfinite(sys, 'elem')
```

### **Description**

B = isfinite(sys) returns a logical value of 1 (true) if the model sys has finite coefficients, and a logical value of 0 (false) otherwise. If sys is a model array, then B = 1 if all models in sys have finite coefficients.

B = isfinite(sys,'elem') checks each model in the model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys have finite coefficients.

### **Examples**

#### **Check Model for Finite Coefficients**

Create model and check whether its coefficients are all finite.

```
sys = rss(3);
B = isfinite(sys)
B = logical
1
```

The model, sys, has finite coefficients.

#### **Check Each Model in Array**

Create a 1-by-5 array of models, and check each model for finite coefficients.

isfinite checks each model in the model array, sys, and returns a logical array indicating which models have all finite coefficients.

### **Input Arguments**

#### sys — Model or array to check

input-output model | model array

Model or array to check, specified as an input-output model or model array. Input-output models include dynamic system models such as numeric LTI models and generalized models. Input-output models also include static models such as tunable parameters or generalized matrices.

### **Output Arguments**

#### B — Flag indicating whether model has finite coefficients

logical | logical array

Flag indicating whether model has finite coefficients, returned as a logical value or logical array.

#### See Also

isreal

#### Introduced in R2013a

#### **isParametric**

Determine if model has tunable parameters

### **Syntax**

bool = isParametric(M)

### **Description**

bool = isParametric(M) returns a logical value of 1 (true) if the model M contains parametric (tunable) "Control Design Blocks". The function returns a logical value of 0 (false) otherwise.

# **Input Arguments**

М

A Dynamic System model or Static model, or an array of such models.

# **Output Arguments**

#### bool

Logical value indicating whether M contains tunable parameters.

bool = 1 (true) if the model M contains parametric (tunable) "Control Design Blocks" such as realp or tunableSS. If M does not contain parametric Control Design Blocks, bool = 0 (false).

#### See Also

nblocks

**Topics**"Control Design Blocks"
"Dynamic System Models"
"Static Models"

#### Introduced in R2011a

#### **isPassive**

Check passivity of linear systems

### **Syntax**

```
pf = isPassive(G)
pf = isPassive(G,nu,rho)
[pf,R] = isPassive(G,  )
```

# **Description**

pf = isPassive(G) returns a logical value of 1 (true) if the dynamic system model G is passive, and a logical value of 0 (false) otherwise. A system is passive if all its I/O trajectories (u(t), y(t)) satisfy:

$$\int_0^T y(t)^T u(t) dt > 0,$$

for all T > 0. Equivalently, a system is passive if its frequency response is positive real, which means that for all  $\omega > 0$ ,

$$G(j\omega) + G(j\omega)^H > 0$$

(or the discrete-time equivalent). If G is a model array, then <code>isPassive</code> returns a logical array of the same array dimensions as G, where each entry in the array reflects the passivity of the corresponding entry in G.

For more information about the notion of passivity, see "About Passivity and Passivity Indices".

pf = isPassive(G,nu,rho) returns 1 (true) if G is passive with index nu at the
inputs, and index rho at the outputs. Such systems satisfy:

$$\int_{0}^{T} y(t)^{T} u(t) dt > \nu \int_{0}^{T} u(t)^{T} u(t) dt + \rho \int_{0}^{T} y(t)^{T} y(t) dt,$$

for all T > 0.

- Use rho = 0 to check whether a system is input passive with index nu at the inputs.
- Use nu = 0 to check whether a system is output passive with index rho at the outputs.

For more information about input and output passivity, see "About Passivity and Passivity Indices".

[pf,R] =  $isPassive(G, ___)$  also returns the relative index for the corresponding passivity bound (see getPassiveIndex). R measures the amount by which the passivity property is satisfied (R < 1) or violated (R > 1). You can use this syntax with any of the previous combinations of input arguments.

## **Examples**

#### **Check Passivity of Dynamic System**

Test whether the following transfer function is passive:

$$G(s) = \frac{s+1}{s+2}.$$

$$G = tf([1,1],[1,2]);$$

$$[pf,R] = isPassive(G)$$

$$pf = logical$$

$$1$$

$$R = 0.3333$$

pf = 1 indicates that G is passive. R = 0.3333 indicates that R has a relative excess of passivity.

#### **Check Input and Output Passivity**

Test whether the transfer function G is input passive with index 0.25. To do so, use nu = 0.25 and rho = 0.

The result shows that G is input passive with this nu value and has some excess passivity.

Test whether G is output passive with index 2.

```
[pfout,Rout] = isPassive(G,0,2)
pfout = logical
0

Rout = 2.6180
```

Here, the result pfout = 0 shows that G is not output passive with this rho value. The R value gives a relative measure of the shortage of passivity.

#### **Check Passivity of Models in Array**

You can use isPassive to evaluate the passivity of multiple models in a model array simultaneously. For this example, generate a random array of transfer function models.

```
G = rss(3,1,1,1,5);
```

G is a 1-by-5 array of 3-state SISO models. Check the passivity of all the models in G.

```
[pf,R] = isPassive(G)
pf = 1x5 logical array
     0     0     1     0
R = 1×5
```

```
35.3759 Inf Inf 0.1130 4.3096
```

pf and R are also 1-by-5 arrays. Each pf entry indicates whether the corresponding model in G is passive. Likewise, each R value gives the relative excess or shortage of passivity in the corresponding model in G. For instance, examine the passivity of the second entry in G, and compare the result with the second entries in pf and R.

```
[pf2,R2] = isPassive(G(:,:,2))
pf2 = logical
0
R2 = Inf
```

# **Input Arguments**

#### **G** — Model to analyze

dynamic system model | model array

Model to analyze for passivity, specified as a dynamic system model such as a tf, ss, or genss model. G can be MIMO, if the number of inputs equals the number of outputs. G can be continuous or discrete. If G is a generalized model with tunable or uncertain blocks, isPassive evaluates passivity of the current, nominal value of G.

#### nu — Input passivity index

0 (default) | real scalar

Input passivity index, specified as a real scalar value. Use nu and rho to specify particular passivity bounds. To check whether a system is passive with a particular index at the inputs, set nu to that value and set rho = 0.

#### rho — Output passivity index

0 (default) | real scalar

Output passivity index, specified as a real scalar value. Use nu and rho to specify particular passivity bounds. To check whether a system is passive with a particular passivity index at the outputs, set rho to that value and set nu = 0.

## **Output Arguments**

#### pf — Passivity indicator

1 (true) | 0 (false) | logical array

Passivity indicator, returned as a boolean value:

- 1 (true) if G is passive.
- 0 (false) if G is not passive.

If you specify input and output passivity indices nu and rho, then pf indicates passivity with respect to the corresponding passivity bound.

If G is a model array, then pf is an array of the same size, where pf(k) indicates the passivity of the kth entry in G, G(:,:,k).

#### R — Relative passivity index

positive real scalar

Relative passivity index, returned as a positive real scalar. R measures the excess (R < 1) or shortage (R > 1) of passivity in the system.

If you specify  $nu \neq 0$  or  $rho \neq 0$ , then R measures how much the specified passivity properties are satisfied or violated.

For more information about the notion of relative passivity index, see "About Passivity and Passivity Indices".

#### See Also

getPassiveIndex | getPeakGain | getSectorCrossover | getSectorIndex |
passiveplot | sectorplot

#### **Topics**

"Passivity Indices"

"About Passivity and Passivity Indices"

#### Introduced in R2016a

# isproper

Determine if dynamic system model is proper

## **Syntax**

```
B = isproper(sys)
B = isproper(sys,'elem')
[B,sysr] = isproper(sys)
```

# **Description**

B = isproper(sys) returns a logical value of 1 (true) if the dynamic system model sys is proper and a logical value of 0 (false) otherwise.

A proper model has relative degree  $\leq 0$  and is causal. SISO transfer functions and zero-pole-gain models are proper if the degree of their numerator is less than or equal to the degree of their denominator (in other words, if they have at least as many poles as zeroes). MIMO transfer functions are proper if all their SISO entries are proper. Regular state-space models (state-space models having no E matrix) are always proper. A descriptor state-space model that has an invertible E matrix is always proper. A descriptor state-space model having a singular (non-invertible) E matrix is proper if the model has at least as many poles as zeroes.

If sys is a model array, then B is 1 if all models in the array are proper.

B = isproper(sys,'elem') checks each model in a model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys are proper.

[B,sysr] = isproper(sys) also returns an equivalent model sysr with fewer states (reduced order) and a non-singular E matrix, if sys is a proper descriptor state-space model with a non-invertible E matrix. If sys is not proper, sysr = sys.

# **Examples**

B1 = logical

#### **Examine Whether Models are Proper**

```
Create a SISO continuous-time transfer function, H_1 = s

H1 = tf([1 0],1);

Check whether H1 is proper.

B1 = isproper(H1)
```

SISO transfer functions are proper if the degree of their numerator is less than or equal to the degree of their denominator That is, if the transfer function has at least as many poles as zeroes. Since H1 has one zero and no poles, the isproper command returns 0.

Now create a transfer function with one pole and one zero,  $H_2 = s/(s+1)$ 

```
H2 = tf([1 0],[1 1]);
Check whether H2 is proper.
B2 = isproper(H2)
B2 = logical
1
```

Since H2 has equal number of poles and zeros, isproper returns 1.

#### **Compute Equivalent Lower-Order Model**

Combining state-space models sometimes yields results that include more states than necessary. Use <code>isproper</code> to compute an equivalent lower-order model.

```
H1 = ss(tf([1 1],[1 2 5]));
H2 = ss(tf([1 7],[1]));
H = H1*H2;
size(H)
State-space model with 1 outputs, 1 inputs, and 4 states.
```

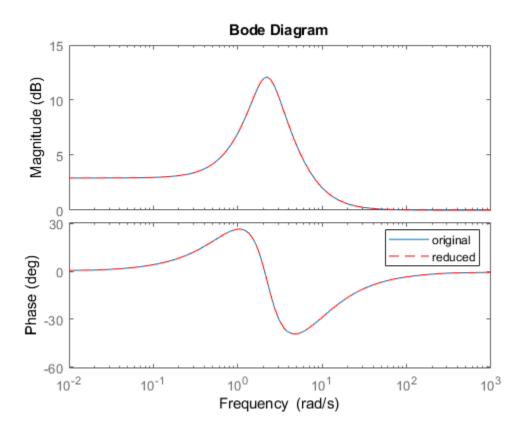
H is proper and reducible. isproper returns the reduced model.

```
[isprop,Hr] = isproper(H);
size(Hr)
```

State-space model with 1 outputs, 1 inputs, and 2 states.

H and Hr are equivalent, as a Bode plot demonstrates.

```
bodeplot(H,Hr,'r--')
legend('original','reduced')
```



# **See Also**

dss|ss

**Introduced before R2006a** 

### isreal

Determine if model has real-valued coefficients

## **Syntax**

```
B = isreal(sys)
B = isreal(sys, 'elem')
```

# **Description**

B = isreal(sys) returns a logical value of 1 (true) if the model sys has real-valued coefficients, and a logical value of 0 (false) otherwise. If sys is a model array, then B = 1 if all models in sys have real-valued coefficients.

B = isreal(sys, 'elem') checks each model in the model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys have real coefficients.

## **Examples**

#### **Check Model for Real-Valued Coefficients**

Create a model and check whether its coefficients are all real-valued.

```
sys = rss(3);
B = isreal(sys)
B = logical
1
```

The model, sys, has real-valued coefficients.

#### **Check Model Array for Real-Valued Coefficients**

Create a 1-by-5 array of models, and check each model for real-valued coefficients.

isreal checks each model in the model array, sys, and returns a logical array indicating which models have all real-valued coefficients.

## **Input Arguments**

#### sys — Model or array to check

input-output model | model array

Model or array to check, specified as an input-output model or model array. Input-output models include dynamic system models such as numeric LTI models and generalized models. Input-output models also include static models such as tunable parameters or generalized matrices.

# **Output Arguments**

#### B — Flag indicating whether model has real-valued coefficients

logical | logical array

Flag indicating whether model has real-valued coefficients, returned as a logical value or logical array.

#### See Also

isfinite

#### Introduced in R2013a

## isstable

Determine whether system is stable

### **Syntax**

```
B = isstable(sys)
B = isstable(sys,'elem')
```

# **Description**

B = isstable(sys) returns a logical value of 1 (true) if the dynamic system model sys has stable dynamics, and a logical value of 0 (false) otherwise. If sys is a model array, then B = 1 only if all models in sys are stable.

B = isstable(sys,'elem') returns a logical array of the same dimensions as the model array sys. The logical array indicates which models in sys are stable.

isstable is only supported for analytical models with a finite number of poles.

# **Examples**

#### **Determine Stability of Models in Model Array**

Create an array of SISO transfer function models with poles varying from -2 to 2. To do so, first initialize an array of dimension [1,length(a)] with zero-valued SISO transfer functions.

```
a = [-2:2];
sys = tf(zeros(1,1,1,length(a)));
```

Populate this array with transfer functions of the form 1/(s-a).

```
for j = 1:length(a)
   sys(1,1,1,j) = tf(1,[1 -a(j)]);
```

```
end
sys.SamplingGrid = struct('a',a);
Examine the stability of the model array.
B_all = isstable(sys)
B_all = logical
```

By default, isstable returns a single Boolean value that is 1 (true) only if all models in the array are stable. sys contains some models with nonnegative poles, which are not stable. Therefore, isstable returns 0 (false) for the entire array.

Examine stability of each model in the array, element by element.

```
B_elem = isstable(sys,'elem')
B_elem = 1x5 logical array
1 1 0 0 0
```

The 'elem' flag causes isstable to return an array of Boolean values, which indicate the stability of the corresponding entry in the model array. For example,  $B_elem(2) = 1$ , which indicates that sys(1,1,1,2) is stable. This result is expected, because sys(1,1,1,2) has a = -1.

#### See Also

pole

Introduced in R2012a

# issiso

Determine if dynamic system model is single-input/single-output (SISO)

### **Syntax**

issiso(sys)

# **Description**

issiso(sys) returns a logical value of 1 (true) if the dynamic system model sys is SISO and a logical value of 0 (false) otherwise.

#### See Also

isempty | size

**Introduced before R2006a** 

### isstatic

Determine if model is static or dynamic

## **Syntax**

```
B = isstatic(sys)
B = isstatic(sys, 'elem')
```

# **Description**

B = isstatic(sys) returns a logical value of 1 (true) if the model sys is a static model, and a logical value of 0 (false) if sys has dynamics, such as states or delays. If sys is a model array, then B = 1 if all models in sys are static.

B = isstatic(sys,'elem') checks each model in the model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys are static.

# **Input Arguments**

#### sys — Model or array to check

input-output model | model array

Model or array to check, specified as an input-output model or model array. Input-output models include dynamic system models such as numeric LTI models and generalized models. Input-output models also include static models such as tunable parameters or generalized matrices.

# **Output Arguments**

B — Flag indicating whether input model is static

logical | logical array

Flag indicating whether input model is static, returned as a logical value or logical array.

# **See Also**

hasdelay | pole | zero

### **Topics**

"Types of Model Objects"

Introduced in R2013a

### kalman

Kalman filter design, Kalman estimator

## **Syntax**

```
[kest,L,P] = kalman(sys,Qn,Rn,Nn)
[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known)
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,...,type)
```

# **Description**

kalman designs a Kalman filter or Kalman state estimator given a state-space model of the plant and the process and measurement noise covariance data. The Kalman estimator provides the optimal solution to the following continuous or discrete estimation problems.

#### **Continuous-Time Estimation**

Given the continuous plant

$$\dot{x} = Ax + Bu + Gw$$
 (state equation)  
 $v = Cx + Du + Hw + v$  (measurement equation)

with known inputs u, white process noise w, and white measurement noise v satisfying

$$E(w) = E(v) = 0$$
,  $E(ww^{T}) = Q$ ,  $E(vv^{T}) = R$ ,  $E(wv^{T}) = N$ 

construct a state estimate  $\widehat{\boldsymbol{x}}\left(t\right)$  that minimizes the steady-state error covariance

$$P = \lim_{t \to \infty} E(\{x - \widehat{x}\} \{x - \widehat{x}\}^T)$$

The optimal solution is the Kalman filter with equations

$$\dot{\widehat{x}} = A\widehat{x} + Bu + L(y - C\widehat{x} - Du)$$
$$\begin{vmatrix} \widehat{y} \\ \widehat{x} \end{vmatrix} = \begin{vmatrix} C \\ I \end{vmatrix} \widehat{x} + \begin{vmatrix} D \\ 0 \end{vmatrix} u$$

The filter gain L is determined by solving an algebraic Riccati equation to be

$$L = (PC^T + \overline{N})\overline{R}^{-1}$$

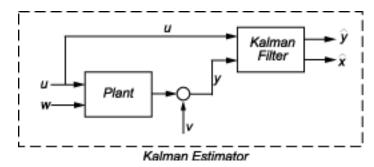
where

$$\bar{R} = R + HN + N^{T}H^{T} + HQH^{T}$$
$$\bar{N} = G(OH^{T} + N)$$

and P solves the corresponding algebraic Riccati equation.

The estimator uses the known inputs u and the measurements y to generate the output and state estimates  $\widehat{y}$  and  $\widehat{x}$ . Note that  $\widehat{y}$  estimates the true plant output

$$y = Cx + Du + Hw + v$$



#### **Discrete-Time Estimation**

Given the discrete plant

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
  
 $y[n] = Cx[n] + Du[n] + Hw[n] + v[n]$ 

and the noise covariance data

$$E(w[n]w[n]^T) = Q$$
,  $E(v[n]v[n]^T) = R$ ,  $E(w[n]v[n]^T) = N$ 

The estimator has the following state equation:

$$\widehat{x}\left[n+1\left|n\right]=A\widehat{x}\left[n\left|n-1\right]+Bu[n]+L(y[n]-C\widehat{x}\left[n\left|n-1\right]-Du[n]\right)$$

The gain matrix *L* is derived by solving a discrete Riccati equation to be

$$L = (APC^{T} + \overline{N})(CPC^{T} + \overline{R})^{-1}$$

where

$$\overline{R} = R + HN + N^{T}H^{T} + HQH^{T}$$

$$\overline{N} = G(QH^{T} + N)$$

There are two variants of discrete-time Kalman estimators:

• The current estimator generates output estimates  $\widehat{y}[n|n]$  and state estimates  $\widehat{x}[n|n]$  using all available measurements up to y[n]. This estimator has the output equation

$$\begin{bmatrix} \widehat{y}[n \mid n] \\ \widehat{x}[n \mid n] \end{bmatrix} = \begin{bmatrix} (I - M_y)C \\ I - M_xC \end{bmatrix} \widehat{x}[n \mid n - 1] + \begin{bmatrix} (I - M_y)D & M_y \\ -M_xD & M_x \end{bmatrix} \begin{bmatrix} u[n] \\ y[n] \end{bmatrix},$$

where the innovation gains  $M_{\nu}$  and  $M_{\nu}$  are defined as:

$$M_{x} = PC^{T} (CPC^{T} + \overline{R})^{-1},$$
  
$$M_{y} = (CPC^{T} + HQH^{T} + HN)(CPC^{T} + \overline{R})^{-1}.$$

 $M_x$  updates the prediction  $\widehat{x}[n|n-1]$  using the new measurement y[n].

$$\widehat{x}[n|n] = \widehat{x}[n|n-1] + M_{X}(y[n] - C\widehat{x}[n|n-1] - Du[n]).$$
innovation

When 
$$H = 0$$
,  $M_v = CM_x$  and  $y[n|n] = Cx[n|n] + Du[n]$ .

• The delayed estimator generates output estimates  $\widehat{y}[n|n-1]$  and state estimates  $\widehat{x}[n|n-1]$  using measurements only up to  $y_v[n-1]$ . This estimator is easier to implement inside control loops and has the output equation

$$\begin{bmatrix} \widehat{y}[n|n-1] \\ \widehat{x}[n|n-1] \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} \widehat{x}[n|n-1] + \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u[n] \\ y[n] \end{bmatrix}$$

[kest,L,P] = kalman(sys,Qn,Rn,Nn) creates a state-space model kest of the Kalman estimator given the plant model sys and the noise covariance data Qn, Rn, Nn (matrices Q, R, N described in "Description" on page 2-607). sys must be a state-space model with matrices A, [BG], C, [DH].

The resulting estimator kest has inputs [u; y] and outputs  $[\widehat{y}; \widehat{x}]$  (or their discrete-time counterparts). You can omit the last input argument Nn when N = 0.

The function kalman handles both continuous and discrete problems and produces a continuous estimator when sys is continuous and a discrete estimator otherwise. In continuous time, kalman also returns the Kalman gain L and the steady-state error covariance matrix P. P solves the associated Riccati equation.

[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known) handles the more general situation when

- Not all outputs of sys are measured.
- The disturbance inputs *w* are not the last inputs of sys.

The index vectors sensors and known specify which outputs y of sys are measured and which inputs u are known (deterministic). All other inputs of sys are assumed stochastic.

[kest,L,P,M,Z] = kalman(sys,Qn,Rn,...,type) specifies the estimator type for discrete-time plants sys. The type argument is either 'current' (default) or 'delayed'. For discrete-time plants, kalman returns the estimator and innovation gains L and M and the steady-state error covariances

$$P = \lim_{n \to \infty} E(e[n \mid n-1]e[^n), \quad e[n \mid n-1] = x[n] - x[n \mid n-1]$$

$$Z = \lim_{n \to \infty} E(e[n \mid n]e[^n), \quad e[n \mid n] = x[n] - x[n \mid n]$$

## **Examples**

See "LQG Design for the x-Axis" and Kalman Filtering for examples that use the kalman function.

#### Limitations

The plant and noise data must satisfy:

• (*C*,*A*) detectable

- $\overline{R} > 0$  and  $\overline{Q} \overline{N}\overline{R}^{-1}\overline{N}^T \ge 0$
- $(A \overline{N}\overline{R}^{-1}C, \overline{Q} \overline{N}\overline{R}^{-1}\overline{N}^T)$  has no uncontrollable mode on the imaginary axis (or unit circle in discrete time) with the notation

$$\begin{split} \overline{Q} &= GQG^T \\ \overline{R} &= R + HN + N^TH^T + HQH^T \\ \overline{N} &= G(QH^T + N) \end{split}$$

#### References

- [1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.
- [2] Lewis, F., Optimal Estimation, John Wiley & Sons, Inc, 1986.
- [3] Deshpande, A.S., "Bridging a Gap in Applied Kalman Filtering: Estimating Outputs When Measurements Are Correlated with the Process Noise." *IEEE Control Systems Magazine*, Vol. 37, Number 3, 2017, pp. 87–93.

#### See Also

Kalman Filter | care | dare | estim | extendedKalmanFilter | kalmd | lqg | lqgreg |
ss | unscentedKalmanFilter

#### **Topics**

"Kalman Filtering"

#### Introduced before R2006a

### kalmd

Design discrete Kalman estimator for continuous plant

## **Syntax**

```
[\text{kest,L,P,M,Z}] = \text{kalmd(sys,Qn,Rn,Ts)}
```

# **Description**

kalmd designs a discrete-time Kalman estimator that has response characteristics similar to a continuous-time estimator designed with kalman. This command is useful to derive a discrete estimator for digital implementation after a satisfactory continuous estimator has been designed.

[kest,L,P,M,Z] = kalmd(sys,Qn,Rn,Ts) produces a discrete Kalman estimator kest with sample time Ts for the continuous-time plant

$$\dot{x} = Ax + Bu + Gw$$
 (state equation)  
 $y_v = Cx + Du + v$  (measurement equation)

with process noise w and measurement noise v satisfying

$$E(w) = E(v) = 0$$
,  $E(ww^{T}) = Q_{n}$ ,  $E(vv^{T}) = R_{n}$ ,  $E(wv^{T}) = 0$ 

The estimator kest is derived as follows. The continuous plant sys is first discretized using zero-order hold with sample time Ts (see c2d entry), and the continuous noise covariance matrices  $Q_n$  and  $R_n$  are replaced by their discrete equivalents

$$Q_d = \int_0^{T_S} e^{A\tau} G Q_n G^T e^{A^T \tau} d\tau$$

$$R_d = R_n / T_S$$

The integral is computed using the matrix exponential formulas in [2]. A discrete-time estimator is then designed for the discretized plant and noise. See kalman for details on discrete-time Kalman estimation.

kalmd also returns the estimator gains L and M, and the discrete error covariance matrices P and Z (see kalman for details).

#### Limitations

The discretized problem data should satisfy the requirements for kalman.

### References

- [1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.
- [2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-15, October 1970.

#### See Also

kalman | lqgreg | lqrd

Introduced before R2006a

## Ift

Generalized feedback interconnection of two models (Redheffer star product)

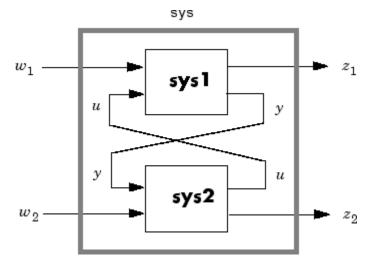
# **Syntax**

```
lft
sys = lft(sys1,sys2,nu,ny)
```

# **Description**

lft forms the star product or linear fractional transformation (LFT) of two model objects or model arrays. Such interconnections are widely used in robust control techniques.

sys = lft(sys1,sys2,nu,ny) forms the star product sys of the two models (or arrays) sys1 and sys2. The star product amounts to the following feedback connection for single models (or for each model in an array).



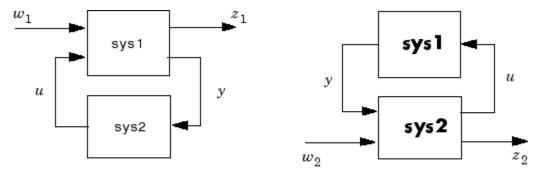
This feedback loop connects the first nu outputs of sys2 to the last nu inputs of sys1 (signals u), and the last ny outputs of sys1 to the first ny inputs of sys2 (signals y). The resulting system sys maps the input vector [ $w_1$ ;  $w_2$ ] to the output vector [ $z_1$ ;  $z_2$ ].

The abbreviated syntax

$$sys = lft(sys1, sys2)$$

#### produces:

- The lower LFT of sys1 and sys2 if sys2 has fewer inputs and outputs than sys1. This amounts to deleting  $w_2$  and  $z_2$  in the above diagram.
- The upper LFT of sys1 and sys2 if sys1 has fewer inputs and outputs than sys2. This amounts to deleting  $w_1$  and  $z_1$  in the above diagram.



Lower LFT connection

Upper LFT connection

#### Limitations

There should be no algebraic loop in the feedback connection.

# **Algorithms**

The closed-loop model is derived by elementary state-space manipulations.

# See Also

connect | feedback

**Introduced before R2006a** 

# **Linear System Analyzer**

Analyze time and frequency responses of linear time-invariant (LTI) systems

# **Description**

The **Linear System Analyzer** app lets you analyze time and frequency responses of LTI systems. Using this app, you can:

- View and compare the response plots of SISO and MIMO systems, or of several linear models at the same time.
- Generate time response plots such as step, impulse, and time response to arbitrary inputs.
- Generate frequency response plots such as Bode, Nyquist, Nichols, singular-value, and pole-zero plots.
- Inspect key response characteristics, such as rise time, maximum overshoot, and stability margins.

#### **Available Plots**

**Linear System Analyzer** can generate the following response plots:

- Step response
- Impulse response
- Simulated time response to specified input signal
- Simulated time response from specified initial conditions (state-space models only)
- Bode diagram (magnitude and phase, or magnitude alone)
- Nyquist plot
- · Nichols plot
- Singular value plot
- Pole/zero map and I/O pole/zero map

## **Open the Linear System Analyzer App**

- MATLAB Toolstrip: On the Apps tab, under Control System Design and Analysis, click the app icon.
- MATLAB command prompt: Enter linearSystemAnalyzer.

# **Examples**

- "Linear Analysis Using the Linear System Analyzer"
- "Joint Time-Domain and Frequency-Domain Analysis"

#### **Programmatic Use**

linearSystemAnalyzer opens the **Linear System Analyzer** app with no LTI systems to analyze. To specify a system to analyze, select **File > Import**.

linearSystemAnalyzer(sys1,sys2,...,sysn) opens **Linear System Analyzer** and displays the step response of one or more dynamic system models, sys1, sys2, ..., sysn. Such models include:

- Numeric LTI models such as tf, zpk, or ss models.
- Identified models such as idtf, idss, or idproc (requires System Identification Toolbox software).
- Generalized LTI models such as genss or uss models. For generalized LTI models
  without uncertainty, Linear System Analyzer plots the response of the nominal value
  of the model. For generalized models with uncertainty, the app plots the responses of
  20 random samples of the uncertain system. (Uncertain models require Robust Control
  Toolbox software.)

linearSystemAnalyzer(sys1,LineSpec1,sys2,LineSpec2...,sysn,LineSpecn) specifies the line style, marker, and color of each response plot. Specify plot styles using one, two, or three characters. For example, the following code uses red asterisks for the response of sys1, and a magenta dotted line for the response of sys2.

```
linearSystemAnalyzer(sys1, 'r-*', sys2, 'm--');
```

For more information about configuring this argument, see the LineSpec input argument of the plot function.

linearSystemAnalyzer(plottype, \_\_\_\_) opens Linear System Analyzer and displays the response types specified by plottype. You can use this syntax with any of the previous input argument combinations. The plottype argument can be any one of the following:

- 'step' Step response.
- 'impulse' Impulse response.
- 'lsim' Linear simulation plot. When you use this plot type, the Linear Simulation Tool dialog box prompts you to specify an input signal for the simulation.
- 'initial' Initial condition plot (state-space models only). You can use the extras argument to specify the initial state. If you do not, the Linear Simulation Tool dialog box opens and prompts you to specify an initial state for the simulation.
- 'bode' Bode diagram.
- 'bodemag' Bode magnitude diagram.
- 'nyquist' Nyquist plot.
- 'nichols' Nichols plot.
- 'sigma' Singular value plot. (See sigma).
- 'pzmap' Pole/zero map.
- 'iopzmap' Pole/zero map of each input/output pair of the LTI system.

To open **Linear System Analyzer** with multiple response plots, use a cell array of up to six of these plot types for the plottype input argument. For example, the following command opens the app with a step response plot and a Nyquist plot for the system sys.

```
linearSystemAnalyzer({'step';'nyquist'},sys)
```

linearSystemAnalyzer(plottype, sys1, sys2,..., sysn, extras) specifies additional input arguments specific to the type of response plot. extras can be one or more of the input arguments available for the function corresponding to the plot type. For example, suppose plottype is 'step'. Then, extras enables you to use the additional arguments that you could use with the step command, such as the desired final time, Tfinal. Thus, the following command opens the app with a step response plot of sys, with a final time of Tfinal.

```
linearSystemAnalyzer('step',sys,Tfinal)
```

If plottype is 'initial', you can use extras to supply the initial conditions x0, and other arguments such as Tfinal. For example:

linearSystemAnalyzer('initial', sys, x0, Tfinal)

To determine appropriate arguments for extras, see the reference pages of the functions corresponding to each plot type, such as step, bode, or initial.

h = linearSystemAnalyzer(\_\_\_\_) returns a handle to the **Linear System Analyzer** figure. You can use this syntax with any of the previous combinations of input arguments. Use the handle to modify previously opened **Linear System Analyzer** instances, as described in the next two syntaxes.

linearSystemAnalyzer('clear',h) clears the plots and data from the **Linear**System Analyzer corresponding to handle h. To clear multiple app instances at once, set h to a vector of handles.

linearSystemAnalyzer('current',sys1,sys2,...,sysn,h) adds the responses of the systems sys1, sys2,..., sysn to the **Linear System Analyzer** corresponding to handle h. To update multiple app instances at once, set h to a vector of handles. If the new systems have different I/O dimensions from the currently displayed systems, the app clears the existing responses and displays only the new ones.

#### See Also

#### **Apps**

**Control System Designer** 

#### **Functions**

bode | bodemag | impulse | initial | iopzmap | lsim | nichols | nyquist | pzmap |
sigma | step

#### **Topics**

"Linear Analysis Using the Linear System Analyzer"

"Joint Time-Domain and Frequency-Domain Analysis"

#### Introduced in R2015a

# lqg

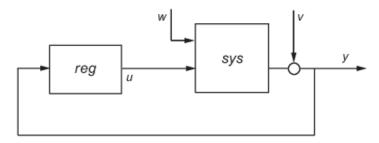
Linear-Quadratic-Gaussian (LQG) design

# **Syntax**

```
reg = lqg(sys,QXU,QWV)
reg = lqg(sys,QXU,QWV,QI)
reg = lqg(sys,QXU,QWV,QI,'1dof')
reg = lqg(sys,QXU,QWV,QI,'2dof')
reg = lqg(___,'current')
[reg,info] = lqg(___)
```

# **Description**

reg = lqg(sys,QXU,QWV) computes an optimal linear-quadratic-Gaussian (LQG) regulator reg given a state-space model sys of the plant and weighting matrices QXU and QWV. The dynamic regulator reg uses the measurements y to generate a control signal u that regulates y around the zero value. Use positive feedback to connect this regulator to the plant output y.



The LQG regulator minimizes the cost function

$$J = E\left\{\lim_{\tau \to \infty} \frac{1}{\tau} \int_{0}^{\tau} \left[x^{T}, u^{T}\right] Q_{xu} \begin{bmatrix} x \\ u \end{bmatrix} dt\right\}$$

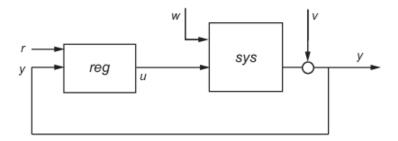
subject to the plant equations

$$dx/dt = Ax + Bu + w$$
$$v = Cx + Du + v$$

where the process noise w and measurement noise v are Gaussian white noises with covariance:

$$E\left(\begin{bmatrix} w \\ v \end{bmatrix} \cdot [w' \ v']\right) = QWV$$

reg = lqg(sys,QXU,QWV,QI) uses the setpoint command r and measurements y to generate the control signal u. reg has integral action to ensure that y tracks the command r.



The LQG servo-controller minimizes the cost function

$$J = E\left\{\lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} \left[ \left[ x^T, u^T \right] Q_{xu} \begin{bmatrix} x \\ u \end{bmatrix} + x_i^T Q_i x_i \right] dt \right\}$$

where  $x_i$  is the integral of the tracking error r - y. For MIMO systems, r, y, and  $x_i$  must have the same length.

reg = lqg(sys,QXU,QWV,QI,'ldof') computes a one-degree-of-freedom servo controller that takes e = r - y rather than [r; y] as input.

reg = lqg(sys,QXU,QWV,QI,'2dof') is equivalent to LQG(sys,QXU,QWV,QI) and produces the two-degree-of-freedom servo-controller shown previously.

reg = lqg(\_\_\_\_, 'current') uses the "current" Kalman estimator, which uses x[n|n] as the state estimate when computing an LQG regulator for a discrete-time system.

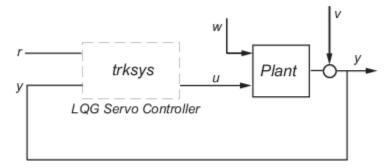
[reg,info] = lqg(\_\_\_\_) returns the controller and estimator gain matrices in the
structure info for any of the previous syntaxes. You can use the controller and estimator

gains to, for example, implement the controller in observer form. For more information, see "Algorithms" on page 2-628.

## **Examples**

#### Linear-Quadratic-Gaussian (LQG) Regulator and Servo Controller Design

This example shows how to design an linear-quadratic-Gaussian (LQG) regulator, a one-degree-of-freedom LQG servo controller, and a two-degree-of-freedom LQG servo controller for the following system.



The plant has three states (x), two control inputs (u), three random inputs (w), one output (y), measurement noise for the output (v), and the following state and measurement equations.

$$\frac{dx}{dt} = Ax + Bu + w$$
$$v = Cx + Du + v$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.3 & 1 \\ 0 & 1 \\ -0.3 & 0.9 \end{bmatrix}$$
$$C = \begin{bmatrix} 1.9 & 1.3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0.53 & -0.61 \end{bmatrix}$$

The system has the following noise covariance data:

$$Q_n = E(ww^T) = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$R_n = E(vv^T) = 0.7$$

For the regulator, use the following cost function to define the tradeoff between regulation performance and control effort:

$$J(u) = \int_0^\infty \left( 0.1 x^T x + u^T \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} u \right) dt$$

For the servo controllers, use the following cost function to define the tradeoff between tracker performance and control effort:

$$J(u) = \int_0^\infty \left( 0.1x^T x + x_i^2 + u^T \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} u \right) dt$$

To design the LQG controllers for this system:

1 Create the state-space system by typing the following in the MATLAB Command Window:

```
A = [0 1 0;0 0 1;1 0 0];
B = [0.3 1;0 1;-0.3 0.9];
C = [1.9 1.3 1];
D = [0.53 -0.61];
sys = ss(A,B,C,D);
```

2 Define the noise covariance data and the weighting matrices by typing the following commands:

```
nx = 3;    %Number of states
ny = 1;    %Number of outputs
Qn = [4 2 0; 2 1 0; 0 0 1];
Rn = 0.7;
R = [1 0; 0 2]
QXU = blkdiag(0.1*eye(nx),R);
QWV = blkdiag(Qn,Rn);
QI = eye(ny);
```

**3** Form the LQG regulator by typing the following command:

```
KLQG = lqg(sys,QXU,QWV)
```

This command returns the following LQG regulator:

Continuous-time model.

Form the one-degree-of-freedom LQG servo controller by typing the following command:

```
KLQG1 = lqg(sys,QXU,QWV,QI,'ldof')
```

This command returns the following LQG servo controller:

```
xi1
         0
                     0
                            0
B =
             e1
  x1 e
         -2.365
  x2 e -1.432
  x3 e -0.7684
  xi1
              1
C =
         x1_e x2_e
                         х3 е
                                    xi1
  u1 -0.5388 -0.4173 -0.2481
                                 0.5578
  u2 -1.492
              -1.388
                        -1.131
                                 0.5869
D =
      e1
       0
  u1
  u2
       0
Input groups:
   Name
            Channels
   Error
               1
Output groups:
     Name
               Channels
   Controls
               1,2
```

5 Form the two-degree-of-freedom LQG servo controller by typing the following command:

Continuous-time model.

This command returns the following LQG servo controller:

```
x1 e
                  2.365
   x2 e
              0 1.432
   х3 е
             0 0.7684
   xi1
              1
                     - 1
C =
          x1 e
                   x2 e
                            х3 е
                                       xi1
   u1 -0.5388
               -0.4173
                         -0.2481
                                    0.5578
   u2
        -1.492
                 -1.388
                          -1.131
                                    0.5869
D =
       r1 y1
   u1
        0
            0
   u2
        0
Input groups:
       Name
                   Channels
     Setpoint
                      1
                      2
   Measurement
Output groups:
      Name
                Channels
    Controls
                  1.2
Continuous-time model.
```

### **Tips**

- lqg can be used for both continuous- and discrete-time plants. In discrete time, lqg uses x[n|n-1] as its state estimate by default. To use x[n|n] as the state estimate and compute the optimal LQG controller, use the 'current' input argument. For details on the state estimators, see kalman.
- To compute the LQG regulator, lqg uses the commands lqr and kalman. To compute the servo-controller, lqg uses the commands lqi and kalman.
- When you want more flexibility for designing regulators, you can use the lqr, kalman, and lqgreg commands. When you want more flexibility for designing servo controllers, you can use the lqi, kalman, and lqgtrack commands. For more information on using these commands and how to decide when to use them, see "Linear-Quadratic-Gaussian (LQG) Design for Regulation" and "Linear-Quadratic-Gaussian (LQG) Design of Servo Controller with Integral Action".

# **Algorithms**

The controller equations are:

• For continuous time:

$$dx_e = Ax_e + Bu + L(y - Cx_e - Du)$$
  
$$u = -K_x x_e - K_i x_i$$

• For discrete time:

$$x[n+1|n] = Ax[n|n-1] + Bu[n] + L(y[n] - Cx[n|n-1] - Du[n])$$

Delayed estimator:

$$u[n] = -K_x x[n|n-1] - K_i x_i[n]$$

• Current estimator:

$$u[n] = -K_{x}x[n|n] - K_{i}x_{i}[n] - K_{w}w[n|n]$$

$$= -K_{x}x[n|n-1] - K_{i}x_{i}[n] - (K_{x}M_{x} + K_{w}M_{w})y_{inn}[n]$$

$$y_{inn}[n] = y[n] - Cx[n|n-1] - Du[n]$$

Here,

- A, B, C, and D are the state-space matrices of the LQG regulator, reg.
- $x_i$  is the integral of the tracking error r y.
- $K_x$ ,  $K_w$ ,  $K_i$ , L,  $M_x$ , and  $M_w$  are the controller and estimator gain matrices returned in info.

#### See Also

care | dare | kalman | lqi | lqr | lqry | ss

**Introduced before R2006a** 

# **Iqgreg**

Form linear-quadratic-Gaussian (LQG) regulator

### **Syntax**

```
rlqg = lqgreg(kest,k)
rlqg = lqgreg(kest,k,controls)
```

# **Description**

rlqg = lqgreg(kest,k) returns the LQG regulator rlqg (a state-space model) given
the Kalman estimator kest and the state-feedback gain matrix k. The same function
handles both continuous- and discrete-time cases. Use consistent tools to design kest
and k:

- Continuous regulator for continuous plant: use lqr or lqry and kalman
- Discrete regulator for discrete plant: use dlqr or lqry and kalman
- Discrete regulator for continuous plant: use lqrd and kalmd

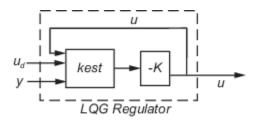
In discrete time, lqgreg produces the regulator

- $u[n] = -K\widehat{x}[n|n]$  when kest is the "current" Kalman estimator
- $u[n] = -K\widehat{x}[n|n-1]$  when kest is the "delayed" Kalman estimator

For more information on Kalman estimators, see the kalman reference page.

rlqg = lqgreg(kest,k,controls) handles estimators that have access to additional deterministic known plant inputs  $u_d$ . The index vector controls then specifies which estimator inputs are the controls u, and the resulting LQG regulator rlqg has  $u_d$  and y as inputs (see the next figure).

**Note** Always use *positive* feedback to connect the LQG regulator to the plant.



### **Examples**

See the example "LQG Regulation: Rolling Mill Case Study".

# **Algorithms**

lqgreg forms the linear-quadratic-Gaussian (LQG) regulator by connecting the Kalman estimator designed with kalman and the optimal state-feedback gain designed with lqr, dlqr, or lqry. The LQG regulator minimizes some quadratic cost function that trades off regulation performance and control effort. This regulator is dynamic and relies on noisy output measurements to generate the regulating commands.

In continuous time, the LQG regulator generates the commands

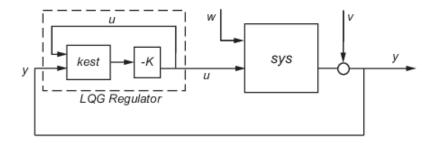
$$u = -K\widehat{x}$$

where  $\hat{x}$  is the Kalman state estimate. The regulator state-space equations are

$$\dot{\widehat{x}} = [A - LC - (B - LD)K]\widehat{x} + Ly$$

$$u = -K\widehat{x}$$

where *y* is the vector of plant output measurements (see kalman for background and notation). The following diagram shows this dynamic regulator in relation to the plant.



In discrete time, you can form the LQG regulator using either the delayed state estimate  $\widehat{x}[n|n-1]$  of x[n], based on measurements up to y[n-1], or the current state estimate  $\widehat{x}[n|n]$ , based on all available measurements including y[n]. While the regulator

$$u[n] = -K\widehat{x}[n|n-1]$$

is always well-defined, the current regulator

$$u[n] = -K\widehat{x}[n|n]$$

is causal only when I-KMD is invertible (see kalman for the notation). In addition, practical implementations of the current regulator should allow for the processing time required to compute u[n] after the measurements y[n] become available (this amounts to a time delay in the feedback loop).

For a discrete-time plant with equations:

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
  
$$y[n] = Cx[n] + Du[n] + Hw[n] + v[n]$$
 {Measurements}

#### See Also

dlqr|kalman|kalmd|lqr|lqrd|lqry|reg

#### Introduced before R2006a

# **Iqgtrack**

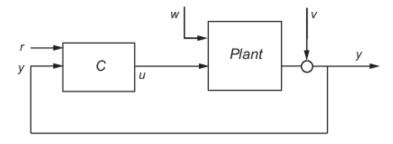
Form Linear-Quadratic-Gaussian (LQG) servo controller

### **Syntax**

```
C = lqgtrack(kest,k)
C = lqgtrack(kest,k,'2dof')
C = lqgtrack(kest,k,'1dof')
C = lqgtrack(kest,k,...CONTROLS)
```

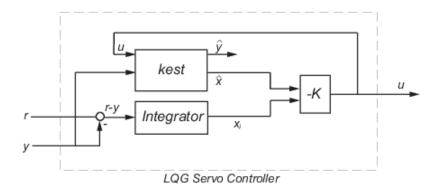
# **Description**

lqgtrack forms a Linear-Quadratic-Gaussian (LQG) servo controller with integral action for the loop shown in the following figure. This compensator ensures that the output y tracks the reference command r and rejects process disturbances w and measurement noise v. lqgtrack assumes that r and y have the same length.



**Note** Always use positive feedback to connect the LQG servo controller C to the plant output y.

C = lqgtrack(kest,k) forms a two-degree-of-freedom LQG servo controller C by connecting the Kalman estimator kest and the state-feedback gain k, as shown in the following figure. C has inputs [r; y] and generates the command  $u = -K[\widehat{x}; x_i]$ , where  $\widehat{x}$  is the Kalman estimate of the plant state, and  $x_i$  is the integrator output.



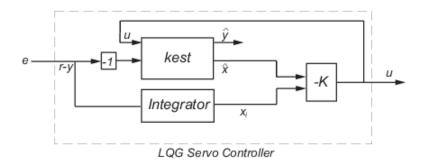
The size of the gain matrix k determines the length of  $x_i$ .  $x_i$ , y, and r all have the same length.

The two-degree-of-freedom LQG servo controller state-space equations are

$$\begin{bmatrix} \hat{x} \\ \hat{x}_i \end{bmatrix} = \begin{bmatrix} A - BK_x - LC + LDK_x & -BK_i + LDK_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ x_i \end{bmatrix} + \begin{bmatrix} 0 & L \\ I & -I \end{bmatrix} \begin{bmatrix} r \\ y \end{bmatrix}$$
$$u = \begin{bmatrix} -K_x & -K_i \end{bmatrix} \begin{bmatrix} \hat{x} \\ x_i \end{bmatrix}$$

**Note** The syntax C = lqgtrack(kest,k,'2dof') is equivalent to C = lqgtrack(kest,k).

C = lqgtrack(kest,k,'ldof') forms a one-degree-of-freedom LQG servo controller C that takes the tracking error e = r - y as input instead of [r;y], as shown in the following figure.



The one-degree-of-freedom LQG servo controller state-space equations are

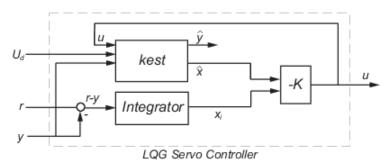
$$\begin{bmatrix} \widehat{x} \\ \widehat{x}_i \end{bmatrix} = \begin{bmatrix} A - BK_X - LC + LDK_X & -BK_i + LDK_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{x} \\ x_i \end{bmatrix} + \begin{bmatrix} -L \\ I \end{bmatrix} e$$

$$u = \begin{bmatrix} -K_X & -K_i \end{bmatrix} \begin{bmatrix} \widehat{x} \\ x_i \end{bmatrix}$$

C = lqgtrack(kest,k,...CONTROLS) forms an LQG servo controller C when the Kalman estimator kest has access to additional known (deterministic) commands  $U_d$  of the plant. In the index vector CONTROLS, specify which inputs of kest are the control channels u. The resulting compensator C has inputs

- $[U_d; r; y]$  in the two-degree-of-freedom case
- $[U_d;e]$  in the one-degree-of-freedom case

The corresponding compensator structure for the two-degree-of-freedom cases appears in the following figure.



### **Examples**

See the example "Design an LQG Servo Controller".

### **Tips**

You can use lqgtrack for both continuous- and discrete-time systems.

In discrete-time systems, integrators are based on forward Euler (see lqi for details). The state estimate  $\widehat{x}$  is either x[n|n] or x[n|n-1], depending on the type of estimator (see kalman for details).

For a discrete-time plant with equations:

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
  
$$y[n] = Cx[n] + Du[n] + Hw[n] + v[n]$$
 {Measurements}

connecting the "current" Kalman estimator to the LQR gain is optimal only when E(w[n]v[n]') = 0 and y[n] does not depend on w[n] (H = 0). If these conditions are not satisfied, compute the optimal LQG controller using lgg.

#### **See Also**

kalman|lqg|lqgreg|lqi|lqr

Introduced in R2008b

# lqi

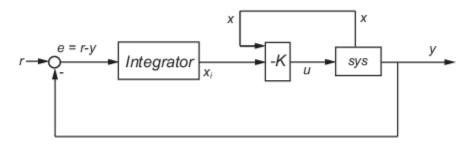
Linear-Quadratic-Integral control

# **Syntax**

$$[K,S,e] = lqi(SYS,Q,R,N)$$

# **Description**

lqi computes an optimal state-feedback control law for the tracking loop shown in the following figure.



For a plant sys with the state-space equations (or their discrete counterpart):

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

the state-feedback control is of the form

$$u = -K[x; x_i]$$

where  $x_i$  is the integrator output. This control law ensures that the output y tracks the reference command r. For MIMO systems, the number of integrators equals the dimension of the output y.

[K,S,e] = lqi(SYS,Q,R,N) calculates the optimal gain matrix K, given a state-space model SYS for the plant and weighting matrices Q, R, N. The control law  $u = -Kz = -K[x;x_i]$  minimizes the following cost functions (for r = 0)

$$J(u) = \int_0^\infty \left\{ z^T Q z + u^T R u + 2 z^T N u \right\} dt \text{ for continuous time}$$

• 
$$J(u) = \sum_{n=0}^{\infty} \left\{ z^T Q z + u^T R u + 2 z^T N u \right\}$$
 for discrete time

In discrete time, lqi computes the integrator output  $x_i$  using the forward Euler formula

$$x_i[n+1] = x_i[n] + Ts(r[n] - y[n])$$

where *Ts* is the sample time of SYS.

When you omit the matrix N, N is set to 0. lqi also returns the solution S of the associated algebraic Riccati equation and the closed-loop eigenvalues e.

#### Limitations

For the following state-space system with a plant with augmented integrator:

$$\frac{\delta z}{\delta t} = A_a z + B_a u$$
$$v = C_a z + D_a u$$

The problem data must satisfy:

- The pair  $(A_a, B_a)$  is stabilizable.
- R > 0 and  $Q NR^{-1}N^T \ge 0$ .
- $(Q NR^{-1}N^T, A_a B_aR^{-1}N^T)$  has no unobservable mode on the imaginary axis (or unit circle in discrete time).

### **Tips**

extstyle ext

$$\frac{dx}{dt} = E^{-1}Ax + E^{-1}Bu$$

#### References

[1] P. C. Young and J. C. Willems, "An approach to the linear multivariable servomechanism problem", *International Journal of Control*, Volume 15, Issue 5, May 1972, pages 961–979.

#### See Also

care | dare | lqg | lqgreg | lqgtrack | lqr

Introduced in R2008b

# lqr

Linear-Quadratic Regulator (LQR) design

# **Syntax**

```
[K,S,P] = lqr(sys,Q,R,N)

[K,S,P] = lqr(A,B,Q,R,N)
```

# **Description**

[K,S,P] = lqr(sys,Q,R,N) calculates the optimal gain matrix K, the solution S of the associated algebraic Riccati equation, and the closed-loop poles P for the continuous-time or discrete-time state-space model sys. Q and R are the weight matrices for states and inputs, respectively. The cross term matrix N is set to zero when omitted.

[K,S,P] = lqr(A,B,Q,R,N) calculates the optimal gain matrix K, the solution S of the associated algebraic Riccati equation and the closed-loop poles P using the continuous-time state-space matrices A and B.

# **Examples**

#### **LQR Control for Inverted Pendulum Model**

pendulumModelCart.mat contains the state-space model of an inverted pendulum on a cart where the outputs are the cart displacement x and the pendulum angle  $\theta$ . The control input u is the horizontal force on the cart.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0 & 5 & 30 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 5 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

First, load the state-space model sys to the workspace.

```
load('pendulumCartModel.mat','sys')
```

Since the outputs are x and  $\theta$ , and there is only one input, use Bryson's rule to determine Q and R.

```
Q = [1,0,0,0; \dots \\ 0,0,0,0; \dots \\ 0,0,1,0; \dots \\ 0,0,0,0];
R = 1;
```

Find the gain matrix K using lqr. Since N is not specified, lqr sets N to 0.

```
[K,S,P] = lqr(sys,Q,R)
K = 1 \times 4
   -1.0000
              -1.7559
                        16.9145
                                     3.2274
S = 4 \times 4
    1.5346
              1.2127
                         -3.2274
                                    -0.6851
              1.5321
                        -4.5626
    1.2127
                                    -0.9640
   -3.2274
              -4.5626
                        26.5487
                                     5.2079
              -0.9640
                         5.2079
   -0.6851
                                     1.0311
P = 4 \times 1 \ complex
  -0.8684 + 0.8523i
```

```
-0.8684 - 0.8523i
-5.4941 + 0.4564i
-5.4941 - 0.4564i
```

Although Bryson's rule usually provides satisfactory results, it is often just the starting point of a trial-and-error iterative design procedure to tune your closed-loop system response based on the design requirements.

#### **LQR Control using State-Space Matrices**

aircraftPitchModel.mat contains the state-space matrices of an aircraft where the input is the elevator deflection angle  $\delta$  and the output is the aircraft pitch angle  $\theta$ .

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} [\delta]$$
$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} [\delta]$$

For a step reference of 0.2 radians, consider the following design criteria:

- Rise time less than 2 seconds
- Settling time less than 10 seconds
- Steady-state error less than 2%

Load the model data to the workspace.

Define the state-cost weighted matrix Q and the control weighted matrix R. Generally, you can use Bryson's Rule to define your initial weighted matrices Q and R. For this example, consider the output vector C along with a scaling factor of 5 for matrix Q and choose R as 1. R is a scalar since the system has only one input.

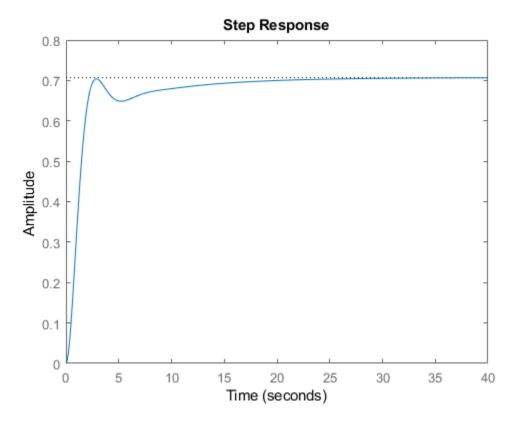
$$R = 1$$
 $R = 1$ 

Compute the gain matrix using lqr.

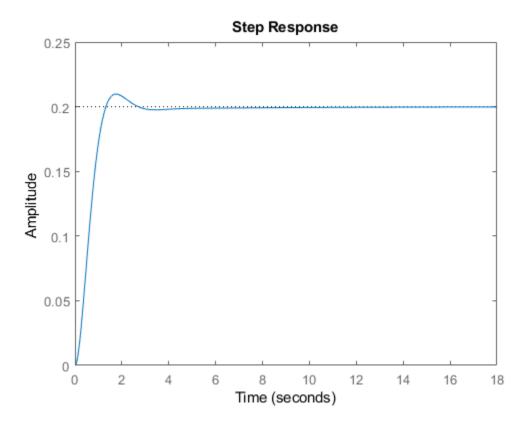
```
[K1,S1,P1] = lqr(A,B,Q1,R);
```

Check the closed-loop step response with the generated gain matrix K1.

```
sys1 = ss(A-B*K1,B,C,D);
step(sys1)
```



Since this response does not meet the design goals, increase the scaling factor to 25, compute the gain matrix K2, and check the closed-loop step response for gain matrix K2.



In the closed-loop step response plot, the rise time, settling time, and steady-state error meet the design goals.

# **Input Arguments**

#### sys — Dynamic system model

ss model object

Dynamic system model, specified as an ss model object.

#### A — State matrix

n-by-n matrix

State matrix, specified as an n-by-n matrix, where n is the number of states.

#### **B** — Input-to-state matrix

n-by-m matrix

Input-to-state matrix, specified as an n-by-m input-to-state matrix, where m is the number of inputs.

#### Q — State-cost weighted matrix

matrix

State-cost weighted matrix, specified as an n-by-n matrix, where n is the number of states. You can use Bryson's rule to set the initial values of Q given by:

$$Q_{i,i} = \frac{1}{\text{maximum acceptable value of } (error_{states})^2}, i \in \{1, 2, ..., n\}$$

$$Q = \begin{bmatrix} Q_{1,1} & 0 & \cdots & 0 \\ 0 & Q_{2,2} & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & Q_{n,n} \end{bmatrix}$$

Here, n is the number of states.

#### R — Input-cost weighted matrix

scalar | matrix

Input-cost weighted matrix, specified as a scalar or a matrix of the same size as D'D. Here, D is the feed-through state-space matrix. You can use Bryson's rule to set the initial values of R given by:

$$R_{j,j} = \frac{1}{\text{maximum acceptable value of } \left(error_{inputs}\right)^2}, j \in \{1, 2, ..., m\}$$

$$R = \begin{bmatrix} R_{1,1} & 0 & \cdots & 0 \\ 0 & R_{2,2} & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & R_{m,m} \end{bmatrix}$$

Here, m is the number of inputs.

#### N — Optional cross term matrix

0 (default) | matrix

Optional cross term matrix, specified as a matrix. If N is not specified, then lqr sets N to 0 by default.

### **Output Arguments**

#### K - Optimal gain

row vector

Optimal gain of the closed-loop system, returned as a row vector of size n, where n is the number of states.

#### S — Solution of the associated algebraic Riccati equation

matrix

Solution of the associated algebraic Riccati equation, returned as an n-by-n matrix, where n is the number of states. In other words, S is the same dimension as state-space matrix A. For more information, see icare and idare.

#### P — Poles of the closed-loop system

column vector

Poles of the closed-loop system, returned as a column vector of size n, where n is the number of states.

#### Limitations

The input data must satisfy the following conditions:

- $\bullet\ \ \,$  The pair A and B must be stabilizable.
- [Q,N;N',R] must be nonnegative definite.
- R>0 and  $Q NR^{-1}N^T \ge 0$ .
- $(Q NR^{-1}N^T, A BR^{-1}N^T)$  has no unobservable mode on the imaginary axis (or unit circle in discrete time).

### **Tips**

• lqr supports descriptor models with nonsingular E. The output S of lqr is the solution of the algebraic Riccati equation for the equivalent explicit state-space model:

$$\frac{dx}{dt} = E^{-1}Ax + E^{-1}Bu$$

### **Algorithms**

For continuous-time systems, lqr computes the state-feedback control u = -Kx that minimizes the quadratic cost function

$$J(u) = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u) dt$$

subject to the system dynamics  $\dot{x} = Ax + Bu$ .

In addition to the state-feedback gain K, lqr returns the solution S of the associated algebraic Riccati equation

$$A^{T}XE + E^{T}XA + E^{T}XGXE - (E^{T}XB + S)R^{-1}(B^{T}XE + S^{T}) + Q = 0$$

and the closed-loop poles P = eig(A - BK). The gain matrix K is derived from S using

$$K = R^{-1}(B^TS + N^T).$$

For discrete-time systems, lqr computes the state-feedback control  $u_n = -Kx_n$  that minimizes

$$J = \sum_{n=0}^{\infty} \left\{ x^T Q x + u^T R u + 2 x^T N u \right\}$$

subject to the system dynamics  $x_{n+1} = Ax_n + Bu_n$ .

In all cases, when you omit the cross term matrix N, lqr sets N to 0.

#### See Also

dlqr|icare|idare|lqg|lqgreg|lqgtrack|lqi|lqrd|lqry

#### **Topics**

"DC Motor Control"

"Regulate Pressure in Drum Boiler"

#### **Introduced before R2006a**

# Iqrd

Design discrete linear-quadratic (LQ) regulator for continuous plant

### **Syntax**

```
lqrd
[Kd,S,e] = lqrd(A,B,Q,R,Ts)
[Kd,S,e] = lqrd(A,B,Q,R,N,Ts)
```

# **Description**

lqrd designs a discrete full-state-feedback regulator that has response characteristics similar to a continuous state-feedback regulator designed using lqr. This command is useful to design a gain matrix for digital implementation after a satisfactory continuous state-feedback gain has been designed.

[Kd,S,e] = lqrd(A,B,Q,R,Ts) calculates the discrete state-feedback law 
$$u[n] = -K_dx[n]$$

that minimizes a discrete cost function equivalent to the continuous cost function

$$J = \int_0^\infty (x^T Q x + u^T R u) dt$$

The matrices A and B specify the continuous plant dynamics

$$\dot{x} = Ax + Bu$$

and Ts specifies the sample time of the discrete regulator. Also returned are the solution S of the discrete Riccati equation for the discretized problem and the discrete closed-loop eigenvalues e = eig(Ad-Bd\*Kd).

[Kd,S,e] = lqrd(A,B,Q,R,N,Ts) solves the more general problem with a cross-coupling term in the cost function.

$$J = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u) dt$$

#### Limitations

The discretized problem data should meet the requirements for dlqr.

# **Algorithms**

The equivalent discrete gain matrix Kd is determined by discretizing the continuous plant and weighting matrices using the sample time Ts and the zero-order hold approximation.

With the notation

$$\begin{split} \Phi(\tau) &= e^{A\tau}, \qquad A_d = \Phi(T_s) \\ \Gamma(\tau) &= \int_0^\tau e^{A\eta} B d\eta, \ B_d = \Gamma(T_s) \end{split}$$

the discretized plant has equations

$$x[n+1] = A_d x[n] + B_d u[n]$$

and the weighting matrices for the equivalent discrete cost function are

$$\begin{bmatrix} Q_d & N_d \\ N_d^T & R_d \end{bmatrix} = \int \!\!\!\!\int^{T_S} \!\! \begin{bmatrix} \Phi^T(\tau) & 0 \\ \Gamma^T(\tau) & I \end{bmatrix} \!\!\! \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \!\!\! \begin{bmatrix} \Phi(\tau) & \Gamma(\tau) \\ 0 & I \end{bmatrix} \!\!\! d\tau$$

The integrals are computed using matrix exponential formulas due to Van Loan (see [2]). The plant is discretized using c2d and the gain matrix is computed from the discretized data using dlqr.

#### References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1980, pp. 439-440.

[2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-23, June 1978.

#### See Also

c2d | dlqr | kalmd | lqr

**Introduced before R2006a** 

# Iqry

Form linear-quadratic (LQ) state-feedback regulator with output weighting

### **Syntax**

$$[K,S,e] = lgry(sys,Q,R,N)$$

# **Description**

Given the plant

$$\dot{x} = Ax + Bu$$

$$v = Cx + Du$$

$$u = -Kx$$

that minimizes the quadratic cost function with output weighting

or its discrete-time counterpart, lqry designs a state-feedback control

$$J(u) = \int_0^\infty (y^T Q y + u^T R u + 2 y^T N u) dt$$

(or its discrete-time counterpart). The function lqry is equivalent to lqr or dlqr with weighting matrices:

$$\begin{bmatrix} \bar{Q} & \bar{N} \\ \bar{N}^T & \bar{R} \end{bmatrix} = \begin{bmatrix} C^T & 0 \\ D^T & I \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}$$

[K,S,e] = lqry(sys,Q,R,N) returns the optimal gain matrix K, the Riccati solution S, and the closed-loop eigenvalues e = eig(A-B\*K). The state-space model sys specifies the continuous- or discrete-time plant data (A,B,C,D). The default value N=0 is assumed when N is omitted.

# **Examples**

See "LQG Design for the x-Axis" for an example.

# **Limitations**

The data  $A,B,\overline{Q},\overline{R},\overline{N}$  must satisfy the requirements for lqr or dlqr.

#### **See Also**

dlqr|kalman|lqgreg|lqr

**Introduced before R2006a** 

#### Isim

Simulate time response of dynamic system to arbitrary inputs

### **Syntax**

```
lsim(sys,u,t)
lsim(sys,u,t,x0)
lsim(sys,u,t,x0,method)
lsim(sys1,...,sysn,u,t)
lsim(sys1,LineSpec1,...,sysN,LineSpecN,u,t)
y = lsim(____)
[y,t,x] = lsim(____)
lsim(sys)
```

# **Description**

lsim simulates the (time) response of continuous or discrete linear systems to arbitrary inputs. When invoked without left-hand arguments, lsim plots the response on the screen.

lsim(sys,u,t) produces a plot of the time response of the dynamic system model sys
to the input history, t,u. The vector t specifies the time samples for the simulation (in
system time units, specified in the TimeUnit property of sys), and consists of regularly
spaced time samples:

```
t = 0:dt:Tfinal
```

The input u is an array having as many rows as time samples (length(t)) and as many columns as system inputs. For instance, if sys is a SISO system, then u is a t-by-1 vector. If sys has three inputs, then u is a t-by-3 array. Each row u(i,:) specifies the input value(s) at the time sample t(i). The signal u also appears on the plot.

The model sys can be continuous or discrete, SISO or MIMO. In discrete time, u must be sampled at the same rate as the system. In this case, the input t is redundant and can be omitted or set to an empty matrix. In continuous time, the time sampling dt = t(2) - t(1) is used to discretize the continuous model. If dt is too large (undersampling), lsim

issues a warning suggesting that you use a more appropriate sample time, but will use the specified sample time. See "Algorithms" on page 2-659 for a discussion of sample times.

lsim(sys,u,t,x0) further specifies an initial condition x0 for the system states. This syntax applies only when sys is a state-space model. x0 is a vector whose entries are the initial values of the corresponding states of sys.

lsim(sys,u,t,x0,method) explicitly specifies how the input values should be interpolated between samples, when sys is a continuous-time system. Specify method as one of the following values:

- 'zoh' Use zero-order hold
- 'foh' Use linear interpolation (first-order hold)

If you do not specify a method, lsim selects the interpolation method automatically based on the smoothness of the signal u.

lsim(sys1,...,sysn,u,t) simulates the responses of several dynamic system models
to the same input history t,u and plots these responses on a single figure. You can also
use the x0 and method input arguments when computing the responses of multiple
models.

lsim(sys1,LineSpec1,...,sysN,LineSpecN,u,t) specifies the line style, marker, and color of each of the system responses in the plot. You can also use the x0 and method input arguments with this syntax. Each LineSpec argument is specified as a vector of one, two, or three characters. The characters can appear in any order. For example, the following code plots the response of sys1 as a yellow dotted line and the response of sys2 as a green dashed line:

For more information about configuring this argument, see the LineSpec input argument of the plot function.

y = lsim(\_\_\_\_) returns the system response y, sampled at the same times as the input (t). The output y is an array having as many rows as time samples (length(t)) and as many columns as system outputs. No plot is drawn on the screen. You can use this syntax with any of the input arguments described in previous syntaxes except the LineSpec arguments.

 $[y,t,x] = lsim(\underline{\underline{\underline{\underline{}}}})$  also returns the time vector t used for simulation and the state trajectories x (for state-space models only). The output x has as many rows as time

samples (length(t)) and as many columns as system states. You can use this syntax with any of the input arguments described in previous syntaxes except the LineSpec arguments.

lsim(sys) opens the Linear Simulation Tool GUI. For more information about working with this GUI, see Working with the Linear Simulation Tool.

### **Examples**

#### **Simulate Response to Square Wave**

Simulate and plot the response of the following system to a square wave with period of four seconds:

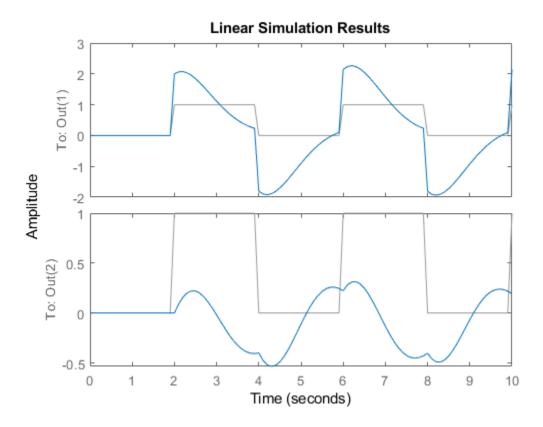
$$H(s) = \begin{bmatrix} \frac{2s^2 + 5s + 1}{s^2 + 2s + 3} \\ \frac{s - 1}{s^2 + s + 5} \end{bmatrix}.$$

Create the transfer function, and generate the square wave with gensig. Sample every 0.1 second during 10 seconds.

```
H = [tf([2 5 1],[1 2 3]);tf([1 -1],[1 1 5])];
[u,t] = gensig('square',4,10,0.1);
```

Then simulate with lsim.

lsim(H,u,t)



The plot displays both the applied signal and the response.

#### **Simulate Response of Identified Model**

Load estimation data to estimate a model.

```
load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
z = iddata(y,u,0.1, 'Name', 'DC-motor');
```

z is an iddata object that stores the 1-input 2-output estimation data with sample time 0.1 seconds.

Estimate a state-space model of order 4 using estimation data z.

```
[sys,x0] = n4sid(z,4);
```

sys is the estimated model and x0 are the estimated initial states.

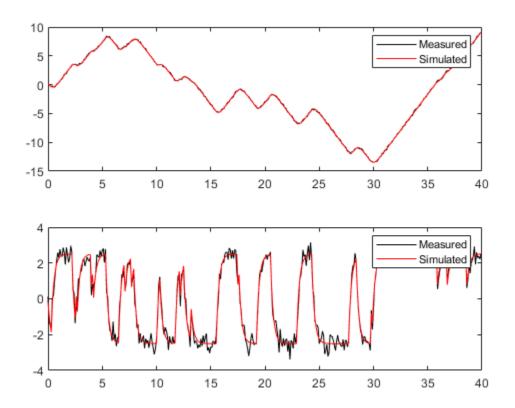
Simulate the response of sys using the same input data as the one used for estimation and the initial states returned by the estimation command.

```
[y,t,x] = lsim(sys,z.InputData,[],x0);
```

Here, y is the system response, t is the time vector used for simulation, and x is the state trajectory.

Compare the simulated response y to measured response z .  $\mathtt{OutputData}$  for both outputs.

```
subplot(211), plot(t,z.OutputData(:,1),'k',t,y(:,1),'r')
legend('Measured','Simulated')
subplot(212), plot(t,z.OutputData(:,2),'k',t,y(:,2),'r')
legend('Measured','Simulated')
```



# **Algorithms**

Discrete-time systems are simulated with ltitr (state space) or filter (transfer function and zero-pole-gain).

Continuous-time systems are discretized with c2d using either the 'zoh' or 'foh' method ('foh' is used for smooth input signals and 'zoh' for discontinuous signals such as pulses or square waves). The sample time is set to the spacing dt between the user-supplied time samples t.

The choice of sample time can drastically affect simulation results. To illustrate why, consider the second-order model

$$H(s) = \frac{\omega^2}{s^2 + 2s + \omega^2}, \ \omega = 62.83$$

To simulate its response to a square wave with period 1 second, you can proceed as follows:

```
w2 = 62.83^2;

h = tf(w2,[1 2 w2]);

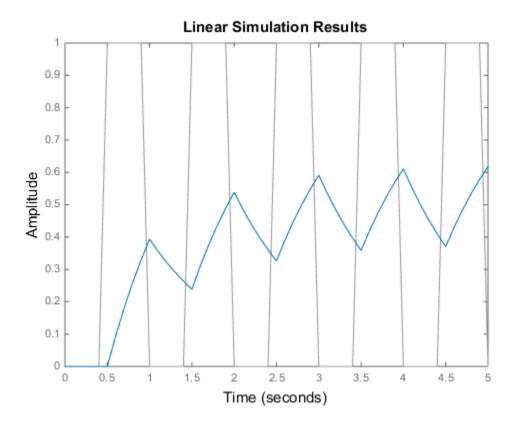
t = 0:0.1:5; % vector of time samples

u = (rem(t,1) >= 0.5); % square wave values

lsim(h,u,t)
```

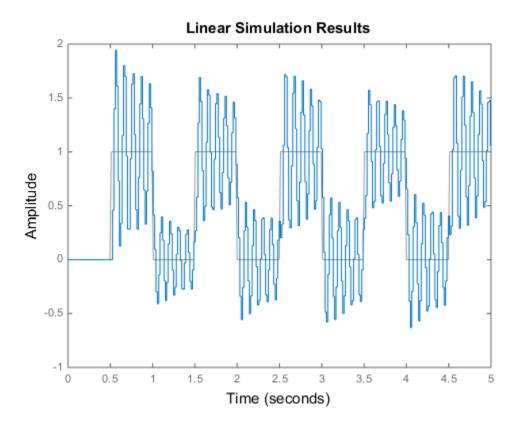
lsim evaluates the specified sample time, and issues a warning:

Warning: Input signal is undersampled. Sample every  $0.016\ \text{sec}$  or faster.



To improve on this response, discretize H(s) using the recommended sample time:

```
dt = 0.016;
ts = 0:dt:5;
us = (rem(ts,1) >= 0.5);
hd = c2d(h,dt);
lsim(hd,us,ts)
```



This response exhibits strong oscillatory behavior that is hidden in the undersampled version.

#### **See Also**

Linear System Analyzer | gensig | impulse | initial | lsiminfo | step

**Introduced before R2006a** 

### Isiminfo

Compute linear response characteristics

## **Syntax**

```
S = lsiminfo(y,t,yfinal)
S = lsiminfo(y,t)
S = lsiminfo(...,'SettlingTimeThreshold',ST)
```

## **Description**

S = lsiminfo(y,t,yfinal) takes the response data (t,y) and a steady-state value yfinal and returns a structure S containing the following performance indicators:

- SettlingTime Settling time
- Min Minimum value of Y
- MinTime Time at which the minimum value is reached
- Max Maximum value of Y
- MaxTime Time at which the maximum value is reached

For SISO responses, t and y are vectors with the same length NS. For responses with NY outputs, you can specify y as an NS-by-NY array and yfinal as a NY-by-1 array. lsiminfo then returns an NY-by-1 structure array S of performance metrics for each output channel.

S = lsiminfo(y,t) uses the last sample value of y as steady-state value yfinal. S = lsiminfo(y) assumes t = 1:NS.

S = lsiminfo(..., 'SettlingTimeThreshold', ST) lets you specify the threshold ST used in the settling time calculation. The response has settled when the error |y(t) - yfinal| becomes smaller than a fraction ST of its peak value. The default value is ST=0.02 (2%).

## **Examples**

#### **Compute Response Characteristics of a Transfer Function**

Create the following continuous-time transfer function:

$$H(s) = \frac{s-1}{s^3 + 2s^2 + 3s + 4}$$

sys = tf([1 -1],[1 2 3 4]);

Calculate the impulse response.

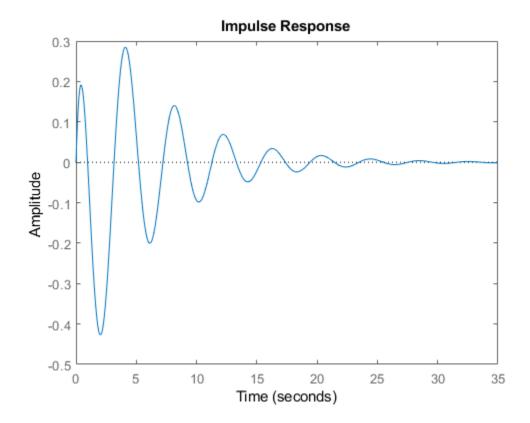
```
[y,t] = impulse(sys);
```

impulse returns the output response y and the time vector t used for simulation.

Compute the response characteristics using a final response value of 0.

You can plot the impulse response and verify these response characteristics. For example, the time at which the minimum response value (MinTime) is reached is approximately 2 seconds.

```
impulse(sys)
```



See Also

impulse|initial|lsim|stepinfo

Introduced in R2006a

# **Isimplot**

Simulate response of dynamic system to arbitrary inputs and return plot handle

## **Syntax**

```
h = lsimplot(sys)
lsimplot(sys1,sys2,...)
lsimplot(sys,u,t)
lsimplot(sys,u,t,x0)
lsimplot(sys1,sys2,...,u,t,x0)
lsimplot(AX,...)
lsimplot(..., plotoptions)
lsimplot(sys,u,t,x0,'zoh')
lsimplot(sys,u,t,x0,'foh')
```

## **Description**

h = lsimplot(sys) opens the Linear Simulation Tool for the dynamic system model sys, which enables interactive specification of driving input(s), the time vector, and initial state. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

```
help timeoptions
```

for a list of available plot options.

lsimplot(sys1,sys2,...) opens the Linear Simulation Tool for multiple models sys1,sys2,.... Driving inputs are common to all specified systems but initial conditions can be specified separately for each.

lsimplot(sys,u,t) plots the time response of the model sys to the input signal described by u and t. The time vector t consists of regularly spaced time samples (in system time units, specified in the TimeUnit property of sys). For MIMO systems, u is a matrix with as many columns as inputs and whose ith row specifies the input value at time t(i). For SISO systems u can be specified either as a row or column vector. For example,

```
t = 0:0.01:5;
u = sin(t);
lsimplot(sys,u,t)
```

simulates the response of a single-input model sys to the input u(t)=sin(t) during 5 seconds.

For discrete-time models, u should be sampled at the same rate as sys (t is then redundant and can be omitted or set to the empty matrix).

For continuous-time models, choose the sampling period t(2)-t(1) small enough to accurately describe the input u. lsim issues a warning when u is undersampled, and hidden oscillations can occur.

lsimplot(sys,u,t,x0) specifies the initial state vector x0 at time t(1) (for state-space models only). x0 is set to zero when omitted.

lsimplot(sys1,sys2,...,u,t,x0) simulates the responses of multiple LTI models sys1,sys2,... on a single plot. The initial condition x0 is optional. You can also specify a color, line style, and marker for each system, as in

```
lsimplot(sys1,'r',sys2,'y--',sys3,'gx',u,t)
```

lsimplot(AX,...) plots into the axes with handle AX.

lsimplot(..., plotoptions) plots the initial condition response with the options
specified in plotoptions. Type

help timeoptions

for more detail.

For continuous-time models, lsimplot(sys,u,t,x0,'zoh') or lsimplot(sys,u,t,x0,'foh') explicitly specifies how the input values should be interpolated between samples (zero-order hold or linear interpolation). By default, lsimplot selects the interpolation method automatically based on the smoothness of the signal u.

### See Also

getoptions | lsim | setoptions

### **Introduced before R2006a**

# looptune

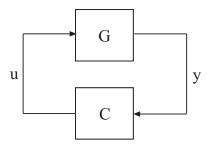
Tune fixed-structure feedback loops

## **Syntax**

```
[G,C,gam] = looptune(G0,C0,wc)
[G,C,gam] = looptune(G0,C0,wc,Req1,...,ReqN)
[G,C,gam] = looptune(...,options)
[G,C,gam,info] = looptune(...)
```

## **Description**

[G,C,gam] = looptune(G0,C0,wc) tunes the feedback loop



to meet the following default requirements:

- Bandwidth Gain crossover for each loop falls in the frequency interval wc
- Performance Integral action at frequencies below wc
- Robustness Adequate stability margins and gain roll-off at frequencies above wc

The tunable genss model C0 specifies the controller structure, parameters, and initial values. The model G0 specifies the plant. G0 can be a Numeric LTI model, or, for co-tuning the plant and controller, a tunable genss model. The sensor signals y (measurements) and actuator signals u (controls) define the boundary between plant and controller.

**Note** For tuning Simulink models with looptune, use slTuner to create an interface to your Simulink model. You can then tune the control system with looptune for slTuner (requires Simulink Control Design).

[G,C,gam] = looptune(G0,C0,wc,Req1,...,ReqN) tunes the feedback loop to meet additional design requirements specified in one or more tuning goal objects Req1,...,ReqN. Omit wc to use the requirements specified in Req1,...,ReqN instead of an explicit target crossover frequency and the default performance and robustness requirements.

[G,C,gam] = looptune(...,options) specifies further options, including target gain margin, target phase margin, and computational options for the tuning algorithm.

[G,C,gam,info] = looptune(...) returns a structure info with additional information about the tuned result. Use info with the loopview command to visualize tuning constraints and validate the tuned design.

## **Input Arguments**

#### G0

Numeric LTI model or tunable genss model representing plant in control system to tune.

The plant is the portion of your control system whose outputs are sensor signals (measurements) and whose inputs are actuator signals (controls). Use connect to build G0 from individual numeric or tunable components.

#### C0

Generalized LTI model representing controller.  ${\tt C0}$  specifies the controller structure, parameters, and initial values.

The controller is the portion of your control system that receives sensor signals (measurements) as inputs and produces actuator signals (controls) as outputs. Use Control Design Blocks and Generalized LTI models to represent tunable components of the controller. Use connect to build C0 from individual numeric or tunable components.

#### WC

Vector specifying target crossover region [wcmin,wcmax]. The looptune command attempts to tune all loops in the control system so that the open-loop gain crosses 0 dB within the target crossover region.

A scalar wc specifies the target crossover region [wc/2,2\*wc].

#### Req1,...,ReqN

One or more TuningGoal objects specifying design requirements, such as TuningGoal.Tracking, TuningGoal.Gain, or TuningGoal.LoopShape.

#### options

Set of options for looptune algorithm, specified using looptuneOptions. See looptuneOptions for information about the available options, including target gain margin and phase margin.

## **Output Arguments**

#### G

Tuned plant.

If GO is a Numeric LTI model, G is the same as GO.

If GO is a tunable genss model, G is a genss model with Control Design Blocks of the same number and types as GO. The current value of G is the tuned plant.

#### C

Tuned controller. C is a genss model with Control Design Blocks of the same number and types as CO. The current value of C is the tuned controller.

#### gam

Parameter indicating degree of success at meeting all tuning constraints. A value of gam <= 1 indicates that all requirements are satisfied. gam >> 1 indicates failure to meet at least one requirement. Use loopview to visualize the tuned result and identify the unsatisfied requirement.

For best results, use the RandomStart option in looptuneOptions to obtain several minimization runs. Setting RandomStart to an integer N > 0 causes looptune to run the optimization N additional times, beginning from parameter values it chooses randomly. You can examine gam for each run to help identify an optimization result that meets your design requirements.

#### info

Data for validating tuning results, returned as a structure. To use the data in info, use the command loopview(G,C,info) to visualize tuning constraints and validate the tuned design.

info contains the following tuning data:

#### Di, Do

Optimal input and output scalings, returned as state-space models. The scaled plant is given by Do\G\*Di.

#### **Specs**

Design requirements that looptune constructs for its call to systune for tuning (see "Algorithms" on page 2-674), returned as a vector of TuningGoal requirement objects.

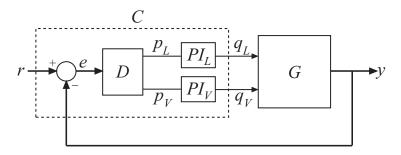
#### Runs

Detailed information about each optimization run performed by systune when called by looptune for tuning (see "Algorithms" on page 2-674), returned as a data structure.

The contents of Runs are the info output of the call to systume. For information about the fields of Runs, see the info output argument description on the systume reference page.

## **Examples**

Tune the control system of the following illustration, to achieve crossover between 0.1 and 1 rad/min.



The 2-by-2 plant **G** is represented by:

$$G(s) = \frac{1}{75s+1} \begin{bmatrix} 87.8 & -86.4\\ 108.2 & -109.6 \end{bmatrix}.$$

The fixed-structure controller, C, includes three components: the 2-by-2 decoupling matrix D and two PI controllers PI\_L and PI\_V. The signals r, y, and e are vector-valued signals of dimension 2.

Build a numeric model that represents the plant and a tunable model that represents the controller. Name all inputs and outputs as in the diagram, so that looptune knows how to interconnect the plant and controller via the control and measurement signals.

```
s = tf('s');
G = 1/(75*s+1)*[87.8 -86.4; 108.2 -109.6];
G.InputName = {'qL','qV'};
G.OutputName = 'y';
D = tunableGain('Decoupler',eye(2));
D.InputName = 'e';
D.OutputName = {'pL', 'pV'};
PI L = tunablePID('PI_L', 'pi');
PI L.InputName = 'pL';
PI_L.OutputName = 'qL';
PI V = tunablePID('PI V','pi');
PI V.InputName = 'pV';
PI V.OutputName = 'qV';
sum1 = sumblk('e = r - y', 2);
C0 = connect(PI L,PI V,D,sum1,{'r','y'},{'qL','qV'});
wc = [0.1,1];
[G,C,gam,info] = looptune(G,C0,wc);
```

C is the tuned controller, in this case a genss model with the same block types as CO.

You can examine the tuned result using loopview.

## **Algorithms**

looptune automatically converts target bandwidth, performance requirements, and additional design requirements into weighting functions that express the requirements as an  $H_{\infty}$  optimization problem. looptune then uses systune to optimize tunable parameters to minimize the  $H_{\infty}$  norm. For more information about the optimization algorithms, see [1].

looptune computes the  $H_{\infty}$  norm using the algorithm of [2] and structure-preserving eigensolvers from the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

### **Alternatives**

For tuning Simulink models with looptune, see slTuner and looptune (requires Simulink Control Design).

# **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### References

- [1] P. Apkarian and D. Noll, "Nonsmooth H-infinity Synthesis." *IEEE Transactions on Automatic Control*, Vol. 51, Number 1, 2006, pp. 71–86.
- [2] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the  $H_{\infty}$ -Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287-293.

# **Extended Capabilities**

## **Automatic Parallel Support**

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox  $^{\text{\tiny TM}}$ .

To run in parallel, set 'UseParallel' to true using looptuneOptions.

For more information, see "Speed Up Tuning with Parallel Computing Toolbox Software".

### See Also

TuningGoal.Gain | TuningGoal.LoopShape | TuningGoal.Tracking | connect | diskmargin | genss | hinfstruct | looptune (for slTuner) | looptuneOptions | loopview | slTuner | systume

### **Topics**

"Tune MIMO Control System for Specified Bandwidth"

"Tuning Feedback Loops with LOOPTUNE"

#### Introduced in R2016a

<sup>&</sup>quot;Decoupling Controller for a Distillation Column"

# **looptuneOptions**

Set options for looptune

## **Syntax**

```
options = looptuneOptions
options = looptuneOptions(Name, Value)
```

## **Description**

options = looptuneOptions returns the default option set for the looptune command.

options = looptuneOptions(Name, Value) creates an option set with the options
specified by one or more Name, Value pair arguments.

## **Input Arguments**

### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

looptuneOptions takes the following Name arguments:

### **GainMargin**

Target gain margin in decibels. GainMargin specifies the required gain margin for the tuned control system. For MIMO control systems, the gain margin is the multiloop disk margin. See "Stability Analysis Using Disk Margins" (Robust Control Toolbox) for the definition of the multiloop disk margin.

#### **Default:** 7.6 dB

#### PhaseMargin

Target phase margin in degrees. PhaseMargin specifies the required phase margin for the tuned control system. For MIMO control systems, the phase margin is the multiloop disk margin. See "Stability Analysis Using Disk Margins" (Robust Control Toolbox) for the definition of the multiloop disk margin.

**Default:** 45 degrees

#### **Display**

Amount of information to display during looptune runs, specified as one of the following values.

- 'off' Run in silent mode, displaying no information during or after the run.
- 'iter' Display optimization progress after each iteration. The display includes the value of the objective parameter gam after each iteration. The display also includes a Progress value, indicating the percent change in gam from the previous iteration.
- 'final' Display a one-line summary at the end of each optimization run. The display includes the minimized value of gam and the number of iterations for each run.

Default: 'final'

#### MaxIter

Maximum number of iterations in each optimization run.

Default: 300

#### RandomStart

Number of additional optimizations starting from random values of the free parameters in the controller.

If RandomStart = 0, looptune performs a single optimization run starting from the initial values of the tunable parameters. Setting RandomStart = N > 0 runs N additional optimizations starting from N randomly generated parameter values.

looptune tunes by finding a local minimum of a gain minimization problem. To increase the likelihood of finding parameter values that meet your design requirements, set

RandomStart > 0. You can then use the best design that results from the multiple optimization runs.

Use with UseParallel = true to distribute independent optimization runs among MATLAB workers (requires Parallel Computing Toolbox<sup>TM</sup> software).

#### Default: 0

#### UseParallel

Parallel processing flag.

Set to true to enable parallel processing by distributing randomized starts among workers in a parallel pool. If there is an available parallel pool, then the software performs independent optimization runs concurrently among workers in that pool. If no parallel pool is available, one of the following occurs:

- If **Automatically create a parallel pool** is selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), then the software starts a parallel pool using the settings in those preferences.
- If **Automatically create a parallel pool** is not selected in your preferences, then the software performs the optimization runs successively, without parallel processing.

If **Automatically create a parallel pool** is not selected in your preferences, you can manually start a parallel pool using parpool before running the tuning command.

Using parallel processing requires Parallel Computing Toolbox software.

#### Default: false

### **TargetGain**

Target value for the objective parameter gam.

The looptune command converts your design requirements into normalized gain constraints. The command then tunes the free parameters of the control system to drive the objective parameter gam below 1 to enforce all requirements.

The default TargetGain = 1 ensures that the optimization stops as soon as gam falls below 1. Set TargetGain to a smaller or larger value to continue the optimization or start sooner, respectively.

#### **Default:** 1

#### TolGain

Relative tolerance for termination.

The optimization terminates when the objective parameter gam decreases by less than TolGain over 10 consecutive iterations. Increasing TolGain speeds up termination, and decreasing TolGain yields tighter final values.

**Default:** 0.001

#### MaxFrequency

Maximum closed-loop natural frequency.

Setting MaxFrequency constrains the closed-loop poles to satisfy |p| < MaxFrequency.

To allow looptune to choose the closed-loop poles automatically, based upon the system's open-loop dynamics, set MaxFrequency = Inf. To prevent unwanted fast dynamics or high-gain control, set MaxFrequency to a finite value.

Specify MaxFrequency in units of 1/TimeUnit, relative to the TimeUnit property of the system you are tuning.

Default: Inf

#### **MinDecay**

Minimum decay rate for closed-loop poles

Constrains the closed-loop poles to satisfy Re(p) < -MinDecay. Increase this value to improve the stability of closed-loop poles that do not affect the closed-loop gain due to pole/zero cancellations.

Specify MinDecay in units of 1/TimeUnit, relative to the TimeUnit property of the system you are tuning.

Default: 1e-7

## **Output Arguments**

#### options

Option set containing the specified options for the looptune command.

## **Examples**

#### **Create Options Set for looptune**

Create an options set for a looptune run using three random restarts. Also, set the target gain and phase margins to 6 dB and 50 degrees, respectively, and limit the closed-loop pole magnitude to 100.

Alternatively, use dot notation to set the values of options.

```
options = looptuneOptions;
options.RandomStart = 3;
options.GainMargin = 6;
options.PhaseMargin = 50;
options.SpecRadius = 100;
```

### **Configure Option Set for Parallel Optimization Runs**

Configure an option set for a looptune run using 20 random restarts. Execute these independent optimization runs concurrently on multiple workers in a parallel pool.

If you have the Parallel Computing Toolbox software installed, you can use parallel computing to speed up looptune tuning of fixed-structure control systems. When you run multiple randomized looptune optimization starts, parallel computing speeds up tuning by distributing the optimization runs among workers.

If **Automatically create a parallel pool** is not selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), manually start a parallel pool using parpool. For example:

```
parpool;
```

If **Automatically create a parallel pool** is selected in your preferences, you do not need to manually start a pool.

Create a looptuneOptions set that specifies 20 random restarts to run in parallel.

```
options = looptuneOptions('RandomStart', 20, 'UseParallel', true);
```

Setting UseParallel to true enables parallel processing by distributing the randomized starts among available workers in the parallel pool.

Use the looptuneOptions set when you call looptune. For example, suppose you have already created a plant model GO and tunable controller CO. In this case, the following command uses parallel computing to tune the control system of GO and CO to the target crossoverwc.

```
[G,C,gamma] = looptune(G0,C0,wc,options);
```

## **Compatibility Considerations**

### **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### See Also

diskmargin|looptune|looptune (for slTuner)

### **Topics**

"Stability Analysis Using Disk Margins" (Robust Control Toolbox)

#### Introduced in R2016a

# **looptuneSetup**

Convert tuning setup for looptune to tuning setup for systune

## **Syntax**

[T0,SoftReqs,HardReqs,sysopt] = looptuneSetup(looptuneInputs)

# **Description**

[T0, SoftReqs, HardReqs, sysopt] = looptuneSetup(looptuneInputs) converts a tuning setup for looptune into an equivalent tuning setup for systume. The argument looptuneInputs is a sequence of input arguments for looptune that specifies the tuning setup. For example,

[T0,SoftRegs,HardRegs,sysopt] = looptuneSetup(G0,C0,wc,Reg1,Reg2,loopopt)

generates a set of arguments such that looptune(G0,C0,wc,Req1,Req2,loopopt) and systume(T0,SoftReqs,HardReqs,sysopt) produce the same results.

Use this command to take advantage of additional flexibility that systune offers relative to looptune. For example, looptune requires that you tune all channels of a MIMO feedback loop to the same target bandwidth. Converting to systune allows you to specify different crossover frequencies and loop shapes for each loop in your control system. Also, looptune treats all tuning requirements as soft requirements, optimizing them but not requiring that any constraint be exactly met. Converting to systune allows you to enforce some tuning requirements as hard constraints, while treating others as soft requirements.

You can also use this command to probe into the tuning requirements used by looptune.

**Note** When tuning Simulink models through an slTuner interface, use looptuneSetup for slTuner (requires Simulink Control Design).

## **Examples**

#### **Convert looptune Problem into systune Problem**

Convert a set of looptune inputs into an equivalent set of inputs for systume.

Suppose you have a numeric plant model, GO, and a tunable controller model, CO. Suppose also that you used looptune to tune the feedback loop between GO and CO to within a bandwidth of WC = [WMin, WMax]. Convert these variables into a form that allows you to use systume for further tuning.

```
[T0,SoftReqs,HardReqs,sysopt] = looptuneSetup(C0,G0,wc);
```

The command returns the closed-loop system and tuning requirements for the equivalent systume command, systume(CL0,SoftReqs,HardReqs,sysopt). The arrays SoftReqs and HardReqs contain the tuning requirements implicitly imposed by looptune. These requirements enforce the target bandwidth and default stability margins of looptune.

If you used additional tuning requirements when tuning the system with looptune, add them to the input list of looptuneSetup. For example, suppose you used a TuningGoal.Tracking requirement, Req1, and a TuningGoal.Rejection requirement, Req2. Suppose also that you set algorithm options for looptune using looptuneOptions. Incorporate these requirements and options into the equivalent systume command.

```
[T0,SoftReqs,HardReqs,sysopt] = looptuneSetup(C0,G0,wc,Req1,Req2,loopopt);
```

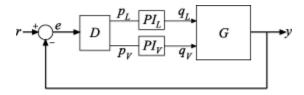
The resulting arguments allow you to construct an equivalent tuning problem for systume. In particular,  $[\sim,C] = looptune(C0,G0,wc,Req1,Req2,loopopt)$  yields the same result as the following commands.

```
T = systune(T0,SoftReqs,HardReqs,sysopt);
C = setBlockValue(C0,T);
```

### **Convert Distillation Column Problem for Tuning With systune**

Set up the following control system for tuning with looptune. Then convert the setup to a systune problem and examine the results. These results reflect the structure of the

control system model that looptune tunes. The results also reflect the tuning requirements implicitly enforced when tuning with looptune.



For this example, the 2-by-2 plant G is represented by:

$$G(s) = \frac{1}{75s+1} \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}.$$

The fixed-structure controller, C, includes three components: the 2-by-2 decoupling matrix D and two PI controllers PI\_L and PI\_V. The signals r, y, and e are vector-valued signals of dimension 2.

Build a numeric model that represents the plant and a tunable model that represents the controller. Name all inputs and outputs as in the diagram, so that looptune and looptuneSetup know how to interconnect the plant and controller via the control and measurement signals.

```
s = tf('s');
G = 1/(75*s+1)*[87.8 -86.4; 108.2 -109.6];
G.InputName = {'qL', 'qV'};
G.OutputName = {'y'};

D = tunableGain('Decoupler',eye(2));
D.InputName = 'e';
D.OutputName = {'pL', 'pV'};
PI_L = tunablePID('PI_L', 'pi');
PI_L.InputName = 'pL';
PI_L.OutputName = 'qL';
PI_V = tunablePID('PI_V', 'pi');
PI_V.InputName = 'pV';
PI_V.InputName = 'pV';
Sum1 = sumblk('e = r - y',2);
C0 = connect(PI_L,PI_V,D,sum1,{'r','y'},{'qL','qV'});
```

This system is now ready for tuning with looptune, using tuning goals that you specify. For example, specify a target bandwidth range. Create a tuning requirement that imposes

reference tracking in both channels of the system with a response time of 15 s, and a disturbance rejection requirement.

```
wc = [0.1,0.5];
TR = TuningGoal.Tracking('r','y',15,0.001,1);
DR = TuningGoal.Rejection({'qL','qV'},1/s);
DR.Focus = [0 0.1];

[G,C,gam,info] = looptune(G,C0,wc,TR,DR);
Final: Peak gain = 1, Iterations = 42
Achieved target gain value TargetGain=1.
```

looptune successfully tunes the system to these requirements. However, you might want to switch to systune to take advantage of additional flexibility in configuring your problem. For example, instead of tuning both channels to a loop bandwidth inside wc, you might want to specify different crossover frequencies for each loop. Or, you might want to enforce the tuning requirements TR and DR as hard constraints, and add other requirements as soft requirements.

Convert the looptune input arguments to a set of input arguments for systume.

```
[T0,SoftReqs,HardReqs,sysopt] = looptuneSetup(G,C0,wc,TR,DR);
```

This command returns a set of arguments you can provide to systune for equivalent results to tuning with looptune. In other words, the following command is equivalent to the previous looptune command.

```
[T,fsoft,ghard,info] = systume(T0,SoftReqs,HardReqs,sysopt);
Final: Peak gain = 1, Iterations = 42
Achieved target gain value TargetGain=1.
```

PI V: Parametric PID controller, 1 occurrences.

Examine the arguments returned by looptuneSetup.

```
T0

T0 =

Generalized continuous-time state-space model with 0 outputs, 2 inputs, 4 states, and APU_: Analysis point, 2 channels, 1 occurrences.

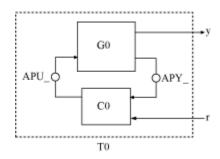
APY_: Analysis point, 2 channels, 1 occurrences.

Decoupler: Parametric 2x2 gain, 1 occurrences.

PI L: Parametric PID controller, 1 occurrences.
```

```
Type "ss(T0)" to see the current value, "get(T0)" to see all properties, and "T0.Blocks
```

The software constructs the closed-loop control system for systune by connecting the plant and controller at their control and measurement signals, and inserting a two-channel AnalysisPoint block at each of the connection locations, as illustrated in the following diagram.



When tuning the control system of this example with looptune, all requirements are treated as soft requirements. Therefore, HardReqs is empty. SoftReqs is an array of TuningGoal requirements. These requirements together enforce the bandwidth and margins of the looptune command, plus the additional requirements that you specified.

#### SoftReqs

```
SoftReqs=5×1 object
5x1 heterogeneous SystemLevel (LoopShape, Tracking, Rejection, ...) array with prope

Models
Openings
Name
```

Examine the first entry in SoftReqs.

#### SoftReqs(1)

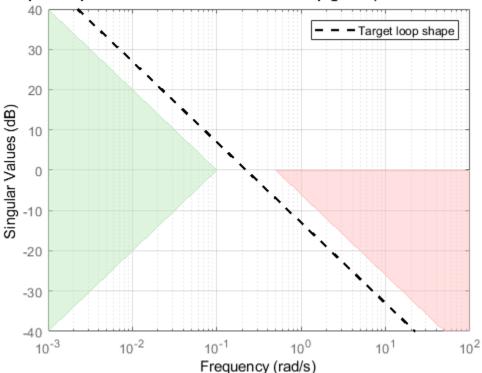
```
ans =
  LoopShape with properties:
    LoopGain: [1x1 zpk]
    CrossTol: 0.3495
    Focus: [0 Inf]
    Stabilize: 1
```

```
LoopScaling: 'on'
Location: {2x1 cell}
Models: NaN
Openings: {0x1 cell}
Name: 'Open loop CG'
```

looptuneSetup expresses the target crossover frequency range wc as a TuningGoal.LoopShape requirement. This requirement constrains the open-loop gain profile to the loop shape stored in the LoopGain property, with a crossover frequency and crossover tolerance (CrossTol) determined by wc. Examine this loop shape.

viewGoal(SoftReqs(1))





SoftReqs(4)

The target crossover is expressed as an integrator gain profile with a crossover between 0.1 and 0.5 rad/s, as specified by wc. If you want to specify a different loop shape, you can alter this TuningGoal.LoopShape requirement before providing it to systume.

looptune also tunes to default stability margins that you can change using
looptuneOptions. For systume, stability margins are specified using
TuningGoal.Margins requirements. Here, looptuneSetup has expressed the default
stability margins of looptune as soft TuningGoal.Margins requirements. For example,
examine the fourth entry in SoftReqs.

```
ans =
  Margins with properties:

    GainMargin: 7.6000
    PhaseMargin: 45
    ScalingOrder: 0
        Focus: [0 Inf]
    Location: {2x1 cell}
        Models: NaN
        Openings: {0x1 cell}
```

Name: 'Margins at plant inputs'

The last entry in SoftReqs is a similar TuningGoal.Margins requirement constraining the margins at the plant outputs. looptune enforces these margins as soft requirements. If you want to convert them to hard constraints, pass them to systune in the input vector HardReqs instead of the input vector SoftReqs.

## **Input Arguments**

**looptuneInputs** — **Plant, controller, and requirement inputs to looptune** valid looptune input sequence

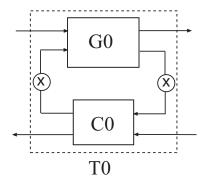
Plant, controller, and requirement inputs to looptune, specified as a valid looptune input sequence. For more information about the arguments in a valid looptune input sequence, see the looptune reference page.

## **Output Arguments**

#### T0 — Closed-loop control system model

generalized state-space model

Closed-loop control system model for tuning with systune, returned as a generalized state-space genss model. To compute T0, the plant, G0, and the controller, C0, are combined in the feedback configuration of the following illustration.



The connections between CO and GO are determined by matching signals using the InputName and OutputName properties of the two models. In general, the signal lines in the diagram can represent vector-valued signals. AnalysisPoint blocks, indicated by X in the diagram, are inserted between the controller and the plant. This allows definition of open-loop and closed-loop requirements on signals injected or measured at the plant inputs or outputs. For example, the bandwidth wc is converted into a TuningGoal.LoopShape requirement that imposes the desired crossover on the open-loop signal measured at the plant input.

For more information on the structure of closed-loop control system models for tuning with systune, see the systune reference page.

### SoftReqs — Soft tuning requirements

vector of TuningGoal requirement objects

Soft tuning requirements for tuning with systune, specified as a vector of TuningGoal requirement objects.

looptune expresses most of its implicit tuning requirements as soft tuning requirements. For example, a specified target loop bandwidth is expressed as a

TuningGoal.LoopShape requirement with integral gain profile and crossover at the target frequency. Additionally, looptune treats all of the explicit requirements you specify (Req1,...ReqN) as soft requirements. SoftReqs contains all of these tuning requirements.

#### **HardReqs** — **Hard tuning requirements**

vector of TuningGoal requirement objects

Hard tuning requirements (constraints) for tuning with systune, specified as a vector of TuningGoal requirement objects.

Because looptune treats most tuning requirements as soft requirements, HardReqs is usually empty. However, if you change the default MaxFrequency option of the looptuneOptions set, loopopt, then this requirement appears as a hard TuningGoal.Poles constraint.

#### sysopt — Algorithm options for systune tuning

systuneOptions options set

Algorithm options for systune tuning, specified as a systuneOptions options set.

Some of the options in the looptuneOptions set, loopopt, are expressed as hard or soft requirements that are returned in HardReqs and SoftReqs. Other options correspond to options in the systtuneOptions set.

### **Alternatives**

When tuning Simulink using an slTuner, interface, convert a looptune problem to systune using looptuneSetup for slTuner.

### See Also

genss | looptune | looptuneOptions | looptuneSetup (for slTuner) | slTuner | systume | systumeOptions

#### Introduced in R2013b

# loopview

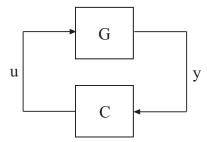
Graphically analyze MIMO feedback loops

## **Syntax**

loopview(G,C)
loopview(G,C,info)

## **Description**

loopview(G,C) plots characteristics of the following positive-feedback, multi-input, multi-output (MIMO) feedback loop with plant G and controller C.



Use loopview to analyze the performance of a tuned control system you obtain using looptune.

**Note** If you are tuning a Simulink model with looptune through an slTuner interface, analyze the performance of your control system using loopview for slTuner (requires Simulink Control Design).

loopview plots the singular values of:

Open-loop frequency responses G\*C and C\*G

- Sensitivity function S = inv(1-G\*C) and complementary sensitivity T = 1-S
- Maximum (target), actual (tuned), and normalized MIMO stability margins. loopview
  plots the multi-loop disk margin (see "Stability Analysis Using Disk Margins" (Robust
  Control Toolbox)). Use this plot to verify that the stability margins of the tuned system
  do not significantly exceed the target value.

For more information about singular values, see sigma.

loopview(G,C,info) uses the info structure returned by looptune. This syntax also plots the target and tuned values of tuning constraints imposed on the system. Additional plots include:

- Singular values of the maximum allowed S and T. The curve marked S/T Max shows
  the maximum allowed S on the low-frequency side of the plot, and the maximum
  allowed T on the high-frequency side. These curves are the constraints that looptune
  imposes on S and T to enforce the target crossover range wc.
- Target and tuned values of constraints imposed by any tuning goal requirements you used with looptune.

Use loopview with the info structure to assist in troubleshooting when tuning fails to meet all requirements.

## **Input Arguments**

#### G

Numeric LTI model or tunable genss model representing the plant in a control system. The plant is the portion of a control system whose outputs are sensor signals (measurements), and whose inputs are actuator signals (controls).

You can obtain G as an output argument from looptune when you tune your control system.

#### C

genss model representing the controller in a control system. The controller is the portion of your control system that receives sensor signals (measurements) as inputs and produces actuator signals (controls) as outputs.

You can obtain C as an output argument from looptune when you tune your control system.

#### info

info structure returned by looptune during control system tuning.

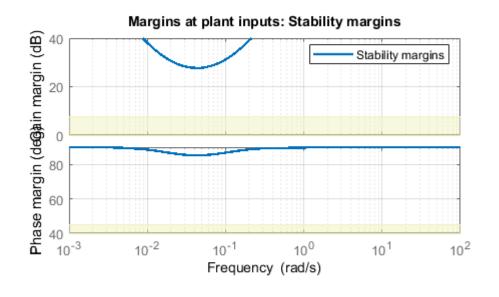
### **Examples**

#### **Examine Performance of Tuned Controller**

Tune a control system, and use loopview to examine the performance of the tuned controller.

```
s = tf('s');
G = 1/(75*s+1)*[87.8 -86.4; 108.2 -109.6];
G.InputName = {'qL','qV'};
G.OutputName = 'y';
D = tunableGain('Decoupler',eye(2));
PI_L = tunablePID('PI_L', 'pi');
PI L.OutputName = 'qL':
PI_V = tunablePID('PI_V','pi');
PI V.OutputName = 'qV';
sum = sumblk('e = r - y', 2);
C0 = (blkdiag(PI_L,PI_V)*D)*sum;
wc = [0.1,1];
options = looptuneOptions('RandomStart',5);
[G,C,gam,info] = looptune(-G,CO,wc,options);
Final: Peak gain = 0.956, Iterations = 28
Achieved target gain value TargetGain=1.
figure('Position',[100,100,520,1000])
loopview(G,C,info)
```

Open loop CG: Minimum and maximum loop gains (CrossTol = 0.5) S Т Singular Values (dB) 20 Loop gain(s) Scaled loop gains Target loop shape 0 -20 -40 10<sup>-2</sup> 10<sup>-3</sup> 10<sup>-1</sup> 10<sup>0</sup> 10<sup>1</sup> 10<sup>2</sup> Frequency (rad/s)



The first plot shows that the open-loop gain crossovers fall close to the specified interval [0.1,1]. This plot also includes the tuned values of the sensitivity function S = inv(1-G\*C) and complementary sensitivity T = 1-S. These curves reflect the constraints that looptune imposes on S and T to enforce the target crossover range wc.

The second and third plots show that the MIMO stability margins of the tuned system fall well within the target range.

### **Alternatives**

For analyzing Simulink models tuned with looptune through an slTuner interface, use loopview for slTuner (requires Simulink Control Design).

### See Also

looptune | looptune (for slTuner) | loopview (for slTuner) | slTuner

### **Topics**

"Tune MIMO Control System for Specified Bandwidth"

"Decoupling Controller for a Distillation Column"

# lyap

Continuous Lyapunov equation solution

## **Syntax**

```
lyap
X = lyap(A,Q)
X = lyap(A,B,C)
X = lyap(A,Q,[],E)
```

## **Description**

lyap solves the special and general forms of the Lyapunov equation. Lyapunov equations arise in several areas of control, including stability theory and the study of the RMS behavior of systems.

X = lyap(A,Q) solves the Lyapunov equation

$$AX + XA^T + Q = 0$$

where A and Q represent square matrices of identical sizes. If Q is a symmetric matrix, the solution X is also a symmetric matrix.

X = lyap(A,B,C) solves the Sylvester equation

$$AX + XB + C = 0$$

The matrices A, B, and C must have compatible dimensions but need not be square.

X = lyap(A,Q,[],E) solves the generalized Lyapunov equation

$$AXE^T + EXA^T + Q = 0$$

where Q is a symmetric matrix. You must use empty square brackets [] for this function. If you place any values inside the brackets, the function errors out.

### Limitations

The continuous Lyapunov equation has a unique solution if the eigenvalues  $\alpha_1, \alpha_2, ..., \alpha_n$  of A and  $\beta_1, \beta_2, ..., \beta_n$  of B satisfy

$$\alpha_i + \beta_j \neq 0$$
 for all pairs  $(i, j)$ 

If this condition is violated, lyap produces the error message:

Solution does not exist or is not unique.

### **Examples**

### **Example 1**

#### **Solve Lyapunov Equation**

Solve the Lyapunov equation

$$AX + XA^T + Q = 0$$

where

$$A = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} \qquad Q = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

The A matrix is stable, and the Q matrix is positive definite.

$$A = [1 \ 2; -3 \ -4];$$
  
 $Q = [3 \ 1; \ 1 \ 1];$   
 $X = lyap(A,Q)$ 

These commands return the following X matrix:

You can compute the eigenvalues to see that X is positive definite.

eig(X)

The command returns the following result:

ans =

0.4359 8.7308

### **Example 2**

### **Solve Sylvester Equation**

Solve the Sylvester equation

$$AX + XB + C = 0$$

where

$$A = 5 \qquad B = \begin{bmatrix} 4 & 3 \\ 4 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

A = 5; B = [4 3; 4 3]; C = [2 1]; X = lyap(A,B,C)

These commands return the following X matrix:

X = -0.2000 -0.0500

# **Algorithms**

lyap uses SLICOT routines SB03MD and SG03AD for Lyapunov equations and SB04MD (SLICOT) and ZTRSYL (LAPACK) for Sylvester equations.

### References

- [1] Bartels, R.H. and G.W. Stewart, "Solution of the Matrix Equation AX + XB = C," *Comm. of the ACM*, Vol. 15, No. 9, 1972.
- [2] Barraud, A.Y., "A numerical algorithm to solve A XA X = Q," *IEEE Trans. Auto. Contr.*, AC-22, pp. 883-885, 1977.
- [3] Hammarling, S.J., "Numerical solution of the stable, non-negative definite Lyapunov equation," *IMA J. Num. Anal.*, Vol. 2, pp. 303–325, 1982.
- [4] Penzl, T., "Numerical solution of generalized Lyapunov equations," *Advances in Comp. Math.*, Vol. 8, pp. 33-48, 1998.
- [5] Golub, G.H., Nash, S. and Van Loan, C.F., "A Hessenberg-Schur method for the problem AX + XB = C," *IEEE Trans. Auto. Contr.*, AC-24, pp. 909-913, 1979.

### See Also

covar | dlyap

Introduced before R2006a

# **lyapchol**

Square-root solver for continuous-time Lyapunov equation

## **Syntax**

```
R = lyapchol(A,B)
X = lyapchol(A,B,E)
```

# **Description**

R = lyapchol(A,B) computes a Cholesky factorization X = R'\*R of the solution X to the Lyapunov matrix equation:

```
A*X + X*A' + B*B' = 0
```

All eigenvalues of matrix A must lie in the open left half-plane for R to exist.

X = lyapchol(A,B,E) computes a Cholesky factorization X = R'\*R of X solving the generalized Lyapunov equation:

```
A*X*E' + E*X*A' + B*B' = 0
```

All generalized eigenvalues of (A,E) must lie in the open left half-plane for R to exist.

## **Algorithms**

lyapchol uses SLICOT routines SB03OD and SG03BD.

### References

[1] Bartels, R.H. and G.W. Stewart, "Solution of the Matrix Equation AX + XB = C," *Comm. of the ACM*, Vol. 15, No. 9, 1972.

- [2] Hammarling, S.J., "Numerical solution of the stable, non-negative definite Lyapunov equation," *IMA J. Num. Anal.*, Vol. 2, pp. 303-325, 1982.
- [3] Penzl, T., "Numerical solution of generalized Lyapunov equations," *Advances in Comp. Math.*, Vol. 8, pp. 33-48, 1998.

### **See Also**

dlyapchol | lyap

Introduced before R2006a

# mag2db

Convert magnitude to decibels (dB)

## **Syntax**

ydb = mag2db(y)

# **Description**

ydb = mag2db(y) returns the corresponding decibel (dB) value ydb for a given magnitude y. The relationship between magnitude and decibels is  $ydb = 20 \log_{10}(y)$ .

### **See Also**

db2mag

Introduced in R2008a

### make1DOF

Convert 2-DOF PID controller to 1-DOF controller

### **Syntax**

```
C1 = make1D0F(C2)
```

# **Description**

C1 = make1D0F(C2) converts the two-degree-of-freedom PID controller C2 to one degree of freedom by removing the terms that depend on coefficients b and c.

# **Examples**

#### Convert 2-DOF PID controller to 1-DOF

Design a 2-DOF PID controller for a plant.

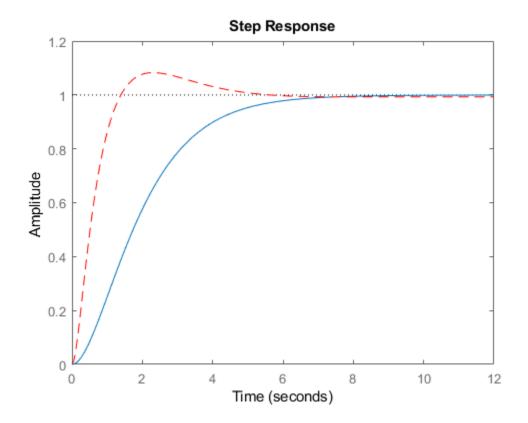
```
G = tf(1,[1\ 0.5\ 0.1]);
C2 = pidtune(G,'pidf2',1.5)
C2 = u = Kp\ (b*r-y) + Ki - (r-y) + Kd - (c*r-y) 
S \qquad Tf*s+1
with\ Kp = 1.12,\ Ki = 0.23,\ Kd = 1.3,\ Tf = 0.122,\ b = 0.664,\ c = 0.0136
Continuous-time\ 2-DOF\ PIDF\ controller\ in\ parallel\ form.
```

Convert the controller to one degree of freedom.

```
C1 = make1D0F(C2)
```

The new controller has the same PID gains and filter constant. However, make1D0F removes the terms involving the setpoint weights b and c. Therefore, in a closed loop with the plant G, the 2-D0F controller C2 yields a different closed-loop response from C1.

```
CM = tf(C2);
T2 = CM(1)*feedback(G,-CM(2));
T1 = feedback(G*C1,1);
stepplot(T2,T1,'r--')
```



# **Input Arguments**

### C2 - 2-DOF PID controller

pid2 object | pidstd2 object

2-DOF PID controller, specified as a  $\verb"pid2"$  object or a  $\verb"pidstd2"$  object.

## **Output Arguments**

#### C1 — 1-DOF PID controller

pid object | pidstd object

1-DOF PID controller, returned as a pid or pidstd object. C1 is in parallel form if C2 is in parallel form, and standard form if C2 is in standard form.

For example, suppose C2 is a continuous-time, parallel-form 2-DOF pid2 controller. The relationship between the inputs, r and y, and the output u of C2 is given by:

$$u=K_p(br-y)+\frac{K_i}{s}(r-y)+\frac{K_ds}{T_fs+1}(cr-y)\,.$$

Then C1 is a parallel-form 1-DOF pid controller of the form:

$$C_1 = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1} \,. \label{eq:continuous}$$

The PID gains  $K_p$ ,  $K_i$ , and  $K_d$ , and the filter time constant  $T_f$  are unchanged. make1D0F removes the terms that depend on the setpoint weights b and c. For more information about 2-D0F PID controllers, see "Two-Degree-of-Freedom PID Controllers".

The conversion also preserves the values of the properties Ts, TimeUnit, Sampling Grid, IFormula, and DFormula.

### See Also

getComponents | make2D0F | pid | pid2 | pidstd | pidstd2

### **Topics**

"Two-Degree-of-Freedom PID Controllers"

#### Introduced in R2015b

### make2DOF

Convert 1-DOF PID controller to 2-DOF controller

## **Syntax**

```
C2 = make2DOF(C1)
C2 = make2DOF(C1,b)
C2 = make2DOF(C1,b,c)
```

# **Description**

C2 = make2DOF(C1) converts the one-degree-of-freedom PID controller C1 to two degrees of freedom. The setpoint weights b and c of the 2-DOF controller are 1, and the remaining PID coefficients do not change.

C2 = make2D0F(C1,b) specifies the setpoint weight for the proportional term.

C2 = make2DOF(C1,b,c) specifies the setpoint weights for both the proportional and derivative terms.

## **Examples**

#### Convert 1-DOF PID controller to 2-DOF

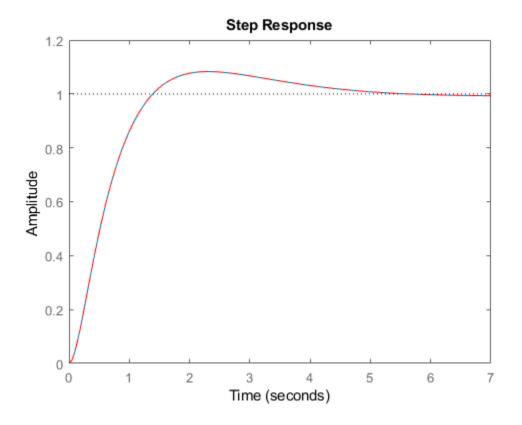
Design a 1-DOF PID controller for a plant.

```
with Kp = 1.12, Ki = 0.23, Kd = 1.3, Tf = 0.122 Continuous-time PIDF controller in parallel form.
```

Convert the controller to two degrees of freedom.

The new controller has the same PID gains and filter constant. It also contains new terms involving the setpoint weights b and c. By default, b = c = 1. Therefore, in a closed loop with the plant G, the 2-DOF controller C2 yields the same response as C1.

```
T1 = feedback(G*C1,1);
CM = tf(C2);
T2 = CM(1)*feedback(G,-CM(2));
stepplot(T1,T2,'r--')
```



Convert C1 to a 2-DOF controller with different b and c values.

$$C2_2 = make2D0F(C1, 0.5, 0.75)$$

$$C2_2 =$$

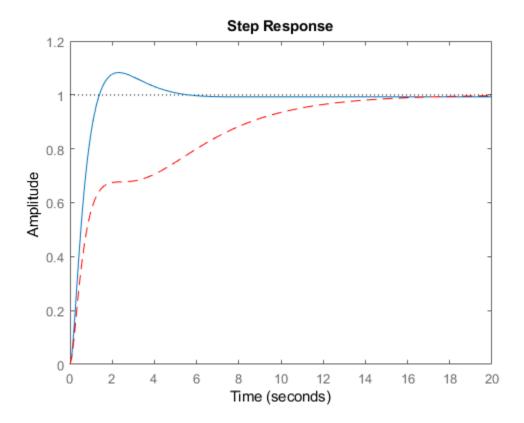
$$u = Kp (b*r-y) + Ki --- (r-y) + Kd ---- (c*r-y)$$
  
 $s Tf*s+1$ 

with Kp = 1.12, Ki = 0.23, Kd = 1.3, Tf = 0.122, b = 0.5, c = 0.75

Continuous-time 2-DOF PIDF controller in parallel form.

The PID gains and filter constant are still unchanged, but the setpoint weights now change the closed-loop response.

```
CM_2 = tf(C2_2);
T2_2 = CM_2(1)*feedback(G,-CM_2(2));
stepplot(T1,T2_2,'r--')
```



# **Input Arguments**

C1 — 1-DOF PID controller
pid object | pidstd object

1-DOF PID controller, specified as a pid object or a pidstd object.

#### b - Setpoint weight on proportional term

1 (default) | real nonnegative scalar

Setpoint weight on proportional term, specified as a real, nonnegative, finite value. If you do not specify b, then C2 has b = 1.

#### c — Setpoint weight on derivative term

1 (default) | real nonnegative scalar

Setpoint weight on derivative term, specified as a real, nonnegative, finite value. If you do not specify c, then C2 has c=1.

## **Output Arguments**

#### C2 — 2-DOF PID controller

pid2 object | pidstd2 object

2-DOF PID controller, returned as a pid2 object or pidstd2 object. C2 is in parallel form if C1 is in parallel form, and standard form if C1 is in standard form.

For example, suppose  ${\tt C1}$  is a continuous-time, parallel-form  ${\tt pid}$  controller of the form:

$$C_1 = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1} \,. \label{eq:continuous}$$

Then C2 is a parallel-form 2-DOF pid2 controller, which has two inputs and one output. The relationship between the inputs, r and y, and the output u of C2 is given by:

$$u=K_p(br-y)+\frac{K_i}{s}(r-y)+\frac{K_ds}{T_fs+1}(cr-y)\,.$$

The PID gains  $K_p$ ,  $K_i$ , and  $K_d$ , and the filter time constant  $T_f$  are unchanged. The setpoint weights b and c are specified by the input arguments b and c, or 1 by default. For more information about 2-DOF PID controllers, see "Two-Degree-of-Freedom PID Controllers".

The conversion also preserves the values of the properties Ts, TimeUnit, Sampling Grid, IFormula, and DFormula.

# **See Also**

getComponents | make1D0F | pid | pid2 | pidstd | pidstd2

### **Topics**

"Two-Degree-of-Freedom PID Controllers"

**Introduced in R2015b** 

# margin

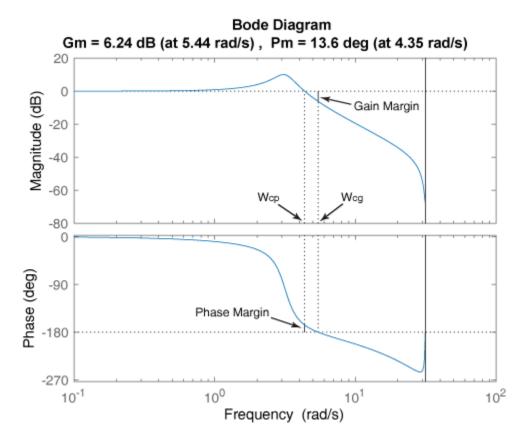
Gain margin, phase margin, and crossover frequencies

## **Syntax**

```
margin(sys)
[Gm,Pm,Wcg,Wcp] = margin(sys)
[Gm,Pm,Wcg,Wcp] = margin(mag,phase,w)
[Gm,Pm] = margin(sys,J1,...,JN)
```

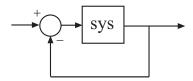
# **Description**

margin(sys) plots the Bode response of sys on the screen and indicates the gain and phase margins on the plot. Gain margins are expressed in dB on the plot.



Solid vertical lines mark the gain margin and phase margin. The dashed vertical lines indicate the locations of Wcp, the frequency where the phase margin is measured, and Wcg, the frequency where the gain margin is measured. The plot title includes the magnitude and location of the gain and phase margin.

Gm and Pm of a system indicate the relative stability of the closed-loop system formed by applying unit negative feedback to sys, as shown in the following figure.



Gm is the amount of gain variance required to make the loop gain unity at the frequency Wcg where the phase angle is  $-180^{\circ}$  (modulo 360°). In other words, the gain margin is 1/g if g is the gain at the  $-180^{\circ}$  phase frequency. Similarly, the phase margin is the difference between the phase of the response and  $-180^{\circ}$  when the loop gain is 1.0.

The frequency Wcp at which the magnitude is 1.0 is called the *unity-gain frequency* or *gain crossover frequency*. Usually, gain margins of three or more combined with phase margins between 30° and 60° result in reasonable tradeoffs between bandwidth and stability.

[Gm,Pm,Wcg,Wcp] = margin(sys) returns the gain margin Gm in absolute units, the phase margin Pm, and the corresponding frequencies Wcg and Wcp, of sys. Wcg is the frequency where the gain margin is measured, which is a -180° phase crossing frequency. Wcp is the frequency where the phase margin is measured, which is a 0-dB gain crossing frequency. These frequencies are expressed in radians/TimeUnit, where TimeUnit is the unit specified in the TimeUnit property of sys. When sys has several crossovers, margin returns the smallest gain and phase margins and corresponding frequencies.

[Gm, Pm, Wcg, Wcp] = margin(mag, phase, w) derives the gain and phase margins from frequency response data. Provide the gain data mag in absolute units, and phase data phase in degrees. You can provide the frequency vector w in any units and margin returns Wcg and Wcp in the same units.

[Gm,Pm] = margin(sys,J1,...,JN) returns the gain margin Gm and phase margin Pm of the entries in model array sys with subscripts (J1,...,JN).

### **Examples**

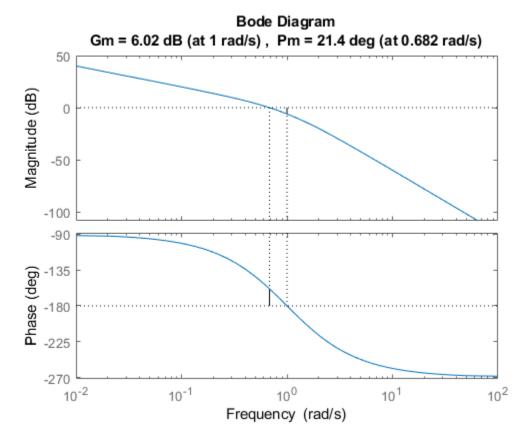
#### Plot Gain and Phase Margins of Transfer Function

For this example, create a continuous transfer function.

Continuous-time transfer function.

Display the gain and phase margins graphically.

margin(sys)



The gain margin (6.02 dB) and phase margin (21.4 deg), displayed in the title, are marked with solid vertical lines. The dashed vertical lines indicate the locations of Wcg, the frequency where the gain margin is measured, and Wcp, the frequency where the phase margin is measured.

#### **Gain and Phase Margins of Transfer Function**

For this example, create a discrete-time transfer function.

Compute the gain margin, phase margin and frequencies.

```
[Gm,Pm,Wcg,Wcp] = margin(sys)

Gm = 2.0517

Pm = 13.5711

Wcg = 5.4374

Wcp = 4.3544
```

The results indicate that a gain variation of over 2.05 dB at the gain crossover frequency of 5.43 rad/s would cause the system to be unstable. Similarly a phase variation of over 13.57 degrees at the phase crossover frequency of 4.35 rad/s will cause the system to lose stability.

#### Gain and Phase Margins using Frequency Response Data

For this example, load the frequency response data of an open loop system, consisting of magnitudes (m) and phase values (p) measured at the frequencies in w.

```
load('openLoopFRD.mat','p','m','w');
```

Compute the gain and phase margins.

```
[Gm,Pm,Wcg,Wcp] = margin(m,p,w)
```

```
Gm = 0.6249
Pm = 48.9853
Wcg = 1.2732
Wcp = 1.5197
```

#### Gain and Phase Margins of Models in an Array

For this example, load invertedPendulumArray.mat, which contains a 3-by-3 array of inverted pendulum models. The mass of the pendulum varies as you move from model to model along a single column of sys, and the length of the pendulum varies as you move along a single row. The mass values used are 100g, 200g and 300g, and the pendulum lengths used are 3m, 2m and 1m respectively.

```
Column 1 Column 2 Column 3

Row 1 100g,3m 100g,2m 100g,1m

Row 2 200g,3m 200g,2m 200g,1m

Row 3 300g,3m 300g,2m 300g,1m

load('invertedPendulumArray.mat','sys');
size(sys)

3x3 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Find gain and phase margin for all models in the array.

```
[Gm,Pm] = margin(sys)

Gm = 3 \times 3

0.9800 0.9800 0.9800
0.9800 0.9800 0.9800
0.9800 0.9800 0.9800

Pm = 3 \times 3

-11.3565 -11.3898 -11.4228
```

```
-11.3842 -11.4088 -11.4333
-11.4020 -11.4208 -11.4396
```

margin returns two arrays, Gm and Pm, in which each entry is the gain and phase margin values of the corresponding entry in sys. For instance, the gain and phase margin of the model with 100g pendulum weight and 2m length is Gm(1,2) and Pm(1,2), respectively.

## **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO dynamic system model, or an array of SISO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, margin returns the gain margin and phase margin of the current or nominal value of sys. If sys is an array of models, margin returns the Gm and Pm of the model corresponding to its subscript J1, ..., JN in sys. For more information on model arrays, see "Model Arrays".

# $J1, \ldots, JN$ — Indices of models in array whose gain and phase margins you want to extract

positive integer

Indices of models in array whose gain and phase margins you want to extract, specified as positive integers. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4-by-5 array of dynamic system models, the following command extracts Gm and Pm for entry (2,3) in the array.

```
[Gm,Pm] = margin(sys,2,3);
```

#### mag — Magnitude of system response

3-D array

Magnitude of the system response in absolute units, specified as a 3-D array. Since margin only accepts SISO systems, mag is a 1-by-1-by-N array, where N is the number of frequency points. For an example, see "Obtain Magnitude and Phase Data" on page 2-70.

#### phase — Phase of system response

3-D array

Phase of the system response in degrees, specified as a 3-D array. Since margin only accepts SISO systems, phase is a 1-by-1-by-N array, where N is the number of frequency points. For an example, see "Obtain Magnitude and Phase Data" on page 2-70.

#### w — Frequencies at which the magnitude and phase values of system response are obtained

column vector

Frequencies at which the magnitude and phase values of system response are obtained, specified as a column vector.

### **Output Arguments**

#### Gm — Gain margin

scalar | array

Gain margin, returned as a scalar or an array. If sys is:

- A single model, then Gm is returned as a scalar.
- A model array, then Gm is an array of gain margins of each model in sys.

Gm is the amount of gain variance required to make the loop gain unity at the frequency Wcg where the phase angle is  $-180^{\circ}$  (modulo 360°). In other words, the gain margin is 1/g if g is the gain at the  $-180^{\circ}$  phase frequency. Negative gain margins indicate that stability is lost by decreasing the gain, while positive gain margins indicate that stability is lost by increasing the gain

The gain margin Gm is computed in absolute units. You can compute the gain margin in dB by,

 $Gm_dB = 20*log10(Gm)$ 

#### Pm — Phase margin

scalar | array

Phase margin, returned as a scalar or an array. If sys is:

- A single model, then Pm is returned as a scalar.
- A model array, then Pm is an array of phase margins of each model in sys.

The phase margin is the difference between the phase of the response and -180° when the loop gain is 1.0.

The phase margin Pm is expressed in degrees.

#### Wcg — Gain crossover frequency

scalar

Gain crossover frequency, returned as a scalar. Wcg is the frequency where the gain margin is measured, which is a -180° phase crossing frequency.

Wcg is expressed in radians/TimeUnit, where TimeUnit is the unit specified in the TimeUnit property of sys.

#### Wcp — Phase crossover frequency

scalar

Phase crossover frequency, returned as a scalar. Wcp is the frequency where the phase margin is measured, which is a 0-dB gain crossing frequency.

Wcp is expressed in radians/TimeUnit, where TimeUnit is the unit specified in the TimeUnit property of sys.

### **Tips**

When you use margin(mag,phase,w), margin relies on interpolation to approximate the margins, which generally produce less accurate results. For example, if there is no 0-dB crossing within the w range, margin returns a phase margin of Inf. Therefore, if you have an analytical model sys, using [Gm,Pm,Wcg,Wcp] = margin(sys) is a more robust way to obtain the margins.

### See Also

Linear System Analyzer | allmargin | bode

#### **Topics**

"Assessing Gain and Phase Margins"

#### **Introduced before R2006a**

### minreal

Minimal realization or pole-zero cancellation

# **Syntax**

```
sysr = minreal(sys)
sysr = minreal(sys,tol)
[sysr,u] = minreal(sys,tol)
... = minreal(sys,tol,false)
... = minreal(sys,[],false)
```

# **Description**

sysr = minreal(sys) eliminates uncontrollable or unobservable state in state-space
models, or cancels pole-zero pairs in transfer functions or zero-pole-gain models. The
output sysr has minimal order and the same response characteristics as the original
model sys.

sysr = minreal(sys,tol) specifies the tolerance used for state elimination or polezero cancellation. The default value is tol = sqrt(eps) and increasing this tolerance forces additional cancellations.

[sysr,u] = minreal(sys,tol) returns, for state-space model sys, an orthogonal matrix U such that (U\*A\*U',U\*B,C\*U') is a Kalman decomposition of (A,B,C)

... = minreal(sys,tol,false) and ... = minreal(sys,[],false) disable the verbose output of the function. By default, minreal displays a message indicating the number of states removed from a state-space model sys.

# **Examples**

The commands

Continuous-time zero/pole/gain model.

# **Algorithms**

Pole-zero cancellation is a straightforward search through the poles and zeros looking for matches that are within tolerance. Transfer functions are first converted to zero-pole-gain form.

# **Alternative Functionality**

### App

**Model Reducer** 

### **Live Editor Task**

Reduce Model Order

### See Also

Model Reducer | balreal | modred | sminreal

### **Topics**

"Pole-Zero Simplification"

"Model Reduction Basics"

#### **Introduced before R2006a**

### modred

Eliminate states from state-space models

# **Syntax**

```
rsys = modred(sys,elim)
rsys = modred(sys,elim,'method')
```

# **Description**

rsys = modred(sys,elim) reduces the order of a continuous or discrete state-space model sys by eliminating the states found in the vector elim. The full state vector X is partitioned as X = [X1;X2] where X1 is the reduced state vector and X2 is discarded.

elim can be a vector of indices or a logical vector commensurate with X where true values mark states to be discarded. This function is usually used in conjunction with balreal. Use balreal to first isolate states with negligible contribution to the I/O response. If sys has been balanced with balreal and the vector  $\mathbf{g}$  of Hankel singular values has M small entries, you can use  $\mathbf{modred}$  to eliminate the corresponding M states. For example:

```
[sys,g] = balreal(sys) % Compute balanced realization
elim = (g<1e-8) % Small entries of g are negligible states
rsys = modred(sys,elim) % Remove negligible states</pre>
```

rsys = modred(sys,elim,'method') also specifies the state elimination method.
Choices for 'method' include

- 'MatchDC' (default): Enforce matching DC gains. The state-space matrices are recomputed as described in "Algorithms" on page 2-730.
- 'Truncate': Simply delete X2.

The 'Truncate' option tends to produces a better approximation in the frequency domain, but the DC gains are not guaranteed to match.

If the state-space model sys has been balanced with balreal and the grammians have m small diagonal entries, you can reduce the model order by eliminating the last m states with modred.

## **Examples**

#### Order Reduction by Matched-DC-Gain and Direct-Deletion Methods

Consider the following continuous fourth-order model.

$$h(s) = \frac{s^3 + 11s^2 + 36s + 26}{s^4 + 14.6s^3 + 74.96s^2 + 153.7s + 99.65}.$$

To reduce its order, first compute a balanced state-space realization with balreal.

```
h = tf([1 \ 11 \ 36 \ 26],[1 \ 14.6 \ 74.96 \ 153.7 \ 99.65]);
[hb,g] = balreal(h);
```

Examine the gramians.

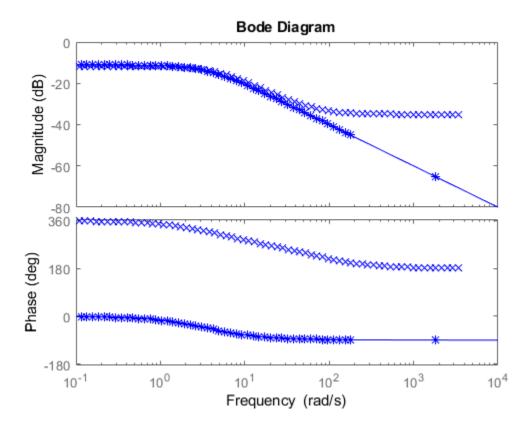
```
g' ans = 1 \times 4 0.1394 0.0095 0.0006 0.0000
```

The last three diagonal entries of the balanced gramians are relatively small. Eliminate these three least-contributing states with modred, using both matched-DC-gain and direct-deletion methods.

```
hmdc = modred(hb,2:4,'MatchDC');
hdel = modred(hb,2:4,'Truncate');
```

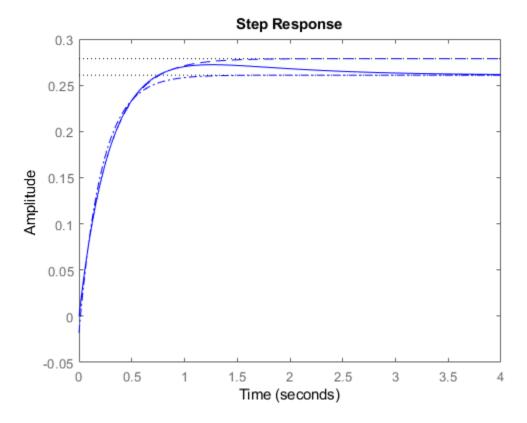
Both hmdc and hdel are first-order models. Compare their Bode responses against that of the original model.

```
bodeplot(h,'-',hmdc,'x',hdel,'*')
```



The reduced-order model hdel is clearly a better frequency-domain approximation of h. Now compare the step responses.

```
stepplot(h,'-',hmdc,'-.',hdel,'--')
```



While hdel accurately reflects the transient behavior, only hmdc gives the true steady-state response.

# **Limitations**

With the matched DC gain method,  $A_{22}$  must be invertible in continuous time, and I –  $A_{22}$  must be invertible in discrete time.

# **Algorithms**

The algorithm for the matched DC gain method is as follows. For continuous-time models

$$\dot{x} = Ax + By$$
$$y = Cx + Du$$

the state vector is partitioned into  $x_1$ , to be kept, and  $x_2$ , to be eliminated.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$
$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} x + Du$$

Next, the derivative of  $x_2$  is set to zero and the resulting equation is solved for  $x_1$ . The reduced-order model is given by

$$\dot{x}_1 = [A_{11} - A_{12}A_{22}^{-1}A_{21}]x_1 + [B_1 - A_{12}A_{22}^{-1}B_2]u$$

$$y = [C_1 - C_2A_{22}^{-1}A_{21}]x + [D - C_2A_{22}^{-1}B_2]u$$

The discrete-time case is treated similarly by setting

$$x_2[n+1] = x_2[n]$$

### See Also

balreal | balred | minreal

**Introduced before R2006a** 

### **Model Reducer**

Reduce complexity of linear time-invariant (LTI) models

# **Description**

The **Model Reducer** app lets you compute reduced-order approximations of high-order models. Working with lower-order models can simplify analysis and control design. Simpler models are also easier to understand and manipulate. You can reduce a plant model to focus on relevant dynamics before designing a controller for the plant. Or, you can use model reduction to simplify a full-order controller.

Using any of the following methods, **Model Reducer** helps you reduce model order while preserving model characteristics that are important to your application:

- Balanced Truncation Remove states with relatively small energy contributions.
- Mode Selection Select modes by specifying a frequency range of interest.
- Pole-Zero Simplification Eliminate canceling or near-canceling pole-zero pairs.

**Model Reducer** provides response plots and error plots to help ensure that the reducedorder model preserves important dynamics. For more information on model reduction and why it is useful, see "Model Reduction Basics".

For an alternative to the **Model Reducer** app that lets you interactively perform model reduction and generate code for a live script, see the Reduce Model Order task in the Live Editor.

# **Open the Model Reducer App**

- MATLAB Toolstrip: On the **Apps** tab, under **Control System Design and Analysis**, click the app icon.
- MATLAB command prompt: Enter modelReducer.

# **Examples**

- "Model Reduction Basics"
- "Reduce Model Order Using the Model Reducer App"
- "Balanced Truncation Model Reduction"
- "Pole-Zero Simplification"
- "Mode-Selection Model Reduction"

### **Parameters**

#### **Balanced Truncation Tab**

#### Model — Currently selected model for reduction

model name

Specify the model you want to reduce by selecting from the **Model** drop-down list. The list includes all models currently in the **Data Browser**. To get a model from the MATLAB

workspace into the **Data Browser**, on the **Model Reducer** tab, click **Import Model**. You can import any:

- tf, ss, or zpk model that is proper. The model can be SISO or MIMO, and continuous or discrete.
  - Continuous-time models must not have time delays. To reduce a continuous-time model with time delays, first use pade to approximate the time delays as model dynamics.
  - Discrete-time models can have time delays. For the Balanced Truncation reduction method, the app uses absorbDelay to convert the delay into poles at z=0 before reducing the model. The additional states are reflected in the response plot and Hankel singular-value plot.
- Generalized model such as a genss model. The Model Reducer app uses the current or nominal value of all control design blocks in model (see getValue).

**Note Model Reducer** assumes that the model time unit (specified in the TimeUnit property of the model) is seconds. If your model does not have TimeUnit = 'seconds', use chgTimeUnit to convert the model to seconds.

#### Reduced model orders — Number of states in reduced model

integer | integer array

Specify the number of states in the reduced-order model. Any value is permitted that falls between the number of unstable states in the model and the number of states in the original model. If you specify a single value, Model Reducer computes and displays the responses of a model of that order. If you specify multiple values, Model Reducer computes models of all specified orders and displays their responses on the same plot. To

store reduced models in the **Data Browser**, click .



For more information, see "Balanced Truncation Model Reduction".

Example: 5 Example: 4:7

Example: [3,7,10]

#### Preserve DC Gain — Match DC gain of reduced model to original model checked (default) | unchecked

When **Preserve DC Gain** is checked, the DC gain of the reduced model equals the DC gain of the original model. When the DC behavior of the model is important in your application, leave this option checked. Uncheck the option to get better matching of higher-frequency behavior.

For more information, see "Balanced Truncation Model Reduction".

#### Select frequency range — Limit analysis to specified frequencies unchecked (default) | checked

By default, **Model Reducer** analyzes Hankel singular values across all frequencies. Restricting this analysis to a particular frequency range is useful when you know the model has modes outside the region of interest to your particular application. When you apply a frequency limit, **Model Reducer** determines which states are the low-energy states to truncate based on their energy contribution within the specified frequency range only.

To limit the analysis of state contributions to a particular frequency range, check **Select** frequency range. Then, drag the vertical cursors on the response plot to specify the frequency range of interest. Alternatively, enter a frequency range in the text box as a vector of the form [fmin, fmax]. Units are rad/TimeUnit, where TimeUnit is the TimeUnit property of the model you are reducing.

#### **Mode Selection Tab**

#### Model — Currently selected model for reduction

model name

Specify the model you want to reduce by selecting from the **Model** drop-down list. The list includes all models currently in the **Data Browser**. To get a model from the MATLAB

workspace into the **Data Browser**, on the **Model Reducer** tab, click **Import Model**. You can import any:

- tf, ss, or zpk model that is proper. The model can be SISO or MIMO, and continuous or discrete.
  - Continuous-time models must not have time delays. To reduce a continuous-time model with time delays, first use pade to approximate the time delays as model dynamics.
  - Discrete-time models can have time delays. For the Balanced Truncation reduction method, the app uses absorbDelay to convert the delay into poles at z=0 before reducing the model. The additional states are reflected in the response plot and Hankel singular-value plot.
- Generalized model such as a genss model. The Model Reducer app uses the current or nominal value of all control design blocks in model (see getValue).

For more information, see "Mode-Selection Model Reduction".

Note Reduce Model Order assumes that the model time unit (specified in the TimeUnit property of the model) is seconds. If your model does not have TimeUnit = 'seconds', use chgTimeUnit to convert the model to seconds.

### Lower Cutoff — Lowest mode frequency

positive scalar

Enter the frequency of the slowest dynamics to preserve in the reduced model. Poles with natural frequency below this cutoff are eliminated from the reduced model.

#### Upper Cutoff — Highest mode frequency

positive scalar

Enter the frequency of the fastest dynamics to preserve in the reduced model. Poles with natural frequency above this cutoff are eliminated from the reduced model.

#### Pole/Zero Simplification Tab

#### Model — Currently selected model for reduction

model name

Specify the model you want to reduce by selecting from the **Model** drop-down list. The list includes all models currently in the **Data Browser**. To get a model from the MATLAB

workspace into the **Data Browser**, on the **Model Reducer** tab, click **Import Model**. You can import any:

- tf, ss, or zpk model that is proper. The model can be SISO or MIMO, and continuous or discrete.
  - Continuous-time models must not have time delays. To reduce a continuous-time model with time delays, first use pade to approximate the time delays as model dynamics.
  - Discrete-time models can have time delays. For the Balanced Truncation reduction method, the app uses absorbDelay to convert the delay into poles at z=0 before reducing the model. The additional states are reflected in the response plot and Hankel singular-value plot.
- Generalized model such as a genss model. The **Model Reducer** app uses the current or nominal value of all control design blocks in model (see getValue).

# **Simplification of Pole-Zero Pairs — Tolerance for pole-zero cancellation** positive scalar

Set the tolerance for pole-zero cancellation by using the slider or entering a value in the text box. The value determines how close together a pole and zero must be for **Model Reducer** to eliminate them from the reduced model. Moving the slider to the left or entering a smaller value in the text box simplifies the model less, by cancelling fewer poles and zeros. Moving the slider to the right, or entering a larger value, simplifies the model more by cancelling poles and zeros that are further apart.

For more information, see "Pole-Zero Simplification".

#### **Programmatic Use**

modelReducer opens the **Model Reducer** app with no models in the **Data Browser**. To import a model from the MATLAB workspace, click Limport Model.

modelReducer(model) opens app and imports the specified LTI model. model can be a:

- tf, ss, or zpk model that is proper. The model can be SISO or MIMO, and continuous or discrete.
  - Continuous-time models must not have time delays. To reduce a continuous-time model with time delays, first use pade to approximate the time delays as model dynamics.
  - Discrete-time models can have time delays. For the Balanced Truncation reduction method, the app uses absorbDelay to convert the delay into poles at z=0 before reducing the model. The additional states are reflected in the response plot and Hankel singular-value plot.
- Generalized model such as a genss model. The **Model Reducer** app uses the current or nominal value of all control design blocks in model (see getValue).

modelReducer(model1,...,modelN) opens the app and imports the specified models.

modelReducer(sessionFile) opens the app and loads a previously saved session. sessionFile is the name of a session data file in the current working directory or on the MATLAB path.

To save session data to disk, in the **Model Reducer** app, on the **Model Reducer** tab, click Save Session. The saved session data includes the current plot configuration and all models in the **Data Browser**.

#### See Also

#### **Functions**

balred | freqsep | minreal

#### **Live Editor Tasks**

Reduce Model Order

#### **Topics**

- "Model Reduction Basics"
- "Reduce Model Order Using the Model Reducer App"
- "Balanced Truncation Model Reduction"
- "Pole-Zero Simplification"

"Mode-Selection Model Reduction"

#### Introduced in R2016a

# modsep

Region-based modal decomposition

### **Syntax**

```
[H,H0] = modsep(G,N,REGIONFCN)
MODSEP(G,N,REGIONFCN,PARAM1,...)
```

## **Description**

[H,H0] = modsep(G,N,REGIONFCN) decomposes the LTI model G into a sum of n simpler models Hj with their poles in disjoint regions Rj of the complex plane:

$$G(s) = H0 + \sum_{j=1}^{N} Hj(s)$$

G can be any LTI model created with ss, tf, or zpk, and N is the number of regions used in the decomposition. modsep packs the submodels Hj into an LTI array H and returns the static gain H0 separately. Use H(:,:,j) to retrieve the submodel Hj(s).

To specify the regions of interest, use a function of the form

```
IR = REGIONFCN(p)
```

that assigns a region index IR between 1 and N to a given pole p. You can specify this function by its name or as a function handle, and use the syntax MODSEP(G,N,REGIONFCN,PARAM1,...) to pass extra input arguments:

```
IR = REGIONFCN(p,PARAM1,...)
```

### **Examples**

To decompose G into G(z) = H0 + H1(z) + H2(z) where H1 and H2 have their poles inside and outside the unit disk respectively, use

```
[H,H0] = modsep(G,2,@udsep)
```

where the function udsep is defined by

```
function r = udsep(p)

if abs(p)<1, r = 1; % assign r=1 to poles inside unit disk

else r = 2; % assign r=2 to poles outside unit disk

end

To extract H1(z) and H2(z) from the LTI array H, use

H1 = H(:,:,1); H2 = H(:,:,2);
```

#### See Also

stabsep

Introduced before R2006a

#### nblocks

Number of blocks in Generalized matrix or Generalized LTI model

### **Syntax**

N = nblocks(M)

## **Description**

N = nblocks(M) returns the number of "Control Design Blocks" in the Generalized LTI model or Generalized matrix M.

## **Input Arguments**

М

A Generalized LTI model (genss or genfrd model), a Generalized matrix (genmat), or an array of such models.

### **Output Arguments**

N

The number of "Control Design Blocks" in M. If a block appears multiple times in M, N reflects the total number of occurrences.

If M is a model array, N is an array with the same dimensions as M. Each entry of N is the number of Control Design Blocks in the corresponding entry of M.

# **Examples**

Number of Control Design Blocks in a Second-Order Filter Model

This example shows how to use nblocks to examine two different ways of parameterizing a model of a second-order filter.

**1** Create a tunable (parametric) model of the second-order filter:

$$F(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2},$$

where the damping  $\zeta$  and the natural frequency  $\omega_n$  are tunable parameters.

```
wn = realp('wn',3);
zeta = realp('zeta',0.8);
F = tf(wn^2,[1 2*zeta*wn wn^2]);
```

F is a genss model with two tunable Control Design Blocks, the realp blocks wn and zeta. The blocks wn and zeta have initial values of 3 and 0.8, respectively.

**2** Examine the number of tunable blocks in the model using nblocks.

```
nblocks(F)
```

This command returns the result:

```
ans = 6
```

F has two tunable parameters, but the parameter wn appears five times — twice in the numerator and three times in the denominator.

**3** Rewrite F for fewer occurrences of wn.

The second-order filter transfer function can be expressed as follows:

$$F(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}.$$

Use this expression to create the tunable filter:

$$F = tf(1, (1/wn)^2 2*zeta*(1/wn) 1)$$

**4** Examine the number of tunable blocks in the new filter model.

```
nblocks(F)
```

This command returns the result:

```
ans =
```

4

In the new formulation, there are only three occurrences of the tunable parameter wn. Reducing the number of occurrences of a block in a model can improve performance time of calculations involving the model. However, the number of occurrences does not affect the results of tuning the model or sampling the model for parameter studies.

#### See Also

genfrd|genmat|genss|getValue

#### **Topics**

- "Control Design Blocks"
- "Generalized Matrices"
- "Generalized and Uncertain LTI Models"

#### Introduced in R2011a

#### ndBasis

Basis functions for tunable gain surface

### **Syntax**

```
shapefcn = ndBasis(F1,F2)
shapefcn = ndBasis(F1,F2,...,FN)
```

## **Description**

You use basis function expansions to parameterize gain surfaces for tuning gain-scheduled controllers, with the tunableSurface command. The complexity of such expansions grows quickly when you have multiple scheduling variables. Use ndBasis to build N-dimensional expansions from low-dimensional expansions. ndBasis is analogous to ndgrid in the way it spatially replicates the expansions along each dimension.

shapefon = ndBasis(F1,F2) forms the outer (tensor) product of two basis function expansions. Each basis function expansion is a function that returns a vector of expansion terms, such as returned by polyBasis. If  $F_1(x_1) = [F_{1,1}(x_1), F_{1,2}(x_1), ..., F_{1,i}(x_1)]$  and  $F_2(x_2) = [F_{2,1}(x_2), F_{2,2}(x_2), ..., F_{2,i}(x_2)]$ , then shapefon is a vector of terms of the form:

$$F_{ij} = F_{1,i}(x_1)F_{2,j}(x_2)$$
.

The terms are listed in a column-oriented fashion, with i varying first, then j.

shapefcn = ndBasis(F1,F2,...,FN) forms the outer product of three or more basis
function expansions. The terms in the vector returned by shapefcn are of the form:

$$F_{i_1...i_N} = F_{1,i_i}(x_1)F_{2,i_2}(x_2)...F_{N,i_N}(x_N)$$
.

These terms are listed in sort order that of an N-dimensional array, with  $i_1$  varying first, then  $i_2$ , and so on. Each Fj can itself be a multi-dimensional basis function expansion.

### **Examples**

#### **Polynomial Basis Functions of Two Variables**

Create a two-dimensional basis of polynomial functions to second-order in both variables.

Define a one-dimensional set of basis functions.

```
F = @(x)[x,x^2];
```

Equivalently, you can use polyBasis to create F.

```
F = polyBasis('canonical',2);
```

Generate a two-dimensional expansion from F.

```
F2D = ndBasis(F,F);
```

F2D is a function of two variables. The function returns a vector containing the evaluated basis functions of those two variables:

$$F2D(x, y) = [x, x^2, y, yx, yx^2, y^2, xy^2, x^2y^2].$$

To confirm this, evaluate F2D for x = 0.2, y = -0.3.

```
F2D(0.2,-0.3) ans = 1 \times 8 0.2000 0.0400 -0.3000 -0.0600 -0.0120 0.0900 0.0180 0.0036
```

The expansion you combine with ndBasis need not have the same order. For instance, combine F with first-order expansion in one variable.

```
G = @(y)[y];
F2D2 = ndBasis(F,G);
```

The array returned by F2D2 is similar to that returned by F2D, without the terms that are quadratic in the second variable.

$$F2D2(x, y) = \left[x, x^2, y, yx, yx^2\right].$$

Evaluate F2D2 for x = 0.2, y = -0.3 to confirm the order of terms.

```
F2D2(0.2,-0.3) ans = 1 \times 5 0.2000 0.0400 -0.3000 -0.0600 -0.0120
```

#### **Mixed Multi-Dimensional Basis Functions**

Create a set of two-dimensional basis functions where the expansion is quadratic in one variable and periodic in the other variable.

First generate the one-dimensional expansions. Name the variables for improved readability.

```
F1 = polyBasis('canonical',2,'x');
F2 = fourierBasis(1,1,'y');
```

For simplicity, this example takes only the first harmonic of the periodic variation. These expansions have basis functions given by:

$$F1(x) = [x, x^2], F2(y) = [\cos(\pi y), \sin(\pi y)].$$

Create the two-dimensional basis function expansion. Note that ndBasis preserves the variable names you assigned to one-dimensional expansions.

```
F = ndBasis(F1,F2)
F = function_handle with value:
    @(x,y)utFcnBasisOuterProduct(FDATA ,x,y)
```

The array returned by F includes all multiplicative combinations of the basis functions:

$$F(x, y) = \left[x, x^2, \cos(\pi y), \cos(\pi y)x, \cos(\pi y)x^2, \sin(\pi y), x\sin(\pi y), x^2\sin(\pi y)\right].$$

To confirm this, evaluate F for x = 0.2, y = -0.3.

```
F(0.2, -0.3)
```

```
ans = 1×8
0.2000 0.0400 0.5878 0.1176 0.0235 -0.8090 -0.1618 -0.0324
```

## **Input Arguments**

#### F — Basis function expansion

function handle

Basis function expansion, specified as a function handle. The function must return a vector of basis functions of one or more scheduling variables. You can define these basis functions explicitly, or using polyBasis or fourierBasis.

```
Example: F = @(x)[x,x^2,x^3]
Example: F = polyBasis(3,2)
```

## **Output Arguments**

#### shapefcn — Basis function expansion

function handle

Basis function expansion, specified as a function handle. shapefcn takes as input arguments the total number of variables in F1, F2, . . . , FN. It returns a vector of functions of those variables, defined on the interval [-1,1] for each input variable. When you use shapefcn to create a gain surface, tunableSurface automatically generates tunable coefficients for each term in the vector.

### **Tips**

• The ndBasis operation is associative:

```
ndBasis(F1, ndBasis(F2, F3)) = ndBasis(ndBasis(F1, F2), F3) = ndBasis(F1, F2, F3)
```

#### See Also

fourierBasis | polyBasis | tunableSurface

#### Introduced in R2015b

#### ndims

Query number of dimensions of dynamic system model or model array

### **Syntax**

```
n = ndims(sys)
```

# **Description**

n = ndims(sys) is the number of dimensions of a dynamic system model or a model array sys. A single model has two dimensions (one for outputs, and one for inputs). A model array has 2 + p dimensions, where  $p \ge 2$  is the number of array dimensions. For example, a 2-by-3-by-4 array of models has 2 + 3 = 5 dimensions.

```
ndims(sys) = length(size(sys))
```

### **Examples**

#### **Determine Dimensions of Model Array**

Create a 3-by-1 array of random state-space models, each with 4 states, 1 input, and 1 output.

```
sys = rss(4,1,1,3);
```

Compute the number of dimensions of the model array.

```
ndims(sys)
ans = 4
```

The number of dimensions is 2+p, where p is the number of array dimensions. In this example, p is 2 because sys is 3-by-1.

# See Also

size

**Introduced before R2006a** 

# ngrid

Superimpose Nichols chart on Nichols plot

## **Syntax**

```
ngrid
ngrid('new')
ngrid(AX,____)
```

## **Description**

ngrid superimposes Nichols chart grid lines over the Nichols frequency response of a SISO LTI system. The range of the Nichols grid lines is set to encompass the entire Nichols frequency response.

The chart relates the complex number H/(1 + H) to H, where H is any complex number. For SISO systems, when H is a point on the open-loop frequency response, then the corresponding value of the closed-loop frequency response assuming unit negative feedback is

$$\frac{H}{1+H}$$

If the current axis is empty, ngrid generates a new Nichols chart grid in the region -40 dB to 40 dB in magnitude and -360 degrees to 0 degrees in phase. ngrid returns a warning if the current axis does not contain a SISO Nichols frequency response.

ngrid('new') clears the current axes first and sets hold on.

ngrid(AX, \_\_\_\_) plots the grid on the Axes or UIAxes object in the current figure with the handle AX. Use this syntax when creating apps with ngrid in the App Designer.

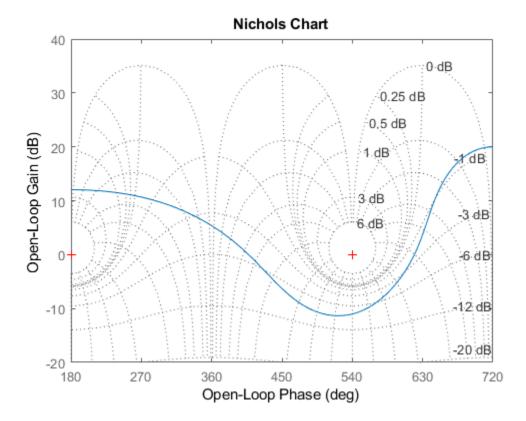
## **Examples**

#### **Nichols Response with Nichols Grid Lines**

Plot the Nichols response with Nichols grid lines for the following system:

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60} \,.$$

 $H = tf([-4\ 48\ -18\ 250\ 600],[1\ 30\ 282\ 525\ 60]);$  nichols(H) ngrid



The context menu for Nichols charts includes the **Tight** option under **Zoom**. You can use this option to clip unbounded branches of the Nichols chart.

# **Input Arguments**

#### **AX** — Object handle

Axes object | UIAxes object

Object handle, specified as an Axes or UIAxes object. Use AX to create apps with ngrid in the App Designer.

#### See Also

nichols

**Introduced before R2006a** 

#### nichols

Nichols chart of frequency response

### **Syntax**

```
nichols(sys)
nichols(sys,w)
nichols(sys1,sys2,...,sysN)
nichols(sys1,sys2,...,sysN,w)
nichols(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
[mag,phase,w] = nichols(sys)
[mag,phase] = nichols(sys,w)
```

## **Description**

nichols creates a Nichols chart of the frequency response. A Nichols chart displays the magnitude (in dB) plotted against the phase (in degrees) of the system response. Nichols charts are useful to analyze open- and closed-loop properties of SISO systems, but offer little insight into MIMO control loops. Use ngrid to superimpose a Nichols chart on an existing SISO Nichols chart.

nichols (sys) creates a Nichols chart of the dynamic system sys. This model can be continuous or discrete, SISO or MIMO. In the MIMO case, nichols produces an array of Nichols charts, each plot showing the response of one particular I/O channel. The frequency range and gridding are determined automatically based on the system poles and zeros.

nichols(sys,w) specifies the frequency range or frequency points to be used for the chart. To focus on a particular frequency interval [wmin,wmax], set w = {wmin,wmax}. To use particular frequency points, set w to the vector of desired frequencies. Use logspace to generate logarithmically spaced frequency vectors. Frequencies must be in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys.

nichols(sys1,sys2,...,sysN) or nichols(sys1,sys2,...,sysN,w) superimposes the Nichols charts of several models on a single figure. All systems must

have the same number of inputs and outputs, but may otherwise be a mix of continuousand discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

nichols(sys1, 'PlotStyle1',...,sysN, 'PlotStyleN').

See bode for an example.

[mag,phase,w] = nichols(sys) or [mag,phase] = nichols(sys,w) returns the
magnitude and phase (in degrees) of the frequency response at the frequencies w (in
rad/TimeUnit). The outputs mag and phase are 3-D arrays similar to those produced by
bode (see the bode reference page). They have dimensions

(number of outputs) × (number of inputs) × (length of w)

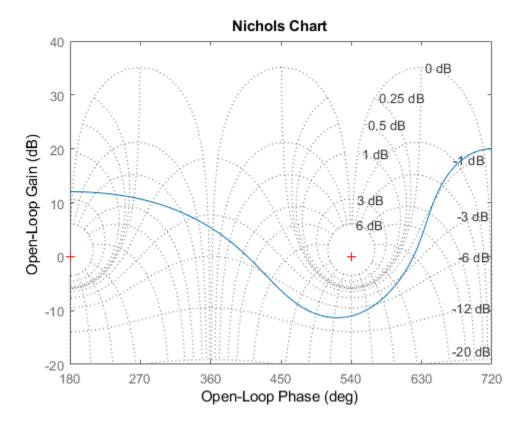
## **Examples**

#### Nichols Response with Nichols Grid Lines

Plot the Nichols response with Nichols grid lines for the following system:

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60} \,.$$

H = tf([-4 48 -18 250 600],[1 30 282 525 60]);
nichols(H)
ngrid



The context menu for Nichols charts includes the **Tight** option under **Zoom**. You can use this option to clip unbounded branches of the Nichols chart.

# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

# **Algorithms**

See bode.

#### See Also

Linear System Analyzer | bode | evalfr | freqresp | ngrid | nyquist | sigma

**Introduced before R2006a** 

# nicholsoptions

Create list of Nichols plot options

# **Syntax**

```
P = nicholsoptions
P = nicholsoptions('cstprefs')
```

## **Description**

P = nicholsoptions returns a list of available options for Nichols plots with default values set. You can use these options to customize the Nichols plot appearance from the command line.

P = nicholsoptions('cstprefs') initializes the plot options with the options you selected in the Control System Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the Nichols plot options.

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid	Show or hide the grid Specified as one of: 'off'   'on' Default: 'off'
GridColor	Color of the grid lines Specified as one of: Vector of RGB values in the range [0,1]  color shorthand such as 'k' or 'r' 'none'.  Default: [0.15,0.15,0.15]
XlimMode, YlimMode	Limit modes

Option	Description
Xlim, Ylim	Axes limits
IOGrouping	Grouping of input-output pairs Specified as one of: 'none'  'inputs' 'outputs' 'all' Default: 'none'
InputLabels, OutputLabels	Input and output label styles.
InputVisible, OutputVisible	Visibility of input and output channels

Option	Description
FreqUnits	Frequency units, specified as one of:
	• 'Hz'
	• 'rad/second'
	• 'rpm'
	• 'kHz'
	• 'MHz'
	• 'GHz'
	• 'rad/nanosecond'
	• 'rad/microsecond'
	• 'rad/millisecond'
	• 'rad/minute'
	• 'rad/hour'
	• 'rad/day'
	• 'rad/week'
	• 'rad/month'
	• 'rad/year'
	• 'cycles/nanosecond'
	• 'cycles/microsecond'
	• 'cycles/millisecond'
	• 'cycles/hour'
	• 'cycles/day'
	• 'cycles/week'
	• 'cycles/month'
	• 'cycles/year'
MagLowerLimMode	Enables a lower magnitude limit Specified as one of: 'auto'   'manual' Default: 'auto'
MagLowerLim	Specifies the lower magnitude limit

Option	Description
PhaseUnits	Phase units Specified as one of: 'deg'   'rad' Default: 'deg'
PhaseWrapping	Enables phase wrapping Specified as one of: 'on'   'off' Default: 'off'
PhaseWrappingBranch	Phase value at which the plot wraps accumulated phase when PhaseWrapping is set to 'on'. <b>Default</b> : -180 (phase wraps into the interval [-180°,180°))
PhaseMatching	Enables phase matching Specified as one of: 'on'   'off' Default: 'off'
PhaseMatchingFreq	Frequency for matching phase
PhaseMatchingValue	The value to make the phase responses close to

# **Examples**

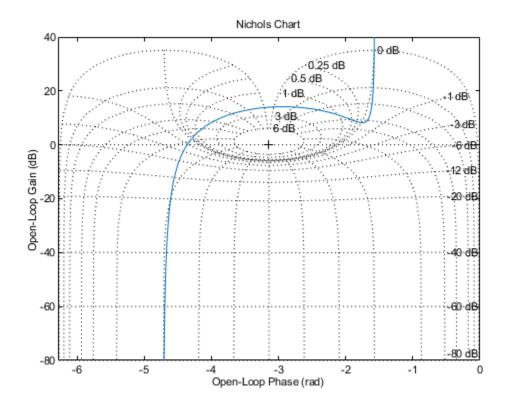
#### **Set Options for Nichols Plot**

Create an options set, and set the phase units and grid option.

```
P = nicholsoptions;
P.PhaseUnits = 'rad';
P.Grid = 'on';
```

Use the options set to generate a Nichols plot. Not the phase units and grid in the plot.

```
h = nicholsplot(tf(1,[1,.2,1,0]),P);
```



# See Also

getoptions | nicholsplot | setoptions

Introduced in R2008a

# nicholsplot

Plot Nichols frequency responses and return plot handle

### **Syntax**

```
h = nicholsplot(sys)
nicholsplot(sys,{wmin,wmax})
nicholsplot(sys,w)
nicholsplot(sys1,sys2,...,w)
nicholsplot(AX,...)
nicholsplot(..., plotoptions)
```

# **Description**

h = nicholsplot(sys) draws the Nichols plot of the dynamic system sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

help nicholsoptions

for a list of available plot options.

The frequency range and number of points are chosen automatically. See **bode** for details on the notion of frequency in discrete time.

nicholsplot(sys, {wmin,wmax}) draws the Nichols plot for frequencies between wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys).

nicholsplot(sys,w) uses the user-supplied vector w of frequencies, in rad/ TimeUnit, at which the Nichols response is to be evaluated. See logspace to generate logarithmically spaced frequency vectors.

nicholsplot(sys1,sys2,...,w) draws the Nichols plots of multiple models sys1,sys2,... on a single plot. The frequency vector w is optional. You can also specify a color, line style, and marker for each system, as in

```
nicholsplot(sys1,'r',sys2,'y--',sys3,'gx').
nicholsplot(AX,...) plots into the axes with handle AX.
nicholsplot(..., plotoptions) plots the Nichols plot with the options specified in plotoptions. Type
help nicholsoptions
for more details.
```

## **Examples**

Generate Nichols plot and use plot handle to change frequency units to Hz

```
sys = rss(5);
h = nicholsplot(sys);
% Change units to Hz
setoptions(h,'FreqUnits','Hz');
```

### **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

#### See Also

getoptions | nichols | nicholsoptions | setoptions

Introduced before R2006a

#### nmodels

Number of models in model array

### **Syntax**

```
N = nmodels(sysarray)
```

# **Description**

N = nmodels (sysarray) returns the number of models in an array of dynamic system models or static models.

# **Examples**

#### **Confirm Number of Models in Array**

```
Create a 2-by-3-by-4 array of state-space models.
```

```
sysarr = rss(2,2,2,2,3,4);
```

Confirm the number of models in the array.

```
N = nmodels(sysarr)
N = 24
```

### **Input Arguments**

#### sysarray — Input model array

model array

Input model array, specified as an array of input-output models such as numeric LTI models, generalized models, or identified LTI models.

# **Output Arguments**

#### N — Number of models in array

positive integer

Number of models in the input model array, returned as a positive integer.

#### **See Also**

ndims | size

Introduced in R2013a

#### norm

Norm of linear model

## **Syntax**

```
n = norm(sys)
n = norm(sys,2)

n = norm(sys,Inf)
[n,fpeak] = norm(sys,Inf)
[n,fpeak] = norm(sys,Inf,tol)
```

# **Description**

n = norm(sys) or n = norm(sys, 2) returns the root-mean-squares of the impulse response of the linear dynamic system model sys. This value is equivalent to the  $H_2$  norm on page 2-768 of sys.

n = norm(sys, Inf) returns the  $L_{\infty}$  norm on page 2-769 of sys, which is the peak gain of the frequency response of sys across frequencies. For MIMO systems, this quantity is the peak gain over all frequencies and all input directions, which corresponds to the peak value of the largest singular value of sys. For stable systems, the  $L_{\infty}$  norm is equivalent to the  $H_{\infty}$  norm. For more information, see hinfnorm.

[n,fpeak] = norm(sys,Inf) also returns the frequency fpeak at which the gain reaches its peak value.

[n,fpeak] = norm(sys,Inf,tol) sets the relative accuracy of the  $L_{\infty}$  norm to tol.

## **Examples**

#### **Compute Norm of Discrete-Time Linear System**

Compute the  $H_2$  and  $L_{\infty}$  norms of the following discrete-time transfer function, with sample time 0.1 second.

$$sys(z) = \frac{z^3 - 2.841z^2 + 2.875z - 1.004}{z^3 - 2.417z^2 + 2.003z - 0.5488}.$$

Compute the  $H_2$  norm of the transfer function. The  $H_2$  norm is the root-mean-square of the impulse response of sys.

```
sys = tf([1 -2.841 2.875 -1.004],[1 -2.417 2.003 -0.5488],0.1);

n2 = norm(sys)

n2 = 1.2438
```

Compute the  $L_{\infty}$  norm of the transfer function.

```
[ninf,fpeak] = norm(sys,Inf)
ninf = 2.5721
fpeak = 3.0178
```

Because sys is a stable system, ninf is the peak gain of the frequency response of sys, and fpeak is the frequency at which the peak gain occurs. Confirm these values using getPeakGain.

```
[gpeak,fpeak] = getPeakGain(sys)
gpeak = 2.5721
fpeak = 3.0178
```

### **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Input dynamic system, specified as any SISO or MIMO linear dynamic system model or model array. sys can be continuous-time or discrete-time.

#### tol — Relative accuracy

0.01 (default) | positive real scalar

Relative accuracy of the  $H_{\infty}$  norm, specified as a positive real scalar value.

## **Output Arguments**

#### $n - H_2$ or $L_{\infty}$ norm

scalar | array

 $H_2$  norm or  $L_{\infty}$  norm of sys, returned as a scalar or an array.

- If sys is a single model, then n is a scalar value.
- If sys is a model array, then n is an array of the same size as sys, where n(k) = norm(sys(:,:,k)).

#### fpeak — Frequency of peak gain

nonnegative real scalar | array of nonnegative real values

Frequency at which the gain achieves the peak value gpeak, returned as a nonnegative real scalar value or an array of nonnegative real values. The frequency is expressed in units of rad/TimeUnit, relative to the TimeUnit property of sys.

- If sys is a single model, then fpeak is a scalar.
- If sys is a model array, then fpeak is an array of the same size as sys, where fpeak(k) is the peak gain frequency of sys(:,:,k).

#### **More About**

#### H<sub>2</sub> norm

The  $H_2$  norm of a stable system H is the root-mean-square of the impulse response of the system. The  $H_2$  norm measures the steady-state covariance (or power) of the output response y = Hw to unit white noise inputs w:

$$||H||_2^2 = \lim_{t \to \infty} \{y(t)^T y(t)\}, \qquad E(w(t)w(\tau)^T) = \delta(t - \tau)I.$$

The  $H_2$  norm of a continuous-time system with transfer function H(s) is given by:

$$||H||_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Trace}[H(j\omega)^H H(j\omega)] d\omega}.$$

For a discrete-time system with transfer function H(z), the  $H_2$  norm is given by:

$$||H||_2 = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \operatorname{Trace}\left[H(e^{j\omega})^H H(e^{j\omega})\right] d\omega}$$
.

The  $H_2$  norm is infinite in the following cases:

- sys is unstable.
- sys is continuous and has a nonzero feedthrough (that is, nonzero gain at the frequency  $\omega = \infty$ ).

Using norm(sys) produces the same result as sqrt(trace(covar(sys,1))).

#### L-infinity norm

The  $L_{\infty}$  norm of a SISO linear system is the peak gain of the frequency response. For a MIMO system, the  $L_{\infty}$  norm is the peak gain across all input/output channels.

For a continuous-time system H(s), this definition means:

$$\|H(s)\|_{L_{\infty}} = \max_{\omega \in R} |H(j\omega)|$$
 (SISO)  
$$\|H(s)\|_{L_{\infty}} = \max_{\omega \in R} \sigma_{\max}(H(j\omega))$$
 (MIMO)

where  $\sigma_{\text{max}}(\cdot)$  denotes the largest singular value of a matrix.

For a discrete-time system H(z), the definition means:

$$\begin{split} \|H(z)\|_{L_{\infty}} &= \max_{\theta \in [0, 2\pi]} \left| H(e^{j\theta}) \right| & \text{(SISO)} \\ \|H(z)\|_{L_{\infty}} &= \max_{\theta \in [0, 2\pi]} \sigma_{\max} \big( H(e^{j\theta}) \big) & \text{(MIMO)} \end{split}$$

For stable systems, the  $L_{\infty}$  norm is equivalent to the  $H_{\infty}$  norm. For more information, see hinfnorm. For a system with unstable poles, the  $H_{\infty}$  norm is infinite. For all systems, norm returns the  $L_{\infty}$  norm, which is the peak gain without regard to system stability.

## **Algorithms**

After converting sys to a state space model, norm uses the same algorithm as covar for the  $H_2$  norm. For the  $L_\infty$  norm, norm uses the algorithm of [1]. norm computes the peak gain using the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

#### References

[1] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the  $H_{\infty}$ -Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287-293.

#### See Also

freqresp|getPeakGain|hinfnorm|sigma

Introduced before R2006a

# nyquist

Nyquist plot of frequency response

#### **Syntax**

```
nyquist(sys)
nyquist(sys,w)
nyquist(sys1,sys2,...,sysN)
nyquist(sys1,sys2,...,sysN,w)
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
[re,im,w,sdre,sdim] = nyquist(sys)
```

## **Description**

nyquist creates a Nyquist plot of the frequency response of a dynamic system model. When invoked without left-hand arguments, nyquist produces a Nyquist plot on the screen. Nyquist plots are used to analyze system properties including gain margin, phase margin, and stability.

nyquist(sys) creates a Nyquist plot of a dynamic system sys. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, nyquist produces an array of Nyquist plots, each plot showing the response of one particular I/O channel. The frequency points are chosen automatically based on the system poles and zeros.

nyquist(sys,w) explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval, set  $w = \{wmin, wmax\}$ . To use particular frequency points, set w to the vector of desired frequencies. Use logspace to generate logarithmically spaced frequency vectors. Frequencies must be in rad/ TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys.

nyquist(sys1,sys2,...,sysN) or nyquist(sys1,sys2,...,sysN,w) superimposes the Nyquist plots of several LTI models on a single figure. All systems must

have the same number of inputs and outputs, but may otherwise be a mix of continuousand discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
nyquist(sys1, 'PlotStyle1',...,sysN, 'PlotStyleN').
```

[re,im,w] = nyquist(sys) and [re,im] = nyquist(sys,w) return the real and imaginary parts of the frequency response at the frequencies w (in rad/TimeUnit). re and im are 3-D arrays (see "Arguments" below for details).

[re,im,w,sdre,sdim] = nyquist(sys) also returns the standard deviations of re and im for the identified system sys.

#### **Arguments**

The output arguments re and im are 3-D arrays with dimensions

```
(number of outputs) \times (number of inputs) \times (length of w)
```

For SISO systems, the scalars re(1,1,k) and im(1,1,k) are the real and imaginary parts of the response at the frequency  $\omega_k = w(k)$ .

```
re(1, 1, k) = Re(h(j\omega_k))

im(1, 1, k) = Im(h(jw_k))
```

For MIMO systems with transfer function H(s), re(:,:,k) and im(:,:,k) give the real and imaginary parts of  $H(j\omega_k)$  (both arrays with as many rows as outputs and as many columns as inputs). Thus,

```
re (i, j, k) = Re(h_{ij}(j\omega_k))
im (i, j, k) = Im(h_{ij}(j\omega_k))
```

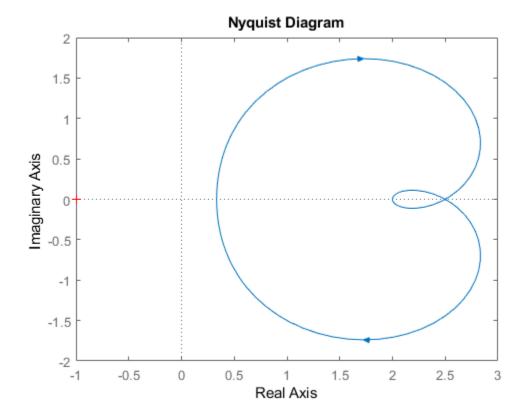
where  $h_{ij}$  is the transfer function from input j to output i.

#### **Examples**

#### Nyquist Plot of Dynamic System

Create the following transfer function and plot its Nyquist response.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}.$$
 H = tf([2 5 1],[1 2 3]); nyquist(H)



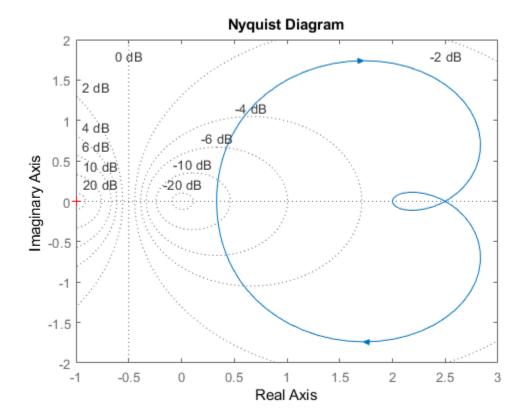
The nyquist function can display a grid of M-circles, which are the contours of constant closed-loop magnitude. M-circles are defined as the locus of complex numbers where the following quantity is a constant value across frequency.

$$T(j\omega) = \left| \frac{G(j\omega)}{1 + G(j\omega)} \right|.$$

Here,  $\omega$  is the frequency in radians/TimeUnit, where TimeUnit is the system time units, and G is the collection of complex numbers that satisfy the constant magnitude requirement.

To display the grid of M-circles, right-click in the plot and select  $\mathbf{Grid}$ . Alternatively, use the  $\mathtt{grid}$  command.

grid on



#### **Create Nyquist Plot of Identified Model With Response Uncertainty**

Compute the standard deviations of the real and imaginary parts of the frequency response of an identified model. Use this data to create a  $3\sigma$  plot of the response uncertainty.

Load estimation data z2.

```
load iddata2 z2:
```

Identify a transfer function model using the data.

```
sys p = tfest(z2,2);
```

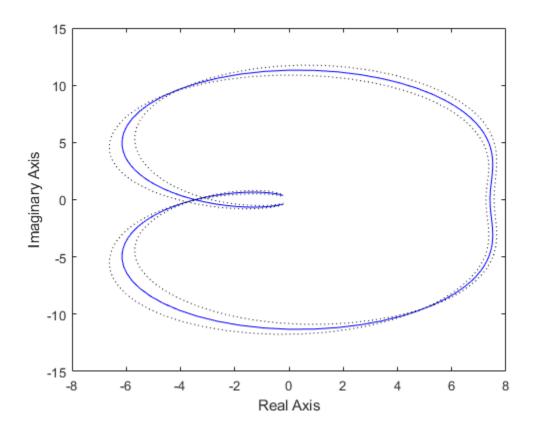
Obtain the standard deviations for the real and imaginary parts of the frequency response for a set of 512 frequencies, w.

```
w = linspace(-10*pi,10*pi,512);
[re,im,wout,sdre,sdim] = nyquist(sys_p,w);
```

Here re and im are the real and imaginary parts of the frequency response, and sdre and sdim are their standard deviations, respectively. The frequencies in wout are the same as the frequencies you specified in w.

Create a Nyquist plot showing the response and its  $3\sigma$  uncertainty.

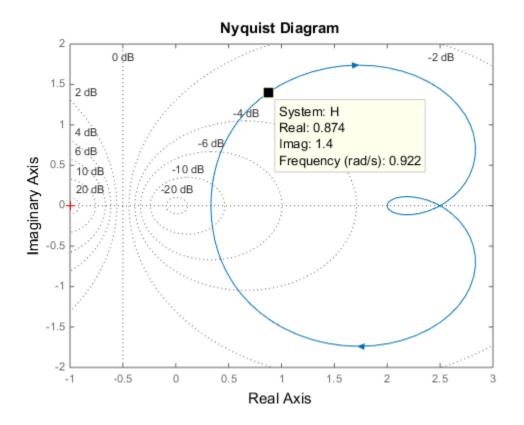
```
re = squeeze(re);
im = squeeze(im);
sdre = squeeze(sdre);
sdim = squeeze(sdim);
plot(re,im,'b',re+3*sdre,im+3*sdim,'k:',re-3*sdre,im-3*sdim,'k:')
xlabel('Real Axis');
ylabel('Imaginary Axis');
```



# **Tips**

- You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots". For the most flexibility in customizing plot properties, use the nyquistplot command instead of nyquist.
- There are two zoom options available from the right-click menu that apply specifically to Nyquist plots:
  - **Full View** Clips unbounded branches of the Nyquist plot, but still includes the critical point (-1, 0).

- **Zoom on (-1,0)** Zooms around the critical point (-1,0). (To access critical-point zoom programmatically, use nyquistplot instead.)
- To activate data markers that display the real and imaginary values at a given frequency, click anywhere on the curve. The following figure shows a nyquist plot with a data marker.



# **Algorithms**

See bode.

## **See Also**

Linear System Analyzer | bode | evalfr | freqresp | nichols | sigma

**Introduced before R2006a** 

# nyquistoptions

List of Nyquist plot options

## **Syntax**

P = nyquistoptions
P = nyquistoptions('cstprefs')

## **Description**

P = nyquistoptions returns the default options for Nyquist plots. You can use these options to customize the Nyquist plot appearance using the command line.

P = nyquistoptions('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

The following table summarizes the Nyquist plot options.

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid	Show or hide the grid Specified as one of the following values: 'off'   'on' Default: 'off'
GridColor	Color of the grid lines Specified as one of the following: Vector of RGB values in the range [0,1]   character vector of color name   'none'. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'.  Default: [0.15,0.15,0.15]
XlimMode, YlimMode	Limit modes

Option	Description
Xlim, Ylim	Axes limits
IOGrouping	Grouping of input-output pairs Specified as one of the following values: 'none'  'inputs' 'outputs' 'all' Default: 'none'
InputLabels, OutputLabels	Input and output label styles
InputVisible, OutputVisible	Visibility of input and output channels

Option	Description
FreqUnits	Frequency units, specified as one of the following values:
	• 'Hz'
	• 'rad/second'
	• 'rpm'
	• 'kHz'
	• 'MHz'
	• 'GHz'
	• 'rad/nanosecond'
	• 'rad/microsecond'
	• 'rad/millisecond'
	• 'rad/minute'
	• 'rad/hour'
	• 'rad/day'
	• 'rad/week'
	• 'rad/month'
	• 'rad/year'
	• 'cycles/nanosecond'
	• 'cycles/microsecond'
	• 'cycles/millisecond'
	• 'cycles/hour'
	• 'cycles/day'
	• 'cycles/week'
	• 'cycles/month'
	• 'cycles/year'
	Default: 'rad/s'
	You can also specify 'auto' which uses frequency units rad/TimeUnit relative to system time units specified in the TimeUnit property. For

Option	Description
	multiple systems with different time units, the units of the first system are used.
MagUnits	Magnitude units Specified as one of the following values: 'dB'   'abs' Default: 'dB'
PhaseUnits	Phase units Specified as one of the following values: 'deg'   'rad' Default: 'deg'
ShowFullContour	Show response for negative frequencies Specified as one of the following values: 'on'   'off' Default: 'on'
ConfidenceRegionNumb erSD	Number of standard deviations to use to plotting the response confidence region (identified models only).  Default: 1.
ConfidenceRegionDispla ySpacing	The frequency spacing of confidence ellipses. For identified models only. <b>Default</b> : 5, which means the confidence ellipses are shown at every 5th frequency sample.

# **Examples**

This example shows how to create a Nyquist plot displaying the full contour (the response for both positive and negative frequencies).

```
P = nyquistoptions;
P.ShowFullContour = 'on';
h = nyquistplot(tf(1,[1,.2,1]),P);
```

#### See Also

getoptions | nyquist | nyquistplot | setoptions

Introduced in R2011a

# nyquistplot

Nyquist plot with additional plot customization options

#### **Syntax**

```
h = nyquistplot(sys)
nyquistplot(sys,{wmin,wmax})
nyquistplot(sys,w)
nyquistplot(sys1,sys2,...,w)
nyquistplot(AX,...)
nyquistplot(..., plotoptions)
```

## **Description**

h = nyquistplot(sys) draws the Nyquist plot of the dynamic system model sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

help nyquistoptions

for a list of available plot options.

The frequency range and number of points are chosen automatically. See **bode** for details on the notion of frequency in discrete time.

nyquistplot(sys, {wmin, wmax}) draws the Nyquist plot for frequencies between
wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input
dynamic system, specified in the TimeUnit property of sys).

nyquistplot(sys,w) uses the user-supplied vector w of frequencies (in rad/ TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys) at which the Nyquist response is to be evaluated. See logspace to generate logarithmically spaced frequency vectors.

nyquistplot(sys1,sys2,...,w) draws the Nyquist plots of multiple models sys1,sys2,... on a single plot. The frequency vector w is optional. You can also specify a color, line style, and marker for each system, as in

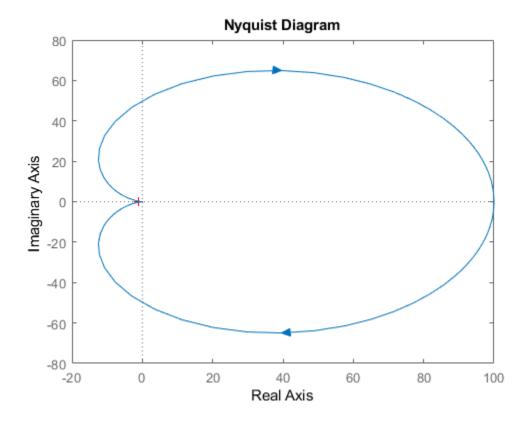
```
nyquistplot(sys1,'r',sys2,'y--',sys3,'gx')
nyquistplot(AX,...) plots into the axes with handle AX.
nyquistplot(..., plotoptions) plots the Nyquist response with the options specified in plotoptions. Type
help nyquistoptions
for more details.
```

# **Examples**

#### **Customize Nyquist Plot Title**

Create a Nyquist plot of a dynamic system model and store a handle to the plot.

```
sys = tf(100,[1,2,1]);
h = nyquistplot(sys);
```



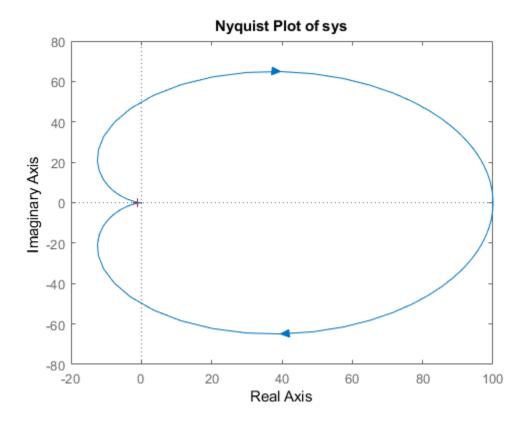
Change the plot title to read "Nyquist Plot of sys." To do so, use getoptions to extract the existing plot options from the plot handle h.

The Title option is a structure with several fields.

#### opt.Title

Change the String field of the Title structure, and use setoptions to apply the change to the plot.

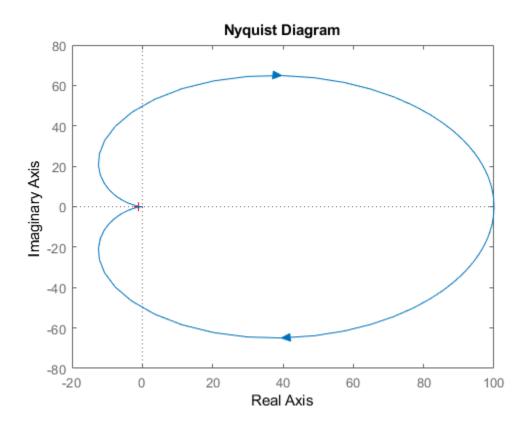
```
opt.Title.String = 'Nyquist Plot of sys';
setoptions(h,opt)
```



#### **Zoom on Critical Point**

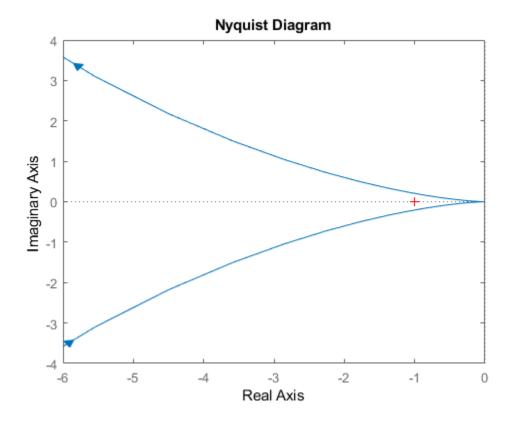
Plot the Nyquist frequency response of a dynamic system. Assign a variable name to the plot handle so that you can access it for further manipulation.

```
sys = tf(100,[1,2,1]);
h = nyquistplot(sys);
```



Zoom in on the critical point, (-1,0). You can do so interactively by right-clicking on the plot and selecting **Zoom on (-1,0)**. Alternatively, use the **zoomcp** command on the plot handle h.

zoomcp(h)



# Plot Identified Models with Confidence Regions at Selected Points

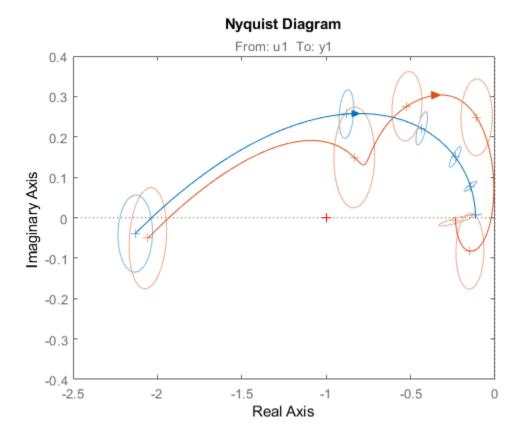
Compare the frequency responses of identified state-space models of order 2 and 6 along with their 1-std confidence regions rendered at every 50th frequency sample.

```
load iddata1
sys1 = n4sid(z1,2); % discrete-time IDSS model of order 2
sys2 = n4sid(z1,6); % discrete-time IDSS model of order 6
```

Both models produce about 76% fit to data. However, sys2 shows higher uncertainty in its frequency response, especially close to Nyquist frequency as shown by the plot. To see

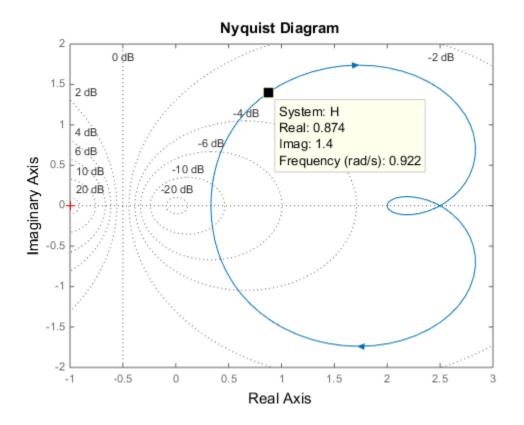
this, generate a Nyquist plot and show the confidence region at a subset of the points at which the Nyquist response is displayed.

To turn on the confidence region display, right-click the plot and select **Characteristics** > **Confidence Region**.



#### **Tips**

- You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".
- There are two zoom options available from the right-click menu that apply specifically to Nyquist plots:
  - **Full View** Clips unbounded branches of the Nyquist plot, but still includes the critical point (-1, 0).
  - **Zoom on (-1,0)** Zooms around the critical point (-1,0). To access critical-point zoom programmatically, use the zoomcp command. See "Zoom on Critical Point" on page 2-787.
- To activate data markers that display the real and imaginary values at a given frequency, click anywhere on the curve. The following figure shows a nyquist plot with a data marker.



**See Also** getoptions | nyquist | setoptions

Introduced before R2006a

#### obsv

Observability matrix

#### **Syntax**

```
obsv(A,C)
Ob = obsv(sys)
```

# **Description**

obsv computes the observability matrix for state-space systems. For an n-by-n matrix A and a p-by-n matrix C, obsv(A,C) returns the observability matrix

$$Ob = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

with n columns and np rows.

0b = obsv(sys) calculates the observability matrix of the state-space model sys. This syntax is equivalent to executing

$$0b = obsv(sys.A, sys.C)$$

The model is observable if 0b has full rank n.

#### **Examples**

Determine if the pair

```
4 -2
C =
    1    0
    0    1

is observable. Type

Ob = obsv(A,C);
% Number of unobservable states
unob = length(A)-rank(Ob)

These commands produce the following result.
unob =
    0
```

#### **Tips**

obsv is here for educational purposes and is not recommended for serious control design. Computing the rank of the observability matrix is not recommended for observability testing. Ob will be numerically singular for most systems with more than a handful of states. This fact is well-documented in the control literature. For example, see Section III in [1].

#### References

[1] Paige, C. C. "Properties of Numerical Algorithms Related to Computing Controllability." *IEEE Transactions on Automatic Control*. Vol. 26, Number 1, 1981, pp. 130-138.

#### See Also

obsvf

Introduced before R2006a

#### obsvf

Compute observability staircase form

#### **Syntax**

[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C) obsvf(A,B,C,tol)

## **Description**

If the observability matrix of (A,C) has rank  $r \le n$ , where n is the size of A, then there exists a similarity transformation such that

$$\bar{A} = TAT^T$$
,  $\bar{B} = TB$ ,  $\bar{C} = CT^T$ 

where T is unitary and the transformed system has a *staircase* form with the unobservable modes, if any, in the upper left corner.

$$\bar{A} = \begin{bmatrix} A_{no} & A_{12} \\ 0 & A_o \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B_{no} \\ B_o \end{bmatrix}, \ \bar{C} = \begin{bmatrix} 0 & C_o \end{bmatrix}$$

where  $(C_o, A_o)$  is observable, and the eigenvalues of  $A_{no}$  are the unobservable modes.

[Abar, Bbar, Cbar, T, k] = obsvf(A, B, C) decomposes the state-space system with matrices A, B, and C into the observability staircase form Abar, Bbar, and Cbar, as described above. T is the similarity transformation matrix and k is a vector of length n, where n is the number of states in A. Each entry of k represents the number of observable states factored out during each step of the transformation matrix calculation [1]. The number of nonzero elements in k indicates how many iterations were necessary to calculate T, and sum(k) is the number of states in  $A_o$ , the observable portion of Abar.

obsvf(A,B,C,tol) uses the tolerance tol when calculating the observable/unobservable subspaces. When the tolerance is not specified, it defaults to 10\*n\*norm(a,1)\*eps.

# **Examples**

Form the observability staircase form of

$$A = \begin{bmatrix} & & & & & & \\ & & 1 & & 1 & \\ & & 4 & & -2 & \\ & & 1 & & -1 & \\ & & 1 & & -1 & \\ & & C & = & & \\ & & & 0 & & 1 & \\ \end{bmatrix}$$

by typing

# **Algorithms**

 ${\tt obsvf} \ implements \ the \ Staircase \ Algorithm \ of \ [1] \ by \ calling \ {\tt ctrbf} \ and \ using \ duality.$ 

#### **References**

[1] Rosenbrock, M.M., State-Space and Multivariable Theory, John Wiley, 1970.

#### See Also

ctrbf|obsv

**Introduced before R2006a** 

#### ord2

Generate continuous second-order systems

#### **Syntax**

```
[A,B,C,D] = ord2(wn,z)

[num,den] = ord2(wn,z)
```

## **Description**

[A,B,C,D] = ord2(wn,z) generates the state-space description (A,B,C,D) of the second-order system

$$h(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

given the natural frequency wn  $(\omega_n)$  and damping factor z  $(\zeta)$ . Use ss to turn this description into a state-space object.

[num,den] = ord2(wn,z) returns the numerator and denominator of the second-order transfer function. Use tf to form the corresponding transfer function object.

#### **Examples**

To generate an LTI model of the second-order transfer function with damping factor  $\zeta = 0.4$  and natural frequency  $\omega_n = 2.4$  rad/sec., type

s^2 + 1.92 s + 5.76

# **See Also**

rss|ss|tf

**Introduced before R2006a** 

#### order

Query model order

#### **Syntax**

```
NS = order(sys)
```

## **Description**

NS = order(sys) returns the model order NS. The order of a dynamic system model is the number of poles (for proper transfer functions) or the number of states (for state-space models). For improper transfer functions, the order is defined as the minimum number of states needed to build an equivalent state-space model (ignoring pole/zero cancellations).

order(sys) is an overloaded method that accepts SS, TF, and ZPK models. For LTI arrays, NS is an array of the same size listing the orders of each model in sys.

#### **Caveat**

order does not attempt to find minimal realizations of MIMO systems. For example, consider this 2-by-2 MIMO system:

```
s=tf('s');
h = [1, 1/(s*(s+1)); 1/(s+2), 1/(s*(s+1)*(s+2))];
order(h)
ans =
6
```

Although h has a 3rd order realization, order returns 6. Use

```
order(ss(h,'min'))
```

to find the minimal realization order.

# **See Also**

balred | pole

Introduced in R2012a

# pade

Padé approximation of model with time delays

#### **Syntax**

```
[num,den] = pade(T,N)
pade(T,N)
sysx = pade(sys,N)
sysx = pade(sys,NU,NY,NINT)
```

# **Description**

pade approximates time delays by rational models. Such approximations are useful to model time delay effects such as transport and computation delays within the context of continuous-time systems. The Laplace transform of a time delay of T seconds is  $\exp(-sT)$ . This exponential transfer function is approximated by a rational transfer function using Padé approximation formulas [1].

[num,den] = pade(T,N) returns the Padé approximation of order N of the continuous-time I/O delay  $\exp(-sT)$  in transfer function form. The row vectors num and den contain the numerator and denominator coefficients in descending powers of s. Both are Nth-order polynomials.

When invoked without output arguments, pade(T,N) plots the step and phase responses of the Nth-order Padé approximation and compares them with the exact responses of the model with I/O delay T. Note that the Padé approximation has unit gain at all frequencies.

sysx = pade(sys,N) produces a delay-free approximation sysx of the continuous
delay system sys. All delays are replaced by their Nth-order Padé approximation. See
"Time Delays in Linear Systems" for more information about models with time delays.

sysx = pade(sys,NU,NY,NINT) specifies independent approximation orders for each
input, output, and I/O or internal delay. Here NU, NY, and NINT are integer arrays such
that

- NU is the vector of approximation orders for the input channel
- NY is the vector of approximation orders for the output channel
- NINT is the approximation order for I/O delays (TF or ZPK models) or internal delays (state-space models)

You can use scalar values for NU, NY, or NINT to specify a uniform approximation order. You can also set some entries of NU, NY, or NINT to Inf to prevent approximation of the corresponding delays.

#### **Examples**

#### **Third-Order Padé Approximation**

Compute a third-order Padé approximation of a 0.1-second I/O delay.

```
s = tf('s');
sys = exp(-0.1*s);
sysx = pade(sys,3)

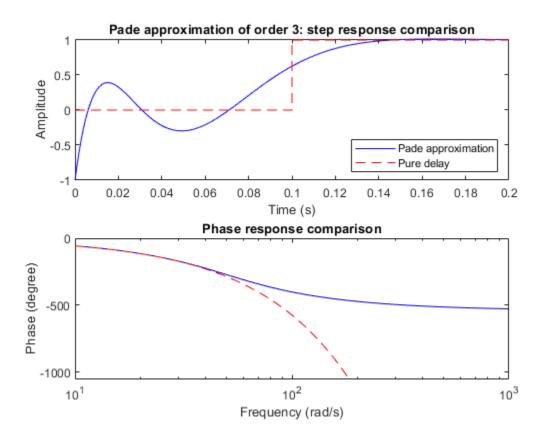
sysx =
    -s^3 + 120 s^2 - 6000 s + 1.2e05
    s^3 + 120 s^2 + 6000 s + 1.2e05

Continuous-time transfer function.
```

Here, sys is a dynamic system representation of the exact time delay of 0.1 s. sysx is a transfer function that approximates that delay.

Compare the time and frequency responses of the true delay and its approximation. Calling the pade command without output arguments generates the comparison plots. In this case the first argument to pade is just the magnitude of the exact time delay, rather than a dynamic system representing the time delay.

```
pade(0.1,3)
```



## **Limitations**

High-order Padé approximations produce transfer functions with clustered poles. Because such pole configurations tend to be very sensitive to perturbations, Padé approximations with order N>10 should be avoided.

#### References

[1] Golub, G. H. and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore, 1989, pp. 557-558.

#### **See Also**

absorbDelay | c2d | thiran

#### **Topics**

"Time-Delay Approximation"

**Introduced before R2006a** 

# parallel

Parallel connection of two models

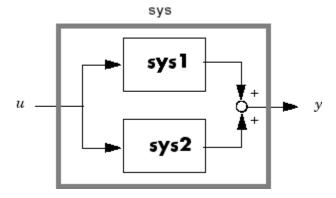
## **Syntax**

```
parallel
sys = parallel(sys1,sys2)
sys = parallel(sys1,sys2,inp1,inp2,out1,out2)
sys = parallel(sys1,sys2,'name')
```

## **Description**

parallel connects two model objects in parallel. This function accepts any type of model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

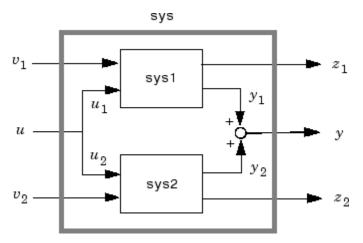
sys = parallel(sys1, sys2) forms the basic parallel connection shown in the following figure.



This command equals the direct addition

$$sys = sys1 + sys2$$

sys = parallel(sys1,sys2,inp1,inp2,out1,out2) forms the more general
parallel connection shown in the following figure.



The vectors inp1 and inp2 contain indexes into the input channels of sys1 and sys2, respectively, and define the input channels  $u_1$  and  $u_2$  in the diagram. Similarly, the vectors out1 and out2 contain indexes into the outputs of these two systems and define the output channels  $y_1$  and  $y_2$  in the diagram. The resulting model sys has  $[v_1; u; v_2]$  as inputs and  $[z_1; y; z_2]$  as outputs.

sys = parallel(sys1,sys2,'name') connects sys1 and sys2 by matching I/O names. You must specify all I/O names of sys1 and sys2. The matching names appear in sys in the same order as in sys1. For example, the following specification:

```
 sys1 = ss(eye(3), 'InputName', {'C', 'B', 'A'}, 'OutputName', {'Z', 'Y', 'X'}); \\ sys2 = ss(eye(3), 'InputName', {'A', 'C', 'B'}, 'OutputName', {'X', 'Y', 'Z'}); \\ parallel(sys1, sys2, 'name')
```

returns this result:

d =

C B A

Z 1 1 0

Y 1 1 0

X 0 0 2

Static gain.

**Note** If sys1 and sys2 are model arrays, parallel returns model array sys of the same size, where sys(:,:,k)=parallel(sys1(:,:,k),sys2(:,:,k),inp1,...).

# **Examples**

See Kalman Filtering for an example.

## See Also

append | feedback | series

**Introduced before R2006a** 

# particleFilter

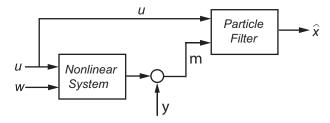
Particle filter object for online state estimation

# **Description**

A particle filter is a recursive, Bayesian state estimator that uses discrete particles to approximate the posterior distribution of an estimated state. It is useful for online state estimation when measurements and a system model, that relates model states to the measurements, are available. The particle filter algorithm computes the state estimates recursively and involves initialization, prediction, and correction steps.

particleFilter creates an object for online state estimation of a discrete-time nonlinear system using the discrete-time particle filter algorithm.

Consider a plant with states x, input u, output m, process noise w, and measurement y. Assume that you can represent the plant as a nonlinear system.



The algorithm computes the state estimates  $\hat{x}$  of the nonlinear system using the state transition and measurement likelihood functions you specify.

The software supports arbitrary nonlinear state transition and measurement models, with arbitrary process and measurement noise distributions.

To perform online state estimation, create the nonlinear state transition function and measurement likelihood function. Then construct the particleFilter object using these nonlinear functions. After you create the object:

- 1 Initialize the particles using the initialize command.
- **2** Predict state estimates at the next step using the predict command.

**3** Correct the state estimates using the correct command.

The prediction step uses the latest state to predict the next state based on the state transition model you provide. The correction step uses the current sensor measurement to correct the state estimate. The algorithm optionally redistributes, or resamples, the particles in the state space to match the posterior distribution of the estimated state. Each particle represents a discrete state hypothesis of these state variables. The set of all particles is used to help determine the state estimate.

## Creation

# **Syntax**

pf = particleFilter(StateTransitionFcn, MeasurementLikelihoodFcn)

## **Object Description**

pf = particleFilter(StateTransitionFcn, MeasurementLikelihoodFcn)
creates a particle filter object for online state estimation of a discrete-time nonlinear
system. StateTransitionFcn is a function that calculates the particles (state
hypotheses) at the next time step, given the state vector at a time step.
MeasurementLikelihoodFcn is a function that calculates the likelihood of each particle
based on sensor measurements.

After creating the object, use the initialize command to initialize the particles with a known mean and covariance or uniformly distributed particles within defined bounds. Then, use the correct and predict commands to update particles (and hence the state estimate) using sensor measurements.

## **Input Arguments**

#### StateTransitionFcn — State transition function

function handle

State transition function, specified as a function handle, determines the transition of particles (state hypotheses) between time steps. Also a property of the particleFilter object. For more information, see "Properties" on page 2-811.

#### MeasurementLikelihoodFcn — Measurement likelihood function

function handle

Measurement likelihood function, specified as a function handle, is used to calculate the likelihood of particles (state hypotheses) from sensor measurements. Also a property of the particleFilter object. For more information, see "Properties" on page 2-811.

# **Properties**

### NumStateVariables — Number of state variables

[] (default) | scalar

Number of state variables, specified as a scalar. This property is read-only and is set using initialize. The number of states is implicit based on the specified matrices for the initial mean of particles, or the state bounds.

## NumParticles — Number of particles used in the filter

[] (default) | scalar

Number of particles used in the filter, specified as a scalar. Each particle represents a state hypothesis. You specify this property only by using initialize.

## StateTransitionFcn — State transition function

function handle

State transition function, specified as a function handle, determines the transition of particles (state hypotheses) between time steps. This function calculates the particles at the next time step, including the process noise, given particles at a time step.

In contrast, the state transition function for the extendedKalmanFilter and unscentedKalmanFilter generates a single state estimate at a given time step.

You write and save the state transition function for your nonlinear system, and specify it as a function handle when constructing the particleFilter object. For example, if vdpParticleFilterStateFcn. m is the state transition function, specify StateTransitionFcn as @vdpParticleFilterStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The function signature is as follows:

function predictedParticles = myStateTransitionFcn(previousParticles, varargin)

The StateTransitionFcn function accepts at least one input argument. The first argument is the set of particles previousParticles that represents the state hypotheses at the previous time step. The optional use of varargin in the function enables you to input any extra parameters that are relevant for predicting the next state, using predict, as follows:

predict(pf,arg1,arg2)

If StateOrientation is 'column', then previousParticles is a NumStateVariables-by-NumParticles array. If StateOrientation is 'row', then previousParticles is a NumParticles-by-NumStateVariables array.

StateTransitionFcn must return exactly one output, predictedParticles, which is the set of predicted particle locations for the current time step (array with same dimensions as previousParticles).

StateTransitionFcn must include the random process noise (from any distribution suitable for your application) in the predictedParticles.

To see an example of a state transition function with the StateOrientation property set to 'column', type edit vdpParticleFilterStateFcn at the command line.

# MeasurementLikelihoodFcn — Measurement likelihood function function handle

Measurement likelihood function, specified as a function handle, is used to calculate the likelihood of particles (state hypotheses) using the sensor measurements. For each state hypothesis (particle), the function first calculates an N-element measurement hypothesis vector. Then the likelihood of each measurement hypothesis is calculated based on the sensor measurement and the measurement noise probability distribution.

In contrast, the measurement function for extendedKalmanFilter and unscentedKalmanFilter takes a single state hypothesis and returns a single measurement estimate.

You write and save the measurement likelihood function based on your measurement model, and use it to construct the object. For example, if vdpMeasurementLikelihoodFcn.m is the measurement likelihood function, specify MeasurementLikelihoodFcn as @vdpMeasurementLikelihoodFcn. You can also specify MeasurementLikelihoodFcn as a function handle to an anonymous function.

The function signature is as follows:

function likelihood = myMeasurementLikelihoodFcn(predictedParticles,measurement,varargin)

The MeasurementLikelihoodFcn function accepts at least two input arguments. The first argument is the set of particles predictedParticles that represents the predicted state hypothesis. If StateOrientation is 'column', then predictedParticles is a NumStateVariables-by-NumParticles array. If StateOrientation is 'row', then predictedParticles is a NumParticles-by-NumStateVariables array. The second argument, measurement, is the N-element sensor measurement at the current time step. You can provide additional input arguments using varargin.

The MeasurementLikelihoodFcn must return exactly one output, likelihood, a vector with NumParticles length, which is the likelihood of the given measurement for each particle (state hypothesis).

To see an example of a measurement likelihood function, type edit vdpMeasurementLikelihoodFcn at the command line.

# IsStateVariableCircular — Whether the state variables have a circular distribution

[] (default) | logical array

Whether the state variables have a circular distribution, specified as a logical array.

This is a read-only property and is set using initialize.

Circular (or angular) distributions use a probability density function with a range of [-pi,pi]. IsStateVariableCircular is a row-vector with NumStateVariables elements. Each vector element indicates whether the associated state variable is circular.

# ResamplingPolicy — Policy settings that determine when to trigger resampling particleResamplingPolicy object

Policy settings that determine when to trigger resampling, specified as a particleResamplingPolicy object.

The resampling of particles is a vital step in estimating states using a particle filter. It enables you to select particles based on the current state, instead of using the particle distribution given at initialization. By continuously resampling the particles around the current estimate, you can get more accurate tracking and improve long-term performance.

You can trigger resampling either at fixed intervals or dynamically, based on the number of effective particles. The minimum effective particle ratio is a measure of how well the

current set of particles approximates the posterior distribution. The number of effective particles is calculated by:

$$N_{eff} = \frac{1}{\sum_{i=1}^{N} \left(w^{i}\right)^{2}}$$

In this equation, N is the number of particles, and w is the normalized weight of each particle. The effective particle ratio is then  $N_{\text{eff}}$  / NumParticles. Therefore, the effective particle ratio is a function of the weights of all the particles. After the weights of the particles reach a low enough value, they are not contributing to the state estimation. This low value triggers resampling, so the particles are closer to the current state estimation and have higher weights.

The following properties of the particleResamplingPolicy object can be modified to control when resampling is triggered:

# **D**escription At is a method to determine when resampling occurs, based on the value chosen. The minterval' value triggers resampling at regular time steps of the particle filter aperation. The 'ratio' value triggers resampling based on the ratio of effective total **p**articles. е ť M ŧ ħ a tl b t n t е r V а ι

# peer tty y Sixed interval between resampling, specified as a scalar. This interval determines during which correction steps the resampling is executed. For example, a value of 2 means the desampling is executed every second correction step. A value of inf means that pesampling is never executed. In This property only applies with the TriggerMethod is set to 'interval'. In tee Tr V a l

```
Bescription
Mis the minimum desired ratio of the effective number of particles to the total number of
particles NumParticles. The effective number of particles is a measure of how well the
Eurrent set of particles approximates the posterior distribution. A lower effective particle
Fatio implies that a lower number of particles are contributing to the estimation and
fesampling is required.
If the ratio of the effective number of particles to the total number of particles
NumParticles falls below the MinEffectiveParticleRatio, a resampling step is
triggered.
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```

## ResamplingMethod — Method used for particle resampling

'multinomial' (default) | 'residual' | 'stratified' | 'systematic'

Method used for particle resampling, specified as one of the following:

'multinomial'

- 'residual'
- 'stratified'
- 'systematic'

# StateEstimationMethod — Method used for extracting a state estimate from particles

```
'mean' (default) | 'maxweight'
```

Method used for extracting a state estimate from particles, specified as one of the following:

- 'mean' The object outputs the weighted mean of the particles, depending on the properties Weights and Particles, as the state estimate.
- 'maxweight' The object outputs the particle with the highest weight as the state estimate.

## Particles — Array of particle values

```
[] (default) | array
```

Array of particle values, specified as an array based on the StateOrientation property:

- If StateOrientation is 'row' then Particles is an NumParticles-by-NumStateVariables array.
- If StateOrientation is 'column' then Particles is an NumStateVariables-by-NumParticles array.

Each row or column corresponds to a state hypothesis (a single particle).

## Weights — Particle weights

```
[](default)|vector
```

Particle weights, defined as a vector based on the value of the StateOrientation property:

- If StateOrientation is 'row' then Weights is a NumParticles-by-1 vector, where each weight is associated with the particle in the same row in the Particles property.
- If StateOrientation is 'column' then Weights is a 1-by-NumParticles vector, where each weight is associated with the particle in the same column in the Particles property.

#### State — Current state estimate

[] (default) | vector

Current state estimate, defined as a vector based on the value of the StateOrientation property:

- If StateOrientation is 'row' then State is a 1-by-NumStateVariables vector
- If StateOrientation is 'column' then State is a NumStateVariables-by-1 vector

State is a read-only property, and is derived from Particles based on the StateEstimationMethod property. Refer to "StateEstimationMethod" on page 2-0 for details on how the value of State is determined.

State along with StateCovariance can also be determined using getStateEstimate.

## StateCovariance — Current estimate of state estimation error covariance

NumStateVariables-by-NumStateVariables array (default) | [] | array

Current estimate of state estimation error covariance, defined as an NumStateVariables-by-NumStateVariables array. StateCovariance is a read-only property and is calculated based on the StateEstimationMethod. If you specify a state estimation method that does not support covariance, then the function returns StateCovariance as [].

StateCovariance and State can be determined together using getStateEstimate.

## **Object Functions**

initialize Initialize the state of the particle filter

predict Predict state and state estimation error covariance at next time step

using extended or unscented Kalman filter, or particle filter

correct Correct state and state estimation error covariance using extended or

unscented Kalman filter, or particle filter and measurements

getStateEstimate Extract best state estimate and covariance from particles

clone Copy online state estimation object

# **Examples**

## **Create Particle Filter Object for Online State Estimation**

To create a particle filter object for estimating the states of your system, create appropriate state transition function and measurement likelihood function for the system.

In this example, the function vdpParticleFilterStateFcn describes a discrete-time approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1. In addition, it models Gaussian process noise. vdpMeasurementLikelihood function calculates the likelihood of particles from the noisy measurements of the first state, assuming a Gaussian measurement noise distribution.

Create the particle filter object. Use function handles to provide the state transition and measurement likelihood functions to the object.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

To initialize and estimate the states and state estimation error covariance from the constructed object, use the initialize, predict, and correct commands.

## **Estimate States Online using Particle Filter**

Load the van der Pol ODE data, and specify the sample time.

vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time dt = 0.05.

```
addpath(fullfile(matlabroot, 'examples', 'control', 'main')) % add example data
load ('vdp0DEdata.mat', 'xTrue', 'dt')
tSpan = 0:dt:5;
```

Get the measurements. For this example, a sensor measures the first state with a Gaussian noise with standard deviation 0.04.

```
sqrtR = 0.04;
yMeas = xTrue(:,1) + sqrtR*randn(numel(tSpan),1);
```

Create a particle filter, and set the state transition and measurement likelihood functions.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF,1000,[2;0],eye(2));
```

Pick the mean state estimation and systematic resampling methods.

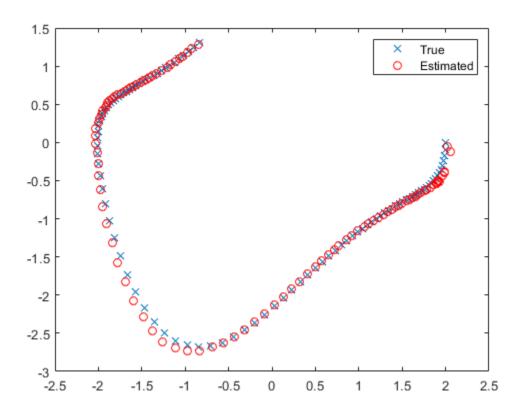
```
myPF.StateEstimationMethod = 'mean';
myPF.ResamplingMethod = 'systematic';
```

Estimate the states using the correct and predict commands, and store the estimated states.

```
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
```

Plot the results, and compare the estimated and true states.

```
figure(1)
plot(xTrue(:,1),xTrue(:,2),'x',xEst(:,1),xEst(:,2),'ro')
legend('True','Estimated')
```



rmpath(fullfile(matlabroot, 'examples', 'control', 'main')) % remove example data

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using MATLAB® Coder  $^{\scriptscriptstyle\mathsf{TM}}.$ 

Usage notes and limitations:

For more information, see "Generate Code for Online State Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

#### **Functions**

clone | correct | extendedKalmanFilter | initialize | predict |
unscentedKalmanFilter

## **Topics**

- "Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
- "Generate Code for Online State Estimation in MATLAB"
- "Validate Online State Estimation at the Command Line"

## Introduced in R2017b

<sup>&</sup>quot;Troubleshoot Online State Estimation"

# passiveplot

Compute or plot passivity index as function of frequency

# **Syntax**

```
passiveplot(G)
passiveplot(G,type)
passiveplot(____,w)
passiveplot(G1,G2,...,GN,____)
passiveplot(G1,LineSpec1,...,GN,LineSpecN,____)

[index,wout] = passiveplot(G)
[index,wout] = passiveplot(G,type)
index = passiveplot(G,w)
index = passiveplot(G,type,w)
```

# **Description**

passiveplot(G) plots the relative passivity indices of the dynamic system G as a function of frequency. When I + G is minimum phase, the relative passivity indices are the singular values of  $(I - G)(I + G)^-1$ . The largest singular value measures the relative excess (R < 1) or shortage (R > 1) at each frequency. See getPassiveIndex for more information about the meaning of the passivity index.

passiveplot automatically chooses the frequency range and number of points for the plot based on the dynamics of G.

passiveplot(G, type) plots the input, output, or I/O passivity index, depending on the value of type: 'input', 'output', or 'io', respectively.

passiveplot( \_\_\_\_, w) plots the passivity index for frequencies specified by w.

- If w is a cell array of the form {wmin,wmax}, then passiveplot plots the passivity index at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then passiveplot plots the passivity index at each specified frequency.

You can use this syntax with any of the previous input-argument combinations.

<code>passiveplot(G1,G2,...,GN, $\_$ )</code> plots the passivity index for multiple dynamic systems <code>G1,G2,...,GN</code> on the same plot. You can also use this syntax with the type input argument, with w to specify frequencies to plot, or both.

passiveplot(G1,LineSpec1,...,GN,LineSpecN,\_\_\_\_) specifies a color, linestyle, and marker for each system in the plot.

[index,wout] = passiveplot(G) and [index,wout] = passiveplot(G,type) return the passivity index at each frequency in the vector wout. The output index is a matrix, and the value index(:,k) gives the passivity indices in descending order at the frequency w(k). This syntax does not draw a plot.

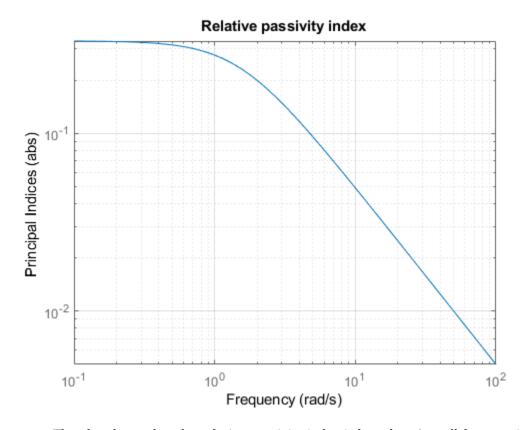
index = passiveplot(G, w) and index = passiveplot(G, type, w) return the passivity indices at the frequencies specified by w.

# **Examples**

## **Plot Passivity Versus Frequency**

Plot the relative passivity index as a function of frequency of the system G = (s + 2)/(s + 1).

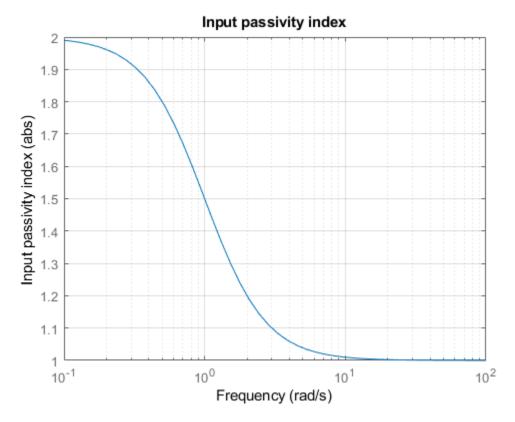
```
G = tf([1 2],[1 1]);
passiveplot(G)
```



The plot shows that the relative passivity index is less than 1 at all frequencies. Therefore, the system  ${\sf G}$  is passive.

Plot the input passivity index of the same system.

passiveplot(G,'input')



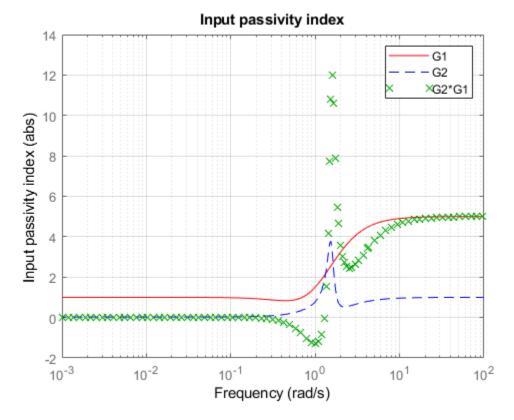
The input passivity index is positive at all frequencies. Therefore, the system is input strictly passive.

## **Plot Passivity of Multiple Systems**

Plot the input passivity index of two dynamic systems and their series interconnection.

```
G1 = tf([5 3 1],[1 2 1]);
G2 = tf([1 1 5 0.1],[1 2 3 4]);
H = G2*G1;
```

```
passiveplot(G1,'r',G2,'b--',H,'gx','input')
legend('G1','G2','G2*G1')
```



The input passivity index of the interconnected system dips below 0 around 1 rad/s. This plot shows that the series interconnection of two passive systems is not necessarily passive. However, passivity is preserved for parallel or feedback interconnections of passive systems.

# **Input Arguments**

## **G** — Model to analyze

dynamic system model | model array

Model to analyze for passivity, specified as a dynamic system model such as a tf, ss, or genss model. G can be MIMO, if the number of inputs equals the number of outputs. G can be continuous or discrete. If G is a generalized model with tunable or uncertain blocks, passiveplot evaluates passivity of the current, nominal value of G.

If G is a model array, then passiveplot plots the passivity index of all models in the array on the same plot. When you use output arguments to get passivity data, G must be a single model.

## type — Type of passivity index

```
'input'|'output'|'io'
```

Type of passivity index, specified as one of the following:

- 'input' Input passivity index (input feedforward passivity). This value is the smallest eigenvalue of  $(G(s) + G(s)^H)/2$ , for  $s = j\omega$  in continuous time, and  $s = e_{j\omega}$  in discrete time.
- 'output' Output passivity index (output feedback passivity). When G is minimum phase, this value is the smallest eigenvalue of  $(G(s)^{-1} + G(s)^{-H})/2$ , for  $s = j\omega$  in continuous time, and  $s = e_{i\omega}$  in discrete time.
- 'io' Combined I/O passivity index. When I + G is minimum phase, this value is the largest  $\tau(\omega)$  such that:

$$G(s) + G(s)^{H} > 2\tau(\omega)(I + G(s)^{H}G(s)),$$

for  $s = j\omega$  in continuous time, and  $s = e_{i\omega}$  in discrete time.

See "About Passivity and Passivity Indices" for details about these indices.

## w — Frequencies

{wmin,wmax} | vector

Frequencies at which to compute and plot indices, specified as the cell array {wmin,wmax} or as a vector of frequency values.

- If w is a cell array of the form {wmin,wmax}, then the function computes the index at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then the function computes the index at each specified frequency. For example, use logspace to generate a row vector with logarithmicallyspaced frequency values.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the model.

## LineSpec — Line style, marker, and color

character vector | string

Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.

```
Example: 'r--' specifies a red dashed line
```

Example: '\*b' specifies blue asterisk markers

Example: 'y' specifies a yellow line

# **Output Arguments**

## index — Passivity indices

matrix

Passivity indices as a function of frequency, returned as a matrix. index contains whichever type of passivity index you specify, computed at the frequencies w if you supplied them, or wout if you did not. index has as many columns as there are values in w or wout, and

- One row, for the input, output, or combined i/o passivity indices.
- As many rows as G has inputs or outputs, for the relative passivity index.

For example, suppose that G is a 3-input, 3-output system, and w is a 1-by-30 vector of frequencies. Then the following syntax returns a 3-by-30 matrix index.

```
index = passiveplot(G,w);
```

The entry index(:,k) contains the relative passivity indices of G, in descending order, at the frequency w(k).

#### wout — Frequencies

vector

Frequencies at which the indices are calculated, returned as a vector. The function automatically chooses the frequency range and number of points based on the dynamics of the model.

## See Also

getPassiveIndex | getSectorIndex | isPassive | sectorplot

## **Topics**

"About Passivity and Passivity Indices"

Introduced in R2016a

# permute

Rearrange array dimensions in model arrays

# **Syntax**

```
newarray = permute(sysarray,order)
```

# **Description**

newarray = permute(sysarray, order) rearranges the array dimensions of a model array so that the dimensions are in the specified order. The input and output dimensions of the model array are not counted as array dimensions for this operation.

# **Examples**

## **Permute Model Array Dimensions**

```
Create a 1-by-2-by-3 array of state-space models.
sysarr = rss(2,2,2,1,2,3);
Rearrange the model array so that the dimensions are 3-by-2-by-1.
newarr = permute(sysarr,[3 2 1]);
size(newarr)
3x2 array of state-space models.
```

Each model has 2 outputs, 2 inputs, and 2 states.

The input and output dimensions of the model array remain unchanged.

# **Input Arguments**

## sysarray — Model array to rearrange

model array

Model array to rearrange, specified as an array of input-output models such as numeric LTI models, generalized models, or identified LTI models.

## order — Dimensions of rearranged model array

vector

Dimensions of rearranged model array, specified as a vector of positive integers. For example, to rearrange a model array into a 3-by-2 array, order is [3 2].

Data Types: double

# **Output Arguments**

## newarray — Rearranged model array

model array

Rearranged model array, returned as an array of input-output models with the new dimensions as specified in order.

## See Also

ndims | reshape | size

Introduced in R2013a

# pid

Create PID controller in parallel form, convert to parallel-form PID controller

## **Syntax**

```
C = pid(Kp,Ki,Kd,Tf)
C = pid(Kp,Ki,Kd,Tf,Ts)
C = pid(sys)
C = pid(Kp)
C = pid(Kp,Ki)
C = pid(Kp,Ki,Kd)
C = pid(...,Name,Value)
C = pid
```

# **Description**

C = pid(Kp, Ki, Kd, Tf) creates a continuous-time PID controller with proportional, integral, and derivative gains Kp, Ki, and Kd and first-order derivative filter time constant Tf:

$$C = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1} \,. \label{eq:constraint}$$

This representation is in parallel form. If all of Kp, Ki, Kd, and Tf are real, then the resulting C is a pid controller object. If one or more of these coefficients is tunable (realp or genmat), then C is a tunable generalized state-space (genss) model object.

C = pid(Kp, Ki, Kd, Tf, Ts) creates a discrete-time PID controller with sample time Ts. The controller is:

$$C = K_p + K_i IF(z) + \frac{K_d}{T_f + DF(z)} \,. \label{eq:constraint}$$

IF(z) and DF(z) are the discrete integrator formulas for the integrator and derivative filter. By default,

$$IF(z) = DF(z) = \frac{T_S}{z - 1}.$$

To choose different discrete integrator formulas, use the IFormula and DFormula properties. (See "Properties" on page 2-838 for more information about IFormula and DFormula). If DFormula = 'ForwardEuler' (the default value) and Tf  $\neq$  0, then Ts and Tf must satisfy Tf > Ts/2. This requirement ensures a stable derivative filter pole.

C = pid(sys) converts the dynamic system sys to a parallel form pid controller object.

C = pid(Kp) creates a continuous-time proportional (P) controller with Ki = 0, Kd = 0, and Tf = 0.

C = pid(Kp,Ki) creates a proportional and integral (PI) controller with Kd = 0 and Tf = 0.

C = pid(Kp, Ki, Kd) creates a proportional, integral, and derivative (PID) controller with Tf = 0.

C = pid(...,Name,Value) creates a controller or converts a dynamic system to a pid controller object with additional options specified by one or more Name, Value pair arguments.

C = pid creates a P controller with Kp = 1.

# **Input Arguments**

#### Kp

Proportional gain.

Kp can be:

- A real and finite value.
- · An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Kp = 0, the controller has no proportional action.

## **Default:** 1

#### Κi

Integral gain.

Ki can be:

- A real and finite value.
- An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Ki = 0, the controller has no integral action.

#### Default: 0

#### Kd

Derivative gain.

Kd can be:

- · A real and finite value.
- An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Kd = 0, the controller has no derivative action.

#### **Default:** 0

#### Τf

Time constant of the first-order derivative filter.

Tf can be:

- A real, finite, and nonnegative value.
- An array of real, finite, and nonnegative values.

- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Tf = 0, the controller has no filter on the derivative action.

Default: 0

#### Ts

Sample time.

To create a discrete-time pid controller, provide a positive real value (Ts > 0). pid does not support discrete-time controller with unspecified sample time (Ts = -1).

Ts must be a scalar value. In an array of pid controllers, each controller must have the same Ts.

**Default:** 0 (continuous time)

#### sys

SISO dynamic system to convert to parallel pid form.

sys must represent a valid PID controller that can be written in parallel form with  $Tf \ge 0$ .

sys can also be an array of SISO dynamic systems.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value syntax to set the numerical integration formulas IFormula and DFormula of a discrete-time pid controller, or to set other object properties such as InputName and OutputName. For information about available properties of pid controller objects, see "Properties" on page 2-838.

# **Output Arguments**

C

PID controller, represented as a pid controller object, an array of pid controller objects, a genss object, or a genss array.

- If all the gains Kp, Ki, Kd, and Tf have numeric values, then C is a pid controller object. When the gains are numeric arrays, C is an array of pid controller objects. The controller type (P, I, PI, PD, PDF, PID, PIDF) depends upon the values of the gains. For example, when Kd = 0, but Kp and Ki are nonzero, C is a PI controller.
- If one or more gains is a tunable parameter (realp), generalized matrix (genmat), or tunable gain surface (tunableSurface), then C is a generalized state-space model (genss).

# **Properties**

Kp, Ki, Kd

PID controller gains.

The Kp, Ki, and Kd properties store the proportional, integral, and derivative gains, respectively. Kp, Ki, and Kd are real and finite.

#### Τf

Derivative filter time constant.

The Tf property stores the derivative filter time constant of the pid controller object. Tf is real, finite, and nonnegative.

#### **IFormula**

Discrete integrator formula IF(z) for the integrator of the discrete-time pid controller C:

$$C = K_p + K_i IF(z) + \frac{K_d}{T_f + DF(z)}.$$

IFormula can take the following values:

• 'ForwardEuler'  $-IF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time

BackwardEuler'  $-IF(z) = \frac{T_s z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

• 'Trapezoidal'  $-IF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

When C is a continuous-time controller, IFormula is ''.

Default: 'ForwardEuler'

#### **DFormula**

Discrete integrator formula DF(z) for the derivative filter of the discrete-time pid controller C:

$$C = K_p + K_i IF(z) + \frac{K_d}{T_f + DF(z)} \,.$$

DFormula can take the following values:

• 'ForwardEuler'  $-DF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula

can result in instability, even when discretizing a system that is stable in continuous time.

BackwardEuler'  $-DF(z) = \frac{T_S z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

'Trapezoidal'  $-DF(z) = \frac{T_S z + 1}{2z - 1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

The Trapezoidal value for DFormula is not available for a pid controller with no derivative filter (Tf = 0).

When C is a continuous-time controller, DFormula is ''.

Default: 'ForwardEuler'

## InputDelay

Time delay on the system input. InputDelay is always 0 for a pid controller object.

## **OutputDelay**

Time delay on the system Output. OutputDelay is always 0 for a pid controller object.

## Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. PID controller models do not support unspecified sample time (Ts = -1).

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

## **Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

#### Default: 'seconds'

#### InputName

Input channel name, specified as a character vector. Use this property to name the input channel of the controller model. For example, assign the name error to the input of a controller model C as follows.

```
C.InputName = 'error';
```

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

Default: Empty character vector, ''

## InputUnit

Input channel units, specified as a character vector. Use this property to track input signal units. For example, assign the concentration units mol/m^3 to the input of a controller model C as follows.

```
C.InputUnit = 'mol/m^3';
```

InputUnit has no effect on system behavior.

Default: Empty character vector, ''

## **InputGroup**

Input channel groups. This property is not needed for PID controller models.

Default: struct with no fields

## **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the <code>OutputName</code> property. For example, <code>C.y</code> is equivalent to <code>C.OutputName</code>.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- · Specifying connection points when interconnecting models

Default: Empty character vector, ''

## OutputUnit

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

Default: Empty character vector, ''

## **OutputGroup**

Output channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

### Name

System name, specified as a character vector. For example, 'system 1'.

Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

# **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

## Default: []

# **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

#### М

```
M(:,:,2,1) [zeta=0.35, w=5] =

25

s^2 + 3.5 s + 25
```

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

### **PDF Controller**

Create a continuous-time controller with proportional and derivative gains and a filter on the derivative term. To do so, set the integral gain to zero. Set the other gains and the filter time constant to the desired values.

```
\label{eq:Kp} \begin{split} &\text{Kp} = 1; \\ &\text{Ki} = 0; \quad \text{% No integrator} \\ &\text{Kd} = 3; \\ &\text{Tf} = 0.5; \\ &\text{C} = \text{pid}(\text{Kp,Ki,Kd,Tf}) \\ &\text{C} = \\ &\text{Kp} + \text{Kd} * \frac{\text{S}}{\text{-----}} \\ &\text{Tf}*\text{s}+1 \\ &\text{with Kp} = 1, \text{Kd} = 3, \text{Tf} = 0.5 \\ &\text{Continuous-time PDF controller in parallel form.} \end{split}
```

The display shows the controller type, formula, and parameter values, and verifies that the controller has no integrator term.

### **Discrete-Time PI Controller**

Create a discrete-time PI controller with trapezoidal discretization formula.

To create a discrete-time PI controller, set the value of Ts and the discretization formula using Name, Value syntax.

Alternatively, you can create the same discrete-time controller by supplying Ts as the fifth input argument after all four PID parameters, Kp, Ki, Kd, and Tf. Since you only want a PI controller, set Kd and Tf to zero.

```
C2 = pid(5,2.4,0,0,0.1,'IFormula','Trapezoidal')

C2 =

Ts*(z+1)

Kp + Ki * ------
2*(z-1)

with Kp = 5, Ki = 2.4, Ts = 0.1

Sample time: 0.1 seconds

Discrete-time PI controller in parallel form.
```

The display shows that C1 and C2 are the same.

# PID Controller with Named Input and Output

When you create a PID controller, set the dynamic system properties InputName and OutputName. This is useful, for example, when you interconnect the PID controller with other dynamic system models using the connect command.

```
C = pid(1,2,3,'InputName','e','OutputName','u')
C =
     Kp + Ki * --- + Kd * s
     with Kp = 1, Ki = 2, Kd = 3
Continuous-time PID controller in parallel form.
```

The display does not show the input and output names for the PID controller, but you can examine the property values. For instance, verify the input name of the controller.

### C.InputName

```
ans = 1x1 cell array
{'e'}
```

# **Array of PID Controllers**

Create a 2-by-3 grid of PI controllers with proportional gain ranging from 1–2 across the array rows and integral gain ranging from 5–9 across columns.

To build the array of PID controllers, start with arrays representing the gains.

```
Kp = [1 \ 1 \ 1;2 \ 2 \ 2];

Ki = [5:2:9;5:2:9];
```

When you pass these arrays to the pid command, the command returns the array.

```
pi_array = pid(Kp,Ki,'Ts',0.1,'IFormula','BackwardEuler');
size(pi_array)
```

```
2x3 array of PID controller.
Each PID has 1 output and 1 input.
```

Alternatively, use the stack command to build an array of PID controllers.

C = pid(1,5,0.1) % PID controller

C =
 Kp + Ki \* 
$$\frac{1}{---}$$
 + Kd \* s
 with Kp = 1, Ki = 5, Kd = 0.1

Continuous-time PID controller in parallel form.

Cf = pid(1,5,0.1,0.5) % PID controller with filter

Cf =

1 s Kp + Ki \* --- + Kd \* -----s Tf\*s+1

with 
$$Kp = 1$$
,  $Ki = 5$ ,  $Kd = 0.1$ ,  $Tf = 0.5$ 

Continuous-time PIDF controller in parallel form.

These commands return a 1-by-2 array of controllers.

# size(pid\_array)

```
1x2 array of PID controller.
Each PID has 1 output and 1 input.
```

All PID controllers in an array must have the same sample time, discrete integrator formulas, and dynamic system properties such as InputName and OutputName.

#### Convert PID Controller from Standard to Parallel Form

Convert a standard form pidstd controller to parallel form.

Standard PID form expresses the controller actions in terms of an overall proportional gain Kp, integral and derivative time constants Ti and Td, and filter divisor N. You can convert any standard-form controller to parallel form using the pid command. For example, consider the following standard-form controller.

Continuous-time PIDF controller in standard form

Convert this controller to parallel form using pid.

```
\label{eq:cpar} \begin{split} &\text{C\_par = pid(C\_std)} \\ &\text{C\_par =} \\ &\text{Kp + Ki * --- + Kd * -------} \\ &\text{s & Tf*s+1} \\ &\text{with Kp = 2, Ki = 0.667, Kd = 8, Tf = 0.08} \\ &\text{Continuous-time PIDF controller in parallel form.} \end{split}
```

## **Convert Dynamic System to Parallel-Form PID Controller**

Convert a continuous-time dynamic system that represents a PID controller to parallel pid form.

The following dynamic system, with an integrator and two zeros, is equivalent to a PID controller.

$$H(s) = \frac{3(s+1)(s+2)}{s} \, .$$

Create a zpk model of H. Then use the pid command to obtain H in terms of the PID gains Kp, Ki, and Kd.

```
H = zpk([-1,-2],0,3);
C = pid(H)

C =

Kp + Ki * --- + Kd * s

with Kp = 9, Ki = 6, Kd = 3

Continuous-time PID controller in parallel form.
```

# Convert Discrete-Time Dynamic System to Parallel-Form PID Controller

Convert a discrete-time dynamic system that represents a PID controller with derivative filter to parallel pid form.

Create a discrete-time zpk model that represents a PIDF controller (two zeros and two poles, including the integrator pole at z = 1).

```
sys = zpk([-0.5, -0.6], [1 -0.2], 3, 'Ts', 0.1);
```

When you convert sys to PID form, the result depends on which discrete integrator formulas you specify for the conversion. For instance, use the default, ForwardEuler, for both the integrator and the derivative.

```
Cfe = pid(sys)
```

```
Cfe =  Ts \qquad 1 \\ Kp + Ki * ---- + Kd * ---- \\ z-1 \qquad Tf+Ts/(z-1)  with Kp = 2.75, Ki = 60, Kd = 0.0208, Tf = 0.0833, Ts = 0.1 
 Sample time: 0.1 seconds 
 Discrete-time PIDF controller in parallel form.
```

Now convert using the Trapezoidal formula.

The displays show the difference in resulting coefficient values and functional form.

For this particular dynamic system, you cannot write sys in parallel PID form using the BackwardEuler formula for the derivative filter. Doing so would result in Tf < 0, which is not permitted. In that case, pid returns an error.

#### Discretize a Continuous-Time PID Controller

Discretize a continuous-time PID controller and set integral and derivative filter formulas.

Create a continuous-time controller and discretize it using the zero-order-hold method of the c2d command.

```
Ccon = pid(1,2,3,4); % continuous-time PIDF controller Cdis1 = c2d(Ccon,0.1,'zoh')
```

The display shows that c2d computes new PID gains for the discrete-time controller.

The discrete integrator formulas of the discretized controller depend on the c2d discretization method, as described in "Tips" on page 2-853. For the zoh method, both IFormula and DFormula are ForwardEuler.

### Cdis1.IFormula

```
ans =
'ForwardEuler'
```

#### Cdis1.DFormula

```
ans =
'ForwardEuler'
```

If you want to use different formulas from the ones returned by c2d, then you can directly set the Ts, IFormula, and DFormula properties of the controller to the desired values.

```
Cdis2 = Ccon;
Cdis2.Ts = 0.1;
Cdis2.IFormula = 'BackwardEuler';
Cdis2.DFormula = 'BackwardEuler';
```

However, these commands do not compute new PID gains for the discretized controller. To see this, examine Cdis2 and compare the coefficients to Ccon and Cdis1.

### Cdis2

Cdis2 =

with 
$$Kp = 1$$
,  $Ki = 2$ ,  $Kd = 3$ ,  $Tf = 4$ ,  $Ts = 0.1$ 

Sample time: 0.1 seconds

Discrete-time PIDF controller in parallel form.

# **Tips**

- Use pid to:
  - Create a pid controller object from known PID gains and filter time constant.
  - Convert a pidstd controller object to a standard-form pid controller object.
  - Convert other types of dynamic system models to a pid controller object.
- To design a PID controller for a particular plant, use pidtune or pidTuner. To create a tunable PID controller as a control design block, use tunablePID.
- Create arrays of pid controller objects by:
  - Specifying array values for Kp,Ki,Kd, and Tf
  - Specifying an array of dynamic systems sys to convert to pid controller objects
  - Using stack to build arrays from individual controllers or smaller arrays

In an array of pid controllers, each controller must have the same sample time Ts and discrete integrator formulas IFormula and DFormula.

• To create or convert to a standard-form controller, use pidstd. Standard form expresses the controller actions in terms of an overall proportional gain  $K_p$ , integral and derivative times  $T_i$  and  $T_d$ , and filter divisor N:

$$C = K_p \left( 1 + \frac{1}{T_i} \frac{1}{s} + \frac{T_d s}{\frac{T_d}{N} s + 1} \right).$$

- There are two ways to discretize a continuous-time pid controller:
  - Use the c2d command. c2d computes new parameter values for the discretized controller. The discrete integrator formulas of the discretized controller depend upon the c2d discretization method you use, as shown in the following table.

c2d Discretization Method	IFormula	DFormula
'zoh'	ForwardEuler	ForwardEuler
'foh'	Trapezoidal	Trapezoidal
'tustin'	Trapezoidal	Trapezoidal
'impulse'	ForwardEuler	ForwardEuler
'matched'	ForwardEuler	ForwardEuler

For more information about c2d discretization methods, see the c2d reference page. For more information about IFormula and DFormula, see "Properties" on page 2-838.

• If you require different discrete integrator formulas, you can discretize the controller by directly setting Ts, IFormula, and DFormula to the desired values. (See "Discretize a Continuous-Time PID Controller" on page 2-851.) However, this method does not compute new gain and filter-constant values for the discretized controller. Therefore, this method might yield a poorer match between the continuous- and discrete-time pid controllers than using c2d.

# See Also

genss | make2DOF | pid2 | pidTuner | piddata | pidstd | pidtune | realp |
tunablePID

# **Topics**

- "Proportional-Integral-Derivative (PID) Controllers"
- "Discrete-Time Proportional-Integral-Derivative (PID) Controllers"

### Introduced in R2010b

<sup>&</sup>quot;What Are Model Objects?"

# pid2

Create 2-DOF PID controller in parallel form, convert to parallel-form 2-DOF PID controller

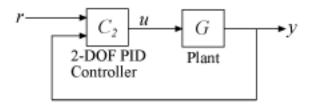
# **Syntax**

```
C2 = pid2(Kp,Ki,Kd,Tf,b,c)
C2 = pid2(Kp,Ki,Kd,Tf,b,c,Ts)
C2 = pid2(sys)
C2 = pid2(____,Name,Value)
```

# **Description**

pid2 controller objects represent two-degree-of-freedom (2-DOF) PID controllers in parallel form. Use pid2 either to create a pid2 controller object from known coefficients or to convert a dynamic system model to a pid2 object.

Two-degree-of-freedom (2-DOF) PID controllers include setpoint weighting on the proportional and derivative terms. A 2-DOF PID controller can achieve fast disturbance rejection without significant increase of overshoot in setpoint tracking. 2-DOF PID controllers are also useful to mitigate the influence of changes in the reference signal on the control signal. The following illustration shows a typical control architecture using a 2-DOF PID controller.



C2 = pid2(Kp,Ki,Kd,Tf,b,c) creates a continuous-time 2-DOF PID controller with proportional, integral, and derivative gains Kp, Ki, and Kd and first-order derivative filter

time constant Tf. The controller also has setpoint weighting b on the proportional term, and setpoint weighting c on the derivative term. The relationship between the 2-DOF controller output (u) and its two inputs (r and y) is given by:

$$u=K_p(br-y)+\frac{K_i}{s}(r-y)+\frac{K_ds}{T_fs+1}(cr-y)\,.$$

This representation is in parallel form. If all coefficients are real-valued, then the resulting C2 is a pid2 controller object. If one or more of these coefficients is tunable (realp or genmat), then C2 is a tunable generalized state-space (genss) model object.

C2 = pid2(Kp,Ki,Kd,Tf,b,c,Ts) creates a discrete-time 2-DOF PID controller with sample time Ts. The relationship between the controller output and inputs is given by:

$$u = K_p(br-y) + K_i IF(z)(r-y) + \frac{K_d}{T_f + DF(z)}(cr-y) \,. \label{eq:update}$$

IF(z) and DF(z) are the discrete integrator formulas for the integrator and derivative filter. By default,

$$IF(z) = DF(z) = \frac{T_s}{z - 1}.$$

To choose different discrete integrator formulas, use the IFormula and DFormula properties. (See "Properties" on page 2-860 for more information). If DFormula = 'ForwardEuler' (the default value) and Tf  $\neq$  0, then Ts and Tf must satisfy Tf > Ts/2. This requirement ensures a stable derivative filter pole.

C2 = pid2(sys) converts the dynamic system sys to a parallel form pid2 controller object.

C2 = pid2(\_\_\_\_, Name, Value) specifies additional properties as comma-separated pairs of Name, Value arguments.

# **Input Arguments**

## Kp

Proportional gain.

Kp can be:

- A real and finite value.
- An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Kp = 0, the controller has no proportional action.

### Default: 1

### Κi

Integral gain.

### Ki can be:

- A real and finite value.
- · An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Ki = 0, the controller has no integral action.

# **Default:** 0

### Kd

Derivative gain.

### Kd can be:

- · A real and finite value.
- An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Kd = 0, the controller has no derivative action.

## **Default:** 0

## Tf

Time constant of the first-order derivative filter.

Tf can be:

- A real, finite, and nonnegative value.
- An array of real, finite, and nonnegative values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Tf = 0, the controller has no filter on the derivative action.

### **Default:** 0

### b

Setpoint weighting on proportional term.

b can be:

- A real, nonnegative, and finite value.
- · An array of real, nonnegative, finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When b = 0, changes in setpoint do not feed directly into the proportional term.

### Default: 1

C

Setpoint weighting on derivative term.

c can be:

- A real, nonnegative, and finite value.
- An array of real, nonnegative, finite values.
- A tunable parameter (realp) or generalized matrix (genmat).

• A tunable surface for gain-scheduled tuning, created using tunableSurface.

When c = 0, changes in setpoint do not feed directly into the derivative term.

### **Default:** 1

### Ts

Sample time.

To create a discrete-time pid2 controller, provide a positive real value (Ts > 0). pid2 does not support discrete-time controllers with unspecified sample time (Ts = -1).

Ts must be a scalar value. In an array of pid2 controllers, each controller must have the same Ts.

**Default:** 0 (continuous time)

### sys

SISO dynamic system to convert to parallel pid2 form.

sys must be a two-input, one-output system. sys must represent a valid 2-DOF PID controller that can be written in parallel form with  $Tf \ge 0$ .

sys can also be an array of SISO dynamic systems.

# **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value syntax to set the numerical integration formulas IFormula and DFormula of a discrete-time pid2 controller, or to set other object properties such as InputName and OutputName. For information about available properties of pid2 controller objects, see "Properties" on page 2-860.

# **Output Arguments**

### **C2**

2-DOF PID controller, returned as a pid2 controller object, an array of pid2 controller objects, a genss object, or a genss array.

- If all the coefficients have scalar numeric values, then C2 is a pid2 controller object.
- If one or more coefficients is a numeric array, C2 is an array of pid2 controller objects. The controller type (such as PI, PID, or PDF) depends upon the values of the gains. For example, when Kd = 0, but Kp and Ki are nonzero, C2 is a PI controller.
- If one or more coefficients is a tunable parameter (realp), generalized matrix (genmat), or tunable gain surface (tunableSurface), then C2 is a generalized state-space model (genss).

# **Properties**

# b, c

Setpoint weights on the proportional and derivative terms, respectively. b and c values are real, finite, and positive. When you use the pid2 command to create a 2-DOF PID controller, the b, and c input arguments, respectively, set the initial values of these properties.

# Kp, Ki, Kd

PID controller gains.

Proportional, integral, and derivative gains, respectively. Kp, Ki, and Kd values are real and finite. When you use the pid2 command to create a 2-DOF PID controller, the Kp, Ki, and Kd input arguments, respectively, set the initial values of these properties.

## Tf

Derivative filter time constant.

The Tf property stores the derivative filter time constant of the pid2 controller object. Tf is real, finite, and greater than or equal to zero. When you create a 2-DOF PID controller using the pid2 command, the Tf input argument sets the initial value of this property.

#### **IFormula**

Discrete integrator formula IF(z) for the integrator of the discrete-time pid2 controller C2. The relationship between the inputs and output of C2 is given by:

$$u=K_p(br-y)+K_iIF(z)(r-y)+\frac{K_d}{T_f+DF(z)}(cr-y)\,.$$

IFormula can take the following values:

'ForwardEuler'  $-IF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time.

'BackwardEuler'  $-IF(z) = \frac{T_s z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

'Trapezoidal'  $-IF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

When C2 is a continuous-time controller, IFormula is ''.

Default: 'ForwardEuler'

### **DFormula**

Discrete integrator formula DF(z) for the derivative filter of the discrete-time pid2 controller C2. The relationship between the inputs and output of C2 is given by:

$$u=K_p(br-y)+K_iIF(z)(r-y)+\frac{K_d}{T_f+DF(z)}(cr-y)\,.$$

DFormula can take the following values:

• 'ForwardEuler'  $-DF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time.

BackwardEuler' —  $DF(z) = \frac{T_S z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

'Trapezoidal'  $-DF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

The Trapezoidal value for DFormula is not available for a pid2 controller with no derivative filter (Tf = 0).

When C2 is a continuous-time controller, DFormula is ''.

Default: 'ForwardEuler'

# InputDelay

Time delay on the system input. InputDelay is always 0 for a pid2 controller object.

# OutputDelay

Time delay on the system Output. OutputDelay is always 0 for a pid2 controller object.

## Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit

specified by the TimeUnit property of the model. PID controller models do not support unspecified sample time (Ts = -1).

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

**Default:** 0 (continuous time)

#### TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### **InputName**

Input channel name, specified as a character vector or a 2-by-1 cell array of character vectors. Use this property to name the input channels of the controller model. For example, assign the names setpoint and measurement to the inputs of a 2-DOF PID controller model C as follows.

```
C.InputName = {'setpoint';'measurement'};
```

Alternatively, use automatic vector expansion to assign both input names. For example:

```
C.InputName = 'C-input';
```

The input names automatically expand to {'C-input(1)';'C-input(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Specifying connection points when interconnecting models

```
Default: { ' ' ; ' ' }
```

## InputUnit

Input channel units, specified as a 2-by-1 cell array of character vectors. Use this property to track input signal units. For example, assign the units Volts to the reference input and the concentration units mol/m^3 to the measurement input of a 2-DOF PID controller model C as follows.

```
C.InputUnit = {'Volts';'mol/m^3'};
```

InputUnit has no effect on system behavior.

```
Default: { ' ' ; ' ' }
```

## **InputGroup**

Input channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

## **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the OutputName property. For example, C.y is equivalent to C.OutputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

Default: Empty character vector, ''

# OutputUnit

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

**Default:** Empty character vector, ''

# OutputGroup

Output channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
```

```
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

**Default:** [0×1 string]

### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

# Default: []

# **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

М

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

#### 2-DOF PDF Controller

Create a continuous-time 2-DOF controller with proportional and derivative gains and a filter on the derivative term. To do so, set the integral gain to zero. Set the other gains and the filter time constant to the desired values.

```
Kp = 1;
Ki = 0; % No integrator
```

The display shows the controller type, formula, and parameter values, and verifies that the controller has no integrator term.

### **Discrete-Time 2-DOF PI Controller**

Create a discrete-time 2-DOF PI controller using the trapezoidal discretization formula. Specify the formula using Name, Value syntax.

Setting Kd = 0 specifies a PI controller with no derivative term. As the display shows, the values of Tf and c are not used in this controller. The display also shows that the trapezoidal formula is used for the integrator.

## 2-DOF PID Controller with Named Inputs and Output

Create a 2-DOF PID controller, and set the dynamic system properties InputName and OutputName. Naming the inputs and the output is useful, for example, when you interconnect the PID controller with other dynamic system models using the connect command.

```
C2 = pid2(1,2,3,0,1,1,'InputName',{'r','y'},'OutputName','u')
C2 =

u = Kp (b*r-y) + Ki --- (r-y) + Kd*s (c*r-y)
s

with Kp = 1, Ki = 2, Kd = 3, b = 1, c = 1

Continuous-time 2-DOF PID controller in parallel form.
```

A 2-DOF PID controller has two inputs and one output. Therefore, the 'InputName' property is an array containing two names, one for each input. The model display does not show the input and output names for the PID controller, but you can examine the property values to see them. For instance, verify the input name of the controller.

#### C2.InputName

```
ans = 2x1 cell array
     {'r'}
     {'y'}
```

# Array of 2-DOF PID Controllers

Create a 2-by-3 grid of 2-DOF PI controllers with proportional gain ranging from 1–2 across the array rows and integral gain ranging from 5–9 across columns.

To build the array of PID controllers, start with arrays representing the gains.

```
Kp = [1 \ 1 \ 1;2 \ 2 \ 2];

Ki = [5:2:9;5:2:9];
```

When you pass these arrays to the pid2 command, the command returns the array of controllers.

```
pi_array = pid2(Kp,Ki,0,0,0.5,0,'Ts',0.1,'IFormula','BackwardEuler');
size(pi_array)

2x3 array of 2-DOF PID controller.
Each PID has 1 output and 2 inputs.
```

If you provide scalar values for some coefficients, pid2 automatically expands them and assigns the same value to all entries in the array. For instance, in this example, Kd = Tf = 0, so that all entries in the array are PI controllers. Also, all entries in the array have b = 0.5.

Access entries in the array using array indexing. For dynamic system arrays, the first two dimensions are the I/O dimensions of the model, and the remaining dimensions are the array dimensions. Therefore, the following command extracts the (2,3) entry in the array.

```
pi23 = pi_array(:,:,2,3)
pi23 =

u = Kp (b*r-y) + Ki ----- (r-y)
    z-1

with Kp = 2, Ki = 9, b = 0.5, Ts = 0.1

Sample time: 0.1 seconds
Discrete-time 2-DOF PI controller in parallel form.
```

You can also build an array of PID controllers using the stack command.

These commands return a 1-by-2 array of controllers.

```
size(pid_array)
```

```
1x2 array of 2-DOF PID controller.
Each PID has 1 output and 2 inputs.
```

All PID controllers in an array must have the same sample time, discrete integrator formulas, and dynamic system properties such as InputName and OutputName.

## **Convert 2-DOF PID Controller from Standard to Parallel Form**

Convert a standard-form pidstd2 controller to parallel form.

Standard PID form expresses the controller actions in terms of an overall proportional gain Kp, integrator and derivative time constants Ti and Td, and filter divisor N. You can convert any 2-DOF standard-form controller to parallel form using the pid2 command. For example, consider the following standard-form controller.

Continuous-time 2-DOF PIDF controller in standard form

Convert this controller to parallel form using pid2.

C2\_par = pid2(C2\_std)

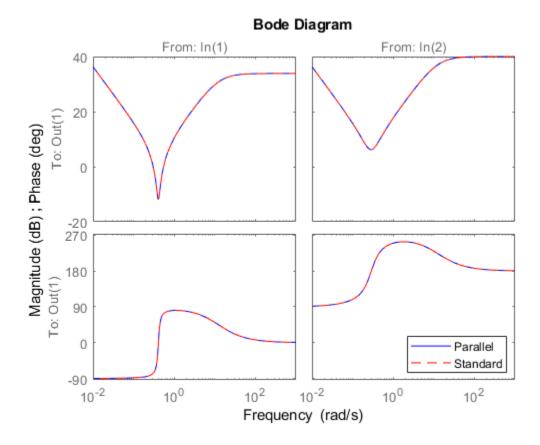
C2\_par = 
$$u = Kp (b*r-y) + Ki - (r-y) + Kd - (c*r-y)$$

$$s Tf*s+1$$

```
with Kp = 2, Ki = 0.667, Kd = 8, Tf = 0.08, b = 0.1, c = 0.5 Continuous-time 2-DOF PIDF controller in parallel form.
```

A response plot confirms that the two forms are equivalent.

```
bodeplot(C2_par, 'b-', C2_std, 'r--')
legend('Parallel', 'Standard', 'Location', 'Southeast')
```



## **Convert Dynamic System to Parallel-Form PID Controller**

Convert a continuous-time dynamic system that represents a PID controller to parallel pid form.

The following dynamic system, with an integrator and two zeros, is equivalent to a PID controller.

$$H(s) = \frac{3(s+1)(s+2)}{s}$$
.

Create a zpk model of H. Then use the pid command to obtain H in terms of the PID gains Kp, Ki, and Kd.

# Convert Dynamic System to 2-DOF Parallel-Form PID Controller

Convert a discrete-time dynamic system that represents a 2-DOF PID controller with derivative filter to parallel pid2 form.

The following state-space matrices represent a discrete-time 2-DOF PID controller with a sample time of  $0.1~\rm s.$ 

```
A = [1,0;0,0.99];
B = [0.1,-0.1; -0.005,0.01];
C = [3,0.2];
D = [2.6,-5.2];
Ts = 0.1;
sys = ss(A,B,C,D,Ts);
```

C2fe = pid2(sys)

When you convert sys to 2-DOF PID form, the result depends on which discrete integrator formulas you specify for the conversion. For instance, use the default, ForwardEuler, for both the integrator and the derivative.

```
C2fe =

U = Kp (b*r-y) + Ki ----- (r-y) + Kd ----- (c*r-y)

with Kp = 5, Ki = 3, Kd = 2, Tf = 10, b = 0.5, c = 0.5, Ts = 0.1

Sample time: 0.1 seconds
Discrete-time 2-DOF PIDF controller in parallel form.

Now convert using the Trapezoidal formula.

C2trap = pid2(sys, 'IFormula', 'Trapezoidal', 'DFormula', 'Trapezoidal')

C2trap =

U = Kp (b*r-y) + Ki ------ (r-y) + Kd ------- (c*r-y)
```

Discrete-time 2-DOF PIDF controller in parallel form.

The displays show the difference in resulting coefficient values and functional form.

#### Discretize a Continuous-Time 2-DOF PID Controller

Discretize a continuous-time 2-DOF PID controller and specify the integral and derivative filter formulas.

Create a continuous-time controller and discretize it using the zero-order-hold method of the c2d command.

The display shows that c2d computes new PID coefficients for the discrete-time controller.

The discrete integrator formulas of the discretized controller depend on the c2d discretization method, as described in "Tips" on page 2-876. For the zoh method, both IFormula and DFormula are ForwardFuler.

#### C2dis1. TFormula

```
ans =
'ForwardEuler'
C2dis1.DFormula
ans =
'ForwardEuler'
```

If you want to use different formulas from the ones returned by c2d, then you can directly set the Ts, IFormula, and DFormula properties of the controller to the desired values.

```
C2dis2 = C2con;
C2dis2.Ts = 0.1;
C2dis2.IFormula = 'BackwardEuler';
C2dis2.DFormula = 'BackwardEuler';
```

However, these commands do not compute new PID gains for the discretized controller. To see this, examine C2dis2 and compare the coefficients to C2con and C2dis1.

# C2dis2

C2dis2 =

Tips

- To design a PID controller for a particular plant, use pidtune or pidTuner. To create a tunable 2-DOF PID controller as a control design block, use tunablePID2.
- To break a 2-DOF controller into two SISO control components, such as a feedback controller and a feedforward controller, use getComponents.
- Create arrays of pid2 controller objects by:
  - Specifying array values for one or more of the coefficients Kp, Ki, Kd, Tf, b, and c.
  - Specifying an array of dynamic systems sys to convert to pid2 controller objects.
  - Using stack to build arrays from individual controllers or smaller arrays.
  - Passing an array of plant models to pidtune.

In an array of pid2 controllers, each controller must have the same sample time Ts and discrete integrator formulas IFormula and DFormula.

• To create or convert to a standard-form controller, use pidstd2. Standard form expresses the controller actions in terms of an overall proportional gain  $K_p$ , integral and derivative times  $T_i$  and  $T_d$ , and filter divisor N. For example, the relationship between the inputs and output of a continuous-time standard-form 2-DOF PID controller is given by:

$$u = K_p \left[ (br - y) + \frac{1}{T_i s} (r - y) + \frac{T_d s}{\frac{T_d}{N} s + 1} (cr - y) \right].$$

- There are two ways to discretize a continuous-time pid2 controller:
  - Use the c2d command. c2d computes new parameter values for the discretized controller. The discrete integrator formulas of the discretized controller depend upon the c2d discretization method you use, as shown in the following table.

c2d Discretization Method	IFormula	DFormula
'zoh'	ForwardEuler	ForwardEuler
'foh'	Trapezoidal	Trapezoidal
'tustin'	Trapezoidal	Trapezoidal
'impulse'	ForwardEuler	ForwardEuler
'matched'	ForwardEuler	ForwardEuler

For more information about c2d discretization methods, See the c2d reference page. For more information about IFormula and DFormula, see "Properties" on page 2-860.

• If you require different discrete integrator formulas, you can discretize the controller by directly setting Ts, IFormula, and DFormula to the desired values. (See "Discretize a Continuous-Time 2-DOF PID Controller" on page 2-874.) However, this method does not compute new gain and filter-constant values for the discretized controller. Therefore, this method might yield a poorer match between the continuous- and discrete-time pid2 controllers than using c2d.

# See Also

genss | getComponents | make1DOF | pid | pidTuner | piddata2 | pidstd2 |
pidtune | realp | tunablePID2

# **Topics**

"Two-Degree-of-Freedom PID Controllers"

"Discrete-Time Proportional-Integral-Derivative (PID) Controllers"

### Introduced in R2015b

<sup>&</sup>quot;What Are Model Objects?"

# piddata

Access coefficients of parallel-form PID controller

# **Syntax**

```
[Kp,Ki,Kd,Tf] = piddata(sys)
[Kp,Ki,Kd,Tf,Ts] = piddata(sys)
[Kp,Ki,Kd,Tf,Ts] = piddata(sys,J1,...,JN)
```

# **Description**

[Kp,Ki,Kd,Tf] = piddata(sys) returns the PID gains Kp,Ki, Kd and the filter time constant Tf of the parallel-form controller represented by the dynamic system sys.

```
[Kp,Ki,Kd,Tf,Ts] = piddata(sys) also returns the sample time Ts.
```

[Kp,Ki,Kd,Tf,Ts] = piddata(sys,J1,...,JN) extracts the data for a subset of entries in sys, where sys is an N-dimensional array of dynamic systems. The indices J specify the array entry to extract.

# **Input Arguments**

## sys

SISO dynamic system or array of SISO dynamic systems. If sys is not a pid object, it must represent a valid PID controller that can be written in parallel PID form.

J

Integer indices of N entries in the array sys of dynamic systems. For example, suppose sys is a 4-by-5 (two-dimensional) array of pid controllers or dynamic system models that represent PID controllers. The following command extracts the data for entry (2,3) in the array.

```
[Kp,Ki,Kd,Tf,Ts] = piddata(sys,2,3);
```

# **Output Arguments**

# Kр

Proportional gain of the parallel-form PID controller represented by dynamic system sys.

If sys is a pid controller object, the output Kp is equal to the Kp value of sys.

If sys is not a pid object, Kp is the proportional gain of a parallel PID controller equivalent to sys.

If sys is an array of dynamic systems, Kp is an array of the same dimensions as sys.

# Κi

Integral gain of the parallel-form PID controller represented by dynamic system sys.

If sys is a pid controller object, then the output Ki is equal to the Ki value of sys.

If sys is not a pid object, then Ki is the integral gain of a parallel PID controller equivalent to sys.

If sys is an array of dynamic systems, then Ki is an array of the same dimensions as sys.

## Kd

Derivative gain of the parallel-form PID controller represented by dynamic system sys.

If sys is a pid controller object, then the output Kd is equal to the Kd value of sys.

If sys is not a pid object, then Kd is the derivative gain of a parallel PID controller equivalent to sys.

If sys is an array of dynamic systems, then Kd is an array of the same dimensions as sys.

## Τf

Filter time constant of the parallel-form PID controller represented by dynamic system sys.

If sys is a pid controller object, the output Tf is equal to the Tf value of sys.

If sys is not a pid object, Tf is the filter time constant of a parallel PID controller equivalent to sys.

If sys is an array of dynamic systems, Tf is an array of the same dimensions as sys.

#### Ts

Sample time of the dynamic system sys. Ts is always a scalar value.

# **Examples**

Extract the proportional, integral, and derivative gains and the filter time constant from a parallel-form pid controller.

For the following pid object:

```
sys = pid(1,4,0.3,10);
```

you can extract the parameter values from sys by entering:

```
[Kp Ki Kd Tf] = piddata(sys);
```

Extract the parallel form proportional and integral gains from an equivalent standard-form PI controller.

For a standard-form PI controller, such as:

```
sys = pidstd(2,3);
```

you can extract the gains of an equivalent parallel-form PI controller by entering:

```
[Kp Ki] = piddata(sys)
```

These commands return the result:

```
Kp =
```

Ki =

0.6667

2

Extract parameters from a dynamic system that represents a PID controller.

The dynamic system

$$H(z) = \frac{(z - 0.5)(z - 0.6)}{(z - 1)(z + 0.8)}$$

represents a discrete-time PID controller with a derivative filter. Use piddata to extract the parallel-form PID parameters.

```
H = zpk([0.5 \ 0.6],[1,-0.8],1,0.1); % sample time Ts = 0.1s [Kp Ki Kd Tf Ts] = piddata(H);
```

the piddata function uses the default ForwardEuler discrete integrator formula for IFormula and DFormula to compute the parameter values.

Extract the gains from an array of PI controllers.

```
sys = pid(rand(2,3), rand(2,3)); % 2-by-3 array of PI controllers
[Kp Ki Kd Tf] = piddata(sys);
```

The parameters Kp, Ki, Kd, and Tf are also 2-by-3 arrays.

Use the index input  ${\bf J}$  to extract the parameters of a subset of  ${\bf sys}$ .

```
[Kp Ki Kd Tf] = piddata(sys,5);
```

# **Tips**

If sys is not a pid controller object, piddata returns the PID gains Kp, Ki, Kd and the filter time constant Tf of a parallel-form controller equivalent to sys.

For discrete-time sys, piddata returns the parameters of an equivalent parallel-form controller. This controller has discrete integrator formulas IFormula and DFormula set to ForwardEuler. See the pid reference page for more information about discrete integrator formulas.

# See Also

get | pid | pidstd

# Introduced in R2010b

# piddata2

Access coefficients of parallel-form 2-DOF PID controller

# **Syntax**

```
[Kp,Ki,Kd,Tf,b,c] = piddata2(sys)
[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys)
[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys,J1,...,JN)
```

# **Description**

[Kp,Ki,Kd,Tf,b,c] = piddata2(sys) returns the PID gains Kp,Ki, Kd, the filter time constant Tf, and the setpoint weights b and c of the parallel-form 2-DOF PID controller represented by the dynamic system sys.

If sys is a pid2 controller object, then each output argument is the corresponding coefficient in sys.

If sys is not a pid2 object, then each output argument is the corresponding coefficient of the parallel-form 2-DOF PID controller that is equivalent to sys.

If sys is an array of dynamic systems, then each output argument is an array of the same dimensions as sys.

[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys) also returns the sample time Ts. For discrete-time sys that is not a pid2 object, piddata2 calculates the coefficient values using the default ForwardEuler discrete integrator formula for both IFormula and DFormula. See the pid2 reference page for more information about discrete integrator formulas.

[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys,J1,...,JN) extracts the data for a subset of entries in sys, where sys is an N-dimensional array of dynamic systems. The indices J specify the array entry to extract.

# **Examples**

## **Extract Coefficients from Parallel-Form 2-DOF PID Controller**

Typically, you extract coefficients from a controller obtained from another function, such as pidtune or getBlockValue. For this example, create a 2-DOF PID controller that has random coefficients.

```
rng('default'); % for reproducibility
C2 = pid2(rand,rand,rand,rand,rand,rand);
```

Extract the PID coefficients, filter time constant, and setpoint weights.

```
[Kp,Ki,Kd,Tf,b,c] = piddata2(C2);
```

# **Extract Parallel-Form Gains from Standard-Form Controller**

Create a 2-DOF PID controller in standard form.

```
C2 = pidstd2(2,3,4,10,0.5,0.5)

C2 = 
 u = Kp * [(b*r-y) + \frac{1}{-1} * (r-y) + Td * \frac{s}{(Td/N)*s+1} * (c*r-y)] 
with Kp = 2, Ti = 3, Td = 4, N = 10, b = 0.5, c = 0.5
```

Continuous-time 2-DOF PIDF controller in standard form

Compute the coefficients of an equivalent parallel-form PID controller.

```
[Kp,Ki,Kd,Tf,b,c] = piddata2(C2);
```

Check some of the coefficients to confirm that they are different from the standard-form coefficients.

```
Ki = 0.6667
```

```
Kd
```

Kd = 8

# **Extract 2-DOF PID Coefficients from Equivalent System**

Extract coefficients from a two-input, one-output dynamic system that represents a valid 2-DOF parallel-form PID controller.

The following A, B, C, and D matrices form a discrete-time state-space model that represents a 2-DOF PID controller.

```
A = [1,0;0.09975,0.995];
B = [0.00625, -0.00625; 0.1245, -0.1241];
C = [0,4];
D = [2.875, -5.75];
sys = ss(A,B,C,D,0.1)
sys =
  A =
                     x2
            x1
             1
                       0
   x1
   x2
       0.09975
                  0.995
  B =
             u1
                       u2
   x1
        0.00625
                -0.00625
   x2
         0.1245
                 -0.1241
  C =
       x1 x2
        0
            4
   у1
  D =
          u1
                 u2
   y1 2.875 -5.75
Sample time: 0.1 seconds
```

Discrete-time state-space model.

Extract the PID gains, filter time constant, and setpoint weights of the model.

```
[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys);
```

For a discrete-time system, piddata2 calculates the coefficient values using the default ForwardEuler discrete integrator formula for both IFormula and DFormula.

# **Extract Coefficients from 2-DOF PI Controller Array**

Typically, you obtain an array of controllers by using pidtune on an array of plant models. For this example, create an 2-by-3 array of 2-DOF PI controllers with random values of Kp, Ki, and b.

```
rng('default');
C2 = pid2(rand(2,3),rand(2,3),0,0,rand(2,3),0);
```

Extract all the coefficients from the array.

```
[Kp,Ki,Kd,Tf,b,c] = piddata2(C2);
```

Each of the outputs is itself a 2-by-3 array. For example, examine Ki.

Κi

```
Ki = 2×3

0.2785 0.9575 0.1576

0.5469 0.9649 0.9706
```

Extract only the coefficients of entry (2,1) in the array.

```
[Kp21, Ki21, Kd21, Tf21, b21, c21] = piddata2(C2, 2, 1);
```

Each of these outputs is a scalar.

Ki21

```
Ki21 = 0.5469
```

# **Input Arguments**

# sys — 2-DOF PID controller

pid2 controller object | dynamic system model | dynamic system array

2-DOF PID controller in parallel form, specified as a pid2 controller object, a dynamic system model, or a dynamic system array. If sys is not a pid2 controller object, it must be a two-input, one-output model that represents a valid 2-DOF PID controller that can be written in parallel form.

## J - Indices

positive integers

Indices of entry to extract from a model array sys, specified as positive integers. Provide as many indices as there are array dimensions in sys. For example, suppose sys is a 4-by-5 (two-dimensional) array of pid2 controllers or dynamic system models that represent 2-DOF PID controllers. The following command extracts the data for entry (2,3) in the array.

[Kp,Ki,Kd,Tf,b,c,Ts] = piddata2(sys,2,3);

# **Output Arguments**

# **Kp** — Proportional gain

scalar | array

Proportional gain of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

If sys is a pid2 controller object, then Kp is the Kp value of sys.

If sys is not a pid2 object, then Kp is the proportional gain of the parallel-form 2-DOF PID controller that is equivalent to sys.

If sys is an array of dynamic systems, then Kp is an array of the same dimensions as sys.

# Ki — Integral gain

scalar | array

Integral gain of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# Kd — Derivative gain

scalar | array

Derivative gain of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# Tf — Filter time constant

scalar | array

Filter time constant of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# b — Setpoint weight on proportional term

scalar | array

Setpoint weight on the proportional term of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# c — Setpoint weight on derivative term

scalar | array

Setpoint weight on the derivative term of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# Ts — Sample time

scalar

Sample time of the pid2 controller, dynamic system sys, or dynamic system array, returned as a scalar.

# See Also

pid2 | piddata | pidstddata2

## Introduced in R2015b

# pidstd

Create a PID controller in standard form, convert to standard-form PID controller

# **Syntax**

```
C = pidstd(Kp,Ti,Td,N)
C = pidstd(Kp,Ti,Td,N,Ts)
C = pidstd(sys)
C = pidstd(Kp)
C = pidstd(Kp,Ti)
C = pidstd(Kp,Ti,Td)
C = pidstd(...,Name,Value)
C = pidstd
```

# **Description**

C = pidstd(Kp,Ti,Td,N) creates a continuous-time PIDF (PID with first-order derivative filter) controller object in standard form. The controller has proportional gain Kp, integral and derivative times Ti and Td, and first-order derivative filter divisor N:

$$C = K_p \left( 1 + \frac{1}{T_i} \frac{1}{s} + \frac{T_d s}{\frac{T_d}{N} s + 1} \right).$$

C = pidstd(Kp,Ti,Td,N,Ts) creates a discrete-time controller with sample time Ts.
The discrete-time controller is:

$$C = K_p \left( 1 + \frac{1}{T_i} IF(z) + \frac{T_d}{\frac{T_d}{N} + DF(z)} \right).$$

IF(z) and DF(z) are the discrete integrator formulas for the integrator and derivative filter. By default,

$$IF(z) = DF(z) = \frac{T_s}{z - 1}.$$

To choose different discrete integrator formulas, use the IFormula and DFormula inputs. (See "Properties" on page 2-893 for more information about IFormula and DFormula). If DFormula = 'ForwardEuler' (the default value) and  $N \neq Inf$ , then Ts, Td, and N must satisfy Td/N > Ts/2. This requirement ensures a stable derivative filter pole.

C = pidstd(sys) converts the dynamic system sys to a standard form pidstd controller object.

C = pidstd(Kp) creates a continuous-time proportional (P) controller with Ti = Inf, Td = 0, and N = Inf.

C = pidstd(Kp,Ti) creates a proportional and integral (PI) controller with Td = 0 and N = Inf.

C = pidstd(Kp,Ti,Td) creates a proportional, integral, and derivative (PID) controller with N = Inf.

C = pidstd(...,Name,Value) creates a controller or converts a dynamic system to a pidstd controller object with additional options specified by one or more Name,Value pair arguments.

C = pidstd creates a P controller with Kp = 1.

# **Input Arguments**

# Kp

Proportional gain.

Kp can be:

- · A real and finite value.
- · An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

## **Default:** 1

### Τi

Integrator time.

#### Ti can be:

- · A real and positive value.
- · An array of real and positive values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

# **Default: Inf**

## Td

Derivative time.

#### Td can be:

- · A real, finite, and nonnegative value.
- An array of real, finite, and nonnegative values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Td = 0, the controller has no derivative action.

# **Default:** 0

#### Ν

Derivative filter divisor.

## N can be:

- A real and positive value.
- An array of real and positive values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When N = Inf, the controller has no filter on the derivative action.

## Default: Inf

#### Ts

Sample time.

To create a discrete-time pidstd controller, provide a positive real value (Ts >  $\theta$ ).pidstd does not support discrete-time controller with undetermined sample time (Ts = -1).

Ts must be a scalar value. In an array of pidstd controllers, each controller must have the same Ts.

**Default:** 0 (continuous time)

## sys

SISO dynamic system to convert to standard pidstd form.

sys must represent a valid controller that can be written in standard form with Ti > 0,  $Td \ge 0$ , and N > 0.

sys can also be an array of SISO dynamic systems.

# Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value syntax to set the numerical integration formulas IFormula and DFormula of a discrete-time pidstd controller, or to set other object properties such as InputName and OutputName. For information about available properties of pidstd controller objects, see "Properties" on page 2-893.

# **Output Arguments**

## C

pidstd object representing a single-input, single-output PID controller in standard form.

The controller type (P, PI, PD, PDF, PID, PIDF) depends upon the values of Kp, Ti, Td, and N. For example, when Td = Inf and Kp and Ti are finite and nonzero, C is a PI controller. Enter getType(C) to obtain the controller type.

When the inputs Kp,Ti, Td, and N or the input sys are arrays, C is an array of pidstd objects.

# **Properties**

# Kp

Proportional gain. Kp must be real and finite.

## Τi

Integral time. Ti must be real, finite, and greater than or equal to zero.

## Td

Derivative time. Td must be real, finite, and greater than or equal to zero.

## N

Derivative filter divisor. N must be real, and greater than or equal to zero.

## **IFormula**

Discrete integrator formula IF(z) for the integrator of the discrete-time pidstd controller C:

$$C = K_p \left( 1 + \frac{1}{T_i} IF(z) + \frac{T_d}{\frac{T_d}{N} + DF(z)} \right).$$

IFormula can take the following values:

• 'ForwardEuler'  $-IF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula

can result in instability, even when discretizing a system that is stable in continuous time.

BackwardEuler'  $-IF(z) = \frac{T_S z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

• 'Trapezoidal'  $-IF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

When C is a continuous-time controller, IFormula is ''.

Default: 'ForwardEuler'

#### **DFormula**

Discrete integrator formula DF(z) for the derivative filter of the discrete-time pidstd controller C:

$$C = K_p \left( 1 + \frac{1}{T_i} IF(z) + \frac{T_d}{\frac{T_d}{N} + DF(z)} \right).$$

DFormula can take the following values:

• 'ForwardEuler'  $-DF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time.

BackwardEuler' —  $DF(z) = \frac{T_S z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

'Trapezoidal'  $-DF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

The Trapezoidal value for DFormula is not available for a pidstd controller with no derivative filter (N = Inf).

When C is a continuous-time controller, DFormula is ''.

Default: 'ForwardEuler'

# **InputDelay**

Time delay on the system input. InputDelay is always 0 for a pidstd controller object.

# **OutputDelay**

Time delay on the system Output. OutputDelay is always 0 for a pidstd controller object.

# Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. PID controller models do not support unspecified sample time (Ts = -1).

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

**Default:** 0 (continuous time)

## **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

# **InputName**

Input channel name, specified as a character vector. Use this property to name the input channel of the controller model. For example, assign the name error to the input of a controller model C as follows.

```
C.InputName = 'error';
```

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Specifying connection points when interconnecting models

Default: Empty character vector, ''

# InputUnit

Input channel units, specified as a character vector. Use this property to track input signal units. For example, assign the concentration units mol/m^3 to the input of a controller model C as follows.

```
C.InputUnit = 'mol/m^3';
```

InputUnit has no effect on system behavior.

**Default:** Empty character vector, ''

# **InputGroup**

Input channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

# **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the OutputName property. For example, C.y is equivalent to C.OutputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

Default: Empty character vector, ''

# **OutputUnit**

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

**Default:** Empty character vector, ''

# OutputGroup

Output channel groups. This property is not needed for PID controller models.

Default: struct with no fields

# Name

System name, specified as a character vector. For example, 'system 1'.

Default: ''

## **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

# Default: [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

# Default: []

# SamplingGrid

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

#### М

```
M(:,:,1,1) [zeta=0.3, w=5] =

25

s^2 + 3 s + 25

M(:,:,2,1) [zeta=0.35, w=5] =

25
```

```
s^2 + 3.5 s + 25
```

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

# Create Continuous-Time Standard-Form PDF Controller

Create a continuous-time standard-form PDF controller with proportional gain 1, derivative time 3, and a filter divisor of 6.

The display shows the controller type, formula, and coefficient values.

# Create Discrete-Time PI Controller with Trapezoidal Discretization Formula

To create a discrete-time controller, set the value of Ts using Name, Value syntax.

```
C = pidstd(1, 0.5, 'Ts', 0.1, 'IFormula', 'Trapezoidal') % Ts = 0.1s
```

# This command produces the result:

Discrete-time PI controller in standard form:

Alternatively, you can create the same discrete-time controller by supplying Ts as the fifth argument after all four PID parameters Kp, Ti, Td, and N.

```
C = pidstd(5,2.4,0,Inf,0.1,'IFormula','Trapezoidal');
```

# **Create PID Controller and Set System Properties**

Create a PID controller and set dynamic system properties InputName and OutputName.

```
C = pidstd(1,0.5,3,'InputName','e','OutputName','u');
```

# **Create Grid of Standard-Form PID Controllers**

Create a 2-by-3 grid of PI controllers with proportional gain ranging from 1–2 and integral time ranging from 5–9.

Create a grid of PI controllers with proportional gain varying row to row and integral time varying column to column. To do so, start with arrays representing the gains.

```
Kp = [1 1 1;2 2 2];
Ti = [5:2:9;5:2:9];
pi_array = pidstd(Kp,Ti,'Ts',0.1,'IFormula','BackwardEuler');
```

These commands produce a 2-by-3 array of discrete-time pidstd objects. All pidstd objects in an array must have the same sample time, discrete integrator formulas, and dynamic system properties (such as InputName and OutputName).

Alternatively, you can use the stack command to build arrays of pidstd objects.

```
C = pidstd(1,5,0.1) % PID controller
Cf = pidstd(1,5,0.1,0.5) % PID controller with filter
pid_array = stack(2,C,Cf); % stack along 2nd array dimension
```

These commands produce a 1-by-2 array of controllers. Enter the command:

```
size(pid array)
```

to see the result

```
1x2 array of PID controller.
Each PID has 1 output and 1 input.
```

# **Convert Parallel-Form pid Controller to Standard Form**

Parallel PID form expresses the controller actions in terms of an proportional, integral, and derivative gains  $K_p$ ,  $K_i$ , and  $K_d$ , and a filter time constant  $T_f$ . You can convert a parallel form controller parsys to standard form using pidstd, provided that:

- parsys is not a pure integrator (I) controller.
- The gains Kp, Ki, and Kd of parsys all have the same sign.

```
parsys = pid(2,3,4,5); % Standard-form controller
stdsys = pidstd(parsys)
```

These commands produce a parallel-form controller:

Continuous-time PIDF controller in standard form:

# Create pidstd Controller from Continuous-Time Dynamic System

The dynamic system

$$H(s) = \frac{3(s+1)(s+2)}{s}$$

represents a PID controller. Use pidstd to obtain H(s) to in terms of the standard-form PID parameters  $K_p$ ,  $T_i$ , and  $T_d$ .

$$H = zpk([-1,-2],0,3);$$
  
 $C = pidstd(H)$ 

These commands produce the result:

Continuous-time PID controller in standard form:

with Kp = 9, Ti = 1.5, Td = 0.33333

# Create pidstd Controller from Discrete-Time Dynamic System

You can convert a discrete-time dynamic system that represents a PID controller with derivative filter to standard pidstd form.

```
% PIDF controller expressed in zpk form
sys = zpk([-0.5,-0.6],[1 -0.2],3,'Ts',0.1);
```

The resulting pidstd object depends upon the discrete integrator formula you specify for IFormula and DFormula.

For example, if you use the default ForwardEuler for both formulas:

$$C = pidstd(sys)$$

you obtain the result:

Discrete-time PIDF controller in standard form:

Kp \* 
$$(1 + \frac{1}{1 +$$

For this particular sys, you cannot write sys in standard PID form using the BackwardEuler formula for the DFormula. Doing so would result in N < 0, which is not permitted. In that case, pidstd returns an error.

Similarly, you cannot write sys in standard form using the Trapezoidal formula for both integrators. Doing so would result in negative Ti and Td, which also returns an error.

# **Discretize Continuous-Time pidstd Controller**

First, discretize the controller using the 'zoh' method of c2d.

```
Cc = pidstd(1,2,3,4); % continuous-time pidf controller Cd1 = c2d(Cc,0.1,'zoh')
```

Discrete-time PIDF controller in standard form:

with 
$$Kp = 1$$
,  $Ti = 2$ ,  $Td = 3.2044$ ,  $N = 4$ ,  $Ts = 0.1$ 

The resulting discrete-time controller uses ForwardEuler  $(T_s/(z-1))$  for both IFormula and DFormula.

The discrete integrator formulas of the discretized controller depend upon the c2d discretization method, as described in "Tips" on page 2-905. To use a different IFormula and DFormula, directly set Ts, IFormula, and DFormula to the desired values:

```
Cd2 = Cc;
Cd2.Ts = 0.1;
Cd2.IFormula = 'BackwardEuler';
Cd2.DFormula = 'BackwardEuler';
```

These commands do not compute new parameter values for the discretized controller. To see this, enter:

Cd2

to obtain the result:

Discrete-time PIDF controller in standard form:

with 
$$Kp = 1$$
,  $Ti = 2$ ,  $Td = 3$ ,  $N = 4$ ,  $Ts = 0.1$ 

# **Tips**

- Use pidstd either to create a pidstd controller object from known PID gain, integral
  and derivative times, and filter divisor, or to convert a dynamic system model to a
  pidstd object.
- To tune a PID controller for a particular plant, use pidtune or pidTuner.
- Create arrays of pidstd controllers by:
  - Specifying array values for Kp,Ti,Td, and N
  - Specifying an array of dynamic systems sys to convert to standard PID form
  - Using stack to build arrays from individual controllers or smaller arrays

In an array of pidstd controllers, each controller must have the same sample time Ts and discrete integrator formulas IFormula and DFormula.

• To create or convert to a parallel-form controller, use pid. Parallel form expresses the controller actions in terms of proportional, integral, and derivative gains  $K_p$ ,  $K_i$  and  $K_d$ , and a filter time constant  $T_f$ :

$$C = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}.$$

- There are two ways to discretize a continuous-time pidstd controller:
  - Use the c2d command. c2d computes new parameter values for the discretized controller. The discrete integrator formulas of the discretized controller depend upon the c2d discretization method you use, as shown in the following table.

c2d Discretization Method	IFormula	DFormula
'zoh'	ForwardEuler	ForwardEuler
'foh'	Trapezoidal	Trapezoidal
'tustin'	Trapezoidal	Trapezoidal
'impulse'	ForwardEuler	ForwardEuler

c2d Discretization Method	IFormula	DFormula
'matched'	ForwardEuler	ForwardEuler

For more information about c2d discretization methods, See the c2d reference page. For more information about IFormula and DFormula, see "Properties" on page 2-893.

• If you require different discrete integrator formulas, you can discretize the controller by directly setting Ts, IFormula, and DFormula to the desired values. (For more information, see "Discretize Continuous-Time pidstd Controller" on page 2-904.) However, this method does not compute new gain and filter-constant values for the discretized controller. Therefore, this method might yield a poorer match between the continuous-time and discrete-time pidstd controllers than using c2d.

# See Also

pidTuner|pidstd2|pidstddata|pidtune

# **Topics**

"Proportional-Integral-Derivative (PID) Controllers"

"Discrete-Time Proportional-Integral-Derivative (PID) Controllers"

# Introduced in R2010b

<sup>&</sup>quot;What Are Model Objects?"

# pidstd2

Create 2-DOF PID controller in standard form, convert to standard-form 2-DOF PID controller

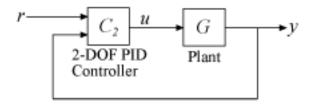
# **Syntax**

```
C2 = pidstd2(Kp,Ti,Td,N,b,c)
C2 = pidstd2(Kp,Ti,Td,N,b,c,Ts)
C2 = pidstd2(sys)
C2 = pid2(____,Name,Value)
```

# **Description**

pid2 controller objects represent two-degree-of-freedom (2-DOF) PID controllers in parallel form. Use pid2 either to create a pid2 controller object from known coefficients or to convert a dynamic system model to a pid2 object.

Two-degree-of-freedom (2-DOF) PID controllers include setpoint weighting on the proportional and derivative terms. A 2-DOF PID controller is capable of fast disturbance rejection without significant increase of overshoot in setpoint tracking. 2-DOF PID controllers are also useful to mitigate the influence of changes in the reference signal on the control signal. The following illustration shows a typical control architecture using a 2-DOF PID controller.



C2 = pidstd2(Kp,Ti,Td,N,b,c) creates a continuous-time 2-DOF PID controller with proportional gain Kp, integrator and derivative time constants Ti, and Td, and derivative

filter divisor N. The controller also has setpoint weighting b on the proportional term, and setpoint weighting c on the derivative term. The relationship between the 2-DOF controller's output (u) and its two inputs (r and y) is given by:

$$u = K_p \left[ (br - y) + \frac{1}{T_i s} (r - y) + \frac{T_d s}{\frac{T_d}{N} s + 1} (cr - y) \right].$$

This representation is in standard form. If all of the coefficients are real-valued, then the resulting C2 is a pidstd2 controller object. If one or more of these coefficients is tunable (realp or genmat), then C2 is a tunable generalized state-space (genss) model object.

C2 = pidstd2(Kp,Ti,Td,N,b,c,Ts) creates a discrete-time 2-DOF PID controller with sample time Ts. The relationship between the controller's output and inputs is given by:

$$u = K_p \left[ (br - y) + \frac{1}{T_i} IF(z)(r - y) + \frac{T_d}{\frac{T_d}{N} + DF(z)} (cr - y) \right].$$

IF(z) and DF(z) are the discrete integrator formulas for the integrator and derivative filter. By default,

$$IF(z) = DF(z) = \frac{T_S}{z - 1}.$$

To choose different discrete integrator formulas, use the IFormula and DFormula properties. (See "Properties" on page 2-912 for more information). If DFormula = 'ForwardEuler' (the default value) and  $N \neq Inf$ , then Ts, Td, and N must satisfy Td/N > Ts/2. This requirement ensures a stable derivative filter pole.

C2 = pidstd2(sys) converts the dynamic system sys to a standard form pidstd2 controller object.

C2 = pid2(\_\_\_\_, Name, Value) specifies additional properties as comma-separated pairs of Name, Value arguments.

# **Input Arguments**

# Кp

Proportional gain.

Kp can be:

- A real and finite value.
- · An array of real and finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

## Default: 1

#### Τi

Integrator time.

Ti can be:

- · A real and positive value.
- An array of real and positive values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Ti = Inf, the controller has no integral action.

## Default: Inf

## Td

Derivative time.

Td can be:

- · A real, finite, and nonnegative value.
- $\bullet \hspace{0.4cm}$  An array of real, finite, and nonnegative values.
- A tunable parameter (realp) or generalized matrix (genmat).

• A tunable surface for gain-scheduled tuning, created using tunableSurface.

When Td = 0, the controller has no derivative action.

## **Default:** 0

## N

Derivative filter divisor.

N can be:

- · A real and positive value.
- · An array of real and positive values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When N = Inf, the controller has no filter on the derivative action.

### Default: Inf

## b

Setpoint weighting on proportional term.

b can be:

- A real, nonnegative, and finite value.
- An array of real, nonnegative, finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When b = 0, changes in setpoint do not feed directly into the proportional term.

# **Default:** 1

#### C

Setpoint weighting on derivative term.

c can be:

- A real, nonnegative, and finite value.
- An array of real, nonnegative, finite values.
- A tunable parameter (realp) or generalized matrix (genmat).
- A tunable surface for gain-scheduled tuning, created using tunableSurface.

When c = 0, changes in setpoint do not feed directly into the derivative term.

#### Default: 1

## Ts

Sample time.

To create a discrete-time pidstd2 controller, provide a positive real value (Ts > 0).pidstd2 does not support discrete-time controller with undetermined sample time (Ts = -1).

Ts must be a scalar value. In an array of pidstd2 controllers, each controller must have the same Ts.

**Default:** 0 (continuous time)

## sys

SISO dynamic system to convert to standard pidstd2 form.

sys be a two-input, one-output system. sys must represent a valid 2-DOF controller that can be written in standard form with Ti > 0,  $Td \ge 0$ , and N > 0.

sys can also be an array of SISO dynamic systems.

# Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value syntax to set the numerical integration formulas IFormula and DFormula of a discrete-time pidstd2 controller, or to set other object properties such as

InputName and OutputName. For information about available properties of pidstd2 controller objects, see "Properties" on page 2-912.

# **Output Arguments**

#### **C2**

2-DOF PID controller, returned as a pidstd2 controller object, an array of pidstd2 controller objects, a genss object, or a genss array.

- If all the coefficients have scalar numeric values, then C2 is a pidstd2 controller object.
- If one or more coefficients is a numeric array, C2 is an array of pidstd2 controller objects. The controller type (such as PI, PID, or PDF) depends upon the values of the gains. For example, when Td = 0, but Kp and Ti are nonzero and finite, C2 is a PI controller.
- If one or more coefficients is a tunable parameter (realp), generalized matrix (genmat), or tunable gain surface (tunableSurface), then C2 is a generalized state-space model (genss).

# **Properties**

# b, c

Setpoint weights on the proportional and derivative terms, respectively. b and c values are real, finite, and positive. When you create a 2-DOF PID controller using the pidstd2 command, the initial values of these properties are set by the b, and c input arguments, respectively.

# Кp

Proportional gain.

The value of Kp is real and finite. When you create a 2-DOF PID controller using the pidstd2 command, the initial value of this property is set by the Kp input argument.

### Τi

Integrator time. Ti is real and positive. When you create a 2-DOF PID controller using the pidstd2 command, the initial value of this property is set by the Ti input argument. When Ti = Inf, the controller has no integral action.

## Td

Derivative time. Td is real, finite, and nonnegative. When you create a 2-DOF PID controller using the pidstd2 command, the initial value of this property is set by the Td input argument. When Td = 0, the controller has no derivative action.

#### N

Derivative filter divisor. N must be real and positive. When you create a 2-DOF PID controller using the pidstd2 command, the initial value of this property is set by the N input argument.

#### **IFormula**

Discrete integrator formula IF(z) for the integrator of the discrete-time pidstd2 controller C2. The relationship between the inputs and output of C2 is given by:

$$u = K_p \left[ (br - y) + \frac{1}{T_i} IF(z)(r - y) + \frac{T_d}{\frac{T_d}{N} + DF(z)} (cr - y) \right].$$

IFormula can take the following values:

• 'ForwardEuler' 
$$-IF(z) = \frac{T_s}{z-1}$$
.

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time.

'BackwardEuler' 
$$-IF(z) = \frac{T_s z}{z-1}$$
.

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result. • 'Trapezoidal'  $-IF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

When C2 is a continuous-time controller, IFormula is ''.

Default: 'ForwardEuler'

### **DFormula**

Discrete integrator formula DF(z) for the derivative filter of the discrete-time pidstd2 controller C2. The relationship between the inputs and output of C2 is given by:

$$u=K_p\left[(br-y)+\frac{1}{T_i}IF(z)(r-y)+\frac{T_d}{\frac{T_d}{N}+DF(z)}(cr-y)\right].$$

DFormula can take the following values:

'ForwardEuler'  $-DF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample time, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample time, the ForwardEuler formula can result in instability, even when discretizing a system that is stable in continuous time.

• 'BackwardEuler'  $-DF(z) = \frac{T_S z}{z-1}$ .

An advantage of the BackwardEuler formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result.

'Trapezoidal'  $-DF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuoustime system using this formula always yields a stable discrete-time result. Of all available integration formulas, the Trapezoidal formula yields the closest match between frequency-domain properties of the discretized system and the corresponding continuous-time system.

The Trapezoidal value for DFormula is not available for a pidstd2 controller with no derivative filter (N = Inf).

When C2 is a continuous-time controller, DFormula is ''.

Default: 'ForwardEuler'

## **InputDelay**

Time delay on the system input. InputDelay is always 0 for a pidstd2 controller object.

### **OutputDelay**

Time delay on the system Output. OutputDelay is always 0 for a pidstd2 controller object.

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. PID controller models do not support unspecified sample time (Ts = -1).

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

**Default:** 0 (continuous time)

#### TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'

- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### InputName

Input channel name, specified as a character vector or a 2-by-1 cell array of character vectors. Use this property to name the input channels of the controller model. For example, assign the names setpoint and measurement to the inputs of a 2-DOF PID controller model C as follows.

```
C.InputName = {'setpoint';'measurement'};
```

Alternatively, use automatic vector expansion to assign both input names. For example:

```
C.InputName = 'C-input';
```

The input names automatically expand to {'C-input(1)';'C-input(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

```
Default: { ' ' ; ' ' }
```

### InputUnit

Input channel units, specified as a 2-by-1 cell array of character vectors. Use this property to track input signal units. For example, assign the units Volts to the reference input and the concentration units mol/m^3 to the measurement input of a 2-DOF PID controller model C as follows.

```
C.InputUnit = {'Volts';'mol/m^3'};
```

InputUnit has no effect on system behavior.

```
Default: { ' ' ; ' ' }
```

### **InputGroup**

Input channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

### **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the OutputName property. For example, C.y is equivalent to C.OutputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- · Specifying connection points when interconnecting models

Default: Empty character vector, ''

## OutputUnit

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

Default: Empty character vector, ''

## OutputGroup

Output channel groups. This property is not needed for PID controller models.

Default: struct with no fields

### Name

System name, specified as a character vector. For example, 'system 1'.

Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

## Default: [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

## Default: []

### SamplingGrid

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

М

```
M(:,:,1,1) [zeta=0.3, w=5] =

25

s^2 + 3 s + 25

M(:,:,2,1) [zeta=0.35, w=5] =

25
```

```
s^2 + 3.5 s + 25
```

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

# **Examples**

#### 2-DOF PDF Controller

Create a continuous-time 2-DOF PDF controller in standard form. To do so, set the integral time constant to Inf. Set the other gains and the filter divisor to the desired values.

The display shows the controller type, formula, and parameter values, and verifies that the controller has no integrator term.

#### Discrete-Time 2-DOF PI Controller in Standard Form

Create a discrete-time 2-DOF PI controller in standard form, using the trapezoidal discretization formula. Specify the formula using Name, Value syntax.

Setting Td = 0 specifies a PI controller with no derivative term. As the display shows, the values of N and c are not used in this controller. The display also shows that the trapezoidal formula is used for the integrator.

## 2-DOF PID Controller with Named Inputs and Output

Create a 2-DOF PID controller in standard form, and set the dynamic system properties InputName and OutputName. Naming the inputs and the output is useful, for example, when you interconnect the PID controller with other dynamic system models using the connect command.

```
C2 = pidstd2(1,2,3,10,1,1,'InputName',{'r','y'},'OutputName','u')
C2 =
```

Continuous-time 2-DOF PIDF controller in standard form

A 2-DOF PID controller has two inputs and one output. Therefore, the 'InputName' property is an array containing two names, one for each input. The model display does not show the input and output names for the PID controller, but you can examine the property values to see them. For instance, verify the input name of the controller.

### C2.InputName

```
ans = 2x1 cell array {'r'} {'y'}
```

## Array of 2-DOF PID Controllers

Create a 2-by-3 grid of 2-DOF PI controllers in standard form. The proportional gain ranges from 1–2 across the array rows, and the integrator time constant ranges from 5–9 across columns.

To build the array of PID controllers, start with arrays representing the gains.

```
Kp = [1 \ 1 \ 1;2 \ 2 \ 2];

Ti = [5:2:9;5:2:9];
```

When you pass these arrays to the pidstd2 command, the command returns the array of controllers.

```
pi_array = pidstd2(Kp,Ti,0,Inf,0.5,0,'Ts',0.1,'IFormula','BackwardEuler');
size(pi_array)

2x3 array of 2-DOF PID controller.
Each PID has 1 output and 2 inputs.
```

If you provide scalar values for some coefficients, pidstd2 automatically expands them and assigns the same value to all entries in the array. For instance, in this example, Td =

0, so that all entries in the array are PI controllers. Also, all entries in the array have b = 0.5.

Access entries in the array using array indexing. For dynamic system arrays, the first two dimensions are the I/O dimensions of the model, and the remaining dimensions are the array dimensions. Therefore, the following command extracts the (2,3) entry in the array.

You can also build an array of PID controllers using the stack command.

```
 \begin{array}{lll} \text{C2} &=& \text{pidstd2}(1,5,0.1,\text{Inf},0.5,0.5); & \text{\% PID controller} \\ \text{C2f} &=& \text{pidstd2}(1,5,0.1,0.5,0.5,0.5); & \text{\% PID controller with filter} \\ \text{pid\_array} &=& \text{stack}(2,\text{C2},\text{C2f}); & \text{\% stack along 2nd array dimension} \\ \end{array}
```

These commands return a 1-by-2 array of controllers.

```
size(pid_array)

1x2 array of 2-DOF PID controller.
Each PID has 1 output and 2 inputs.
```

All PID controllers in an array must have the same sample time, discrete integrator formulas, and dynamic system properties such as InputName and OutputName.

#### Convert 2-DOF PID Controller from Parallel to Standard Form

Convert a parallel-form pid2 controller to standard form.

Parallel PID form expresses the controller actions in terms of proportional, integral, and derivative gains Kp, Ki, and Kd, and filter time constant Tf. You can convert a parallel-

form pid2 controller to standard form using the pidstd2 command, provided that both of the following are true:

- The pid2 controller can be expressed in valid standard form.
- The gains Kp, Ki, and Kd of the pid2 controller all have the same sign.

For example, consider the following parallel-form controller.

Convert this controller to norallel form using nidetd?

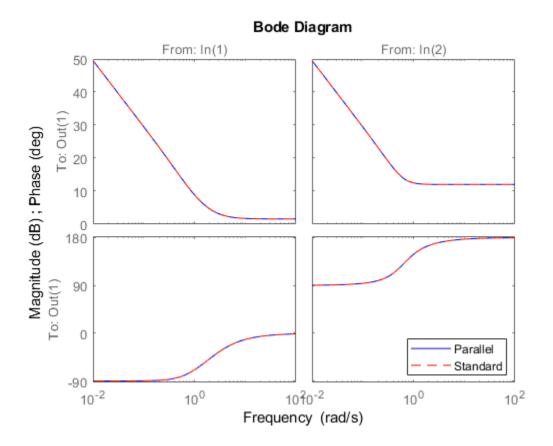
Convert this controller to parallel form using pidstd2.

```
 \begin{aligned} &\text{C2\_std} = \text{pidstd2(C2\_par)} \\ &\text{C2\_std} = \\ &\text{u} = \text{Kp} * [(b*r-y) + \frac{1}{-\cdots} * \frac{1}{-\cdots} * (r-y) + \text{Td} * \frac{s}{-\cdots} * (c*r-y)] \\ &\text{Ti} & s & (\text{Td/N})*s+1 \end{aligned}  with Kp = 2, Ti = 0.667, Td = 2, N = 1, b = 0.1, c = 0.5
```

Continuous-time 2-DOF PIDF controller in standard form

The display confirms the new standard form. A response plot confirms that the two forms are equivalent.

```
bodeplot(C2_par, 'b-', C2_std, 'r--')
legend('Parallel', 'Standard', 'Location', 'Southeast')
```



# Convert Dynamic System to Standard-Form 2-DOF PID Controller

Convert a two-input, one-output continuous-time dynamic system that represents a 2-DOF PID controller to a standard-form pidstd2 controller.

The following state-space matrices represent a 2-DOF PID controller.

```
A = [0,0;0,-8.181];
B = [1,-1;-0.1109,8.181];
C = [0.2301,10.66];
D = [0.8905,-11.79];
sys = ss(A,B,C,D);
```

Rewrite sys in terms of the standard-form PID parameters Kp, Ti, Td, and N, and the setpoint weights b and c.

```
C2 = pidstd2(sys)

C2 = 
 u = Kp * [(b*r-y) + \frac{1}{---} * \frac{1}{---} * (r-y) + Td * \frac{s}{-----} * (c*r-y)] 
with Kp = 1.13, Ti = 4.91, Td = 1.15, N = 9.43, b = 0.66, c = 0.0136

Continuous-time 2-DOF PIDF controller in standard form
```

## Convert Discrete-Time Dynamic System to 2-DOF Standard-Form PID Controller

Convert a discrete-time dynamic system that represents a 2-DOF PID controller with derivative filter to standard pidstd2 form.

The following state-space matrices represent a discrete-time 2-DOF PID controller with a sample time of  $0.05\ s.$ 

```
A = [1,0;0,0.6643];

B = [0.05,-0.05; -0.004553,0.3357];

C = [0.2301,10.66];

D = [0.8905,-11.79];

Ts = 0.05;

sys = ss(A,B,C,D,Ts);
```

When you convert sys to 2-DOF PID form, the result depends on which discrete integrator formulas you specify for the conversion. For instance, use the default, ForwardEuler, for both the integrator and the derivative.

```
with Kp = 1.13, Ti = 4.91, Td = 1.41, N = 9.43, b = 0.66, c = 0.0136, Ts = 0.05
Sample time: 0.05 seconds
Discrete-time 2-DOF PIDF controller in standard form
```

Now convert using the Trapezoidal formula.

The displays show the difference in resulting coefficient values and functional form.

For some dynamic systems, attempting to use the Trapezoidal or BackwardEuler integrator formulas yields invalid results, such as negative Ti, Td, or N values. In such cases, pidstd2 returns an error.

### Discretize a Standard-Form 2-DOF PID Controller

Discretize a continuous-time standard-form 2-DOF PID controller and specify the integral and derivative filter formulas.

Create a continuous-time pidstd2 controller and discretize it using the zero-order-hold method of the c2d command.

```
with Kp = 10, Ti = 5, Td = 3.03, N = 0.5, b = 1, c = 1, Ts = 0.1 Sample time: 0.1 seconds Discrete-time 2-DOF PIDF controller in standard form
```

The display shows that c2d computes new PID coefficients for the discrete-time controller.

The discrete integrator formulas of the discretized controller depend on the c2d discretization method, as described in "Tips" on page 2-929. For the zoh method, both IFormula and DFormula are ForwardEuler.

#### C2dis1.IFormula

```
ans =
'ForwardEuler'
C2dis1.DFormula
ans =
'ForwardEuler'
```

If you want to use different formulas from the ones returned by c2d, then you can directly set the Ts, IFormula, and DFormula properties of the controller to the desired values.

```
C2dis2 = C2con;
C2dis2.Ts = 0.1;
C2dis2.IFormula = 'BackwardEuler';
C2dis2.DFormula = 'BackwardEuler';
```

However, these commands do not compute new coefficients for the discretized controller. To see this, examine C2dis2 and compare the coefficients to C2con and C2dis1.

#### C2dis2

```
C2dis2 =  u = Kp * [(b*r-y) + \frac{1}{---} * \frac{r-y}{---} * (r-y) + Td * \frac{1}{(Td/N) + Ts*z/(z-1)}  with Kp = 10, Ti = 5, Td = 3, N = 0.5, b = 1, c = 1, Ts = 0.1 Sample time: 0.1 seconds Discrete-time 2-DOF PIDF controller in standard form
```

# **Tips**

- To design a PID controller for a particular plant, use pidtune or pidTuner. To create a tunable 2-DOF PID controller as a control design block, use tunablePID2.
- To break a 2-DOF controller into two SISO control components, such as a feedback controller and a feedforward controller, use getComponents.
- Create arrays of pidstd2 controllers by:
  - Specifying array values for one or more of the coefficients Kp, Ti, Td, N, b, and c.
  - Specifying an array of dynamic systems sys to convert to pid2 controller objects.
  - Using stack to build arrays from individual controllers or smaller arrays.
  - Passing an array of plant models to pidtune.

In an array of pidstd2 controllers, each controller must have the same sample time Ts and discrete integrator formulas IFormula and DFormula.

• To create or convert to a parallel-form controller, use pid2. Parallel form expresses the controller actions in terms of proportional, integral, and derivative gains  $K_p$ ,  $K_i$  and  $K_d$ , and a filter time constant  $T_f$ . For example, the relationship between the inputs and output of a continuous-time parallel-form 2-DOF PID controller is given by:

$$u=K_p(br-y)+\frac{K_i}{s}(r-y)+\frac{K_ds}{T_fs+1}(cr-y)\,.$$

- There are two ways to discretize a continuous-time pidstd2 controller:
  - Use the c2d command. c2d computes new parameter values for the discretized controller. The discrete integrator formulas of the discretized controller depend upon the c2d discretization method you use, as shown in the following table.

c2d Discretization Method	IFormula	DFormula
'zoh'	ForwardEuler	ForwardEuler
'foh'	Trapezoidal	Trapezoidal
'tustin'	Trapezoidal	Trapezoidal
'impulse'	ForwardEuler	ForwardEuler

c2d Discretization Method	IFormula	DFormula
'matched'	ForwardEuler	ForwardEuler

For more information about c2d discretization methods, See the c2d reference page. For more information about IFormula and DFormula, see "Properties" on page 2-912.

• If you require different discrete integrator formulas, you can discretize the controller by directly setting Ts, IFormula, and DFormula to the desired values. (See "Discretize a Standard-Form 2-DOF PID Controller" on page 2-927.) However, this method does not compute new gain and filter-constant values for the discretized controller. Therefore, this method might yield a poorer match between the continuous- and discrete-time pidstd2 controllers than using c2d.

# See Also

getComponents | pid2 | pidTuner | pidstddata2 | pidtune

# **Topics**

"Two-Degree-of-Freedom PID Controllers"

"Discrete-Time Proportional-Integral-Derivative (PID) Controllers"

### Introduced in R2015b

<sup>&</sup>quot;What Are Model Objects?"

# pidstddata

Access coefficients of standard-form PID controller

# **Syntax**

```
[Kp,Ti,Td,N] = pidstddata(sys)
[Kp,Ti,Td,N,Ts] = pidstddata(sys)
[Kp,Ti,Td,N,Ts] = pidstddata(sys, J1,...,JN)
```

# **Description**

[Kp, Ti, Td, N] = pidstddata(sys) returns the proportional gain Kp, integral time Ti, derivative time Td, and filter divisor N of the standard-form controller represented by the dynamic system sys.

```
[Kp,Ti,Td,N,Ts] = pidstddata(sys) also returns the sample time Ts.
```

[Kp,Ti,Td,N,Ts] = pidstddata(sys, J1,...,JN) extracts the data for a subset of entries in the array of sys dynamic systems. The indices J specify the array entries to extract.

# **Input Arguments**

### sys

SISO dynamic system or array of SISO dynamic systems. If sys is not a pidstd object, it must represent a valid PID controller that can be written in standard PID form.

J

Integer indices of N entries in the array sys of dynamic systems.

# **Output Arguments**

### Кp

Proportional gain of the standard-form PID controller represented by dynamic system sys.

If sys is a pidstd controller object, the output Kp is equal to the Kp value of sys.

If sys is not a pidstd object, Kp is the proportional gain of a standard-form PID controller equivalent to sys.

If sys is an array of dynamic systems, Kp is an array of the same dimensions as sys.

#### Τi

Integral time constant of the standard-form PID controller represented by dynamic system sys.

If sys is a pidstd controller object, the output Ti is equal to the Ti value of sys.

If sys is not a pidstd object, Ti is the integral time constant of a standard-form PID controller equivalent to sys.

If sys is an array of dynamic systems, Ti is an array of the same dimensions as sys.

### Td

Derivative time constant of the standard-form PID controller represented by dynamic system sys.

If sys is a pidstd controller object, the output Td is equal to the Td value of sys.

If sys is not a pidstd object, Td is the derivative time constant of a standard-form PID controller equivalent to sys.

If sys is an array of dynamic systems, Td is an array of the same dimensions as sys.

#### N

Filter divisor of the standard-form PID controller represented by dynamic system sys.

If sys is a pidstd controller object, the output N is equal to the N value of sys.

If sys is not a pidstd object, N is the filter time constant of a standard-form PID controller equivalent to sys.

If sys is an array of dynamic systems, N is an array of the same dimensions as sys.

#### Ts

Sample time of the dynamic system sys. Ts is always a scalar value.

# **Examples**

Extract the proportional, integral, and derivative gains and the filter time constant from a standard-form pidstd controller.

For the following pidstd object:

```
sys = pidstd(1,4,0.3,10);
```

you can extract the parameter values from sys by entering:

```
[Kp Ti Td N] = pidstddata(sys);
```

Extract the standard-form proportional and integral gains from an equivalent parallel-form PI controller.

For a standard-form PI controller, such as:

```
sys = pid(2,3);
```

you can extract the gains of an equivalent parallel-form PI controller by entering:

```
[Kp Ti] = pidstddata(sys)
```

These commands return the result:

```
Kp =
```

2

Ti =

0.6667

Extract parameters from a dynamic system that represents a PID controller.

The dynamic system

$$H(z) = \frac{(z - 0.5)(z - 0.6)}{(z - 1)(z + 0.8)}$$

represents a discrete-time PID controller with a derivative filter. Use pidstddata to extract the standard-form PID parameters.

```
H = zpk([0.5 \ 0.6],[1,-0.8],1,0.1); % sample time Ts = 0.1s [Kp Ti Td N Ts] = pidstddata(H);
```

the pidstddata function uses the default ForwardEuler discrete integrator formula for Iformula and Dformula to compute the parameter values.

Extract the gains from an array of PI controllers.

```
sys = pidstd(rand(2,3),rand(2,3)); % 2-by-3 array of PI controllers
[Kp Ti Td N] = pidstddata(sys);
```

The parameters Kp, Ti, Td, and N are also 2-by-3 arrays.

Use the index input  ${\tt J}$  to extract the parameters of a subset of  ${\tt sys}$ .

```
[Kp Ti Td N] = pidstddata(sys,5);
```

# **Tips**

If sys is not a pidstd controller object, pidstddata returns Kp, Ti, Td and N values of a standard-form controller equivalent to sys.

For discrete-time sys, piddata returns parameters of an equivalent pidstd controller. This controller has discrete integrator formulas Iformula and Dformula set to ForwardEuler. See the pidstd reference page for more information about discrete integrator formulas.

# See Also

get | pid | pidstd

## Introduced in R2010b

# pidstddata2

Access coefficients of standard-form 2-DOF PID controller

# **Syntax**

```
[Kp,Ti,Td,N,b,c] = pidstddata2(sys)
[Kp,Ti,Td,N,b,c,Ts] = pidstddata2(sys)
[Kp,Ti,Td,N,b,c,Ts] = pidstddata2(sys,J1,...,JN)
```

# **Description**

[Kp,Ti,Td,N,b,c] = pidstddata2(sys) returns the proportional gain Kp, integral time Ti, derivative time Td, the filter divisor N, and the setpoint weights b and c of the standard-form 2-DOF PID controller represented by the dynamic system sys.

If sys is a pidstd2 controller object, then each output argument is the corresponding coefficient in sys.

If sys is not a pidstd2 object, then each output argument is the corresponding coefficient of the standard-form 2-DOF PID controller that is equivalent to sys.

If sys is an array of dynamic systems, then each output argument is an array of the same dimensions as sys.

[Kp,Ti,Td,N,b,c,Ts] = pidstddata2(sys) also returns the sample time Ts. For discrete-time sys that is not a pidstd2 object, pidstddata2 calculates the coefficient values using the default ForwardEuler discrete integrator formula for both IFormula and DFormula. See the pidstd2 reference page for more information about discrete integrator formulas.

[Kp,Ti,Td,N,b,c,Ts] = pidstddata2(sys,J1,...,JN) extracts the data for a subset of entries in sys, where sys is an N-dimensional array of dynamic systems. The indices J specify the array entry to extract.

# **Examples**

### Extract Coefficients from Standard-Form 2-DOF PID Controller

Typically, you extract coefficients from a controller obtained from another function, such as pidtune or getBlockValue. For this example, create a standard-form 2-DOF PID controller that has random coefficients.

```
rng('default'); % for reproducibility
C2 = pidstd2(rand, rand, rand, rand, rand, rand);
```

Extract the PID coefficients, filter divisor, and setpoint weights.

```
[Kp,Ti,Td,N,b,c] = pidstddata2(C2);
```

### **Extract Standard-Form Coefficients from Parallel-Form Controller**

Create a 2-DOF PID controller in parallel form.

Compute the coefficients of an equivalent parallel-form PID controller.

```
[Kp,Ti,Td,N,b,c] = pidstddata2(C2);
```

Check some of the coefficients to confirm that they are different from the parallel-form coefficients.

```
Ti = 0.6667
```

Td Td = 2

## **Extract Standard-Form 2-DOF PID Coefficients from Equivalent System**

Extract coefficients from a two-input, one-output dynamic system that represents a valid 2-DOF standard-form PID controller.

The following A, B, C, and D matrices form a discrete-time state-space model that represents a 2-DOF PID controller in standard form.

```
A = [1,0;0,0.5];
B = [0.1, -0.1; -0.25, 0.5];
C = [4,400];
D = [220, -440];
sys = ss(A,B,C,D,0.1)
sys =
 A =
       x1 x2
  x1
        1 0
  x2
        0 0.5
  B =
         u1
               u2
  x1
        0.1
              -0.1
      -0.25
  x2
             0.5
  C =
       x1 x2
  у1
       4 400
  D =
        u1
             u2
       220 -440
  ٧1
Sample time: 0.1 seconds
Discrete-time state-space model.
```

Extract the PID coefficients, filter divisor, and setpoint weights of the model.

```
[Kp,Ti,Td,N,b,c,Ts] = pidstddata2(sys);
```

For a discrete-time system, pidstddata2 calculates the coefficient values using the default ForwardEuler discrete integrator formula for both IFormula and DFormula.

## **Extract Standard-Form Coefficients from 2-DOF PI Controller Array**

Typically, you obtain an array of controllers by using pidtune on an array of plant models. For this example, create an 2-by-3 array of standard-form 2-DOF PI controllers with random values of Kp, Ti, and b.

```
rng('default');
C2 = pidstd2(rand(2,3),rand(2,3),0,10,rand(2,3),0);
```

Extract all the coefficients from the array.

```
[Kp,Ti,Td,N,b,c] = pidstddata2(C2);
```

Each of the outputs is itself a 2-by-3 array. For example, examine Ki.

Τi

```
Ti = 2×3

0.2785 0.9575 0.1576

0.5469 0.9649 0.9706
```

Extract only the coefficients of entry (2,1) in the array.

```
[Kp21,Ti21,Td21,N21,b21,c21] = pidstddata2(C2,2,1);
```

Each of these outputs is a scalar.

Ti21

```
Ti21 = 0.5469
```

# **Input Arguments**

### sys — 2-DOF PID controller

pidstd2 controller object | dynamic system model | dynamic system array

2-DOF PID controller in standard form, specified as a pidstd2 controller object, a dynamic system model, or a dynamic system array. If sys is not a pidstd2 controller object, it must be a two-input, one-output model that represents a valid 2-DOF PID controller that can be written in standard form.

### J - Indices

positive integers

Indices of entry to extract from a model array sys, specified as positive integers. Provide as many indices as there are array dimensions in sys. For example, suppose sys is a 4-by-5 (two-dimensional) array of pidstd2 controllers or dynamic system models that represent 2-DOF PID controllers. The following command extracts the data for entry (2,3) in the array.

[Kp,Ti,Td,N,b,c,Ts] = piddstdata2(sys,2,3);

# **Output Arguments**

## **Kp** — Proportional gain

scalar | array

Proportional gain of the standard-form 2-DOF PID controller represented by sys, returned as a scalar or array.

If sys is a pidstd2 controller object, then Kp is the Kp value of sys.

If sys is not a pidstd2 object, then Kp is the proportional gain of the standard-form 2-DOF PID controller that is equivalent to sys.

If sys is an array of dynamic systems, then Kp is an array of the same dimensions as sys.

# Ti — Integral time constant

scalar | array

Integral time constant of the standard-form 2-DOF PID controller represented by sys, returned as a scalar or array.

### Td — Derivative time constant

scalar | array

Derivative time constant of the standard-form 2-DOF PID controller represented by sys, returned as a scalar or array.

#### N — Filter divisor

scalar | array

Filter divisor of the parallel-form 2-DOF PID controller represented by sys, returned as a scalar or array.

### b — Setpoint weight on proportional term

scalar | array

Setpoint weight on the proportional term of the standard-form 2-DOF PID controller represented by sys, returned as a scalar or array.

### c — Setpoint weight on derivative term

scalar | array

Setpoint weight on the derivative term of the standard-form 2-DOF PID controller represented by sys, returned as a scalar or array.

# Ts — Sample time

scalar

Sample time of the pidstd2 controller, dynamic system sys, or dynamic system array, returned as a scalar.

# See Also

piddata2|pidstd2|pidstddata

Introduced in R2015b

# pidtool

Open PID Tuner for PID tuning

**Note** pidtool has been removed. Use pidTuner instead.

Introduced in R2010b

# pidtune

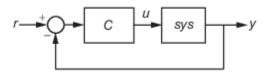
PID tuning algorithm for linear plant model

# **Syntax**

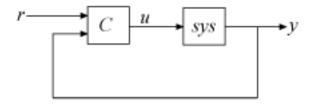
```
C = pidtune(sys,type)
C = pidtune(sys,C0)
C = pidtune(sys,type,wc)
C = pidtune(sys,C0,wc)
C = pidtune(sys,...,opts)
[C,info] = pidtune(...)
```

# **Description**

C = pidtune(sys,type) designs a PID controller of type type for the plant sys. If type specifies a one-degree-of-freedom (1-DOF) PID controller, then the controller is designed for the unit feedback loop as illustrated:



If type specifies a two-degree-of-freedom (2-DOF) PID controller, then pidtune designs a 2-DOF controller as in the feedback loop of this illustration:



pidtune tunes the parameters of the PID controller C to balance performance (response time) and robustness (stability margins).

C = pidtune(sys,C0) designs a controller of the same type and form as the controller C0. If sys and C0 are discrete-time models, C has the same discrete integrator formulas as C0.

C = pidtune(sys,type,wc) and C = pidtune(sys,C0,wc) specify a target value wc for the first 0 dB gain crossover frequency of the open-loop response.

C = pidtune(sys,...,opts) uses additional tuning options, such as the target phase margin. Use pidtuneOptions to specify the option set opts.

[C,info] = pidtune(...) returns the data structure info, which contains information about closed-loop stability, the selected open-loop gain crossover frequency, and the actual phase margin.

# **Input Arguments**

#### sys

Single-input, single-output dynamic system model of the plant for controller design. sys can be:

- Any type of SISO dynamic system model, including Numeric LTI models and identified models. If sys is a tunable or uncertain model, pidtune designs a controller for the current or nominal value of sys.
- A continuous- or discrete-time model.
- Stable, unstable, or integrating. A plant with unstable poles, however, might not be stabilizable under PID control.
- A model that includes any type of time delay. A plant with long time delays, however, might not achieve adequate performance under PID control.
- An array of plant models. If sys is an array, pidtune designs a separate controller for each plant in the array.

If the plant has unstable poles, and sys is one of the following:

• A frd model

A ss model with internal time delays that cannot be converted to I/O delays

you must use pidtuneOptions to specify the number of unstable poles in the plant, if any.

### type

Controller type of the controller to design, specified as a character vector. The term controller type refers to which terms are present in the controller action. For example, a PI controller has only a proportional and an integral term, while a PIDF controller contains proportional, integrator, and filtered derivative terms. type can take the values summarized below. For more detailed information about these controller types, see "PID Controller Types for Tuning"

#### 1-DOF Controllers

- 'P' Proportional only
- 'I' Integral only
- 'PI' Proportional and integral
- 'PD' Proportional and derivative
- 'PDF' Proportional and derivative with first-order filter on derivative term
- 'PID' Proportional, integral, and derivative
- 'PIDF' Proportional, integral, and derivative with first-order filter on derivative term

#### 2-DOF Controllers

- 'PI2' 2-DOF proportional and integral
- 'PD2' 2-DOF proportional and derivative
- $\bullet$  'PDF2' 2-DOF proportional and derivative with first-order filter on derivative term
- 'PID2' 2-DOF proportional, integral, and derivative
- 'PIDF2' 2-DOF proportional, integral, and derivative with first-order filter on derivative term

For more information about 2-DOF PID controllers generally, see "Two-Degree-of-Freedom PID Controllers".

### 2-DOF Controllers with Fixed Setpoint Weights

- 'I-PD' 2-DOF PID with b = 0, c = 0
- 'I-PDF' 2-DOF PIDF with b = 0, c = 0
- 'ID-P' 2-DOF PID with b = 0, c = 1
- 'IDF-P' 2-DOF PIDF with b = 0, c = 1
- 'PI-D' 2-DOF PID with b = 1, c = 0
- 'PI-DF' 2-DOF PIDF with b = 1, c = 0

For more detailed information about fixed-setpoint-weight 2-DOF PID controllers, see "PID Controller Types for Tuning".

#### **Controller Form**

When you use the type input, pidtune designs a controller in parallel (pid or pid2) form. Use the input C0 instead of type if you want to design a controller in standard (pidstd or pidstd2) form.

If sys is a discrete-time model with sample time Ts, pidtune designs a discrete-time controller with the same Ts. The controller has the ForwardEuler discrete integrator formula for both integral and derivative actions. Use the input C0 instead of type if you want to design a controller having a different discrete integrator formula.

For more information about PID controller forms and formulas, see:

- "Proportional-Integral-Derivative (PID) Controllers"
- "Two-Degree-of-Freedom PID Controllers"
- "Discrete-Time Proportional-Integral-Derivative (PID) Controllers"

### C<sub>0</sub>

PID controller setting properties of the designed controller, specified as a pid, pidstd, pid2, or pidstd2 object. If you provide C0, pidtune:

- Designs a controller of the type represented by CO.
- Returns a pid controller, if CO is a pid controller.
- Returns a pidstd controller, if CO is a pidstd controller.
- Returns a 2-DOF pid2 controller, if C0 is a pid2 controller.

- Returns a 2-DOF pidstd2 controller, if C0 is a pidstd2 controller.
- Returns a controller with the same Iformula and Dformula values as C0, if sys is a
  discrete-time system. See the pid, pid2, pidstd, and pidstd2 reference pages for
  more information about Iformula and Dformula.

#### WC

Target value for the 0 dB gain crossover frequency of the tuned open-loop response. Specify wc in units of radians/TimeUnit, where TimeUnit is the time unit of sys. The crossover frequency wc roughly sets the control bandwidth. The closed-loop response time is approximately 1/wc.

Increase wc to speed up the response. Decrease wc to improve stability. When you omit wc, pidtune automatically chooses a value, based on the plant dynamics, that achieves a balance between response and stability.

### opts

Option set specifying additional tuning options for the pidtune design algorithm, such as target phase margin or design focus. Use pidtuneOptions to create opts.

# **Output Arguments**

### C

Controller designed for sys. If sys is an array of linear models, pidtune designs a controller for each linear model and returns an array of PID controllers.

#### Controller form:

- If the second argument to pidtune is type, C is a pid or pid2 controller.
- If the second argument to pidtune is C0:
  - C is a pid controller, if CO is a pid object.
  - C is a pidstd controller, if CO is a pidstd object.
  - C is a pid2 controller, if C0 is a pid2 object.
  - C is a pidstd2 controller, if C0 is a pidstd2 object.

## **Controller type:**

- If the second argument to pidtune is type, C generally has the specified type.
- If the second argument to pidtune is CO, C generally has the same type as CO.

In either case, however, where the algorithm can achieve adequate performance and robustness using a lower-order controller than specified with type or C0, pidtune returns a C having fewer actions than specified. For example, C can be a PI controller even though type is 'PIDF'.

#### Time domain:

- C has the same time domain as sys.
- If sys is a discrete-time model, C has the same sample time as sys.
- If you specify C0, C has the same Iformula and Dformula as C0. If no C0 is specified, both Iformula and Dformula are Forward Euler. See the pid, pid2, pidstd, and pidstd2 reference pages for more information about Iformula and Dformula.

If you specify C0, C also obtains model properties such as InputName and OutputName from C0. For more information about model properties, see the reference pages for each type of dynamic system model.

#### info

Data structure containing information about performance and robustness of the tuned PID loop. The fields of info are:

- Stable Boolean value indicating closed-loop stability. Stable is 1 if the closed loop is stable, and 0 otherwise.
- CrossoverFrequency First 0 dB crossover frequency of the open-loop system C\*sys, in rad/TimeUnit, where TimeUnit is the time units specified in the TimeUnit property of sys.
- PhaseMargin Phase margin of the tuned PID loop, in degrees.

If sys is an array of plant models, info is an array of data structures containing information about each tuned PID loop.

# **Examples**

# **PID Controller Design at the Command Line**

This example shows how to design a PID controller for the plant given by:

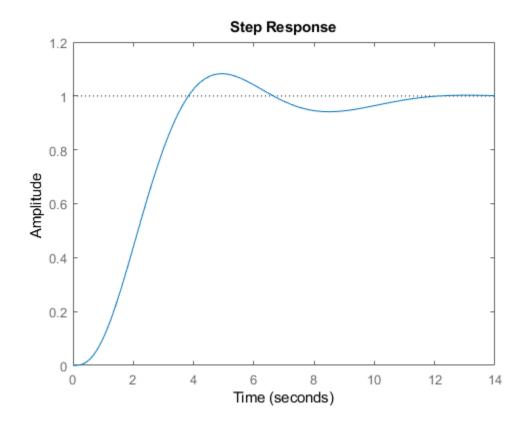
$$sys = \frac{1}{\left(s+1\right)^3}.$$

As a first pass, create a model of the plant and design a simple PI controller for it.

C\_pi is a pid controller object that represents a PI controller. The fields of info show that the tuning algorithm chooses an open-loop crossover frequency of about 0.52 rad/s.

Examine the closed-loop step response (reference tracking) of the controlled system.

```
T_pi = feedback(C_pi*sys, 1);
step(T_pi)
```



To improve the response time, you can set a higher target crossover frequency than the result that pidtune automatically selects, 0.52. Increase the crossover frequency to 1.0.

$$C_pi_fast =$$

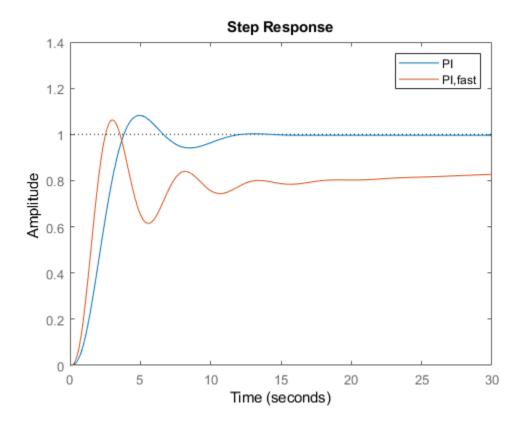
with Kp = 2.83, Ki = 0.0495

Continuous-time PI controller in parallel form.

The new controller achieves the higher crossover frequency, but at the cost of a reduced phase margin.

Compare the closed-loop step response with the two controllers.

```
T_pi_fast = feedback(C_pi_fast*sys,1);
step(T_pi,T_pi_fast)
axis([0 30 0 1.4])
legend('PI','PI,fast')
```



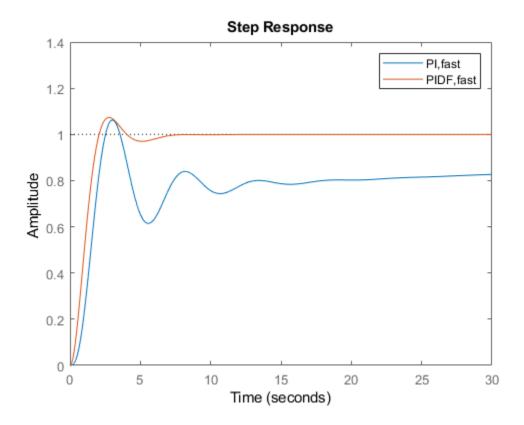
This reduction in performance results because the PI controller does not have enough degrees of freedom to achieve a good phase margin at a crossover frequency of 1.0 rad/s. Adding a derivative action improves the response.

Design a PIDF controller for Gc with the target crossover frequency of 1.0 rad/s.

The fields of info show that the derivative action in the controller allows the tuning algorithm to design a more aggressive controller that achieves the target crossover frequency with a good phase margin.

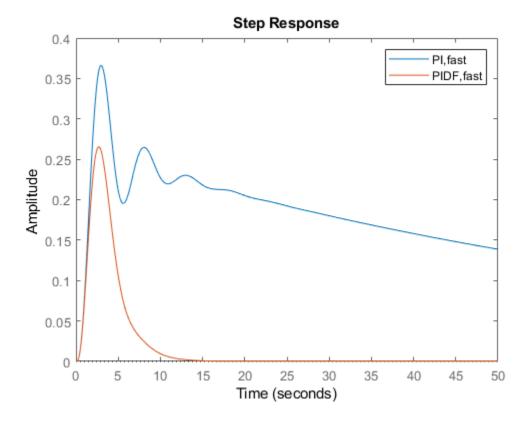
Compare the closed-loop step response and disturbance rejection for the fast PI and PIDF controllers.

```
T_pidf_fast = feedback(C_pidf_fast*sys,1);
step(T_pi_fast, T_pidf_fast);
axis([0 30 0 1.4]);
legend('PI,fast','PIDF,fast');
```



You can compare the input (load) disturbance rejection of the controlled system with the fast PI and PIDF controllers. To do so, plot the response of the closed-loop transfer function from the plant input to the plant output.

```
S_pi_fast = feedback(sys,C_pi_fast);
S_pidf_fast = feedback(sys,C_pidf_fast);
step(S_pi_fast,S_pidf_fast);
axis([0 50 0 0.4]);
legend('PI,fast','PIDF,fast');
```



This plot shows that the PIDF controller also provides faster disturbance rejection.

# **Design Standard-Form PID Controller**

Design a PID controller in standard form for the plant defined by  $% \left\{ \left\{ 1\right\} \right\} =\left\{ 1\right\} =\left\{$ 

$$sys = \frac{1}{\left(s+1\right)^3}.$$

To design a controller in standard form, use a standard-form controller as the  ${\tt CO}$  argument to  ${\tt pidtune}.$ 

```
sys = zpk([],[-1 -1 -1],1);
C0 = pidstd(1,1,1);
C = pidtune(sys,C0)

C =

    Kp * (1 + --- * --- + Td * s)
        Ti     s

with Kp = 2.18, Ti = 2.36, Td = 0.591

Continuous-time PID controller in standard form
```

## **Specify Integrator Discretization Method**

Design a discrete-time PI controller using a specified method to discretize the integrator.

If your plant is in discrete time, pidtune automatically returns a discrete-time controller using the default Forward Euler integration method. To specify a different integration method, use pid or pidstd to create a discrete-time controller having the desired integration method.

Using C0 as an input causes pidtune to design a controller C of the same form, type, and discretization method as C0. The display shows that the integral term of C uses the Backward Euler integration method.

Specify a Trapezoidal integrator and compare the resulting controller.

## **Design 2-DOF PID Controller**

Design a 2-DOF PID Controller for the plant given by the transfer function:

$$G(s) = \frac{1}{s^2 + 0.5s + 0.1}.$$

Use a target bandwidth of 1.5 rad/s.

```
wc = 1.5;
G = tf(1,[1 0.5 0.1]);
C2 = pidtune(G,'PID2',wc)

C2 =

u = Kp (b*r-y) + Ki --- (r-y) + Kd*s (c*r-y)
s

with Kp = 1.26, Ki = 0.255, Kd = 1.38, b = 0.665, c = 0

Continuous-time 2-DOF PID controller in parallel form.
```

Using the type 'PID2' causes pidtune to generate a 2-DOF controller, represented as a pid2 object. The display confirms this result. The display also shows that pidtune tunes all controller coefficients, including the setpoint weights b and c, to balance performance and robustness.

# **Tips**

- By default, pidtune with the type input returns a pid controller in parallel form. To
  design a controller in standard form, use a pidstd controller as input argument CO.
  For more information about parallel and standard controller forms, see the pid and
  pidstd reference pages.
- For interactive PID tuning in the Live Editor, see the Tune PID Controller Live Editor
  task. This task lets you interactively design a PID controller and automatically
  generates MATLAB code for your live script.

# **Algorithms**

For information about the MathWorks® PID tuning algorithm, see "PID Tuning Algorithm".

## **Alternatives**

- For interactive PID tuning in the Live Editor, see the Tune PID Controller Live Editor
  task. This task lets you interactively design a PID controller and automatically
  generates MATLAB code for your live script. For an example, see "PID Controller
  Design in the Live Editor"
- For interactive PID tuning in a standalone app, use PID Tuner. See "PID Controller Design for Fast Reference Tracking" for an example of designing a controller using the app.

## References

Aström, K. J. and Hägglund, T. *Advanced PID Control*, Research Triangle Park, NC: Instrumentation, Systems, and Automation Society, 2006.

## See Also

#### **Functions**

pidTuner | pidtuneOptions

# **Apps** PID Tuner

**Live Editor Tasks Tune PID Controller** 

### **Objects**

pid | pid2 | pidstd | pidstd2

## **Topics**

"Tune 2-DOF PID Controller (Command Line)"

"Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (Command Line)"  $\,$ 

"Designing Cascade Control System with PI Controllers"

"PID Controller Types for Tuning"

"PID Tuning Algorithm"

### Introduced in R2010b

# pidtuneOptions

Define options for pidtune command

# **Syntax**

```
opt = pidtuneOptions
opt = pidtuneOptions(Name, Value)
```

# **Description**

opt = pidtuneOptions returns the default option set for the pidtune command.

opt = pidtuneOptions(Name, Value) creates an option set with the options specified
by one or more Name, Value pair arguments.

# **Input Arguments**

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

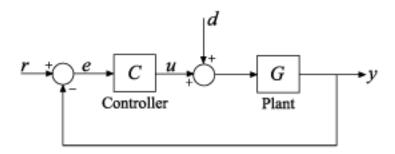
### **PhaseMargin**

Target phase margin in degrees. pidtune attempts to design a controller such that the phase margin is at least the value specified for PhaseMargin. The selected crossover frequency could restrict the achievable phase margin. Typically, higher phase margin improves stability and overshoot, but limits bandwidth and response speed.

Default: 60

### DesignFocus

Closed-loop performance objective to favor in the design. For a given target phase margin, pidtune chooses a controller design that balances the two measures of performance, reference tracking and disturbance rejection. When you change the DesignFocus option, the tuning algorithm attempts to adjust the PID gains to favor either reference tracking or disturbance rejection while achieving the same target phase margin. In the control architecture assumed by pidtune, shown in the following diagram, reference tracking is the response at y to signals at r, and disturbance rejection is the suppression at y of signals at d.



The DesignFocus option can take the following values:

- 'balanced' (default) For a given robustness, tune the controller to balance reference tracking and disturbance rejection.
- 'reference-tracking' Tune the controller to favor reference tracking, if possible.
- 'disturbance-rejection' Tune the controller to favor disturbance rejection, if possible.

The more tunable parameters there are in the system, the more likely it is that the PID algorithm can achieve the desired design focus without sacrificing robustness. For example, setting the design focus is more likely to be effective for PID controllers than for P or PI controllers. In all cases, how much you can fine-tune the performance of the system depends strongly on the properties of your plant.

For an example illustrating the effect of this option, see "Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (Command Line)".

Default: 'balanced'

### **NumUnstablePoles**

Number of unstable poles in the plant. When your plant is a frd model or a state-space model with internal delays, you must specify the number of open-loop unstable poles (if any). Incorrect values might result in PID controllers that fail to stabilize the real plant. (pidtune ignores this option for other model types.)

Unstable poles are poles located at:

- Re(s) > 0, for continuous-time plants
- |z| > 1, for discrete-time plants

A pure integrator in the plant (s=0) or (|z|>1) does not count as an unstable pole for NumUnstablePoles. If your plant is a frd model of a plant with a pure integrator, for best results, ensure that your frequency response data covers a low enough frequency to capture the integrator slope.

Default: 0

# **Output Arguments**

### opt

Object containing the specified options for pidtune.

# **Examples**

Tune a PIDF controller with a target phase margin of 45 degrees, favoring the disturbance-rejection measure of performance.

```
sys = tf(1,[1 3 3 1]);
opts = pidtuneOptions('PhaseMargin',45,'DesignFocus','disturbance-rejection');
[C,info] = pidtune(sys,'pid',opts);
```

# **Tips**

- When using the pidtune command to design a PID controller for a plant with unstable poles, if your plant model is one of the following:
  - A frd model
  - A ss model with internal delays that cannot be converted to I/O delays

then use pidtuneOptions to specify the number of unstable poles in the plant.

## See Also

pidtune

## **Topics**

"PID Tuning Algorithm"

"Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (Command Line)"  ${\bf r}$ 

### Introduced in R2010b

## **PID Tuner**

Tune PID controllers

# **Description**

The **PID Tuner** app automatically tunes the gains of a PID controller for a SISO plant to achieve a balance between performance and robustness. You can specify the controller type, such as PI, PID with derivative filter, or two-degree-of-freedom (2-DOF) PID controllers. Analysis plots let you examine controller performance in time and frequency domains. You can interactively refine the performance of the controller to adjust loop bandwidth and phase margin, or to favor setpoint tracking or disturbance rejection.

You can use **PID Tuner** with a plant represented by a numeric LTI model such as a transfer function (tf) or state-space (ss) model. If you have Simulink Control Design software, you can use **PID Tuner** to tune a PID Controller or PID Controller (2DOF) block in a Simulink model. If you have System Identification Toolbox software, you can use the app to estimate a plant from measured or simulated data and design a controller for the estimated plant.

## Interactive Tuning in the Live Editor

For interactive PID tuning in the Live Editor, see the **Tune PID Controller** Live Editor task. This task lets you interactively design a PID controller and automatically generates MATLAB code for your live script.

## Limitations

When using MATLAB Online, the redesigned **PID Tuner** app and system identification features introduced in R2014a are not available. For information on using the previous **PID Tuner**, see PID Controller Tuning in the R2013b documentation.

# **Open the PID Tuner App**

 MATLAB Toolstrip: On the Apps tab, under Control System Design and Analysis, click the app icon.

- MATLAB command prompt: Enter pidTuner.
- Simulink model: In the PID Controller or PID Controller (2DOF) block dialog box, click Tune.

# **Examples**

- "Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (PID Tuner)"
- "PID Controller Tuning in Simulink" (Simulink Control Design)
- "Designing PID Controllers with PID Tuner"
- "Introduction to Model-Based PID Tuning in Simulink" (Simulink Control Design)

## **Parameters**

### **Plant** — Current plant

LTI model in Data Browser | Import | ...

The **Plant** menu displays the name of the current plant that **PID Tuner** is using for controller design.

Change the current plant using the following menu options:

- A list of the LTI models present in the **PID Tuner** Data Browser.
- Import Import a new LTI model from the MATLAB workspace.
- Re-Linearize Closed Loop Linearize the plant at a different snapshot time. See "Tune at a Different Operating Point" (Simulink Control Design). This option is available only when tuning a PID Controller or PID Controller (2DOF) block in a Simulink model.
- Identify New Plant Use system identification to obtain a plant from measured or simulated system response data (requires System Identification Toolbox software).
   See:
  - "Interactively Estimate Plant Parameters from Response Data", when tuning a PID controller for an LTI model.
  - "Interactively Estimate Plant from Measured or Simulated Response Data" (Simulink Control Design), when tuning a PID Controller block in a Simulink model.

If you are tuning a PID controller for a plant represented by an LTI model, the default plant is:

- Plant = 1, if you opened **PID Tuner** from the **Apps** tab in the MATLAB Toolstrip, or if you used the pidTuner command without an input argument.
- The plant you specified as an input argument to pidTuner.

If you are tuning a PID Controller or PID Controller (2DOF) block in a Simulink model, then the default plant is linearized at the operating point specified by the model initial conditions. See "What Plant Does PID Tuner See?" (Simulink Control Design)

```
Type — Controller type 'PI' | 'PIDF' | 'PID2' | ...
```

The controller type specifies which terms are present in the PID controller. For instance, a PI controller has a proportional and an integral term. A PDF controller has a proportional term and a filtered derivative term.

- If you are tuning a controller for a plant represented by an LTI model, use the Type
  menu to specify controller type. When you change controller type, PID Tuner
  automatically designs a new controller. Available controller types include 2-DOF PID
  controllers for more flexibility in the trade-off between disturbance rejection and
  reference tracking. For details on available controller types, see "PID Controller Types
  for Tuning".
- If you are tuning a PID Controller or PID Controller (2DOF) block in a Simulink model, the **Type** field displays the controller type specified in the block dialog box.

# Form — Controller form 'Parallel' | 'Standard'

This field displays the controller form.

- If you are tuning a controller for a plant represented by an LTI model, use the Form
  menu to specify controller form. For information about parallel and standard forms,
  see the pid and pidstd reference pages.
- If you are tuning a PID Controller or PID Controller (2DOF) block in a Simulink model, the Form field displays the controller form specified in the block dialog box.

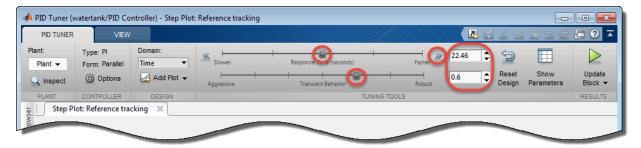
# Domain — Domain for specifying performance targets

```
'Time'|'Frequency'
```

The **Domain** menu enables you to switch the domain in which PID Tuner displays the target performance parameters.

- Time Sliders set the **Response Time** and **Transient Behavior**.
- Frequency Sliders set the **Bandwidth** and **Phase Margin**.

To refine the controller design, you adjust the target performance parameters using the sliders or the corresponding numeric values.



For more information, see:

- "Refine the Design" (tuning a controller for an LTI model)
- "Refine the Design" (Simulink Control Design) (tuning PID Controller or PID Controller (2DOF) block in Simulink model)

## Add Plot — Create analysis plots

Reference Tracking | Input Disturbance Rejection | Controller Effort | ...

Create time-domain and frequency-domain analysis plots to help analyze the performance of the PID controller. For detailed information about the available response plots, see:

- "Analyze Design in PID Tuner" (tuning a controller for an LTI model)
- "Analyze Design in PID Tuner" (Simulink Control Design) (tuning PID Controller or PID Controller (2DOF) block in Simulink model)

## **Programmatic Use**

pidTuner

# **Tips**

- For PID tuning at the command line, use pidtune. The pidtune command can design a controller for multiple plants at once.
- For interactive PID tuning in the Live Editor, see the Tune PID Controller Live Editor
  task. This task lets you interactively design a PID controller and automatically
  generates MATLAB code for your live script.

## See Also

**Functions** pidtune

**Live Editor Tasks Tune PID Controller** 

## **Topics**

"Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (PID Tuner)" "PID Controller Tuning in Simulink" (Simulink Control Design)

"Designing PID Controllers with PID Tuner"

"Introduction to Model-Based PID Tuning in Simulink" (Simulink Control Design)

#### Introduced in R2010b

# pidTuner

Open PID Tuner for PID tuning

# **Syntax**

pidTuner(sys,type)
pidTuner(sys,Cbase)
pidTuner(sys)
pidTuner

# **Description**

pidTuner(sys,type) launches the PID Tuner app and designs a controller of type type for plant sys.

pidTuner(sys,Cbase) launches PID Tuner with a baseline controller Cbase so that you can compare performance between the designed controller and the baseline controller. If Cbase is a pid, pidstd, pid2 or pidstd2 controller object, PID Tuner designs a controller of the same form, type, and discrete integrator formulas as Cbase.

pidTuner(sys) designs a parallel-form PI controller.

pidTuner launches PID Tuner with default plant of 1 and proportional (P) controller of 1.

# **Input Arguments**

### sys

Plant model for controller design. sys can be:

- Any SISO LTI system (such as ss, tf, zpk, or frd).
- Any System Identification Toolbox SISO linear model (idtf, idfrd, idgrey, idpoly, idproc, or idss).

- A continuous- or discrete-time model.
- Stable, unstable, or integrating. However, you might not be able to stabilize a plant with unstable poles under PID control.
- A model that includes any type of time delay. A plant with long time delays, however, might not achieve adequate performance under PID control.

If the plant has unstable poles, and sys is either:

- · A frd model
- A ss model with internal time delays that cannot be converted to I/O delays

then you must specify the number of unstable poles in the plant. To do this, after opening

PID Tuner, in the **Plant** menu, select Import. In the Import Linear System dialog box, reimport sys, specifying the number of unstable poles where prompted.

### type

Controller type of the controller to design, specified as a character vector. The term controller type refers to which terms are present in the controller action. For example, a PI controller has only a proportional and an integral term, while a PIDF controller contains proportional, integrator, and filtered derivative terms. type can take the values summarized below. For more detailed information about these controller types, see "PID Controller Types for Tuning"

#### 1-DOF Controllers

- 'P' Proportional only
- 'I' Integral only
- 'PI' Proportional and integral
- 'PD' Proportional and derivative
- ullet 'PDF' Proportional and derivative with first-order filter on derivative term
- 'PID' Proportional, integral, and derivative
- 'PIDF' Proportional, integral, and derivative with first-order filter on derivative term

### 2-DOF Controllers

• 'PI2' - 2-DOF proportional and integral

- 'PD2' 2-DOF proportional and derivative
- 'PDF2' 2-DOF proportional and derivative with first-order filter on derivative term
- 'PID2' 2-DOF proportional, integral, and derivative
- 'PIDF2' 2-DOF proportional, integral, and derivative with first-order filter on derivative term

For more information about 2-DOF PID controllers generally, see "Two-Degree-of-Freedom PID Controllers".

### 2-DOF Controllers with Fixed Setpoint Weights

- 'I-PD' 2-DOF PID with b = 0, c = 0
- 'I-PDF' 2-DOF PIDF with b = 0, c = 0
- 'ID-P' 2-DOF PID with b = 0, c = 1
- 'IDF-P' 2-DOF PIDF with b = 0, c = 1
- 'PI-D' 2-DOF PID with b = 1, c = 0
- 'PI-DF' 2-DOF PIDF with b = 1, c = 0

For more detailed information about fixed-setpoint-weight 2-DOF PID controllers, see "PID Controller Types for Tuning".

### **Controller Form**

When you use the type input, PID Tuner designs a controller in parallel form. If you want to design a controller in standard form, Use the input Cbase instead of type, or select Standard from the **Form** menu. For more information about parallel and standard forms, see the pid and pidstd reference pages.

If sys is a discrete-time model with sample time Ts, PID Tuner designs a discrete-time pid controller using the ForwardEuler discrete integrator formula. To design a controller having a different discrete integrator formula:

- Use the input argument Cbase instead of type. PID Tuner reads controller type, form, and discrete integrator formulas from the baseline controller Cbase.
- In PID Tuner, click **Options** to open the Controller Options dialog box. Select discrete integrator formulas from the **Integral Formula** and **Derivative Formula** menus.

For more information about discrete integrator formulas, see the pid and pidstd reference pages.

### Cbase

A dynamic system representing a baseline controller, permitting comparison of the performance of the designed controller to the performance of Cbase.

If Cbase is a pid or pidstd object, PID Tuner also uses it to configure the type, form, and discrete integrator formulas of the designed controller. The designed controller:

- Is the type represented by Cbase.
- Is a parallel-form controller, if Cbase is a pid controller object.
- Is a standard-form controller, if Cbase is a pidstd controller object.
- Is a parallel-form 2-DOF controller, if Cbase is a pid2 controller object.
- Is a standard-form 2-DOF controller, if Cbase is a pidstd2 controller object.
- Has the same Iformula and Dformula values as Cbase. For more information about Iformula and Dformula, see the pid and pidstd reference pages.

If Cbase is any other dynamic system, PID Tuner designs a parallel-form PI controller. You can change the controller form and type using the **Form** and **Type** menus after launching PID Tuner.

# **Examples**

### **Interactive PID Tuning of Parallel-Form Controller**

Launch PID Tuner to design a parallel-form PIDF controller for a discrete-time plant:

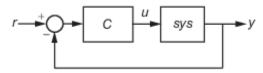
# **Interactive PID Tuning of Standard-Form Controller Using Integrator Discretization Method**

Design a standard-form PIDF controller using BackwardEuler discrete integrator formula:

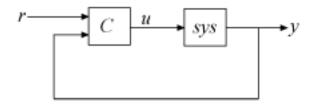
PID Tuner designs a controller for Gd having the same form, type, and discrete integrator formulas as Cbase. For comparison, you can display the response plots of Cbase with the response plots of the designed controller by clicking the **Show baseline** checkbox in PID Tuner.

# **Tips**

• If type or Cbase specifies a one-degree-of-freedom (1-DOF) PID controller, then pidTuner designs a controller for the unit feedback loop as illustrated:



• If type or Cbase specifies a two-degree-of-freedom (2-DOF) PID controller, then pidTuner designs a 2-DOF controller as in the feedback loop of this illustration:



PID Tuner has a default target phase margin of 60 degrees and automatically tunes
the PID gains to balance performance (response time) and robustness (stability
margins). Use the Response time or Bandwidth and Phase Margin sliders to tune
the controller's performance to your requirements. Increasing performance typically
decreases robustness, and vice versa.

- Select response plots from the **Response** menu to analyze the controller's performance.
- If you provide Cbase, check Show baseline to display the response of the baseline controller.
- For more detailed information about using PID Tuner, see "Designing PID Controllers with PID Tuner".
- For interactive PID tuning in the Live Editor, see the **Tune PID Controller** Live Editor task. This task lets you interactively design a PID controller and automatically generates MATLAB code for your live script.

# **Algorithms**

For information about the MathWorks PID tuning algorithm, see "PID Tuning Algorithm".

## **Alternatives**

You can open PID Tuner from the MATLAB desktop, in the **Apps** tab. When you do so, use the **Plant** menu in PID Tuner to specify your plant model.

For PID tuning at the command line, use pidtune. The pidtune command can design a controller for multiple plants at once.

For interactive PID tuning in the Live Editor, see the **Tune PID Controller** Live Editor task. This task lets you interactively design a PID controller and automatically generates MATLAB code for your live script.

## References

Äström, K. J. and Hägglund, T. *Advanced PID Control*, Research Triangle Park, NC: Instrumentation, Systems, and Automation Society, 2006.

## **See Also**

### **Functions**

pidtune

### **Objects**

pid | pid2 | pidstd | pidstd2

**Live Editor Tasks Tune PID Controller** 

## **Topics**

"Tune PID Controller to Favor Reference Tracking or Disturbance Rejection (PID Tuner)"

"Tune 2-DOF PID Controller (PID Tuner)"

"Designing PID Controllers with PID Tuner"

"PID Controller Types for Tuning"

### Introduced in R2014b

# place

Pole placement design

# **Syntax**

```
K = place(A,B,p)
[K,prec,message] = place(A,B,p)
```

# **Description**

Given the single- or multi-input system

```
\dot{x} = Ax + Bu
```

and a vector  $\mathbf{p}$  of desired self-conjugate closed-loop pole locations,  $\mathbf{place}$  computes a gain matrix K such that the state feedback u = -Kx places the closed-loop poles at the locations  $\mathbf{p}$ . In other words, the eigenvalues of A – BK match the entries of  $\mathbf{p}$  (up to the ordering).

K = place(A,B,p) places the desired closed-loop poles p by computing a state-feedback gain matrix K. All the inputs of the plant are assumed to be control inputs. The length of p must match the row size of A. place works for multi-input systems and is based on the algorithm from [1]. This algorithm uses the extra degrees of freedom to find a solution that minimizes the sensitivity of the closed-loop poles to perturbations in A or B.

[K,prec,message] = place(A,B,p) returns prec, an estimate of how closely the eigenvalues of A - BK match the specified locations p (prec measures the number of accurate decimal digits in the actual closed-loop poles). If some nonzero closed-loop pole is more than 10% off from the desired location, message contains a warning message.

You can also use place for estimator gain selection by transposing the A matrix and substituting C' for B.

```
l = place(A',C',p).'
```

# **Examples**

### **Pole Placement Design**

Consider a state-space system (a,b,c,d) with two inputs, three outputs, and three states. You can compute the feedback gain matrix needed to place the closed-loop poles at  $p = [-1 \ -1.23 \ -5.0]$  by

```
p = [-1 -1.23 -5.0];

K = place(a,b,p)
```

# **Algorithms**

place uses the algorithm of [1] which, for multi-input systems, optimizes the choice of eigenvectors for a robust solution.

In high-order problems, some choices of pole locations result in very large gains. The sensitivity problems attached with large gains suggest caution in the use of pole placement techniques. See [2] for results from numerical testing.

## References

- [1] Kautsky, J., N.K. Nichols, and P. Van Dooren, "Robust Pole Assignment in Linear State Feedback," *International Journal of Control*, 41 (1985), pp. 1129-1155.
- [2] Laub, A.J. and M. Wette, *Algorithms and Software for Pole Assignment and Observers*, UCRL-15646 Rev. 1, EE Dept., Univ. of Calif., Santa Barbara, CA, Sept. 1984.

## See Also

lqr|rlocus

Introduced before R2006a

# pole

Poles of dynamic system

# **Syntax**

```
P = pole(sys)
P = pole(sys,J1,...,JN)
```

# **Description**

P = pole(sys) returns the poles of the SISO or MIMO dynamic system model sys. The output is expressed as the reciprocal of the time units specified in sys.TimeUnit. The poles of a dynamic system determine the stability and response of the system.

An open-loop linear time-invariant system is stable if:

- In continuous-time, all the poles of the transfer function have negative real parts. When the poles are visualized on the complex s-plane, then they must all lie in the left-half plane (LHP) to ensure stability.
- In discrete-time, all the poles must have a magnitude strictly smaller than one, that is they must all lie inside the unit circle.

P = pole(sys, J1, ..., JN) returns the poles P of the entries in model array sys with subscripts (J1, ..., JN).

# **Examples**

### Poles of Discrete-Time Transfer Function

Compute the poles of the following discrete-time transfer function:

$$sys(z) = \frac{0.0478z - 0.0464}{z^2 - 1.81z + 0.9048}$$

```
sys = tf([0.04798 0.0464],[1 -1.81 0.9048],0.1);
P = pole(sys)

P = 2×1 complex
    0.9050 + 0.2929i
    0.9050 - 0.2929i
```

For stable discrete systems, all their poles must have a magnitude strictly smaller than one, that is they must all lie inside the unit circle. The poles in this example are a pair of complex conjugates, and lie inside the unit circle. Hence, the system sys is stable.

### **Poles of Transfer Function**

Calculate the poles of following transfer function:

$$sys(s) = \frac{4.2s^2 + 0.25s - 0.004}{s^2 + 9.6s + 17}$$

$$sys = tf([4.2,0.25,-0.004],[1,9.6,17]);$$

$$P = pole(sys)$$

$$P = 2 \times 1$$

$$-7.2576$$

$$-2.3424$$

For stable continuous systems, all their poles must have negative real parts. sys is stable since the poles are negative, that is, they lie in the left half of the complex plane.

## Poles of Models in an Array

For this example, load invertedPendulumArray.mat, which contains a 3-by-3 array of inverted pendulum models. The mass of the pendulum varies as you move from model to model along a single column of sys, and the length of the pendulum varies as you move along a single row. The mass values used are 100g, 200g and 300g, and the pendulum lengths used are 3m, 2m and 1m respectively.

```
Column 1 Column 2 Column 3

Row 1 100g, 3m 100g, 2m 100g, 1m

Row 2 200g, 3m 200g, 2m 200g, 1m

Row 3 300g, 3m 300g, 2m 300g, 1m

load('invertedPendulumArray.mat', 'sys');
size(sys)

3x3 array of transfer functions.
Each model has 1 outputs and 1 inputs.

Find poles of the model array.

P = pole(sys);
P(:,:,2,1)

ans = 3x1

2.1071
-2.1642
-0.1426
```

P(:,:,2,1) corresponds to the poles of the model with 200g pendulum weight and 3m length.

# **Input Arguments**

### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO or MIMO dynamic system model, or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, pole returns the poles of the current or nominal value of sys. If sys is an array of models, pole returns the poles of the model corresponding to its subscript J1,...,JN in sys. For more information on model arrays, see "Model Arrays".

# J1, . . . , JN — Indices of models in array whose poles you want to extract positive integer

Indices of models in array whose poles you want to extract, specified as a positive integer. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4-by-5 array of dynamic system models, the following command extracts the poles for entry (2,3) in the array.

```
P = pole(sys, 2, 3);
```

# **Output Arguments**

### P — Poles of the dynamic system

column vector | array

Poles of the dynamic system, returned as a scalar or an array. If sys is:

- A single model, then P is a column vector of poles of the dynamic system model sys
- A model array, then P is an array of poles of each model in sys

P is expressed as the reciprocal of the time units specified in sys.TimeUnit. For example, pole is expressed in 1/minute if sys.TimeUnit = 'minutes'.

Depending on the type of system model, poles are computed in the following way:

- For state-space models, the poles are the eigenvalues of the A matrix, or the generalized eigenvalues of  $A \lambda E$  in the descriptor case.
- For SISO transfer functions or zero-pole-gain models, the poles are the denominator roots. For more information, see roots.
- For MIMO transfer functions (or zero-pole-gain models), the poles are returned as the union of the poles for each SISO entry. If some I/O pairs have a common denominator, the roots of such I/O pair denominator are counted only once.

## Limitations

• Multiple poles are numerically sensitive and cannot be computed with high accuracy. A pole  $\lambda$  with multiplicity m typically results in a cluster of computed poles distributed on a circle with center  $\lambda$  and radius of order

$$\rho \approx \varepsilon^{1/m}$$
,

where  $\varepsilon$  is the relative machine precision (eps).

For more information on multiple poles, see "Sensitivity of Multiple Roots".

• If sys has internal delays, poles are obtained by first setting all internal delays to zero so that the system has a finite number of poles, thereby creating a zero-order Padé approximation. For some systems, setting delays to zero creates singular algebraic loops, which result in either improper or ill-defined, zero-delay approximations. For these systems, pole returns an error.

To assess the stability of models with internal delays, use step or impulse.

## See Also

damp | dsort | esort | impulse | pzmap | pzplot | step | zero

## **Topics**

"Pole and Zero Locations"

"Sensitivity of Multiple Roots"

### **Introduced before R2006a**

# polyBasis

Polynomial basis functions for tunable gain surface

# **Syntax**

```
shapefcn = polyBasis('canonical',degree)
shapefcn = polyBasis('chebyshev',degree)
shapefcn = polyBasis(____,nvars)
shapefcn = polyBasis(____,varnames)
```

# **Description**

You use basis function expansions to parameterize gain surfaces for tuning gainscheduled controllers. polyBasis generates standard polynomial expansions in any number of scheduling variables. Use the resulting functions to create tunable gain surfaces with tunableSurface.

shapefcn = polyBasis('canonical', degree) generates a function that evaluates the powers of an input variable, x, up to degree:

$$shapefcn(x) = [x, x^2, ..., x^{order}].$$

shapefcn = polyBasis('chebyshev', degree) generates a function that evaluates
Chebyshev polynomials up to degree:

$$shapefcn(x) = [T_1(x), ..., T_{order}(x)].$$

The Chebyshev polynomials are defined recursively by:

$$T_0(x) = 1$$
;  $T_1(x) = x$ ;  $T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x)$ .

shapefcn = polyBasis(\_\_\_\_, nvars) generates an nvars-dimensional polynomial expansion by taking the outer product of nvars 1-D polynomial expansions. The resulting function shapefcn takes nvars input arguments and returns a vector with (degree +1)^(nvars-1) entries. For example, for nvars = 3 and canonical polynomials,

```
shapefcn(x,y,z) = \left[ x^i y^j z^k : \ 0 \le i,j,k \le order, \ i+j+k > 0 \right].
```

Thus, to specify a bilinear function in two scheduling variables, use:

```
shapefcn = polyBasis('canonical',1,2);
```

Using the resulting function with tunableSurface defines a variable gain of the form:

$$K(x, y) = K_0 + K_1 x + K_2 y + K_3 x y$$
.

Here, *x* and *y* are the normalized scheduling variables, whose values lie in the range [-1,1]. (See tunableSurface for more information.)

To specify basis functions in multiple scheduling variables where the expansions are different for each variable, use ndBasis.

shapefcn = polyBasis(\_\_\_\_, varnames) specifies variable names. Use this syntax with any of the previous syntaxes to name the variables in shapefcn. Using variable names improves readability of the tunableSurface object display and of any MATLAB code you generate using codegen.

# **Examples**

## **Polynomial Basis Functions of One Scheduling Variable**

Create basis functions for a gain that varies as a cubic function of one scheduling variable.

```
shapefcn = polyBasis('canonical',3);
```

shapefcn is a handle to a function of one variable that returns an array of values corresponding to the first three powers of its input. In other words,  $shapefcn(x) = [x x^2 x^3]$ . For example, examine shapefcn(-0.2).

```
x = -0.2;
shapefcn(x)
ans = 1×3
-0.2000    0.0400   -0.0080
```

Evaluating [x  $x^2$  x3] for x = -0.2 returns the same result.

```
[x x^2 x^3]

ans = 1 \times 3

-0.2000 0.0400 -0.0080
```

Use shapefcn as an input argument to tunableSurface to define a polynomial gain surface. This shapefcn is equivalent to using:

```
shapefcn = @(x) [x x^2 x^3];
```

### **Chebyshev Basis Functions**

Create a set of basis functions that are Chebyshev polynomials of a single variable, up to third degree.

```
shapefcn = polyBasis('chebyshev',3);
```

### **Bilinear Function of Two Variables**

Create basis functions for a bilinear gain surface, [x, y, xy]. Name the variables to make the function more readable.

```
shapefcn = polyBasis('canonical',1,2,{'x','y'})
shapefcn = function_handle with value:
    @(x,y)utFcnBasisOuterProduct(FDATA_,x,y)
```

Confirm the values returned by shapefon for a particular (x, y) pair.

```
x = 0.2;

y = -0.5;

shapefcn(x,y)

ans = 1 \times 3
```

```
0.2000 -0.5000 -0.1000
```

This shapefcn is equivalent to:

```
shapefcn = @(x,y)[x,y,x*y];
```

The basis functions of shapefcn are first-order in each of the two variables. To create a set of basis functions in different degrees for each variable, use ndBasis.

# **Input Arguments**

### degree — Degree of expansion

positive integer

Degree of the polynomial expansion, specified as a positive integer.

### nvars — Number of variables

1 (default) | positive integer

Number of scheduling variables, specified as a positive integer.

### varnames — Variable names

```
\{ 'x1', 'x2', \ldots \} (default) | character vector | cell array of character vectors
```

Variable names in the generated function shapefcn, specified as a:

- Character vector, for monovariable basis functions.
- Cell array of character vectors, for multivariable basis functions.

If you do not specify varnames, then the variables in shapefcn are named  $\{'x1', 'x2', \ldots\}$ .

```
Example: {'alpha','V'}
```

# **Output Arguments**

### shapefcn — Polynomial expansion

function handle

Polynomial expansion, specified as a function handle. shapefcn takes as input arguments the number of variables specified by nvars. The function evaluates polynomials in those variables up to the specified degree, and returns the resulting values in a vector. When you use shapefcn to create a gain surface, tunableSurface automatically generates tunable coefficients for each polynomial term in the vector.

### **See Also**

fourierBasis | ndBasis | tunableSurface

Introduced in R2015b

# predict

Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter

# **Syntax**

```
[PredictedState,PredictedStateCovariance] = predict(obj)
[PredictedState,PredictedStateCovariance] = predict(obj,Us1,...Usn)
```

# **Description**

The predict command predicts the state and state estimation error covariance of an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object at the next time step. To implement the extended or unscented Kalman filter algorithms, use the predict and correct commands together. If the current output measurement exists, you can use predict and correct. If the measurement is missing, you can only use predict. For information about the order in which to use the commands, see "Using predict and correct Commands" on page 2-997.

[PredictedState, PredictedStateCovariance] = predict(obj) predicts state estimate and state estimation error covariance of an extended or unscented Kalman filter, or particle filter object obj at the next time step.

You create obj using the extendedKalmanFilter, unscentedKalmanFilter or particleFilter commands. You specify the state transition function and measurement function of your nonlinear system in obj. You also specify whether the process and measurement noise terms are additive or nonadditive in these functions. The State property of the object stores the latest estimated state value. Assume that at time step k, obj. State is  $\widehat{x}[k|k]$ . This value is the state estimate for time k, estimated using measured outputs until time k. When you use the predict command, the software returns  $\widehat{x}[k+1|k]$  in the PredictedState output. Where  $\widehat{x}[k+1|k]$  is the state estimate for time k+1, estimated using measured output until time k. The command returns the state estimation error covariance of  $\widehat{x}[k+1|k]$  in the PredictedStateCovariance output. The software also updates the State and StateCovariance properties of obj with these corrected values.

Use this syntax if the state transition function f that you specified in obj.StateTransitionFcn has one of the following forms:

- x(k) = f(x(k-1)) for additive process noise.
- x(k) = f(x(k-1), w(k-1)) for nonadditive process noise.

Where x and w are the state and process noise of the system. The only inputs to f are the states and process noise.

[PredictedState, PredictedStateCovariance] = predict(obj, Us1,...Usn) specifies additional input arguments, if the state transition function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if your state transition function *f* has one of the following forms:

- x(k) = f(x(k-1), Us1, ... Usn) for additive process noise.
- x(k) = f(x(k-1), w(k-1), Us1, ... Usn) for nonadditive process noise.

# **Examples**

#### **Estimate States Online Using Unscented Kalman Filter**

Estimate the states of a van der Pol oscillator using an unscented Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an unscented Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with nonlinearity parameter, mu, equal to 1. The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as [1;0]. This is the guess for the state value at initial time k, using knowledge of system outputs until time k-1,  $\hat{x}[k|k-1]$ .

```
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
```

Load the measured output data, y, from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp\_data.mat file.

```
load vdp_data.mat y
```

Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the unscented Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct  $\hat{\chi}[k \mid k-1]$  using measurements at time k to get  $\hat{\chi}[k \mid k]$ . Then, you predict the state value at next time step,  $\hat{\chi}[k+1 \mid k]$ , using  $\hat{\chi}[k \mid k]$ , the state estimate at time step k that is estimated using measurements until time k.

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual, ResidualCovariance] = residual(obj,y(k));
    [CorrectedState, CorrectedStateCovariance] = correct(obj,y(k));
    [PredictedState, PredictedStateCovariance] = predict(obj);

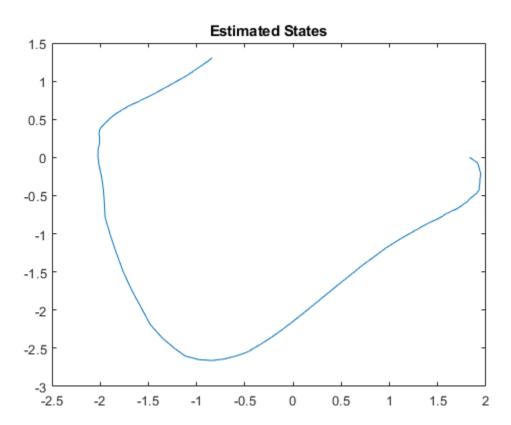
    residBuf(k,:) = Residual;
    xcorBuf(k,:) = CorrectedState';
    xpredBuf(k,:) = PredictedState';
end
```

When you use the correct command, obj.State and obj.StateCovariance are updated with the corrected state and state estimation error covariance values for time step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj.State and obj.StateCovariance are updated with the predicted values for time step k+1, PredictedState and PredictedStateCovariance.

In this example, you used correct before predict because the initial state value was  $\hat{x}[k \mid k-1]$ , a guess for the state value at initial time k using system outputs until time k-1. If your initial state value is  $\hat{x}[k-1 \mid k-1]$ , the value at previous time k-1 using measurement until k-1, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 2-997.

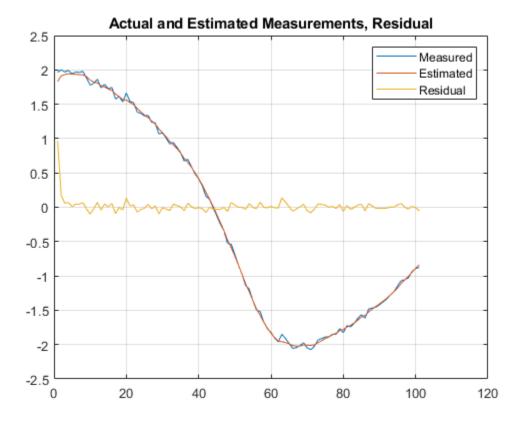
Plot the estimated states, using the postcorrection values.

```
plot(xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')
```



Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.

```
M = [y,xcorBuf(:,1),residBuf];
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')
```



The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

#### **Estimate States Online using Particle Filter**

Load the van der Pol ODE data, and specify the sample time.

vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time dt = 0.05.

```
addpath(fullfile(matlabroot, 'examples', 'control', 'main')) % add example data
load ('vdpODEdata.mat', 'xTrue', 'dt')
tSpan = 0:dt:5;
```

Get the measurements. For this example, a sensor measures the first state with a Gaussian noise with standard deviation 0.04.

```
sqrtR = 0.04;
yMeas = xTrue(:,1) + sqrtR*randn(numel(tSpan),1);
```

Create a particle filter, and set the state transition and measurement likelihood functions.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF, 1000, [2;0], eye(2));
```

Pick the mean state estimation and systematic resampling methods.

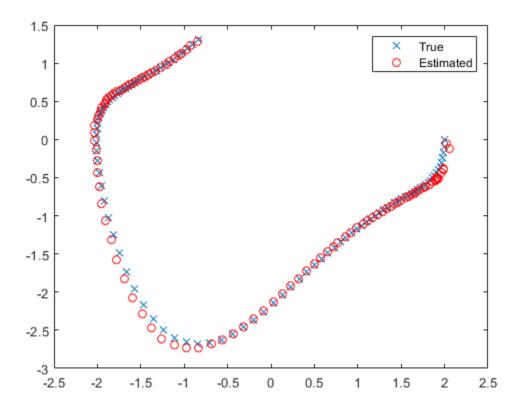
```
myPF.StateEstimationMethod = 'mean';
myPF.ResamplingMethod = 'systematic';
```

Estimate the states using the correct and predict commands, and store the estimated states.

```
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
```

Plot the results, and compare the estimated and true states.

```
figure(1)
plot(xTrue(:,1),xTrue(:,2),'x',xEst(:,1),xEst(:,2),'ro')
legend('True','Estimated')
```



rmpath(fullfile(matlabroot, 'examples', 'control', 'main')) % remove example data

### Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input u whose state x and measurement y evolve according to the following state transition and measurement equations:

$$x[k] = \sqrt{x[k-1] + u[k-1]} + w[k-1]$$

$$y[k] = x[k] + 2 * u[k] + v[k]^{2}$$

The process noise  $\boldsymbol{v}$  of the system is additive while the measurement noise  $\boldsymbol{v}$  is nonadditive.

Create the state transition function and measurement function for the system. Specify the functions with an additional input u.

```
f = @(x,u)(sqrt(x+u));

h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.

```
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
```

Specify the measurement noise covariance.

```
obj.MeasurementNoise = 0.01;
```

You can now estimate the state of the system using the predict and correct commands. You pass the values of u to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement y[k]=0.8 and input u[k]=0.2 at time step k.

```
correct(obj, 0.8, 0.2)
```

Predict the state at the next time step, given u[k]=0.2.

```
predict(obj,0.2)
```

Retrieve the error, or residual, between the prediction and the measurement.

```
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);
```

# **Input Arguments**

#### obj — Extended or unscented Kalman filter, or particle filter object

extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter
object

Extended or unscented Kalman filter, or particle filter object for online state estimation, created using one of the following commands:

- extendedKalmanFilter Uses the extended Kalman filter algorithm.
- unscentedKalmanFilter Uses the unscented Kalman filter algorithm.
- particleFilter Uses the particle filter algorithm.

# Us1,...Usn — Additional input arguments to state transition function input arguments of any type

Additional input arguments to state transition function, specified as input arguments of any type. The state transition function, f, is specified in the StateTransitionFcn property of the object. If the function requires input arguments in addition to the state and process noise values, you specify these inputs in the predict command syntax.

For example, suppose that your state transition function calculates the predicted state x at time step k using system inputs u(k-1) and time k-1, in addition to the state x(k-1):

$$x(k) = f(x(k-1), u(k-1), k-1)$$

Then when you perform online state estimation at time step k, specify these additional inputs in the predict command syntax:

[PredictedState, PredictedStateCovariance] = predict(obj,u(k-1),k-1);

# **Output Arguments**

#### PredictedState — Predicted state estimate

vector

Predicted state estimate, returned as a vector of size M, where M is the number of states of the system. If you specify the initial states of  $\operatorname{obj}$  as a column vector then M is returned as a column vector, otherwise M is returned as a row vector.

For information about how to specify the initial states of the object, see the extendedKalmanFilter, unscentedKalmanFilter and particleFilter reference pages.

# PredictedStateCovariance — Predicted state estimation error covariance matrix

Predicted state estimation error covariance, returned as an M-by-M matrix, where M is the number of states of the system.

#### **More About**

### Using predict and correct Commands

After you have created an extended or unscented Kalman filter, or particle filter object, obj, to implement the estimation algorithms, use the correct and predict commands together.

At time step k, correct command returns the corrected value of states and state estimation error covariance using measured system outputs y[k] at the same time step. If your measurement function has additional input arguments  $U_m$ , you specify these as inputs to the correct command. The command passes these values to the measurement function.

[CorrectedState,CorrectedCovariance] = correct(obj,y,Um)

The correct command updates the State and StateCovariance properties of the object with the estimated values, CorrectedState and CorrectedCovariance.

The predict command returns the prediction of state and state estimation error covariance at the next time step. If your state transition function has additional input arguments  $U_s$ , you specify these as inputs to the predict command. The command passes these values to the state transition function.

[PredictedState,PredictedCovariance] = predict(obj,Us)

The predict command updates the State and StateCovariance properties of the object with the predicted values, PredictedState and PredictedCovariance.

If the current output measurement exists at a given time step, you can use correct and predict. If the measurement is missing, you can only use predict. For details about

how these commands implement the algorithms, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

The order in which you implement the commands depends on the availability of measured data y,  $U_s$ , and  $U_m$  for your system:

• correct then predict — Assume that at time step k, the value of obj. State is  $\widehat{x}[k|k-1]$ . This value is the state of the system at time k, estimated using measured outputs until time k-1. You also have the measured output y[k] and inputs U<sub>s</sub>[k] and U<sub>m</sub>[k] at the same time step.

Then you first execute the correct command with measured system data y[k] and additional inputs  $U_m[k]$ . The command updates the value of obj. State to be  $\widehat{x}[k|k]$ , the state estimate for time k, estimated using measured outputs up to time k. When you then execute the predict command with input  $U_s[k]$ , obj. State now stores  $\widehat{x}[k+1|k]$ . The algorithm uses this state value as an input to the correct command in the next time step.

• predict then correct — Assume that at time step k, the value of obj. State is  $\widehat{x}[k-1|k-1]$ . You also have the measured output y[k] and input  $U_m[k]$  at the same time step but you have  $U_s[k-1]$  from the previous time step.

Then you first execute the predict command with input  $U_s[k-1]$ . The command updates the value of obj.State to  $\widehat{x}[k|k-1]$ . When you then execute the correct command with input arguments y[k] and  $U_m[k]$ , obj.State is updated with  $\widehat{x}[k|k]$ . The algorithm uses this state value as an input to the predict command in the next time step.

Thus, while in both cases the state estimate for time k,  $\widehat{x}[k|k]$  is the same, if at time k you do not have access to the current state transition function inputs  $U_s[k]$ , and instead have  $U_s[k-1]$ , then use predict first and then correct.

For an example of estimating states using the predict and correct commands, see "Estimate States Online Using Unscented Kalman Filter" on page 2-989 or "Estimate States Online using Particle Filter" on page 2-992.

#### See Also

clone | correct | extendedKalmanFilter | initialize | particleFilter |
residual | unscentedKalmanFilter

### **Topics**

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"

#### Introduced in R2016b

<sup>&</sup>quot;Generate Code for Online State Estimation in MATLAB"

<sup>&</sup>quot;Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

# prescale

Optimal scaling of state-space models

# **Syntax**

```
scaledsys = prescale(sys)
scaledsys = prescale(sys,focus)
[scaledsys,info] = prescale(...)
prescale(sys)
```

# **Description**

scaledsys = prescale(sys) scales the entries of the state vector of a state-space model sys to maximize the accuracy of subsequent frequency-domain analysis. The scaled model scaledsys is equivalent to sys.

scaledsys = prescale(sys, focus) specifies a frequency interval focus =
{fmin,fmax} (in rad/TimeUnit, where TimeUnit is the system's time units specified in
the TimeUnit property of sys) over which to maximize accuracy. This is useful when sys
has a combination of slow and fast dynamics and scaling cannot achieve high accuracy
over the entire dynamic range. By default, prescale attempts to maximize accuracy in
the frequency band with dominant dynamics.

[scaledsys,info] = prescale(...) also returns a structure info with the fields shown in the following table.

SL Left scaling factors
SR Right scaling factors

Freqs Frequencies used to test accuracy

RelAcc Guaranteed relative accuracy at these

frequencies

The test frequencies lie in the frequency interval focus when specified. The scaled state-space matrices are

$$A_S = T_L A T_R$$

$$B_s = T_L B$$

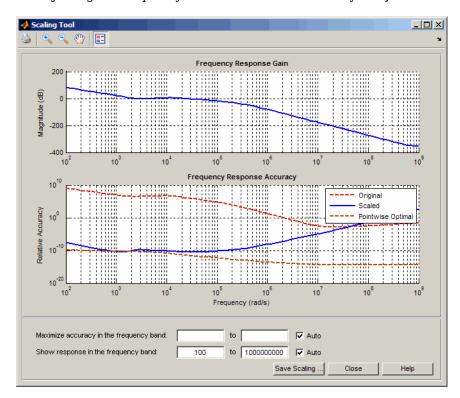
$$C_S = CT_R$$

$$E_S = T_L E T_R$$

where  $T_L = diag(SL)$  and  $T_R = diag(SR)$ .  $T_L$  and  $T_R$  are inverse of each other for explicit models  $(E = [\ ])$ .

prescale(sys) opens an interactive GUI for:

- Visualizing accuracy trade-offs for sys.
- Adjusting the frequency interval where the accuracy of sys is maximized.



For more information on scaling and using the Scaling Tool GUI, see "Scaling State-Space Models".

# **Tips**

Most frequency-domain analysis commands perform automatic scaling equivalent to scaledsys = prescale(sys).

You do not need to scale for time-domain simulations and doing so may invalidate the initial condition x0 used in initial and lsim simulations.

### **See Also**

SS

Introduced in R2008b

### pzmap

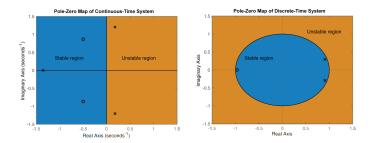
Pole-zero plot of dynamic system

# **Syntax**

```
pzmap(sys)
pzmap(sys1,sys2,...,sysN)
[p,z] = pzmap(sys)
```

# **Description**

pzmap(sys) creates a pole-zero plot of the continuous or discrete-time dynamic system model sys. x and o indicates the poles and zeros respectively, as shown in the following figure.



From the figure above, an open-loop linear time-invariant system is stable if:

- In continuous-time, all the poles on the complex s-plane must be in the left-half plane (blue region) to ensure stability. The system is marginally stable if distinct poles lie on the imaginary axis, that is, the real parts of the poles are zero.
- In discrete-time, all the poles in the complex z-plane must lie inside the unit circle (blue region). The system is marginally stable if it has one or more poles lying on the unit circle.

pzmap(sys1,sys2,...,sysN) creates the pole-zero plot of multiple models on a single figure. The models can have different numbers of inputs and outputs and can be a mix of continuous and discrete systems. For SISO systems, pzmap plots the system poles and zeros. For MIMO systems, pzmap plots the system poles and transmission zeros.

[p,z] = pzmap(sys) returns the system poles and transmission zeros as column vectors p and z. The pole-zero plot is not displayed on the screen.

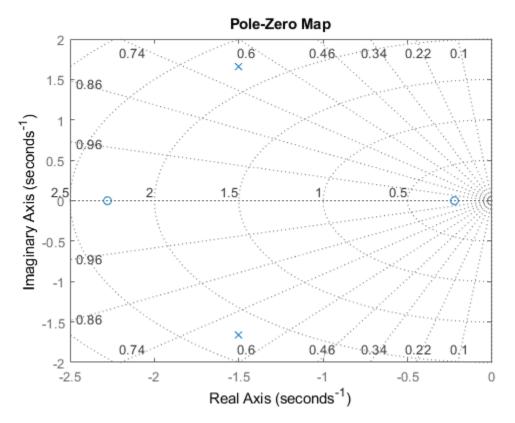
# **Examples**

grid on

#### Pole-Zero Plot of Dynamic System

Plot the poles and zeros of the continuous-time system represented by the following transfer function:

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 3s + 5} \,.$$
 H = tf([2 5 1],[1 3 5]); pzmap(H)

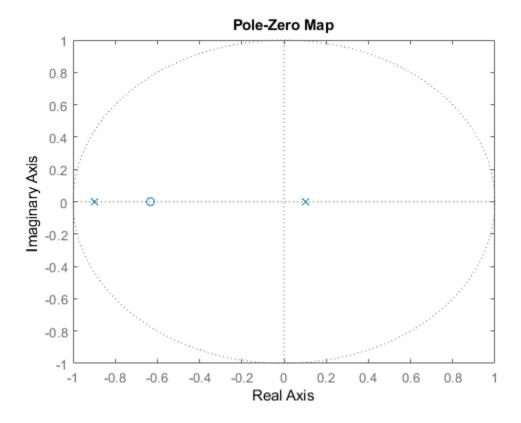


Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by o.

#### Pole-Zero Plot of Identified System

Plot the pole-zero map of a discrete time identified state-space (idss) model. In practice you can obtain an idss model by estimation based on input-output measurements of a system. For this example, create one from state-space data.

```
A = [0.1 0; 0.2 -0.9];
B = [.1; 0.1];
C = [10 5];
D = [0];
sys = idss(A,B,C,D,'Ts',0.1);
Examine the pole-zero map.
pzmap(sys)
```



System poles are marked by x, and zeros are marked by o.

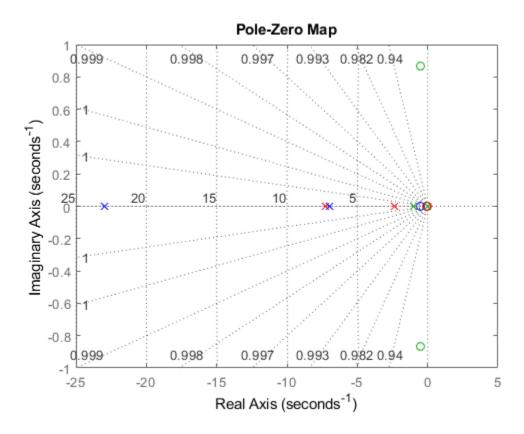
#### **Pole-Zero Map of Multiple Models**

For this example, load a 3-by-1 array of transfer function models.

```
load('tfArray.mat','sys');
size(sys)
3x1 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Plot the poles and zeros of each model in the array with distinct colors. For this example, use red for the first model, green for the second and blue for the third model in the array.

```
pzmap(sys(:,:,1),'r',sys(:,:,2),'g',sys(:,:,3),'b') sgrid
```



sgrid plots lines of constant damping ratio and natural frequency in the s-plane of the pole-zero plot.

#### **Poles and Zeros of Transfer Function**

Use pzmap to calculate the poles and zeros of the following transfer function:

```
sys(s) = \frac{4.2s^2 + 0.25s - 0.004}{s^2 + 9.6s + 17}
sys = tf([4.2,0.25,-0.004],[1,9.6,17]);
[p,z] = pzmap(sys)
p = 2 \times 1
-7.2576
-2.3424
z = 2 \times 1
-0.0726
0.0131
```

#### **Identify Near-Cancelling Pole-Zero Pairs**

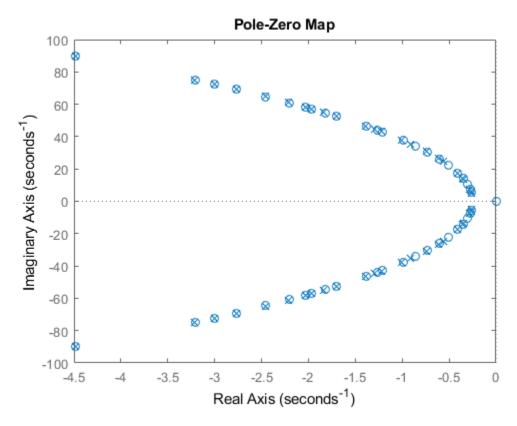
This example uses a model of a building with eight floors, each with three degrees of freedom: two displacements and one rotation. The I/O relationship for any one of these displacements is represented as a 48-state model, where each state represents a displacement or its rate of change (velocity).

Load the building model.

```
load('building.mat');
size(G)
State-space model with 1 outputs, 1 inputs, and 48 states.
```

Plot the poles and zeros of the system.

pzmap(G)



From the plot, observe that there are numerous near-canceling pole-zero pairs that could be potentially eliminated to simplify the model, with no effect on the overall model response. pzmap is useful to visually identify such near-canceling pole-zero pairs to perform pole-zero simplification.

# **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a dynamic system model or model array. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is an array of models, pzmap plots all the poles and zeros of every model in the array on the same plot.

# **Output Arguments**

#### p - Poles of the system

column vector

Poles of the system, returned as a column vector, in order of its increasing natural frequency. p is the same as the output of pole(sys), except for the order.

#### z — Transmission zeros of the system

column vector

Transmission zeros of the system, returned as a column vector. z is the same as the output of tzero(sys).

### **Tips**

- Use the functions sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the *s* or *z*-plane on the pole-zero plot.
- For MIMO models, pzmap displays all system poles and transmission zeros on a single plot. To map poles and zeros for individual I/O pairs, use iopzmap.
- For additional options to customize the appearance of the pole-zero plot, use pzplot.

#### See Also

damp | dsort | esort | iopzmap | pole | pzplot | rlocus | sgrid | zero | zgrid

#### **Introduced before R2006a**

# pzplot

Pole-zero plot of dynamic system model with plot customization options

# **Syntax**

```
h = pzplot(sys)
pzplot(sys)
pzplot(sys1,sys2,...,sysN)
pzplot(ax,...)
pzplot(...,plotoptions)
```

# **Description**

h = pzplot(sys) plots the poles and transmission zeros of the dynamic system model sys and returns the plot handle h to the plot. Use this function to generate a pole-zero map with customizable plot options such as FreqUnits, TimeUnits, and IOGrouping. For more information about using and interpreting pole-zero maps, see pzmap.

pzplot(sys) plots the poles and transmission zeros of the dynamic system model sys. x and o indicates poles and zeros respectively.

pzplot(sys1,sys2,...,sysN) displays the poles and transmission zeros of multiple models on a single plot. You can specify distinct colors for each model individually.

pzplot(ax,...) plots into the axes specified by ax instead of the current axis gca.

pzplot(...,plotoptions) plots the poles and transmission zeros with the options specified in plotoptions. For more information on the ways to change properties of your plots, see "Ways to Customize Plots".

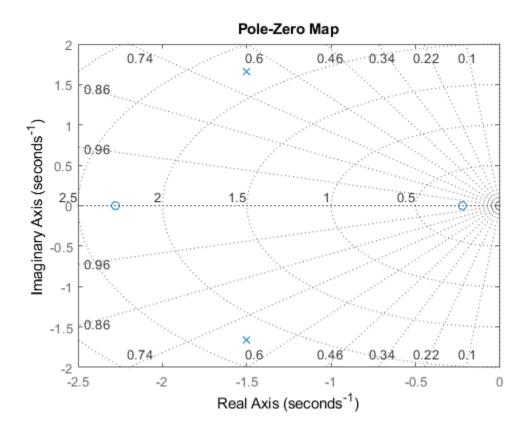
# **Examples**

#### **Pole-Zero Plot with Custom Plot Title**

Plot the poles and zeros of the continuous-time system represented by the following transfer function:

$$sys(s) = \frac{2s^2 + 5s + 1}{s^2 + 3s + 5}.$$

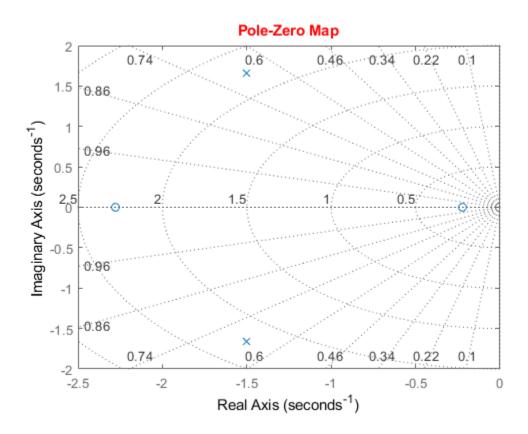
sys = tf([2 5 1],[1 3 5]);
h = pzplot(sys);
grid on



Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by x.

Change the color of the plot title. To do so, use the plot handle, h.

```
p = getoptions(h);
p.Title.Color = [1,0,0];
setoptions(h,p);
```



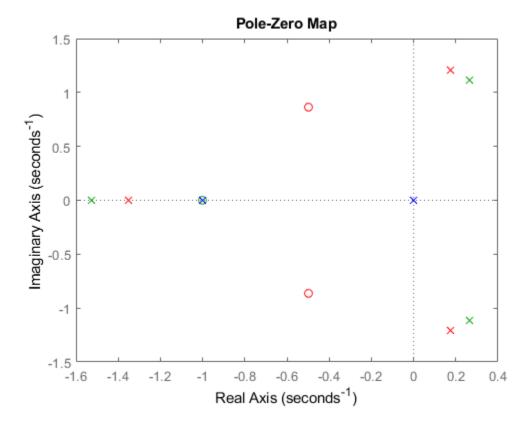
### **Pole-Zero Plot of Multiple Models**

For this example, load a 3-by-1 array of transfer function models.

```
load('tfArrayMargin.mat','sys');
size(sys)
3x1 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Plot the poles and zeros of the model array. Define the colors for each model. For this example, use red for the first model, green for the second and blue for the third model in the array.

```
pzplot(sys(:,:,1),'r',sys(:,:,2),'g',sys(:,:,3),'b');
```



#### **Pole-Zero Plot with Custom Options**

Plot the poles and zeros of the continuous-time system represented by the following transfer function with a custom option set:

$$sys(s) = \frac{2s^2 + 5s + 1}{s^2 + 3s + 5}.$$

Create the custom option set using pzoptions.

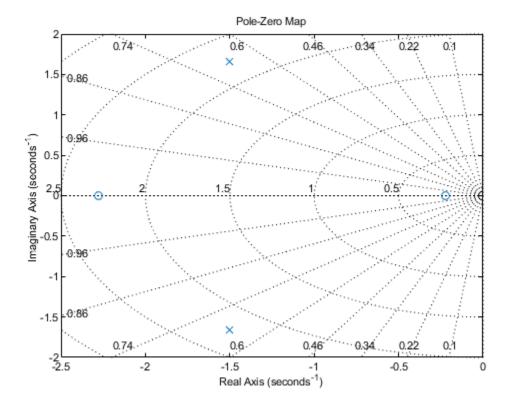
```
plotoptions = pzoptions;
```

For this example, specify the grid to be visible.

```
plotoptions.Grid = 'on';
```

Use the specified options to create a pole-zero map of the transfer function.

```
h = pzplot(tf([2 5 1],[1 3 5]),plotoptions);
```



Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by x.

# **Input Arguments**

### ${\bf sys-Dynamic\ system}$

dynamic system model | model array

Dynamic system, specified as a SISO or MIMO dynamic system model, or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, pzplot returns the poles and transmission of the current or nominal value of sys. If sys is an array of models, pzplot plots the poles and zeros of each model in the array on the same diagram.

#### ax — Axes handle

axes object

Axes handle, specified as an axes object. If you do not specify the axes object, then pzplot uses the current axes gca to plot the poles and zeros of the system.

#### plotoptions — Pole-zero plot options

options object

Pole-zero plot options, specified as an options object. See pzoptions for a list of available plot options.

# **Output Arguments**

#### h — Pole-zero plot options handle

scalar

Pole-zero plot options handle, returned as a scalar. Use h to query and modify properties of your pole-zero plot. You can use this handle to customize the plot with the getoptions and setoptions commands.

# **Tips**

 Use sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the s- or z-plane.

#### See Also

getoptions | iopzplot | pzmap | pzoptions | setoptions

**Topics** "Ways to Customize Plots"

**Introduced before R2006a** 

# pzoptions

Create list of pole/zero plot options

# **Syntax**

```
P = pzoptions
P = pzoption('cstprefs')
```

# **Description**

P = pzoptions returns a list of available options for pole/zero plots (pole/zero, inputoutput pole/zero and root locus) with default values set.. You can use these options to customize the pole/zero plot appearance from the command line.

P = pzoption('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the available pole/zero plot options.

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid	Show or hide the grid, specified as one of the following values: 'off'   'on'  Default: 'off'
GridColor	Color of the grid lines, specified as one of the following: Vector of RGB values in the range [0,1]  character vector of color name   'none'. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'.  Default: [0.15,0.15,0.15]

Option	Description
XlimMode, YlimMode	Limit modes
Xlim, Ylim	Axes limits
IOGrouping	Grouping of input-output pairs, specified as one of the following values: 'none'  'inputs' 'outputs' 'all'    Default: 'none'
InputLabels, OutputLabels	Input and output label styles
InputVisible, OutputVisible	Visibility of input and output channels

Option	Description
FreqUnits	Frequency units, specified as one of the following values:
	• 'Hz'
	• 'rad/second'
	• 'rpm'
	• 'kHz'
	• 'MHz'
	• 'GHz'
	• 'rad/nanosecond'
	• 'rad/microsecond'
	• 'rad/millisecond'
	• 'rad/minute'
	• 'rad/hour'
	• 'rad/day'
	• 'rad/week'
	• 'rad/month'
	• 'rad/year'
	• 'cycles/nanosecond'
	• 'cycles/microsecond'
	• 'cycles/millisecond'
	• 'cycles/hour'
	• 'cycles/day'
	• 'cycles/week'
	• 'cycles/month'
	• 'cycles/year'
	Default: 'rad/s'
	You can also specify 'auto' which uses frequency units rad/TimeUnit relative to system time units specified in the

Option	Description	
	TimeUnit property. For multiple systems with different time units, the units of the first system are used.	
TimeUnits	Time units, specified as one of the following values:	
	• 'nanoseconds'	
	• 'microseconds'	
	• 'milliseconds'	
	• 'seconds'	
	• 'minutes'	
	• 'hours'	
	• 'days'	
	• 'weeks'	
	• 'months'	
	• 'years'	
	Default: 'seconds'	
	You can also specify 'auto' which uses time units specified in the TimeUnit property of the input system. For multiple systems with different time units, the units of the first system is used.	
ConfidenceRegionNumberSD	Number of standard deviations to use when displaying the confidence region characteristic for identified models (valid only iopzplot).	

# **Examples**

### **Display Grid in Root Locus Plot**

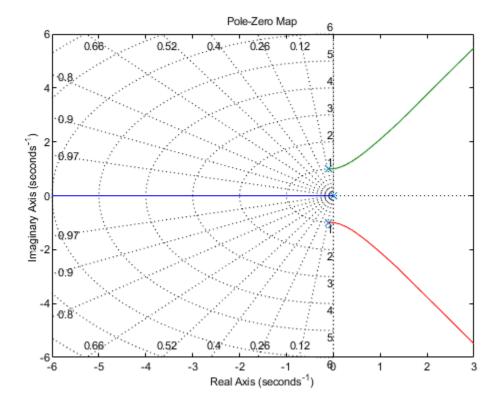
Create a default option set for root locus plots.

```
opt = pzoptions;
```

In the plot options, specify the grid as visible.

```
opt.Grid = 'on';
```

Use the specified options to plot the root locus of a transfer function.



The root locus plot is generated with the grid enabled.

# **See Also**

 $\verb|getoptions|| \verb|iopzplot|| \verb|pzplot|| setoptions|$ 

Introduced in R2008a

# realp

Real tunable parameter

## **Syntax**

p = realp(paramname,initvalue)

# **Description**

p = realp(paramname,initvalue) creates a tunable real-valued parameter with
name specified by paramname and initial value initvalue. Tunable real parameters can
be scalar- or matrix- valued.

## **Input Arguments**

#### paramname

Name of the realp parameter p, specified as a character vector such as 'a' or 'zeta'. This input argument sets the value of the Name property of p.

### initvalue

Initial numeric value of the parameter p. initvalue can be a real scalar value or a 2-dimensional matrix.

## **Output Arguments**

p

realp parameter object.

## **Properties**

#### Name

Name of the realp parameter object, stored as a character vector. The value of Name is set by the paramname input argument to realp and cannot be changed.

#### Value

Value of the tunable parameter.

Value can be a real scalar value or a 2-dimensional matrix. The initial value is set by the initvalue input argument. The dimensions of Value are fixed on creation of the realp object.

#### Minimum

Lower bound for the parameter value. The dimension of the Minimum property matches the dimension of the Value property.

For matrix-valued parameters, use indexing to specify lower bounds on individual elements:

```
p = realp('K',eye(2));
p.Minimum([1 4]) = -5;
```

Use scalar expansion to set the same lower bound for all matrix elements:

```
p.Minimum = -5;
```

**Default:** - Inf for all entries

#### Maximum

Upper bound for the parameter value. The dimension of the Maximum property matches the dimension of the Value property.

For matrix-valued parameters, use indexing to specify upper bounds on individual elements:

```
p = realp('K',eye(2));
p.Maximum([1 4]) = 5;
```

Use scalar expansion to set the same upper bound for all matrix elements:

```
p.Maximum = 5;
```

**Default:** Inf for all entries

#### Free

Boolean value specifying whether the parameter is free to be tuned. Set the Free property to 1 (true) for tunable parameters, and 0 (false) for fixed parameters.

The dimension of the Free property matches the dimension of the Value property.

**Default:** 1 (true) for all entries

## **Examples**

#### **Tunable Low-Pass Filter**

In this example, you will create a low-pass filter with one tunable parameter *a*:

$$F = \frac{a}{s+a}$$

Since the numerator and denominator coefficients of a tunableTF block are independent, you cannot use tunableTF to represent F. Instead, construct F using the tunable real parameter object realp.

Create a real tunable parameter with an initial value of 10.

```
a = realp('a',10)
a =
          Name: 'a'
          Value: 10
          Minimum: -Inf
          Maximum: Inf
          Free: 1
Real scalar parameter.
```

Use tf to create the tunable low-pass filter F.

```
denominator = [1,a];
F = tf(numerator,denominator)
F =
   Generalized continuous-time state-space model with 1 outputs, 1 inputs, 1 states, and a: Scalar parameter, 2 occurrences.
Type "ss(F)" to see the current value, "get(F)" to see all properties, and "F.Blocks" to see the current value, "get(F)" to see all properties.
```

F is a genss object which has the tunable parameter a in its Blocks property. You can connect F with other tunable or numeric models to create more complex control system models. For an example, see "Control System with Tunable Components".

### **Create Parametric Diagonal Matrix**

Create a matrix with tunable diagonal elements and with off-diagonal elements fixed to zero.

Create a parametric matrix whose initial value is the identity matrix.

```
p = realp('P', eye(2));
```

numerator = a;

p is a 2-by-2 parametric matrix. Since the initial value is the identity matrix, the off-diagonal initial values are zero.

Fix the values of the off-diagonal elements by setting the Free property to false.

```
p.Free(1,2) = false;
p.Free(2,1) = false;
```

## **Tips**

• Use arithmetic operators (+, -, \*, /, \, and ^) to combine realp objects into rational expressions or matrix expressions. You can use the resulting expressions in model-creation functions such as tf, zpk, and ss to create tunable models. For more information about tunable models, see "Models with Tunable Coefficients" in the Control System Toolbox User's Guide.

# **See Also**

genmat | genss | ss | tf

### **Topics**

"Models with Tunable Coefficients"

Introduced in R2011a

### **Reduce Model Order**

Reduce complexity of linear time-invariant (LTI) models in the Live Editor

# **Description**

The **Reduce Model Order** task lets you interactively compute reduced-order approximations of high-order models while preserving model characteristics that are important to your application. The task automatically generates MATLAB code for your live script. For more information about Live Editor tasks generally, see "Add Interactive Tasks to a Live Script" (MATLAB).

Working with lower order models can simplify analysis and control design. Simpler models are also easier to understand and manipulate. You can reduce a plant model to focus on relevant dynamics before designing a controller for the plant. You can also use model reduction to simplify a full-order controller. For more information about model reduction and when it is useful, see "Model Reduction Basics".

To get started, select a model to reduce and a model-reduction method. For each method, the task gives you controls and plots that help you ensure that your reduced model preserves dynamics that are important for your application.

- Balanced Truncation Compute a lower order approximation of your model by removing states with relatively small energy contributions.
- Mode Selection Select modes by specifying a frequency range of interest.
- Pole-Zero Simplification Eliminate canceling or near-canceling pole-zero pairs.

### **Related Functions**

The model-reduction code that **Reduce Model Order** generates uses the following functions.

- balred
- freqsep
- minreal

# **Description (collapsed portion)**

#### **Related Functions**

The model-reduction code that **Reduce Model Order** generates uses the following functions.

- balred
- freqsep
- minreal

## Open the Task

To add the **Reduce Model Order** task to a live script in the MATLAB Editor:

- On the **Live Editor** tab, select **Task > Reduce Model Order**.
- In a code block in your script, type a relevant keyword, such as reduce, balred, or minreal. Select Reduce Model Order from the suggested command completions.

### **Parameters**

#### Model — Model to reduce

numeric LTI model

Choose the model to reduce. The list of available models includes proper tf, ss, or zpk models in the MATLAB workspace. The model can be SISO or MIMO, and continuous or discrete.

- Continuous-time models cannot have time delays. To reduce a continuous-time model with time delays, first use pade to approximate the time delays as model dynamics.
- Discrete-time models can have time delays. For the Balanced Truncation reduction method, the task uses absorbDelay to convert the delay into poles at z = 0 before reducing the model.

**Note Reduce Model Order** assumes that the model time unit (specified in the TimeUnit property of the model) is seconds. For the Balanced Truncation and Mode

Selection methods, if your model does not have TimeUnit = 'seconds', use chgTimeUnit to convert the model to seconds.

#### Method — Model reduction method

Balanced Truncation (default) | Mode Selection | Pole-Zero Simplification

For each method, the **Reduce Model Order** task gives you controls and plots that help you ensure that your reduced model preserves dynamics that are important for your application.

- Balanced Truncation Compute a lower order approximation of your model by removing states with relatively small energy contributions. To use this method, specify the number of states (order) in the reduced model. The Hankel singular-value plot visualizes the relative energy contribution of each state in the original model. The task discards states with lower energy than the state you select in this plot. This method generates code that uses the balred command.
  - For discrete-time model that has time delays, **Reduce Model Order** uses absorbDelay to convert the delay into poles at z=0 before reducing the model by balanced truncation. The additional states are reflected in the response plot and Hankel singular-value plot.
- Mode Selection Select modes by specifying a frequency range of interest. The task discards dynamics that fall outside the region you specify on the frequency-response plot. This method generates code that uses the freqsep command.
- Pole-Zero Simplification Eliminate canceling or near-canceling pole-zero pairs. The task discards pole-zero pairs that cancel with the threshold specified by the Tolerance parameter. Increase the tolerance to discard more states. This method generates code that uses the minreal command.

#### **Balanced Truncation Parameters**

# **Reduced Order — Number of states in reduced model** positive integer

Specify the number of states in the reduced-order model. You can use any value that falls between the number of unstable states in the model and the number of states in the original model. For more information, see "Balanced Truncation Model Reduction".

Preserve DC Gain — Match DC gain of reduced model to that of original model on (default) | off

Match the DC gain of the reduced model to that of the original model. Select **Preserve DC Gain** when the DC behavior of the model is important in your application. Clear the parameter to get better matching of higher frequency behavior. For more information, see "Balanced Truncation Model Reduction".

# Frequency Range — Limit analysis to specified frequencies off (default) | on

By default, **Reduce Model Order** analyzes Hankel singular values across all frequencies. Restricting this analysis to a particular frequency range is useful when you know the model has modes outside the region of interest to your particular application. When you apply a frequency limit, **Reduce Model Order** determines which states are the low-energy states to truncate based on their energy contribution within the specified frequency range only.

To limit the analysis of state contributions to a particular frequency range, check **Frequency range**. Then, drag the vertical cursors on the response plot to specify the frequency range of interest. Alternatively, enter the minimum and maximum frequencies in the text boxes. The unit is rad/s. If your model does not have TimeUnit = 'seconds', use chqTimeUnit to convert the model to seconds.

# **Comparison Plot** — **How to compare original and reduced models**Model response (default) | Absolute error | Relative error

**Reduce Model Order** shows you a comparison of the frequency responses between the original and reduced models. You can use this plot to monitor the match between the original and reduced-order models while you experiment with model-reduction parameter values. Available comparison plots are:

- Model response Frequency response of the original and reduced models, shown as a Bode plot for SISO models and a singular-value plot for MIMO models.
- Absolute error plot Singular values of G-Gr, where G is the original model and Gr is the current reduced model. (For SISO models, the singular-value plot is the magnitude of the frequency response.)
- Relative error plot Singular values of (G-Gr)/G. This plot is useful when the model has very high or very low gain in the region that is important to your application. In such regions, absolute error can be misleading.

#### **Mode Selection Parameters**

# **Cutoff Frequency — Frequency range in which to preserve dynamics** positive scalar values

Specify the lower and upper bounds of the frequency range in which to preserve dynamics. You can also use the vertical cursors on the response plot to specify the range. **Reduce Model Order** discards dynamics outside the specified range.

For more information about this method, see "Mode-Selection Model Reduction".

#### Comparison Plot — How to compare original and reduced models

Model response (default) | Absolute error | Relative error

**Reduce Model Order** shows you a comparison of the frequency responses between the original and reduced models. You can use this plot to monitor the match between the original and reduced-order models while you experiment with model-reduction parameter values. Available comparison plots are:

- Model response Frequency response of the original and reduced models, shown as a Bode plot for SISO models and a singular-value plot for MIMO models.
- Absolute error plot Singular values of G-Gr, where G is the original model and Gr is the current reduced model. (For SISO models, the singular-value plot is the magnitude of the frequency response.)
- Relative error plot Singular values of (G-Gr)/G. This plot is useful when the model has very high or very low gain in the region that is important to your application. In such regions, absolute error can be misleading.

#### **Pole-Zero Simplification Parameters**

### Tolerance — Margin for pole-zero cancellation

positive scalar

Specify the margin for pole-zero cancellation. Pole-zero pairs that cancel within this tolerance are removed from the reduced model. You can use the slider to change the tolerance and observe the results in a response plot.

#### **Results Parameters**

### **Output Plot — Type of response plot to generate**

None (default) | Step | Impulse | Bode | ...

**Reduce Model Order** generates code that shows the response of the original and reduced systems on the plot type you specify. Available plots include:

Step response

- · Impulse response
- Bode plot
- Singular value (sigma) plot
- Pole-zero plot

### **See Also**

#### **Functions**

balred | freqsep | minreal

### **Apps**

**Model Reducer** 

### **Topics**

"Model Reduction in the Live Editor"

"Model Reduction Basics"

### Introduced in R2019b

### reg

Form regulator given state-feedback and estimator gains

## **Syntax**

```
rsys = reg(sys,K,L)
rsys = reg(sys,K,L,sensors,known,controls)
```

# **Description**

rsys = reg(sys,K,L) forms a dynamic regulator or compensator rsys given a state-space model sys of the plant, a state-feedback gain matrix K, and an estimator gain matrix L. The gains K and L are typically designed using pole placement or LQG techniques. The function reg handles both continuous- and discrete-time cases.

This syntax assumes that all inputs of sys are controls, and all outputs are measured. The regulator rsys is obtained by connecting the state-feedback law u = -Kx and the state estimator with gain matrix L (see estim). For a plant with equations

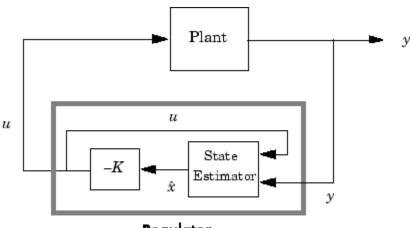
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

this yields the regulator

$$\hat{\vec{x}} = [A - LC - (B - LD)K]\hat{x} + Ly$$

$$u = -K\hat{x}$$

This regulator should be connected to the plant using *positive* feedback.

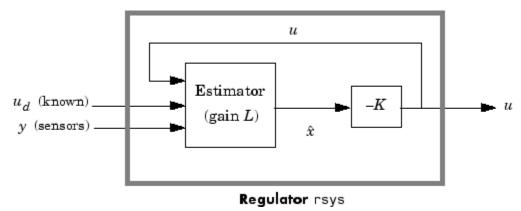


Regulator

rsys = reg(sys,K,L,sensors,known,controls) handles more general regulation
problems where:

- The plant inputs consist of controls u, known inputs  $u_d$ , and stochastic inputs w.
- Only a subset *y* of the plant outputs is measured.

The index vectors sensors, known, and controls specify y,  $u_d$ , and u as subsets of the outputs and inputs of sys. The resulting regulator uses  $[u_d; y]$  as inputs to generate the commands u (see next figure).



## **Examples**

Given a continuous-time state-space model

```
sys = ss(A,B,C,D)
```

with seven outputs and four inputs, suppose you have designed:

- A state-feedback controller gain K using inputs 1, 2, and 4 of the plant as control inputs
- A state estimator with gain L using outputs 4, 7, and 1 of the plant as sensors, and input 3 of the plant as an additional known input

You can then connect the controller and estimator and form the complete regulation system by

```
controls = [1,2,4];
sensors = [4,7,1];
known = [3];
regulator = reg(sys,K,L,sensors,known,controls)
```

### See Also

dlqr|estim|kalman|lqgreg|lqr|place

Introduced before R2006a

# replaceBlock

Replace or update Control Design Blocks in Generalized LTI model

## **Syntax**

```
Mnew = replaceBlock(M,Block1,Value1,...,BlockN,ValueN)
Mnew = replaceBlock(M,blockvalues)
Mnew = replaceBlock(...,mode)
```

## **Description**

```
Mnew = replaceBlock(M,Block1,Value1,...,BlockN,ValueN) replaces the Control Design Blocks Block1,...,BlockN of M with the specified values Value1,...,ValueN. M is a Generalized LTI model or a Generalized matrix.
```

Mnew = replaceBlock(M,blockvalues) specifies the block names and replacement values as field names and values of the structure blockvalues.

Mnew = replaceBlock(...,mode) performs block replacement on an array of models M using the substitution mode specified by mode.

## **Input Arguments**

#### М

Generalized LTI model, Generalized matrix, or array of such models.

### Block1,...,BlockN

Names of Control Design Blocks in M. The replaceBlock command replaces each listed block of M with the corresponding values Value1, . . . , ValueN that you supply.

If a specified Block is not a block of M, replaceBlock that block and the corresponding value.

#### Value1, ..., ValueN

Replacement values for the corresponding blocks Block1,...,BlockN.

The replacement value for a block can be any value compatible with the size of the block, including a different Control Design Block, a numeric matrix, or an LTI model. If any value is [], the corresponding block is replaced by its nominal (current) value.

#### blockvalues

Structure specifying blocks of M to replace and the values with which to replace those blocks.

The field names of blockvalues match names of Control Design Blocks of M. Use the field values to specify the replacement values for the corresponding blocks of M. The replacement values may be numeric values, Numeric LTI models, Control Design Blocks, or Generalized LTI models.

#### mode

Block replacement mode for an input array M of Generalized matrices or LTI models, specified as one of the following values:

 '-once' (default) — Vectorized block replacement across the model array M. Each block is replaced by a single value, but the value may change from model to model across the array.

For vectorized block replacement, use a structure array for the input blockvalues, or cell arrays for the Value1,..., ValueN inputs. For example, if M is a 2-by-3 array of models:

- Mnew = replaceBlock(M, blockvalues, '-once'), where blockvalues is a 2-by-3 structure array, specifies one set of block values blockvalues(k) for each model M(:,:,k) in the array.
- Mnew = replaceBlock(M,Block,Value,'-once'), where Value is a 2-by-3 cell array, replaces Block by Value{k} in the model M(:,:,k) in the array.
- '-batch' Batch block replacement. Each block is replaced by an array of values, and the same array of values is used for each model in M. The resulting array of model Mnew is of size [size(M) Asize], where Asize is the size of the replacement value.

When the input M is a single model, '-once' and '-batch' return identical results.

Default: '-once'

# **Output Arguments**

#### Mnew

Matrix or linear model or matrix where the specified blocks are replaced by the specified replacement values.

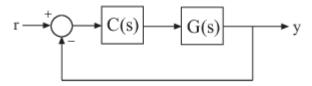
Mnew is a numeric array or numeric LTI model when all the specified replacement values are numeric values or numeric LTI models.

## **Examples**

### **Replace Control Design Block with Numeric Values**

This example shows how to replace a tunable PID controller (tunablePID) in a Generalized LTI model by a pure gain, a numeric PI controller, or the current value of the tunable controller.

**1** Create a Generalized LTI model of the following system:



where the plant  $G(s) = \frac{(s-1)}{(s+1)^3}$ , and C is a tunable PID controller.

```
G = zpk(1,[-1,-1,-1],1);
C = tunablePID('C','pid');
Try = feedback(G*C,1)
```

2 Replace C by a pure gain of 5.

T1 = replaceBlock(Try, 'C',5);

T1 is a ss model that equals feedback(G\*5,1).

**3** Replace C by a PI controller with proportional gain of 5 and integral gain of 0.1.

```
C2 = pid(5,0.1);
T2 = replaceBlock(Try, 'C',C2);
```

T2 is a ss model that equals feedback(G\*C2,1).

**4** Replace C by its current (nominal) value.

```
T3 = replaceBlock(Try, 'C',[]);
```

T3 is a ss model where C has been replaced by getValue(C).

### **Sample Tunable Model Over Grid of Values**

Consider the second-order filter represented by:

$$F(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

Sample this filter at varying values of the damping constant  $\zeta$  and the natural frequency  $\omega_n$ . Create a tunable model of the filter by using tunable elements for  $\zeta$  and  $\omega_n$ .

```
wn = realp('wn',3);
zeta = realp('zeta',0.8);
F = tf(wn^2,[1 2*zeta*wn wn^2])
F =

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 2 states, and wn: Scalar parameter, 5 occurrences.
    zeta: Scalar parameter, 1 occurrences.
Type "ss(F)" to see the current value, "get(F)" to see all properties, and "F.Blocks" "
```

Create a grid of sample values.

```
wnvals = [3;5];
zetavals = [0.6 0.8 1.0];
[wngrid,zetagrid] = ndgrid(wnvals,zetavals);
Fsample = replaceBlock(F,'wn',wngrid,'zeta',zetagrid);
size(Fsample)
```

```
2x3 array of state-space models. Each model has 1 outputs, 1 inputs, and 2 states.
```

The ndgrid command produces a full 2-by-3 grid of parameter combinations. Thus, Fsample is a 2-by-3 array of state-space models. Each entry in the array is a state-space model that represents F evaluated at the corresponding (wn, zeta) pair. For example, Fsample(:,:,2,3) has wn = 5 and zeta = 1.0.

damp(Fsample(:,:,2,3))

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-5.00e+00	1.00e+00	5.00e+00	2.00e-01
-5.00e+00	1.00e+00	5.00e+00	2.00e-01

# **Tips**

- Use replaceBlock to perform parameter studies by sampling Generalized LTI models across a grid of parameters, or to evaluate tunable models for specific values of the tunable blocks. See "Examples" on page 2-1042.
- For additional options for sampling control design blocks, including concurrent sampling, use sampleBlock.
- To take random samples of control design blocks, see rsampleBlock

### **See Also**

genmat | genss | getValue | nblocks | rsampleBlock | sampleBlock

### **Topics**

- "Generalized Matrices"
- "Generalized and Uncertain LTI Models"
- "Models with Tunable Coefficients"

#### Introduced in R2011a

## repsys

Replicate and tile models

## **Syntax**

```
rsys = repsys(sys,[M N])
rsys = repsys(sys,N)
rsys = repsys(sys,[M N S1,...,Sk])
```

# **Description**

rsys = repsys(sys,[M N]) replicates the model sys into an M-by-N tiling pattern. The resulting model rsys has size(sys,1)\*M outputs and size(sys,2)\*N inputs.

```
rsys = repsys(sys,N) creates an N-by-N tiling.
```

```
rsys = repsys(sys,[M N S1,...,Sk]) replicates and tiles sys along both I/O and array dimensions to produce a model array. The indices S specify the array dimensions. The size of the array is [size(sys,1)*M, size(sys,2)*N, size(sys,3)*S1, ...].
```

# **Input Arguments**

#### sys

Model to replicate.

#### М

Number of replications of sys along the output dimension.

#### N

Number of replications of sys along the input dimension.

S

Numbers of replications of sys along array dimensions.

## **Output Arguments**

#### rsys

Model having size(sys,1)\*M outputs and size(sys,2)\*N inputs.

If you provide array dimensions  $S1, \ldots, Sk$ , rsys is an array of dynamic systems which each have size(sys,1)\*M outputs and size(sys,2)\*N inputs. The size of rsys is  $[size(sys,1)*M, size(sys,2)*N, size(sys,3)*S1, \ldots]$ .

# **Examples**

#### **Replicate SISO Transfer Function to Create MIMO Transfer Function**

Create a single-input single-output (SISO) transfer function.

```
sys = tf(2,[1 3])
sys =
    2
    ....
s + 3
```

Continuous-time transfer function.

Replicate the SISO transfer function to create a MIMO transfer function that has three inputs and two outputs.

```
rsys = repsys(sys,[2 3])
rsys =
From input 1 to output...
2
1: -----
```

Continuous-time transfer function.

Alternatively, you can obtain the MIMO transfer function as follows:

### **Replicate SISO Transfer Function to Create Array of Transfer Functions**

Create a SISO transfer function.

$$sys = tf(2,[1 3]);$$

Replicate the transfer function into a 3-by-4 array of two-input, one-output transfer functions.

```
rsys = repsys(sys,[1 2 3 4]);
```

Check the size of rsys.

#### size(rsys)

3x4 array of transfer functions. Each model has 1 outputs and 2 inputs.

# **Tips**

rsys = repsys(sys,N) produces the same result as rsys = repsys(sys,[N N]). To produce a diagonal tiling, use rsys = sys\*eye(N).

### See Also

append

Introduced in R2010b

# reshape

Change shape of model array

## **Syntax**

```
sys = reshape(sys, s1, s2, ..., sk)
sys = reshape(sys,[s1 s2 ... sk])
```

# **Description**

sys = reshape(sys,s1,s2,...,sk) (or, equivalently, sys = reshape(sys,[s1 s2 ... sk])) reshapes the LTI array sys into an s1-by-s2-by-...-by-sk model array. With either syntax, there must be s1\*s2\*...\*sk models in sys to begin with.

## **Examples**

### **Change Shape of Model Array**

6x1 array of state-space models.

Generate a 2-by-3 array of SISO models with four states each.

```
sys = rss(4,1,1,2,3);
size(sys)
2x3 array of state-space models.
Each model has 1 outputs, 1 inputs, and 4 states.
Change the shape of the array to create a 6-by-1 model array.
sys1 = reshape(sys, 6, 1);
size(sys1)
```

Each model has 1 outputs, 1 inputs, and 4 states.

2-1049

# **See Also**

ndims|size

Introduced before R2006a

### residual

Return measurement residual and residual covariance when using extended or unscented Kalman filter

### **Syntax**

```
[Residual, ResidualCovariance] = residual(obj,y)
[Residual, ResidualCovariance] = residual(obj,y,Um1,...,Umn)
```

# **Description**

The residual command returns the difference between the actual and predicted measurements for extendedKalmanFilter and unscentedKalmanFilter objects. Viewing the residual provides a way for you to validate the performance of the filter. Residuals, also known as innovations, quantify the prediction error and drive the correction step in the extended and unscented Kalman filter update sequence. When using correct and predict to update the estimated Kalman filter state, use the residual command immediately before using the correct command.

[Residual, ResidualCovariance] = residual(obj,y) returns the residual Residual between a measurement y and a predicted measurement produced by the Kalman filter obj. The function also returns the covariance of the residual ResidualCovariance.

You create obj using the extendedKalmanFilter or unscentedKalmanFilter commands. You specify the state transition function f and measurement function h of your nonlinear system in obj. The State property of the object stores the latest estimated state value. At each time step, you use correct and predict together to update the state x. The residual f is the difference between the actual and predicted measurements for the time step, and is expressed as f is the covariance of the residual f is the sum f is the measurement noise matrix set by the MeasurementNoise property of the filter and f is the state covariance matrix projected onto the measurement space.

Use this syntax if the measurement function h that you specified in obj.MeasurementFcn has one of the following forms:

- y(k) = h(x(k)) for additive measurement noise
- y(k) = h(x(k), v(k)) for nonadditive measurement noise

Here, y(k), x(k), and v(k) are the measured output, states, and measurement noise of the system at time step k. The only inputs to h are the states and measurement noise.

[Residual, ResidualCovariance] = residual(obj,y,Um1,...,Umn) specifies additional input arguments if the measurement function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if the measurement function h has one of the following forms:

- y(k) = h(x(k), Um1, ..., Umn) for additive measurement noise
- y(k) = h(x(k), v(k), Um1, ..., Umn) for nonadditive measurement noise

## **Examples**

### **Estimate States Online Using Extended Kalman Filter**

Estimate the states of a van der Pol oscillator using an extended Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an extended Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with the nonlinearity parameter mu equal to 1. The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as [1;0]. This is the guess for the state value at initial time k, based on knowledge of system outputs until time k-1,  $\hat{\chi}[k|k-1]$ .

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
```

Load the measured output data y from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp data.mat file.

```
load vdp_data.mat y
```

Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the extended Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct  $\hat{x}[k \mid k-1]$  using measurements at time k to get  $\hat{x}[k \mid k]$ . Then, you predict the state value at the next time step  $\hat{x}[k+1 \mid k]$  using  $\hat{x}[k \mid k]$ , the state estimate at time step k that is estimated using measurements until time k.

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual, ResidualCovariance] = residual(obj,y(k));
    [CorrectedState, CorrectedStateCovariance] = correct(obj,y(k));
    [PredictedState, PredictedStateCovariance] = predict(obj);

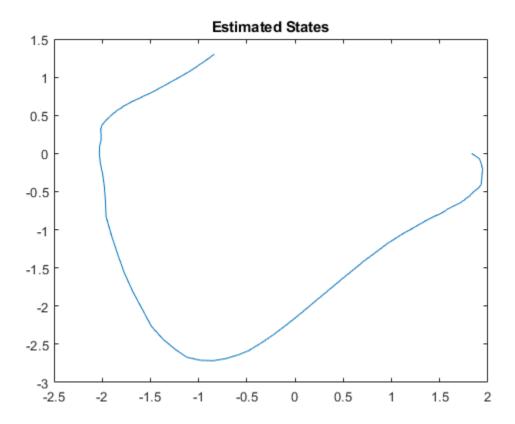
    residBuf(k,:) = Residual;
    xcorBuf(k,:) = CorrectedState';
    xpredBuf(k,:) = PredictedState';
end
```

When you use the correct command, obj.State and obj.StateCovariance are updated with the corrected state and state estimation error covariance values for time step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj.State and obj.StateCovariance are updated with the predicted values for time step k+1, PredictedState and PredictedStateCovariance. When you use the residual command, you do not modify any obj properties.

In this example, you used correct before predict because the initial state value was  $\hat{x}[k \mid k-1]$ , a guess for the state value at initial time k based on system outputs until time k-1. If your initial state value is  $\hat{x}[k-1 \mid k-1]$ , the value at previous time k-1 based on measurements until k-1, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 2-191.

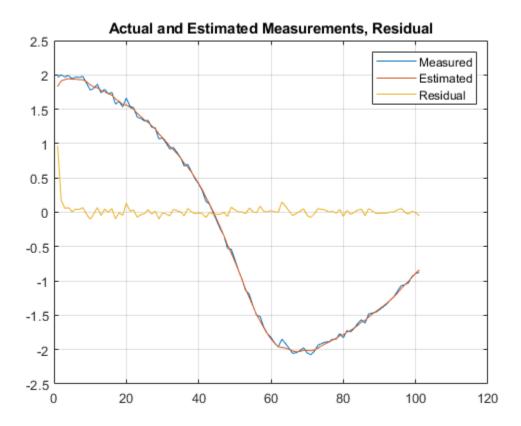
Plot the estimated states, using the postcorrection values.

```
plot(xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')
```



Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.

```
M = [y,xcorBuf(:,1),residBuf];
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')
```



The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

### Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input u whose state x and measurement y evolve according to the following state transition and measurement equations:

$$x[k] = \sqrt{x[k-1] + u[k-1]} + w[k-1]$$
$$v[k] = x[k] + 2 * u[k] + v[k]^{2}$$

The process noise  $\boldsymbol{v}$  of the system is additive while the measurement noise  $\boldsymbol{v}$  is nonadditive.

Create the state transition function and measurement function for the system. Specify the functions with an additional input u.

```
f = @(x,u)(sqrt(x+u));

h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.

```
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
```

Specify the measurement noise covariance.

```
obj.MeasurementNoise = 0.01;
```

You can now estimate the state of the system using the predict and correct commands. You pass the values of u to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement y[k]=0.8 and input u[k]=0.2 at time step k.

```
correct(obj, 0.8, 0.2)
```

Predict the state at the next time step, given u[k]=0.2.

```
predict(obj,0.2)
```

Retrieve the error, or *residual*, between the prediction and the measurement.

```
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);
```

### **Input Arguments**

#### obj — Extended or unscented Kalman filter

extendedKalmanFilter object | unscentedKalmanFilter object

Extended or unscented Kalman filter, created using one of the following commands:

- extendedKalmanFilter Uses the extended Kalman filter algorithm
- unscentedKalmanFilter Uses the unscented Kalman filter algorithm

### y — Measured system output

vector

Measured system output at the current time step, specified as an N-element vector, where N is the number of measurements.

# Um1, . . . , Umn — Additional input arguments to measurement function input arguments of any type

Additional input arguments to the measurement function of the system, specified as input arguments of any type. The measurement function h is specified in the MeasurementFcn or MeasurementLikelihoodFcn property of obj. If the function requires input arguments in addition to the state and measurement noise values, you specify these inputs in the residual command syntax. The residual command passes these inputs to the measurement or the measurement likelihood function to calculate estimated outputs. You can specify multiple arguments.

For instance, suppose that your measurement or measurement likelihood function calculates the estimated system output y using system inputs u and current time k, in addition to the state x. The  $U_{m1}$  and  $U_{m2}$  terms are therefore u(k) and k. These inputs result in the estimated output

$$y(k) = h(x(k), u(k), k)$$

Before you perform online state estimation correction at time step k, specify these additional inputs in the residual command syntax:

```
[Residual, ResidualCovariance] = residual(obj,y,u(k),k);
```

For an example showing how to use additional input arguments, see "Specify State Transition and Measurement Functions with Additional Inputs" on page 2-1055.

## **Output Arguments**

# **Residual — Residual between current and predicted measurement** scalar | vector

Residual between current and predicted measurement, returned as a:

- Scalar for a single-output system
- Vector of size N for a multiple-output system, where N is the number of measured outputs

#### ResidualCovariance — Residual covariance

matrix

Residual covariance, returned as an N-by-N matrix where N is the number of measured outputs.

### See Also

correct | extendedKalmanFilter | predict | unscentedKalmanFilter

### **Topics**

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"

"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

#### Introduced in R2019b

<sup>&</sup>quot;Generate Code for Online State Estimation in MATLAB"

<sup>&</sup>quot;Validate Online State Estimation at the Command Line"

## rlocus

Root locus plot of dynamic system

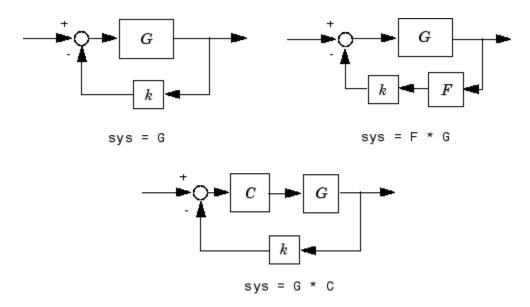
# **Syntax**

```
rlocus(sys)
rlocus(sys1,sys2,...)
[r,k] = rlocus(sys)
r = rlocus(sys,k)
```

# **Description**

rlocus computes the root locus of a SISO open-loop model. The root locus gives the closed-loop pole trajectories as a function of the feedback gain k (assuming negative feedback). Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency responses.

rlocus(sys) calculates and plots the root locus of the open-loop SISO model sys. This function can be applied to any of the following *negative* feedback loops by setting sys appropriately.



If sys has transfer function

$$h(s) = \frac{n(s)}{d(s)}$$

the closed-loop poles are the roots of

$$d(s) + kn(s) = 0$$

rlocus adaptively selects a set of positive gains k to produce a smooth plot. Alternatively, rlocus (sys,k)

uses the user-specified vector  $\boldsymbol{k}$  of gains to plot the root locus.

rlocus (sys1, sys2,...) draws the root loci of multiple LTI models sys1, sys2,... on a single plot. You can specify a color, line style, and marker for each model, as in

[r,k] = rlocus(sys) and r = rlocus(sys,k) return the vector k of selected gains and the complex root locations r for these gains. The matrix r has length(k) columns and its jth column lists the closed-loop roots for the gain k(j).

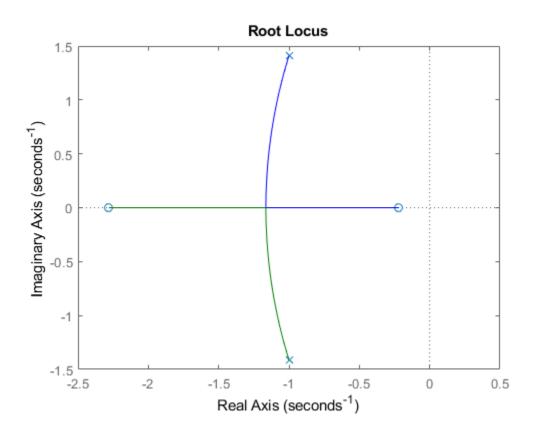
# **Examples**

### **Root Locus Plot of Dynamic System**

Plot the root-locus of the following system.

$$h(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3} \,.$$

h = tf([2 5 1],[1 2 3]); rlocus(h)



You can use the right-click menu for rlocus to add grid lines, zoom in or out, and invoke the Property Editor to customize the plot. Also, click anywhere on the curve to activate a data marker that displays the gain value, pole, damping, overshoot, and frequency at the selected point.

# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

## See Also

pole | pzmap

Introduced before R2006a

# rlocusplot

Plot root locus and return plot handle

# **Syntax**

```
h = rlocusplot(sys)
rlocusplot(sys,k)
rlocusplot(sys1,sys2,...)
rlocusplot(AX,...)
rlocusplot(..., plotoptions)
```

# **Description**

h = rlocusplot(sys) computes and plots the root locus of the single-input, singleoutput LTI model sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

```
help pzoptions
```

for a list of available plot options.

See rlocus for a discussion of the feedback structure and algorithms used to calculate the root locus.

rlocusplot(sys,k) uses a user-specified vector k of gain values.

rlocusplot(sys1,sys2,...) draws the root loci of multiple LTI models sys1, sys2,... on a single plot. You can specify a color, line style, and marker for each model, as in

```
rlocusplot(sys1,'r',sys2,'y:',sys3,'gx')
```

rlocusplot(AX,...) plots into the axes with handle AX.

rlocusplot(..., plotoptions) plots the root locus with the options specified in plotoptions. Type

help proptions

for more details.

# **Examples**

Use the plot handle to change the title of the plot.

```
sys = rss(3);
h = rlocusplot(sys);
p = getoptions(h); % Get options for plot.
p.Title.String = 'My Title'; % Change title in options.
setoptions(h,p); % Apply options to plot.
```

# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

## See Also

getoptions | pzoptions | rlocus | setoptions

Introduced before R2006a

# rsampleBlock

Randomly sample Control Design blocks in generalized model

# **Syntax**

```
Msamp = rsampleBlock(M,names,N)
Msamp = rsampleBlock(M,names1,N1,names2,N2,...,namesM,NM)
[Msamp,samples] = rsampleBlock( )
```

# **Description**

Msamp = rsampleBlock(M, names, N) randomly samples a subset of the Control Design blocks in the generalized model M. The names argument specifies which blocks to sample, and N specifies how many samples to take. The result Msamp is a model array of size [size(M) N] obtained by replacing the sampled blocks with their randomized values.

Msamp = rsampleBlock(M, names1, N1, names2, N2, ..., namesM, NM) takes N1 samples of the blocks listed in names1, N2 samples of the blocks listed in names2, and so on. The result Msamp is a model array of size [size(M) N1 N2 ... NM].

[Msamp, samples] = rsampleBlock( \_\_\_\_) also returns a data structure containing the block replacement values for each sampling point. You can use this syntax with any of the preceding input argument combinations.

# **Examples**

### **Randomly Sample Parameter in Tunable Model**

Create the first-order model  $G(s) = 1/(\tau s + 1)$ , where  $\tau$  is a tunable real parameter.

```
tau = realp('tau',5);
G = tf(1,[tau 1]);
```

Restrain tau to nonnegative values only.

```
G.Blocks.tau.Minimum = 0;
```

Generate 20 random samples of G. The result is a 20-by-1 array of first-order models with random values of tau taken from the range of tau.

```
Gs = rsampleBlock(G, 'tau', 20);
size(Gs)
20x1 array of state-space models.
Each model has 1 outputs, 1 inputs, and 1 states.
```

#### **Randomly Sample Multiple Parameters**

Take random samples of a model with both tunable and uncertain blocks. Using uncertain blocks requires Robust Control Toolbox $^{\text{\tiny{TM}}}$ . Random sampling of tunable blocks works the same way as shown in this example.

Create an uncertain model of  $G(s) = a/(\tau s + 1)$ , where a is an uncertain parameter that varies in the interval [3,5], and  $\tau = 0.5$  +/- 30%. Also, create a tunable PI controller, and form a closed-loop system from the tunable controller and uncertain system.

```
a = ureal('a',4);
tau = ureal('tau',.5,'Percentage',30);
G = tf(a,[tau 1]);
C = tunablePID('C','pi');
T = feedback(G*C,1);
```

T is a generalized state-space model with two uncertain blocks, a and tau, and one tunable block, C. Sample T at 20 random (a,tau) pairs.

```
[Ts,samples] = rsampleBlock(T,{'a','tau'},20);
```

Ts is a 20-by-1 array of genss models. The tunable block C, which is not sampled, is preserved in Ts. The structure samples has fields samples.a and samples.tau that contain the values at which those blocks are sampled.

Grouping a and tau into a cell array causes rsampleBlock to sample them together, as (a,tau) pairs. Sampling the blocks independently generates a higher-dimensionality arrays. For example, independently taking 10 random samples of a and 5 samples of tau generates a 10-by-5 model array.

```
[TsInd, samples] = rsampleBlock(T, 'a', 10, 'tau', 5);
TsInd

TsInd =

10x5 array of generalized continuous-time state-space models.
Each model has 1 outputs, 1 inputs, 2 states, and the following blocks:
    C: Parametric PID controller, 1 occurrences.

Type "ss(TsInd)" to see the current value, "get(TsInd)" to see all properties, and "Tsi
```

In this array, a varies along one dimension and tau varies along the other.

# **Input Arguments**

#### **M** — Model to sample

generalized model | uncertain model | generalized matrix | uncertain matrix

Model to sample, specified as a:

- Generalized model (genss or genfrd)
- Generalized matrix (genmat)
- Uncertain model (uss or ufrd)
- Uncertain matrix (umat)

## names — Control Design blocks

character vector | cell array of character vectors

Control Design blocks to sample, specified as a character vector or cell array of character vectors. The entries in names correspond to the names of at least a subset of the Control Design blocks in M. For example, suppose that M is a genss model with tunable blocks t1 and t2, and uncertain blocks u1 and u2. Then,  $\{'t1', 'u2'\}$  is one possible value for names.

Grouping block names together in a cell array generates samples of the group rather than independent samples of each block. For example, the following code generates a 10-by-1 array of models, where each entry in the array has a random value for the pair (t1,u2).

```
Msamp = rsampleBlock(M,{'t1','u2'},10);
```

To sample parameters independently, do not group them. For example, the following code generates a 10-by-20 array of models, where t1 varies along the first dimension and u2 varies along the second dimension.

```
Msamp = rsampleBlock(M, 't1', 10, 'u2', 20);
```

rsampleBlock ignores any entry in names that does not appear in M.

#### N — Number of samples

positive integer

Number of samples to take of the preceding block or blocks, specified as a positive integer.

# **Output Arguments**

### Msamp — Array of model samples

generalized model array | ss array | frd array | numeric array

Array of model samples, returned as a generalized model array, ss array, frd array, or numeric array. Msamp is of the same type as M, unless all blocks are sampled. In that case, Msamp is a numeric array, ss array, or frd array. For example, suppose that M is a uss model with uncertain blocks u1 and u2. The following command returns an array of uss models, with uncertain block u2.

```
Msamp1 = rsampleBlock(M, 'u1', 10);
```

The following command samples both blocks and returns an array of ss models.

```
Msamp2 = rsampleBlock(M, \{'u1', 'u2'\}, 10);
```

rsampleBlock uses values that fall within the uncertainty range when sampling uncertain blocks, and within the maximum and minimum parameter values when sampling tunable blocks.

## samples — Block sample values

structure

Block sample values, returned as a structure. The fields of samples are the names of the sampled blocks. The values are arrays containing the corresponding random values used to generate the entries in Msamp. For instance, suppose that you run the following command, where M is a genss model with tunable blocks t1 and t2.

```
[Msamp,samples] = rsampleBlock(M,{'t1','t2'},10);
```

Then, samples.tl contains the 10 values of tl and samples.t2 contains the 10 values of t2. If you sample a block that is not scalar valued, the corresponding field of samples contains values compatible with the block. For instance, if you sample a tunablePID block, samples contains an array of state-space models that represent PID controllers.

## See Also

genmat | genss | getValue | replaceBlock | sampleBlock | uss

## **Topics**

"Generalized Models"

Introduced in R2016a

#### rss

Generate random continuous test model

# **Syntax**

```
rss(n)
rss(n,p)
rss(n,p,m,s1,...,sn)
```

# **Description**

rss(n) generates an n-th order model with one input and one output and returns the model in the state-space object sys. The poles of sys are random and stable with the possible exception of poles at s = 0 (integrators).

rss(n,p) generates an nth order model with one input and p outputs, and rss(n,p,m) generates an n-th order model with m inputs and p outputs. The output sys is always a state-space model.

rss(n,p,m,s1,...,sn) generates an s1-by-...-by-sn array of n-th order state-space models with m inputs and p outputs.

Use tf, frd, or zpk to convert the state-space object sys to transfer function, frequency response, or zero-pole-gain form.

# **Examples**

### **Generate State-Space Models**

Generate a random SISO state-space model with two states.

```
sys2 = rss(2)
sys2 =
```

Continuous-time state-space model.

Generate a model with four states, three outputs, and two inputs. The input arguments to rss are arranged in the order states, outputs, inputs.

Continuous-time state-space model.

#### **Generate Array of Random Models**

Generate a 4-by-5 array of SISO models with three states each.

```
sysarray = rss(3,1,1,4,5);
size(sysarray)

4x5 array of state-space models.
Each model has 1 outputs, 1 inputs, and 3 states.
```

## See Also

 ${\tt drss} \mid {\tt frd} \mid {\tt tf} \mid {\tt zpk}$ 

#### Introduced before R2006a

# sampleBlock

Sample Control Design blocks in generalized model

# **Syntax**

```
Msamp = sampleBlock(M,name,vals)
Msamp = sampleBlock(M,nameset,valset)
Msamp = sampleBlock(
M,nameset1,valset1,nameset2,valset2,...,namesetM,valsetM)
[Msamp,samples] = sampleBlock( )
```

# **Description**

Msamp = sampleBlock(M,name,vals) samples one Control Design block in the generalized model M. The result Msamp is a model array of size  $[size(M) \ N]$  obtained by replacing the block with the specified values, where N is the number of values in Vals.

Msamp = sampleBlock(M, nameset, valset) concurrently samples multiple blocks specified as a cell array of block names. valset is a cell array of N sample values for each block. The result Msamp is a model array of size [size(M) N].

```
Msamp = sampleBlock(
M,nameset1,valset1,nameset2,valset2,...,namesetM,valsetM) independently samples multiple blocks. nameset1,nameset2,...,namesetM can each be a single block name (see name) or a cell array of names (see nameset). The model M is sampled over a grid of size [N1 N2 ... NM], where N1 is the number of values in valset1, N2 is the number of values in valset2, and so on. The resulting Msamp is an array of size [size(M) N1 N2 ... NM].
```

[Msamp, samples] = sampleBlock(\_\_\_\_) also returns a data structure containing the block replacement values for each sampling point. You can use this syntax with any of the preceding input argument combinations.

# **Examples**

#### Sample Real Parameter in Tunable Model

Create the first-order model  $G(s) = 1/(\tau s + 1)$ , where  $\tau$  is a tunable real parameter.

```
tau = realp('tau',5);
G = tf(1,[tau 1]);
```

Evaluate this transfer function for  $\tau = 3,4,...,7$ . The result is a 5-by-1 array of first-order models.

```
Gs = sampleBlock(G,'tau',3:7);
size(Gs)

5x1 array of state-space models.
Each model has 1 outputs, 1 inputs, and 1 states.
```

#### **Sample Multiple Parameters in Tunable Model**

Create a model with a pole at s = a and a gain of b\*c, where a, b, and c are tunable scalars.

```
a = realp('a',1);
b = realp('b',3);
c = realp('c',1);
G = tf(b*c,[1 a]);
```

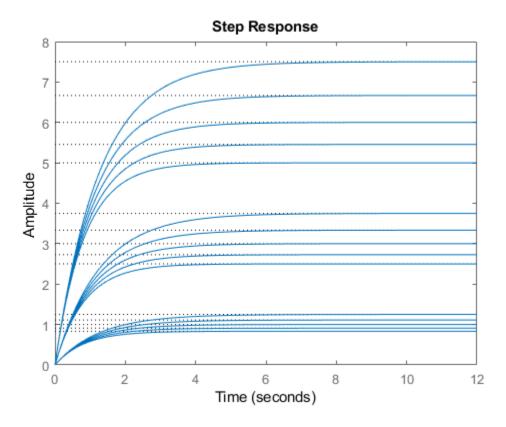
Pick 5 samples for a and 3 samples for (b,c) pairs. Evaluate G over the corresponding 5-by-3 grid of (a,b,c) combinations.

```
as = 0.8:0.1:1.2;
bs = 2:4;
cs = [0.5 1 1.5];
Gs = sampleBlock(G,'a',as,{'b','c'},{bs,cs});
```

Grouping the values for b and c in cell arrays causes sampleBlock to treat them as the (b,c) pairs, (2,0.5), (3,1), and (1,5). Gs is a 5-by-3 array of state-space models, in which a varies along the first dimension and (b,c) varies along the second dimension. Thus, for example, Gs (:,:,3,2) corresponds to a=1, (b,c)=(3,1).

A step plot shows a set of responses for each of the three (b,c) pairs. Each set contains a response for each of the five a values.

#### stepplot(Gs)



If you do not group the values, sampleBlock replaces all values independently, resulting in a 5-by-3-by-3 model array.

```
GsInd = sampleBlock(G,'a',as,'b',bs,'c',cs);
size(GsInd)

5x3x3 array of state-space models.
Each model has 1 outputs, 1 inputs, and 1 states.
```

For example, in GsInd, Gs(:,:,3,2,1) is a model with a=1, b=3, and c=0.5.

# **Input Arguments**

#### **M** — Model to sample

generalized model | uncertain model | generalized matrix | uncertain matrix

Model to sample, specified as a:

- Generalized model (genss or genfrd)
- Generalized matrix (genmat)
- Uncertain model (uss or ufrd)
- Uncertain matrix (umat)

#### name — Control Design block

character vector

Control Design block to sample, specified as a character vector. For example, suppose that M is a genss model with tunable blocks t1 and t2. Then, either 't1' or 't2' is a possible value for name.

### vals — Sample block values

numeric array | model array

Sample block values, specified as a numeric array or a model array. Values must be compatible with the block type. For example, if name is a tunable real parameter (realp), then vals is a numerical array of length N, the number of samples. If name is a tunable PID controller (tunablePID), then vals is an array of LTI models compatible with PID structure.

### nameset — Control Design blocks

cell array of character vectors

Control Design blocks to sample concurrently, specified as cell array of character vectors. The entries in nameset correspond to the names of at least a subset of the Control Design blocks in M. For example, suppose that M is a genss model with tunable blocks t1 and t2, and uncertain blocks u1 and u2. Then, {'t1', 'u2'} is one possible value for nameset.

Grouping block names together in a cell array generates samples of the group rather than independent samples. For example, the following code generates a 10-by-1 array of models, where each entry in the array has the corresponding value for the pair (t1,u2).

```
t1s = 1:10;
u2s = 2:2:20;
valset = {t1s,t2s};
Msamp = sampleBlock(M,{'t1','u2'},valset);
```

sampleBlock ignores any entry in nameset that does not appear in M.

#### valset — Sample block values

cell array

Sample block values, specified as a cell array. Each entry in the cell array is itself an array of N sample values for each block in nameset. For example, the following code samples a model M at the (t1,u2) pairs (1,2), (2,4), ... (10,20).

```
t1s = 1:10;
u2s = 2:2:20;
valset = {t1s,t2s};
Msamp = sampleBlock(M,{'t1','u2'},valset);
```

Values in valset must be compatible with the corresponding block type.

# **Output Arguments**

### **Msamp** — Array of model samples

generalized model array | ss array | frd array | numeric array

Array of model samples, returned as a generalized model array, ss array, frd array, or numeric array. Msamp is of the same type as M, unless all blocks are sampled. In that case, Msamp is a numeric array, ss array, or frd array. For example, suppose that M is a uss model with uncertain blocks u1 and u2. The following command returns an array of uss models, with uncertain block u2.

```
Msamp1 = sampleBlock(M, 'u1',1:10);
```

The following command samples both blocks and returns an array of ss models.

```
Msamp2 = sampleBlock(M, {'u1', 'u2'}, {1:10,2:20});
```

## samples — Block sample values

structure

Block sample values, returned as a structure. The fields of samples are the names of the sampled blocks. The values are arrays containing the corresponding values used to generate the entries in Msamp.

## See Also

genmat | genss | getValue | replaceBlock | rsampleBlock | uss

## **Topics**

"Study Parameter Variation by Sampling Tunable Model"

"Generalized Models"

#### Introduced in R2016a

# sectorplot

Compute or plot sector index as function of frequency

# **Syntax**

```
sectorplot(H,Q)
sectorplot(H,Q,w)
sectorplot(H1,H2,...,HN,Q)
sectorplot(H1,H2,...,HN,Q,w)
sectorplot(H1,LineSpec1,...,HN,LineSpecN,Q)
sectorplot(H1,LineSpec1,...,HN,LineSpecN,Q,w)

[index,wout] = sectorplot(H,Q)
index = sectorplot(H,Q,w)
```

# **Description**

sectorplot(H,Q) plots the relative sector indices for the dynamic system H and a given sector matrix Q. These indices measure by how much the sector bound is satisfied (index less than 1) or violated (index greater than 1) at a given frequency. (See "About Sector Bounds and Sector Indices" for more information about the meaning of the sector index.) sectorplot automatically chooses the frequency range and number of points based on the dynamics of H.

Let the following be an orthogonal decomposition of the symmetric matrix Q into its positive and negative parts.

$$Q = W_1 W_1^T - W_2 W_2^T, \quad W_1^T W_2 = 0.$$

The sector index plot is only meaningful if  $W_2^T H$  has a proper stable inverse. In that case, the sector indices are the singular values of:

$$(W_1^T H(j\omega))(W_2^T H(j\omega))^{-1}$$
.

sectorplot(H,Q,w) plots the sector index for frequencies specified by w.

- If w is a cell array of the form {wmin,wmax}, then sectorplot plots the sector index at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then sectorplot plots the sector index at each specified frequency.

sectorplot(H1, H2, ..., HN, Q) and sectorplot(H1, H2, ..., HN, Q, w) plot the sector index for multiple dynamic systems H1, H2, ..., HN on the same plot.

sectorplot(H1,LineSpec1,...,HN,LineSpecN,Q) and sectorplot(H1, LineSpec1,...,HN,LineSpecN,Q,w) specify a color, linestyle, and marker for each system in the plot.

[index,wout] = sectorplot(H,Q) returns the sector index at each frequency in the vector wout. The output index is a matrix, and the value index(:,k) gives the sector indices in descending order at the frequency w(k). This syntax does not draw a plot.

index = sectorplot(H,Q,w) returns the sector indices at the frequencies specified by w.

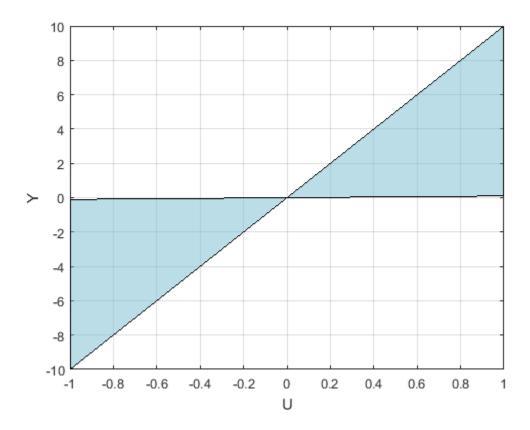
# **Examples**

#### **Plot Sector Index Versus Frequency**

Plot the sector index to visualize the frequencies at which the I/O trajectories of G(s) = (s + 2)/(s + 1) lie within the sector defined by:

$$S = \left\{ (y, u) \colon 0 : 1u^2 < uy < 10u^2 \right\}.$$

In U/Y space, this sector is the shaded region of the following diagram.



The Q matrix for this sector is given by:

$$a = 0.1;$$
  
 $b = 10;$   
 $Q = [1 - (a+b)/2 ; -(a+b)/2 a*b];$ 

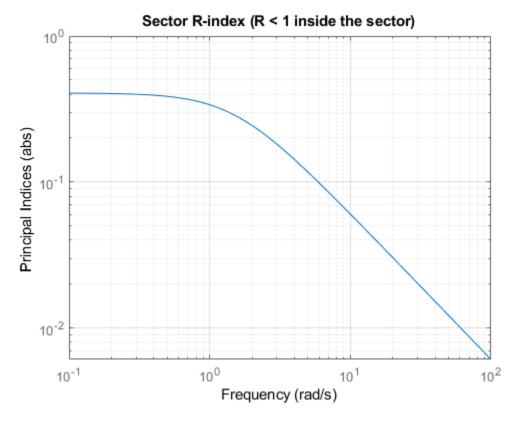
A trajectory y(t) = Gu(t) lies within the sector S when for all T > 0,

$$0.1 \int_{0}^{T} u(t)^{2} < \int_{0}^{T} u(t)y(t)dt < 10 \int_{0}^{T} u(t)^{2}dt.$$

In the frequency domain, this same condition can be expressed as:

$$\binom{G(j\omega)}{1}^HQ\binom{G(j\omega)}{1}<0\;.$$

To check whether G satisfies or violates this condition at any frequency, plot the sector index for H = [G; 1].



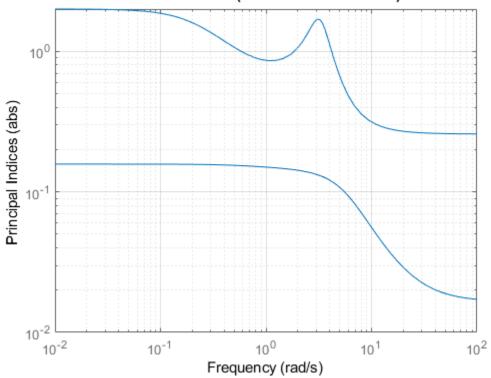
The plot shows that the sector index is less than 1 at all frequencies. Therefore, the trajectories of G(s) fit within in the specified sector Q at all frequencies.

## **Sector Plot with MIMO System**

Examine the sector plot of a 2-output, 2-input system for a particular sector.

```
rng(4,'twister');
H = rss(3,4,2);
Q = [-5.12]
                   -2.04
                           2.17
           2.16
           -1.22
                   -0.28
      2.16
                         -1.11
     -2.04 -0.28
                  -3.35
                           0.00
      2.17 -1.11
                    0.00
                           0.18];
sectorplot(H,Q)
```

### Sector R-index (R < 1 inside the sector)



Because H is 2-by-2, there are two lines on the sector plot. The largest value of the sector index exceeds 1 below about 0.5 rad/s and in a narrow band around 3 rad/s. Therefore, H does not satisfy the sector bound represented by Q.

# **Input Arguments**

#### **H** — Model to analyze

dynamic system model | model array

Model to analyze against sector bounds, specified as a dynamic system model such as a tf, ss, or genss model. H can be continuous or discrete. If H is a generalized model with tunable or uncertain blocks, sectorplot analyzes the current, nominal value of H.

To analyze whether all I/O trajectories (u(t),y(t)) of a linear system G lie in a particular sector, use H = [G;I], where I = eyes(nu), and nu is the number of inputs of G.

If H is a model array, then sectorplot plots the sector index of all models in the array on the same plot. When you use output arguments to get sector-index data, H must be a single model.

### Q — Sector geometry

matrix | LTI model

Sector geometry, specified as:

- A matrix, for constant sector geometry. **Q** is a symmetric square matrix that is ny on a side, where ny is the number of outputs of H.
- An LTI model, for frequency-dependent sector geometry. **Q** satisfies Q(s)' = Q(-s). In other words, Q(s) evaluates to a Hermitian matrix at each frequency.

The matrix  ${\bf Q}$  must be indefinite to describe a well-defined conic sector. An indefinite matrix has both positive and negative eigenvalues.

For more information, see "About Sector Bounds and Sector Indices".

### w — Frequencies

{wmin,wmax} | vector

Frequencies at which to compute and plot indices, specified as the cell array {wmin,wmax} or as a vector of frequency values.

- If w is a cell array of the form {wmin,wmax}, then the function computes the index at frequencies ranging between wmin and wmax.
- If w is a vector of frequencies, then the function computes the index at each specified frequency. For example, use logspace to generate a row vector with logarithmicallyspaced frequency values.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the model.

#### LineSpec — Line style, marker, and color

character vector | string

Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.

```
Example: 'r--' specifies a red dashed line
Example: '*b' specifies blue asterisk markers
Example: 'y' specifies a yellow line
```

# **Output Arguments**

#### index — Sector indices

matrix

Sector indices as a function of frequency, returned as a matrix. index contains the sector indices computed at the frequencies w if you supplied them, or wout if you did not. index has as many columns as there are values in w or wout, and as many rows as H has inputs. Thus the value index(:,k) gives the sector indices in descending order at the frequency w(k).

For example, suppose that G is a 3-input, 3-output system, Q is a suitable sector matrix, and w is a 1-by-30 vector of frequencies, then the following syntax returns a 3-by-30 matrix index.

```
H = [G;eyes(3)]
index = sectorplot(H,Q,w);
```

The entry index(:,k) contains the three sector indices for H, in descending order, at the frequency w(k).

For more information, see "About Sector Bounds and Sector Indices".

#### wout — Frequencies

vector

Frequencies at which the indices are calculated, returned as a vector. The function automatically chooses the frequency range and number of points based on the dynamics of the model.

## See Also

## **Topics**

"About Sector Bounds and Sector Indices"

#### Introduced in R2016a

## series

Series connection of two models

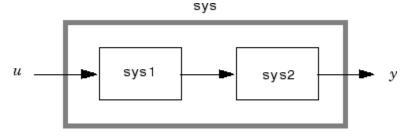
# **Syntax**

```
series
sys = series(sys1,sys2)
sys = series(sys1,sys2,outputs1,inputs2)
```

# **Description**

series connects two model objects in series. This function accepts any type of model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

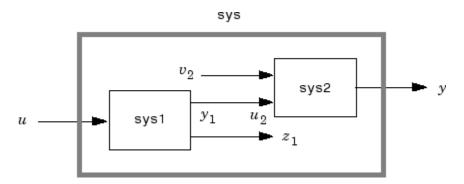
sys = series(sys1,sys2) forms the basic series connection shown below.



This command is equivalent to the direct multiplication

```
sys = sys2 * sys1
```

sys = series(sys1,sys2,outputs1,inputs2) forms the more general series
connection.



The index vectors outputs1 and inputs2 indicate which outputs  $y_1$  of sys1 and which inputs  $u_2$  of sys2 should be connected. The resulting model sys has u as input and y as output.

# **Examples**

Consider a state-space system sys1 with five inputs and four outputs and another system sys2 with two inputs and three outputs. Connect the two systems in series by connecting outputs 2 and 4 of sys1 with inputs 1 and 2 of sys2.

```
outputs1 = [2 4];
inputs2 = [1 2];
sys = series(sys1,sys2,outputs1,inputs2)
```

## **See Also**

append | feedback | parallel

Introduced before R2006a

## set

Set or modify model properties

# **Syntax**

```
set(sys,'Property',Value)
set(sys,'Property1',Value1,'Property2',Value2,...)
sysnew = set(____)
set(sys,'Property')
```

# **Description**

set is used to set or modify the properties of a dynamic system model using property name/property value pairs.

set(sys, 'Property', Value) assigns the value Value to the property of the model
sys. 'Property' can be the full property name (for example, 'UserData') or any
unambiguous case-insensitive abbreviation (for example, 'user'). The specified property
must be compatible with the model type. For example, if sys is a transfer function,
Variable is a valid property but StateName is not. For a complete list of available
system properties for any linear model type, see the reference page for that model type.
This syntax is equivalent to sys.Property = Value.

set(sys,'Property1',Value1,'Property2',Value2,...) sets multiple property values with a single statement. Each property name/property value pair updates one particular property.

sysnew = set(\_\_\_\_) returns the modified dynamic system model, and can be used with
any of the previous syntaxes.

set(sys, 'Property') displays help for the property specified by 'Property'.

# **Examples**

#### **Specify Model Properties**

Create a SISO state-space model with matrices *A*, *B*, *C*, and *D* equal to 1, 2, 3, and 4, respectively.

```
sys = ss(1,2,3,4);
```

Modify the properties of the model. Add an input delay of 0.1 second, label the input as torque, and set the D matrix to 0.

```
set(sys,'InputDelay',0.1,'InputName','torque','D',0);
```

View the model properties, and verify the changes.

get(sys)

```
A: 1
            B: 2
            C: 3
            D: 0
            E: []
       Scaled: 0
    StateName: {''}
   StateUnit: {''}
InternalDelay: [0x1 double]
   InputDelay: 0.1000
  OutputDelay: 0
           Ts: 0
    TimeUnit: 'seconds'
    InputName: {'torque'}
   InputUnit: {''}
   InputGroup: [1x1 struct]
  OutputName: {''}
   OutputUnit: {''}
  OutputGroup: [1x1 struct]
        Notes: [0x1 string]
    UserData: []
        Name: ''
 SamplingGrid: [1x1 struct]
```

# **Tips**

For discrete-time transfer functions, the convention used to represent the numerator and denominator depends on the choice of variable (see tf for details). Like tf, the syntax for set changes to remain consistent with the choice of variable. For example, if the Variable property is set to 'z' (the default),

produces the transfer function

$$h(z) = \frac{z+2}{z^2 + 3z + 4}$$

However, if you change the Variable to 'z^-1' by

the same command

now interprets the row vectors [1 2] and [1 3 4] as the polynomials  $1 + 2z^{-1}$  and  $1 + 3z^{-1} + 4z^{-2}$  and produces:

$$\overline{h}(z^{-1}) = \frac{1 + 2z^{-1}}{1 + 3z^{-1} + 4z^{-2}} = zh(z)$$

**Note** Because the resulting transfer functions are different, make sure to use the convention consistent with your choice of variable.

## See Also

frd | get | ss | tf | zpk

## **Topics**

"Store and Retrieve Model Data"

<sup>&</sup>quot;What Are Model Objects?"

### **Introduced before R2006a**

# setDelayModel

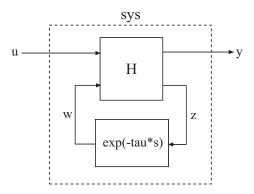
Construct state-space model with internal delays

# **Syntax**

```
sys = setDelayModel(H,tau)
sys = setDelayModel(A,B1,B2,C1,C2,D11,D12,D21,D22,tau)
```

# **Description**

sys = setDelayModel(H,tau) constructs the state-space model sys obtained by LFT
interconnection of the state-space model H with the vector of internal delays tau, as
shown:



sys = setDelayModel(A,B1,B2,C1,C2,D11,D12,D21,D22,tau) constructs the state-space model sys described by the following equations:

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + B_1 u(t) + B_2 w(t) \\ y(t) &= C_1 x(t) + D_{11} u(t) + D_{12} w(t) \\ z(t) &= C_2 x(t) + D_{21} u(t) + D_{22} w(t) \\ w(t) &= z(t - \tau) \,. \end{aligned}$$

tau ( $\tau$ ) is the vector of internal delays in sys.

# **Input Arguments**

#### Н

State-space (ss) model to interconnect with internal delays tau.

#### tau

Vector of internal delays of sys.

For continuous-time models, express tau in seconds.

For discrete-time models, express tau as integer values that represent multiples of the sample time.

#### A, B1, B2, C1, C2, D11, D12, D21, D22

Set of state-space matrices that, with the internal delay vector tau, explicitly describe the state-space model sys.

# **Output Arguments**

#### sys

State-space (ss) model with internal delays tau.

# **Tips**

- setDelayModel is an advanced operation and is not the natural way to construct
  models with internal delays. See "Time Delays in Linear Systems" for recommended
  ways of creating internal delays.
- The syntax sys = setDelayModel(A,B1,B2,C1,C2,D11,D12,D21,D22,tau) constructs a continuous-time model. You can construct the discrete-time model described by the state-space equations

$$x[k+1] = Ax[k] + B_1u[k] + B_2w[k]$$

$$y[k] = C_1x[k] + D_{11}u[k] + D_{12}w[k]$$

$$z[k] = C_2x[k] + D_{21}u[k] + D_{22}w[k]$$

$$w[k] = z[k-\tau].$$

To do so, first construct sys using sys = setDelayModel(A,B1,B2,C1,C2,D11,D12,D21,D22,tau). Then, use sys.Ts to set the sample time.

## See Also

getDelayModel | lft | ss

### **Topics**

"Internal Delays"
"Time Delays in Linear Systems"

Introduced in R2007a

# setoptions

Set plot options for response plot

# **Syntax**

```
setoptions(h, Plot0pts)
setoptions(h, 'Property1', 'value1', ...)
setoptions(h, Plot0pts, 'Property1', 'value1', ...)
```

# **Description**

setoptions(h, PlotOpts) sets preferences for response plot using the plot handle. h
is the plot handle, PlotOpts is a plot options handle containing information about plot
options.

There are two ways to create a plot options handle:

• Use getoptions, which accepts a plot handle and returns a plot options handle.

```
p = getoptions(h)
```

- Create a default plot options handle using one of the following commands:
  - bodeoptions Bode plots
  - hsvoptions Hankel singular values plots
  - nicholsoptions Nichols plots
  - nyquistoptions Nyquist plots
  - pzoptions Pole/zero plots
  - sigmaoptions Sigma plots
  - timeoptions Time plots (step, initial, impulse, etc.)

For example,

```
p = bodeoptions
```

returns a plot options handle for Bode plots.

setoptions(h, 'Property1', 'value1', ...) assigns values to property pairs instead of using Plot0pts. To find out what properties and values are available for a particular plot, type help <function>options. For example, for Bode plots type

help bodeoptions

For a list of the properties and values available for each plot type, see "Properties and Values Reference".

setoptions(h, PlotOpts, 'Property1', 'value1', ...) first assigns plot properties as defined in @PlotOptions, and then overrides any properties governed by the specified property/value pairs.

# **Examples**

### **Change Frequency Units in Response Plot**

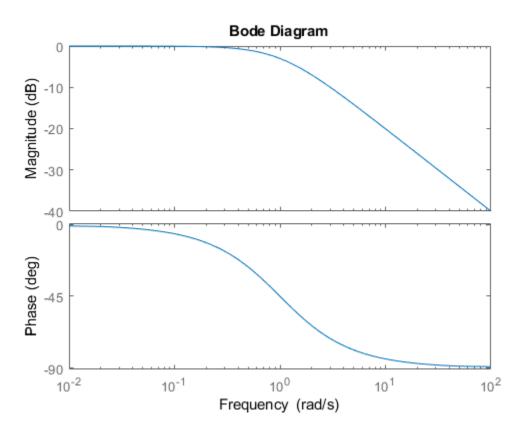
Create the following continuous-time transfer function:

$$H(s) = \frac{1}{s+1}$$

sys = tf(1,[1 1]);

Create a Bode plot with plot handle h.

h = bodeplot(sys);



Create a plot options handle p.

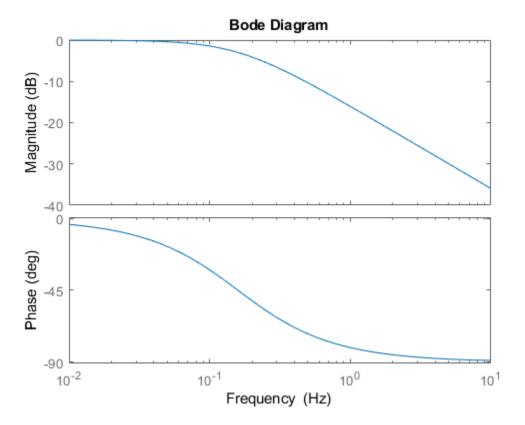
```
p = getoptions(h);
```

Change frequency units of the plot to Hz.

```
p.FreqUnits = 'Hz';
```

Apply the plot options to the Bode plot.

setoptions(h,p);



Alternatively, use setoptions(h, 'FrequencyUnits', 'Hz').

## See Also

getoptions

**Introduced before R2006a** 

## setBlockValue

Modify value of Control Design Block in Generalized Model

# **Syntax**

```
M = setBlockValue(M0,blockname,val)
M = setBlockValue(M0,blockvalues)
M = setBlockValue(M0,Mref)
```

# **Description**

M = setBlockValue(M0, blockname, val) modifies the current or nominal value of the Control Design Block blockname in the Generalized Model M0 to the value specified by val.

M = setBlockValue(M0,blockvalues) modifies the value of several Control Design Blocks at once. The structure blockvalues specifies the blocks and replacement values. Blocks of M0 not listed in blockvalues are unchanged.

M = setBlockValue(M0, Mref) takes replacement values from Control Design blocks in the Generalized Model Mref. This syntax modifies the Control Design Blocks in M0 to match the current values of all corresponding blocks in Mref.

Use this syntax to propagate block values, such as tuned parameter values, from one parametric model to other models that depend on the same parameters.

# **Input Arguments**

#### MΘ

Generalized Model containing the blocks whose current or nominal value is modified to val. For the syntax M = setBlockValue(M0,Mref) M0 can be a single Control Design Block whose value is modified to match the value of the corresponding block in Mref.

#### blockname

Name of the Control Design Block in the model M0 whose current or nominal value is modified.

To get a list of the Control Design Blocks in M0, enter M0.Blocks.

#### val

Replacement value for the current or nominal value of the Control Design Block, blockname. The value val can be any value that is compatible with blockname without changing the size, type, or sample time of blockname.

For example, you can set the value of a tunable PID block (tunablePID) to a pid controller model, or to a transfer function (tf) model that represents a PID controller.

#### blockvalues

Structure specifying Control Design Blocks of M0 to modify, and the corresponding replacement values. The fields of the structure are the names of the blocks to modify. The value of each field specifies the replacement current or nominal value for the corresponding block.

#### Mref

Generalized Model that shares some Control Design Blocks with MO. The values of these blocks in Mref are used to update their counterparts in MO.

# **Output Arguments**

#### М

Generalized Model obtained from M0 by updating the values of the specified blocks.

# **Examples**

### **Update Controller Model with Tuned Values**

Propagate the values of tuned parameters to other Control Design Blocks.

You can use tuning commands such as systune, looptune, or the Robust Control Toolbox™ command hinfstruct to tune blocks in a closed-loop model of a control system. If you do so, the tuned controller parameters are embedded in a generalized model. You can use setBlockValue to propagate those parameters to a controller model.

Create a tunable model of the closed-loop response of a control system, and tune the parameters using systume.

```
s = tf('s');
num = 33000*(s^2 - 200*s + 90000);
den = (s + 12.5)*(s^2 + 25*s + 63000);
G = num/den;

C0 = tunablePID('C0','pi');
a = realp('a',1);
F0 = tf(a,[1 a]);
T0 = feedback(G*C0,F0);
T0.InputName = 'r';
T0.OutputName = 'y';
```

T0 is a generalized model of the closed-loop control system and contains two tunable blocks:

- C0 Tunable PID controller
- a Real tunable parameter

Create a tuning requirement for the output y to track the input r, and tune the system to meet that requirement.

```
Req = TuningGoal.Tracking('r','y',0.05);
[T,fSoft,~] = systume(T0,Req);
Final: Soft = 1.43, Hard = -Inf, Iterations = 59
```

The generalized model T contains the tuned values of C0 and a.

Propagate the tuned values of the controller in  $\mathsf{T}$  to the controller model  $\mathsf{C0}$ .

```
C = setBlockValue(C0,T)
C =
   Parametric continuous-time PID controller "C0" with formula:
```

and tunable parameters Kp, Ki.

Type "pid(C)" to see the current value and "get(C)" to see all properties.

C is still a tunablePID controller. The current PID gains in C are set to the values of the controller in T.

Obtain a numeric LTI model of the tuned controller using getValue.

```
CVal = getValue(C,T);
```

This command returns a numerical state-space model of the tuned controller.

### See Also

genss | getBlockValue | getValue | hinfstruct | looptune | showBlockValue |
systume

Introduced in R2011b

## setData

Set values of tunable-surface coefficients

# **Syntax**

```
Knew = setData(K,Kco)
Knew = getData(K,J,KcoJ)
```

# **Description**

Knew = setData(K,Kco) sets the current values of the tunable coefficients of a tunable surface. K is a tunableSurface object that represents the parametric gain surface:

$$K(n(\sigma)) = \gamma [K_0 + K_1 F_1(n(\sigma)) + \dots + K_M F_M(n(\sigma))],$$

#### where:

- $\sigma$  is a vector of scheduling variables.
- $n(\sigma)$  is a normalization function (see the Normalization property of tunableSurface).
- $\gamma$  is a scaling factor (see the Normalization property of tunableSurface).
- $F_1,...,F_M$  are basis functions.
- $K_0,...,K_M$  are tunable coefficients.

**Kco** is an array of new values for the coefficients  $[K_0,...,K_M]$ .

Knew = getData(K,J,KcoJ) sets the current value of the coefficient of the Jth basis function  $F_J$  to KcoJ. Use J = 0 to set the constant coefficient  $K_0$ .

# **Input Arguments**

#### **K** — Gain surface

tunableSurface object

Gain surface, specified as a tunableSurface object,

#### Kco — New coefficient values

array

New coefficient values of the tunable surface, specified as an array.

If the tunable surface K is a scalar-valued gain, then the length of K is (M+1), where M is the number of basis functions in the parameterization. For example, if K represents the tunable gain surface:

$$K(\alpha, V) = K_0 + K_1 \alpha + K_2 V + K_3 \alpha V,$$

then Kco is the 1-by-4 vector  $[K_0,K_1,K_2,K_3]$ .

For array-valued gains, each coefficient expands to the I/O dimensions of the gain. These expanded coefficients are concatenated horizontally in Kco. (See tunableSurface.) For example, for a two-input, two-output gain surface, Kco has dimensions [2,2(M+1)]. See evalSurf for an example that uses setData on an array-valued gain.

#### J — Index of basis function

nonnegative integer

Index of basis function, specified as a nonnegative integer. To set the constant coefficient  $K_0$ , use J = 0.

### **KcoJ** — Coefficient of Jth basis function

scalar | array

Coefficient of the *J*th basis function in the tunable surface parameterization, specified as a scalar or an array.

If the tunable surface K is a scalar-valued gain, then KcoJ is a scalar. If K is an array-valued gain, then KcoJ is an array that matches the I/O dimensions of the gain.

# **Output Arguments**

#### **Knew** — Gain surface

tunableSurface object

Gain surface with new coefficient values, returned as a tunableSurface object.

# **See Also**

evalSurf|getData|tunableSurface|viewSurf

Introduced in R2015b

### setValue

Modify current value of Control Design Block

## **Syntax**

blk = setValue(blk0,val)

# **Description**

blk = setValue(blk0,val) modifies the parameter values in the tunable Control Design Block, blk0, to best match the values specified by val. An exact match can only occur when val is compatible with the structure of blk0.

# **Input Arguments**

#### blk0

Control Design Block whose value is modified.

#### val

Specifies the replacement parameters values for blk0. The value val can be any value that is compatible with blk0 without changing the size, type, or sample time of blk0. For example, if blk0 is a tunablePID block, valid types for val include tunablePID, a numeric pid controller model, or a numeric tf model that represents a PID controller. setValue uses the parameter values of val to set the current value of blockname.

# **Output Arguments**

#### blk

Control Design Block of the same type as blk0, whose parameters are updated to best match the parameters of val.

# **See Also**

getBlockValue | getValue | setBlockValue

Introduced in R2011b

# sgrid

Generate s-plane grid of constant damping factors and natural frequencies

# **Syntax**

```
sgrid
sgrid(zeta,wn)
sgrid(___,'new')
sgrid(AX, )
```

# **Description**

sgrid generates a grid of constant damping factors from 0 to 1 in steps of 0.1 and natural frequencies from 0 to 10 rad/sec in steps of one rad/sec for pole-zero and root locus plots. sgrid then plots the grid over the current axis. sgrid creates the grid over the plot if the current axis contains a continuous s-plane root locus diagram or pole-zero map.

sgrid(zeta,wn) plots a grid of constant damping factor and natural frequency lines for
the damping factors and natural frequencies in the vectors zeta and wn, respectively.
sgrid(zeta,wn) creates the grid over the plot if the current axis contains a continuous
s-plane root locus diagram or pole-zero map.

Alternatively, you can select **Grid** from the context menu to generate the same s-plane grid.

```
sgrid(____, 'new') clears the current axes first and sets hold on.
```

sgrid(AX, \_\_\_\_) plots the s-plane grid on the Axes or UIAxes object in the current figure with the handle AX. Use this syntax when creating apps with sgrid in the App Designer.

# **Examples**

### **Generate S-Plane Grid on Root Locus Plot**

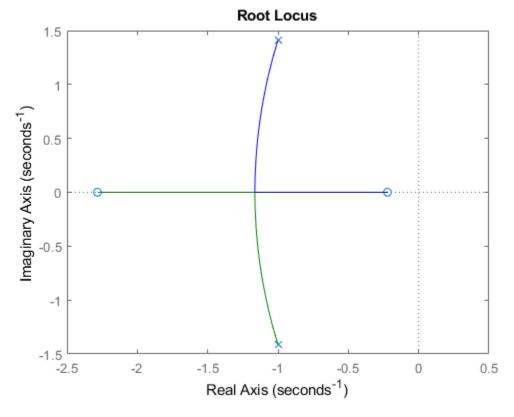
Create the following continuous-time transfer function:

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

H = tf([2 5 1],[1 2 3]);

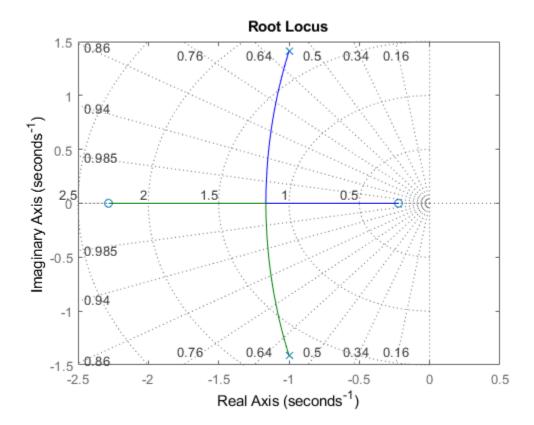
Plot the root locus of the transfer function.

rlocus(H)



Plot s-plane grid lines on the root locus.

### sgrid



# **Input Arguments**

### zeta — Damping ratio

vector

Damping ratio, specified as a vector in the same order as wn.

### wn — Normalized natural frequency

vector

Normalized natural frequency, specified as a vector.

### **AX** — Object handle

Axes object | UIAxes object

Object handle, specified as an Axes or UIAxes object. Use AX to create apps with sgrid in the App Designer.

## See Also

pzmap | rlocus | zgrid

Introduced before R2006a

## showBlockValue

Display current value of Control Design Blocks in Generalized Model

# **Syntax**

showBlockValue(M)

# **Description**

showBlockValue(M) displays the current values of all Control Design Blocks in the Generalized Model, M. (For uncertain blocks, the "current value" is the nominal value of the block.)

# **Input Arguments**

М

Generalized Model.

# **Examples**

Create a tunable genss model, and display the current value of its tunable elements.

```
G = zpk([],[-1,-1],1);
C = tunablePID('C','PID');
a = realp('a',10);
F = tf(a,[1 a]);
T = feedback(G*C,1)*F;
showBlockValue(T)
C =
Continuous-time I-only controller:
```

```
1
Ki * ---
s
With Ki = 0.001
-----
a = 10
```

# **Tips**

- Displaying the current values of a model is useful, for example, after you have tuned the free parameters of the model using a tuning command such as systume.
- showBlockValue displays the current values of all Control Design Blocks in a model, including tunable, uncertain, and switch blocks. To display the current values of only the tunable blocks, use showTunable.

### See Also

genss | getBlockValue | setBlockValue | showTunable

Introduced in R2011b

## showTunable

Display current value of tunable Control Design Blocks in Generalized Model

# **Syntax**

showTunable(M)

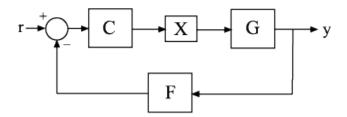
# **Description**

showTunable(M) displays the current values of all tunable Control Design Blocks in a generalized LTI model. Tunable control design blocks are parametric blocks such as realp, tunableTF, and tunablePID.

# **Examples**

### **Display Block Values of Tuned Control System Model**

Tune the following control system using systune, and display the values of the tunable blocks.



The control structure includes a PI controller  ${\sf C}$  and a tunable low-pass filter in the feedback path. The plant  ${\sf G}$  is a third-order system.

Create models of the system components and connect them together to create a tunable closed-loop model of the control system.

```
s = tf('s');
num = 33000*(s^2 - 200*s + 90000);
den = (s + 12.5)*(s^2 + 25*s + 63000);
G = num/den;

C0 = tunablePID('C','pi');
a = realp('a',1);
F0 = tf(a,[1 a]);
X = AnalysisPoint('X');

T0 = feedback(G*X*C0,F0);
T0.InputName = 'r';
T0.OutputName = 'y';
```

T0 is a genss model that has two tunable blocks, the PI controller, C, and the parameter, a. T0 also contains the switch block X.

Create a tuning requirement that forces the output y to track the input r, and tune the system to meet that requirement.

```
Req = TuningGoal.Tracking('r','y',0.05);
[T,fSoft,~] = systune(T0,Req);
Final: Soft = 1.43, Hard = -Inf, Iterations = 59
```

systune finds values for the tunable parameters that optimally meet the tracking requirement. The output T is a genss model with the same Control Design Blocks as T0. The current values of those blocks are the tuned values.

Examine the tuned values of the tunable blocks of the control system.

```
showTunable(T)
```

```
Continuous-time PI controller in parallel form.

a = 67.8
```

showTunable displays the values of the tunable blocks only. If you use showBlockValue instead, the display also includes the switch block X.

# **Input Arguments**

### M - Input model

generalized LTI model

Input model of which to display tunable block values, specified as a generalized LTI model such as a genss model.

# **Tips**

- Displaying the current values of tunable blocks is useful, for example, after you have tuned the free parameters of the model using a tuning command such as systume.
- showTunable displays the current values of the tunable blocks only. To display the current values of all Control Design Blocks in a model, including tunable, uncertain, and switch blocks, use showBlockValue.

### See Also

genss | getBlockValue | setBlockValue | showBlockValue | systume

### **Topics**

"Generalized Models"

"Control Design Blocks"

#### Introduced in R2012b

# sigma

Singular values plot of dynamic system

# **Syntax**

```
sigma(sys)
sigma(sys,w)
sigma(sys,[],type)
sigma(sys,w,type)
sigma(sys1,sys2,...,sysN,w,type)
sigma(sys1,'PlotStyle1',...,sysN,'PlotStyleN',w,type)
sv = sigma(sys,w)
[sv,w] = sigma(sys)
```

# **Description**

sigma calculates the singular values of the frequency response of a dynamic system sys. For an FRD model, sigma computes the singular values of sys.Response at the frequencies, sys.frequency. For continuous-time TF, SS, or ZPK models with transfer function H(s), sigma computes the singular values of  $H(j\omega)$  as a function of the frequency  $\omega$ . For discrete-time TF, SS, or ZPK models with transfer function H(z) and sample time  $T_s$ , sigma computes the singular values of

$$H(e^{j\omega T_S})$$

for frequencies  $\omega$  between 0 and the Nyquist frequency  $\omega_N = \pi/T_s$ .

The singular values of the frequency response extend the Bode magnitude response for MIMO systems and are useful in robustness analysis. The singular value response of a SISO system is identical to its Bode magnitude response. When invoked without output arguments, sigma produces a singular value plot on the screen.

sigma(sys) plots the singular values of the frequency response of a model sys. This model can be continuous or discrete, and SISO or MIMO. The frequency points are chosen automatically based on the system poles and zeros, or from sys.frequency if sys is an FRD.

sigma(sys,w) explicitly specifies the frequency range or frequency points to be used for
the plot. To focus on a particular frequency interval [wmin,wmax], set w =
{wmin,wmax}. To use particular frequency points, set w to the corresponding vector of
frequencies. Use logspace to generate logarithmically spaced frequency vectors.
Frequencies must be in rad/TimeUnit, where TimeUnit is the time units of the input
dynamic system, specified in the TimeUnit property of sys.

sigma(sys,[],type) or sigma(sys,w,type) plots the following modified singular
value responses:

type = 1	Singular values of the frequency response $H^{-1}$ , where $H$ is the frequency response of sys.
type = 2	Singular values of the frequency response $I + H$ .
type = 3	Singular values of the frequency response $I + H^{-1}$ .

These options are available only for square systems, that is, with the same number of inputs and outputs.

sigma(sys1,sys2,...,sysN,w,type) plots the singular value plots of several LTI models on a single figure. The arguments w and type are optional. The models sys1,sys2,...,sysN need not have the same number of inputs and outputs. Each model can be either continuous- or discrete-time.

sigma(sys1, 'PlotStyle1',...,sysN, 'PlotStyleN',w,type) specifies a
distinctive color, linestyle, and/or marker for each system plot. See bode for an example.

sv = sigma(sys,w) and [sv,w] = sigma(sys) return the singular values sv of the frequency response at the frequencies w. For a system with Nu input and Ny outputs, the array sv has min(Nu,Ny) rows and as many columns as frequency points (length of w). The singular values at the frequency w(k) are given by sv(:,k).

# **Examples**

### **Compute and Plot Singular Values**

Consider the following two-input, two-output dynamic system.

$$H(s) = \begin{bmatrix} 0 & \frac{3s}{s^2 + s + 10} \\ \frac{s+1}{s+5} & \frac{2}{s+6} \end{bmatrix}.$$

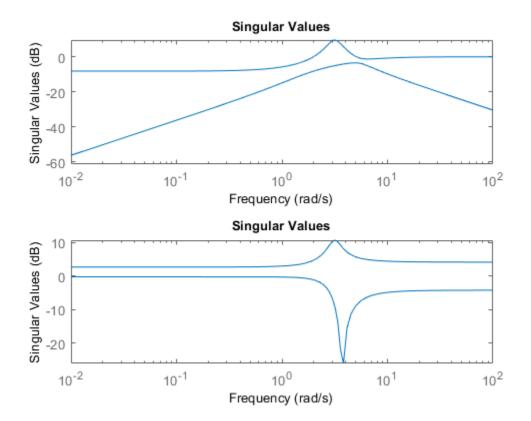
Compute the singular value responses of H(s) and I + H(s).

```
H = [0, tf([3 0],[1 1 10]); tf([1 1],[1 5]), tf(2,[1 6])]; [svH,wH] = sigma(H); [scIH,wIH] = sigma(H,[],2);
```

In the last command, the input 2 selects the second response type, I + H(s). The vectors svH and svIH contain the singular value response data, at the frequencies in wH and wIH.

Plot the singular value responses of both systems.

```
subplot(211)
sigma(H)
subplot(212)
sigma(H,[],2)
```



# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

# **Algorithms**

 ${\tt sigma}$  uses the MATLAB function  ${\tt svd}$  to compute the singular values of a complex matrix.

For TF, ZPK, and SS models, sigma computes the frequency response using the freqresp algorithms. As a result, small discrepancies may exist between the sigma responses for equivalent TF, ZPK, and SS representations of a given model.

## See Also

Linear System Analyzer | bode | evalfr | freqresp | nichols | nyquist

# sigmaoptions

Create list of singular-value plot options

# **Syntax**

```
P = sigmaoptions
P = sigmaoptions('cstprefs')
```

# **Description**

P = sigmaoptions returns a list of available options for singular value plots with default values set. You can use these options to customize the singular value plot appearance from the command line.

P = sigmaoptions('cstprefs') initializes the plot options with the options you selected in the Control System Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the sigma plot options.

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid	Show or hide the grid Specified as one of the following: 'off'   'on' Default: 'off'
GridColor	Color of the grid lines Specified as one of the following: Vector of RGB values in the range [0,1]  color  'none'. Default: [0.15,0.15,0.15]
XlimMode, YlimMode	Limit modes

Option	Description
Xlim, Ylim	Axes limits
IOGrouping	Grouping of input-output pairs Specified as one of the following: 'none'  'inputs' 'outputs' 'all' Default: 'none'
InputLabels, OutputLabels	Input and output label styles
InputVisible, OutputVisible	Visibility of input and output channels

Option	Description
FreqUnits	Frequency units, specified as one of the following:
	• 'Hz'
	• 'rad/second'
	• 'rpm'
	• 'kHz'
	• 'MHz'
	• 'GHz'
	• 'rad/nanosecond'
	• 'rad/microsecond'
	• 'rad/millisecond'
	• 'rad/minute'
	• 'rad/hour'
	• 'rad/day'
	• 'rad/week'
	• 'rad/month'
	• 'rad/year'
	• 'cycles/nanosecond'
	• 'cycles/microsecond'
	• 'cycles/millisecond'
	• 'cycles/hour'
	• 'cycles/day'
	• 'cycles/week'
	• 'cycles/month'
	• 'cycles/year'
	Default: 'rad/s'
	You can also specify 'auto' which uses frequency units rad/TimeUnit relative to system time units specified in the

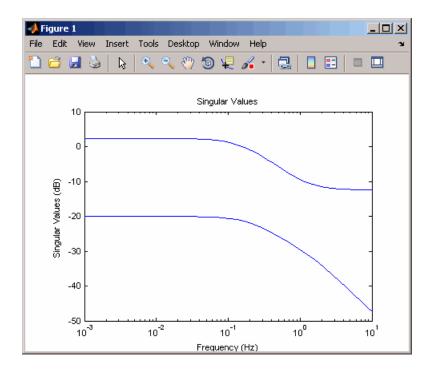
Option	Description
	TimeUnit property. For multiple systems with different time units, the units of the first system are used.
FreqScale	Frequency scale Specified as one of the following: 'linear'   'log' Default: 'log'
MagUnits	Magnitude units Specified as one of the following: 'dB'   'abs' Default: 'dB'
MagScale	Magnitude scale Specified as one of the following: 'linear' 'log' Default: 'linear'

# **Examples**

In this example, set the frequency units to Hz before creating a plot.

```
P= sigmaoptions; % Set the frequency units to Hz in options P.FreqUnits = 'Hz'; % Create plot with the options specified by P h = sigmaplot(rss(2,2,3),P);
```

The following singular value plot is created with the frequency units in Hz.



# **See Also**

getoptions | setoptions | sigmaplot

Introduced in R2008a

# sigmaplot

Plot singular values of frequency response and return plot handle

# **Syntax**

```
h = sigmaplot(sys)
sigmaplot(sys, {wmin, wmax})
sigmaplot(sys, w)
sigmaplot(sys, w, TYPE)
sigmaplot(AX,...)
sigmaplot(..., plotoptions)
```

# **Description**

h = sigmaplot(sys) produces a singular value (SV) plot of the frequency response of the dynamic system sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

help sigmaoptions

for a list of available plot options.

The frequency range and number of points are chosen automatically. See bode for details on the notion of frequency in discrete time.

sigmaplot(sys,{wmin,wmax}) draws the SV plot for frequencies ranging between
wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input
dynamic system, specified in the TimeUnit property of sys).

sigmaplot(sys,w) uses the user-supplied vector w of frequencies, in rad/TimeUnit, at which the frequency response is to be evaluated. See logspace to generate logarithmically spaced frequency vectors.

sigmaplot(sys,w,TYPE) or sigmaplot(sys,[],TYPE) draws the following modified
SV plots depending on the value of TYPE:

```
TYPE = 1 \qquad --> \qquad SV \text{ of inv(SYS)}
TYPE = 2 \qquad --> \qquad SV \text{ of } I + SYS
TYPE = 3 \qquad --> \qquad SV \text{ of } I + inv(SYS)
```

sys should be a square system when using this syntax.

sigmaplot(AX,...) plots into the axes with handle AX.

sigmaplot(..., plotoptions) plots the singular values with the options specified in plotoptions. Type

help sigmaoptions

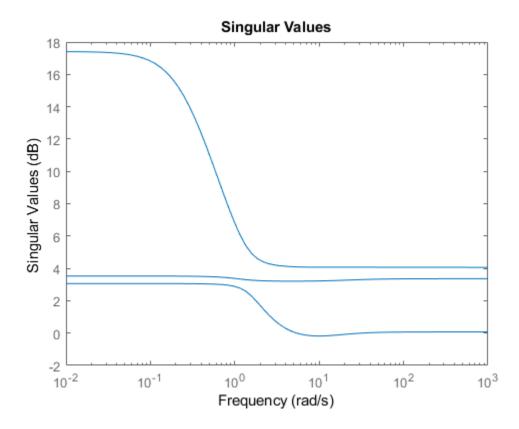
for more details.

# **Examples**

### **Singular Value Response Plot with Custom Plot Options**

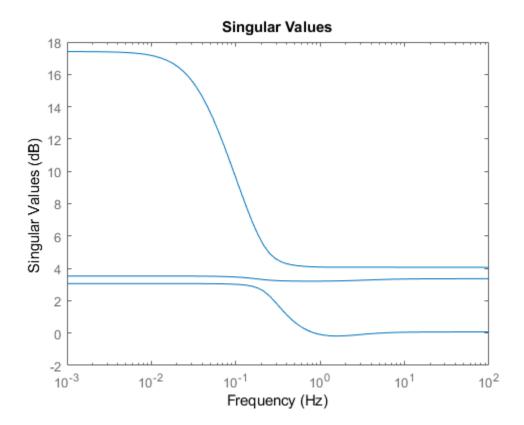
Plot the singular value responses of a dynamic system.

```
sys = rss(3,3,5);
h = sigmaplot(sys);
```



Set properties of the plot handle h to customize the plot. For example, change the plot units to Hz.

setoptions(h,'FreqUnits','Hz');



## **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

## **See Also**

getoptions | setoptions | sigma | sigmaoptions

### **Introduced before R2006a**

## sisoinit

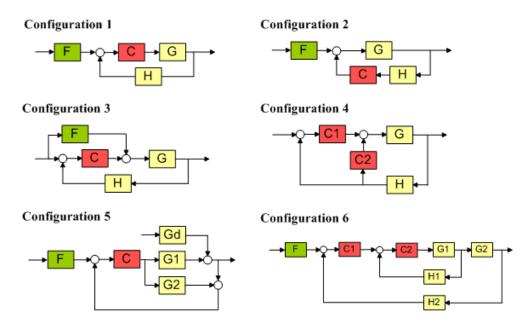
Configure Control System Designer at startup

## **Syntax**

init\_config = sisoinit(config)

## **Description**

init\_config = sisoinit(config) returns a template init\_config for initializing
the Control System Designer with one of the following control system configurations:



For more information about the control system configurations supported by the **Control System Designer**, see "Feedback Control Architectures".

For each configuration, you can specify the plant model G and the sensor dynamics H, initialize the compensator C and prefilter F, and configure the open-loop and closed-loop views by specifying the corresponding fields of the structure <code>init\_config</code>. Then you can start the **Control System Designer** in the specified configuration using <code>controlSystemDesigner(init\_config)</code>.

Output argument init\_config is an object with properties. The following tables list the block and loop properties.

### **Block Properties**

Block	Properties	Values	
F	Name	Character vector	
	Description	Character vector	
	Value	LTI object	
G	Name	Character vector	
	Value	LTI object	
		Row or column array of LTI objects. If the sensor     H is also an array of LTI objects, the lengths of G     and H must match.	
Н	Name	Character vector	
	Value	LTI object	
		Row or column array of LTI objects. If the plant G is also an array of LTI objects, the lengths of H and G must match.	
С	Name	Character vector	
	Description	Character vector	
	Value	LTI object	

### **Loop Properties**

Loops	Properties	Values
0L1	Name	Character vector
	Description	Character vector
	View	'rlocus' 'bode'
CL1	Name	Character vector
	Description	Character vector
	View	'bode'

## **Examples**

### **Initialize Control System Designer**

Create an initialization template for configuration 2, with the compensator in the feedback path.

```
T = sisoinit(2);
```

Specify the fixed plant model.

```
T.G.Value = tf(1, [1 1]);
```

Specify an initial compensator value.

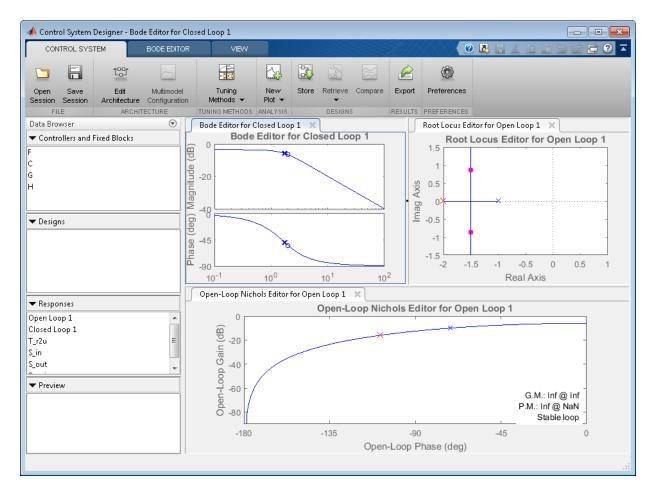
```
T.C.Value = tf(1,[1 2]);
```

Open a root locus Editor and Nichols editor for tuning the open-loop response.

```
T.OL1.View = {'rlocus','nichols'};
```

Open Control System Designer using the specified configuration settings.

```
controlSystemDesigner(T)
```



By default, the template for configuration 2 also opens a Bode editor for tuning the closed-loop response.

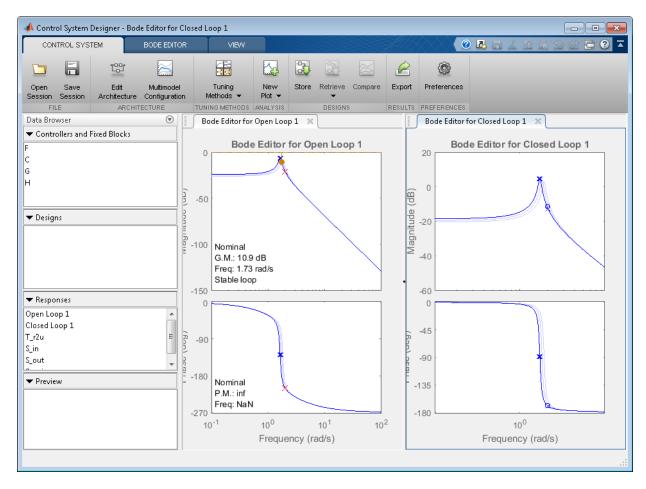
### **Initialize Control System Designer Using Array of Plant Models**

Specify a configuration template.

initconfig = sisoinit(2);

Specify model parameters.

```
m = 3;
b = 0.5;
k = 8:1:10;
T = 0.1:.05:.2;
Create an array of LTI objects to model variations in plant G.
for ct = 1:length(k);
    G(:,:,ct) = tf(1,[m,b,k(ct)]);
end
Assign G to the initial configuration.
initconfig.G.Value = G;
Specify initial compensator value.
initconfig.C.Value = tf(1,[1 2]);
Use a graphical Bode editor to tune the open-loop response.
initconfig.OL1.View = {'bode'};
Open Control System Designer using the specified configuration settings.
controlSystemDesigner(initconfig)
```



By default, the template for configuration 2 also opens a Bode editor for tuning the closed-loop response.

## **See Also**

**Control System Designer** 

## **Topics**

"Feedback Control Architectures"

### Introduced in R2006a

<sup>&</sup>quot;Programmatically Initializing the Control System Designer" "Multimodel Control Design"

## size

Query output/input/array dimensions of input-output model and number of frequencies of FRD model

## **Syntax**

```
size(sys)
d = size(sys)
Ny = size(sys,1)
Nu = size(sys,2)
Sk = size(sys,2+k)
Nf = size(sys,'frequency')
```

## **Description**

When invoked without output arguments, size(sys) returns a description of type and the input-output dimensions of sys. If sys is a model array, the array size is also described. For identified models, the number of free parameters is also displayed. The lengths of the array dimensions are also included in the response to size when sys is a model array.

```
d = size(sys) returns:
```

- The row vector d = [Ny Nu] for a single dynamic model sys with Ny outputs and Nu inputs
- The row vector d = [Ny Nu S1 S2 ... Sp] for an S1-by-S2-by-...-by-Sp array of dynamic models with Ny outputs and Nu inputs

```
Ny = size(sys, 1) returns the number of outputs of sys.
```

```
Nu = size(sys, 2) returns the number of inputs of sys.
```

Sk = size(sys, 2+k) returns the length of the k-th array dimension when sys is a model array.

Nf = size(sys, 'frequency') returns the number of frequencies when sys is a frequency response data model. This is the same as the length of sys.frequency.

## **Examples**

### **Query Dimensions of Model Array**

Create a 3-by-1 model array of random state-space models with 3 outputs, 2 inputs, and 5 states.

```
sys = rss(5,3,2,3);
Verify the size of the model array.
size(sys)
3x1 array of state-space models.
Each model has 3 outputs, 2 inputs, and 5 states.
```

### **Query Dimensions of Identified Model**

Create a 2-input 2-output continuous-time process model with identifiable parameters.

```
type = {'pld','p2';'p3uz','p0'};
sys = idproc(type);
```

Each element of the type cell array describes the model structure for the corresponding input-output pair.

Query the input-output dimensions and number of free parameters in the model.

```
size(sys)
```

```
Process model with 2 outputs, 2 inputs and 12 free parameters.
```

### See Also

```
isempty | issiso | ndims
```

### **Introduced before R2006a**

## **sminreal**

Structural pole/zero cancellations

## **Syntax**

msys = sminreal(sys)

## **Description**

msys = sminreal(sys) eliminates the states of the state-space model sys that don't affect the input/output response. All of the states of the resulting state-space model msys are also states of sys and the input/output response of msys is equivalent to that of sys.

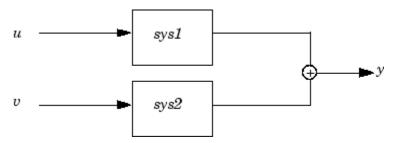
sminreal eliminates only structurally non minimal states, i.e., states that can be discarded by looking only at hard zero entries in the A, B, and C matrices. Such structurally nonminimal states arise, for example, when linearizing a Simulink model that includes some unconnected state-space or transfer function blocks.

## **Examples**

Suppose you concatenate two SS models, sys1 and sys2.

$$sys = [sys1, sys2];$$

This operation is depicted in the diagram below.



If you extract the subsystem sys1 from sys, with

```
sys(1,1)
```

all of the states of sys, including those of sys2 are retained. To eliminate the unobservable states from sys2, while retaining the states of sys1, type

```
sminreal(sys(1,1))
```

## **Tips**

The model resulting from sminreal(sys) is not necessarily minimal, and may have a higher order than one resulting from minreal(sys). However, sminreal(sys) retains the state structure of sys, while, in general, minreal(sys) does not.

## **Alternative Functionality**

### App

Model Reducer

### **Live Editor Task**

Reduce Model Order

## See Also

Model Reducer | Reduce Model Order | minreal

## **Topics**

 ${\it ``Pole-Zero Simplification''}$ 

#### Introduced before R2006a

<sup>&</sup>quot;Model Reduction Basics"

## spectralfact

Spectral factorization of linear systems

## **Syntax**

```
[G,S] = spectralfact(H)
[G,S] = spectralfact(F,R)
G = spectralfact(F,[])
```

## **Description**

[G,S] = spectralfact(H) computes the spectral factorization:

$$H = G'*S*G$$

of an LTI model satisfying H = H'. In this factorization, S is a symmetric matrix and G is a square, stable, and minimum-phase system with unit (identity) feedthrough. G' is the conjugate of G, which has transfer function  $G(-s)^T$  in continuous time, and  $G(1/z)^T$  in discrete time.

[G,S] = spectralfact(F,R) computes the spectral factorization:

$$F'*R*F$$
 =  $G'*S*G$ 

without explicitly forming H = F'\*R\*F. As in the previous syntax, S is a symmetric matrix and G is a square, stable, and minimum-phase system with unit feedthrough.

G = spectralfact(F,[]) computes a stable, minimum-phase system G such that:

$$G'*G = F'*F.$$

## **Examples**

### **Spectral Factorization of System**

Consider the following system.

```
G0 = ss(zpk([-1 -5 1+2i 1-2i],[-100 1+2i 1-2i -10],1e3));

H = G0'*G0;
```

G0 has a mix of stable and unstable dynamics. H is a self-conjugate system whose dynamics consist of the poles and zeros of G0 and their reflections across the imaginary axis. Use spectral factorization to separate the stable poles and zeros into G and the unstable poles and zeros into  $G^{\,\prime}$ .

```
[G,S] = spectralfact(H);
```

Confirm that G is stable and minimum phase, by checking that all its poles and zeros fall in the left half-plane (Re(s) < 0).

```
p = pole(G)
p = 4×1 complex
10² x

   -0.0100 + 0.0200i
   -0.0100 - 0.0200i
   -0.1000 + 0.0000i
   -1.0000 + 0.0000i

z = zero(G)
z = 4×1 complex
   -1.0000 + 2.0000i
   -1.0000 + 0.0000i
   -1.0000 + 0.0000i
   -5.0000 + 0.0000i
```

G also has unit feedthrough.

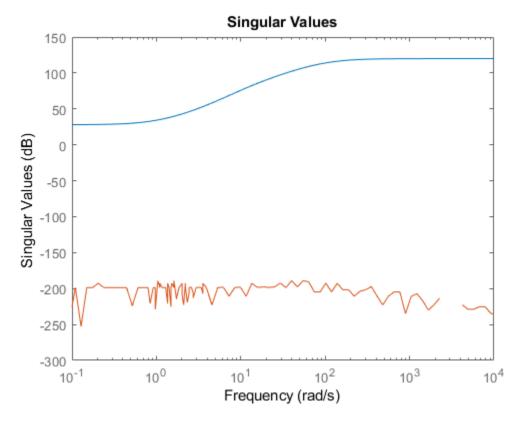
```
G.D ans = 1
```

Because H is SISO, S is a scalar. If H were MIMO, the dimensions of S would match the I/O dimensions of H.

S = 1000000

Confirm that G and S satisfy H = G'\*S\*G by comparing the original system to the difference between the original and factored systems. sigmaplot throws a warning because the difference is very small.

Hf = G'\*S\*G;
sigmaplot(H,H-Hf)



Warning: The frequency response has poor relative accuracy. This may be because the re-

### **Spectral Factorization from Factored Form**

Suppose that you have the following 2-output, 2-input state-space model, F.

```
\begin{array}{lll} A = \begin{bmatrix} -1.1 & 0.37; \\ 0.37 & -0.95 \end{bmatrix}; \\ B = \begin{bmatrix} 0.72 & 0.71; \\ 0 & -0.20 \end{bmatrix}; \\ C = \begin{bmatrix} 0.12 & 1.40 \\ 1.49 & 1.41 \end{bmatrix}; \\ D = \begin{bmatrix} 0.67 & 0.7172; \\ -1.2 & 0 \end{bmatrix}; \\ F = ss(A,B,C,D); \end{array}
```

Suppose further that you have a symmetric 2-by-2 matrix, R.

```
R = \begin{bmatrix} 0.65 & 0.61 \\ 0.61 & -3.42 \end{bmatrix};
```

Compute the spectral factorization of the system given by H = F'\*R\*F, without explicitly computing H.

```
[G,S] = spectralfact(F,R);
```

G is a minimum-phase system with identity feedthrough.

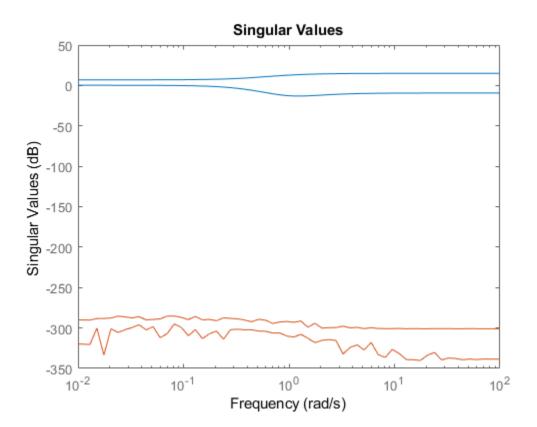
#### G.D

```
ans = 2 \times 2
\begin{array}{ccc}
1 & 0 \\
0 & 1
\end{array}
```

Because F is has two inputs and two outputs, both R and S are 2-by-2 matrices.

Confirm that G'\*S\*G = F'\*R\*F by comparing the original factorization to the difference between the two factorizations. The singular values of the difference are far below those of the original system.

```
Ff = F'*R*F;
Gf = G'*S*G;
sigmaplot(Ff,Ff-Gf)
```



### **Implicit Factorization**

Consider the following discrete-time system.

$$F = zpk(-1.76,[-1+i -1-i],-4,0.002);$$

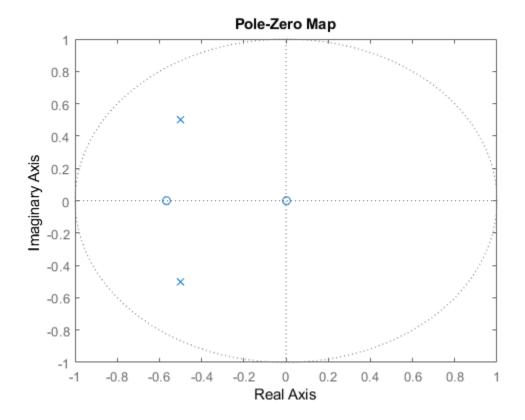
F has poles and zeros outside the unit circle. Use spectral fact to compute a system G with stable poles and zeros, such that G'\*G = F'\*F.

```
G = -3.52 z (z+0.5682)-----(z^2 + z + 0.5)
```

Sample time: 0.002 seconds Discrete-time zero/pole/gain model.

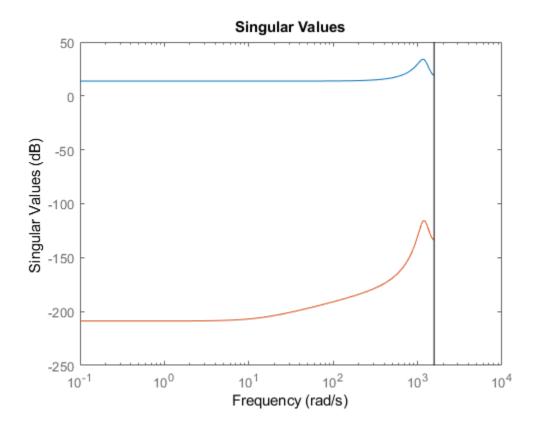
Unlike F, G has no poles or zeroes outside the unit circle. G does have an additional zero at z = 0, which is a reflection of the unstable zero at z = Inf in F.

pzplot(G)



Confirm that G'\*G = F'\*F by comparing the original factorization to the difference between the two factorizations. The singular values of the difference are far below those of the original factorization.

```
Ff = F'*F;
Gf = G'*G;
sigmaplot(Ff,Ff-Gf)
```



## **Input Arguments**

**H — Self-conjugate LTI model** tf | zpk | ss

Self-conjugate LTI model, specified as a tf, ss, or zpk model. Self-conjugate means that is equal to its conjugate, H = H'. The conjugate H' is the transfer function  $H(-s)^T$  in continuous time and  $H(1/z)^T$  in discrete time.

H can be SISO or MIMO, provided it has as many outputs as inputs. H can be continuous or discrete with the following restrictions:

- In continuous time, H must be biproper with no poles or zeros at infinity or on the imaginary axis.
- In discrete time, H must have no poles or zeros on the unit circle.

#### F — F factor

tf | zpk | ss

F factor of the factored form H = F'\*R\*F, specified as a tf, ss, or zpk model. F cannot have more inputs than outputs.

#### R - R factor

square matrix

R factor of the factored form H = F'\*R\*F, specified as a symmetric square matrix with as many rows as there are outputs in F.

## **Output Arguments**

#### G — LTI factor

tf | zpk | ss

LTI factor, returned as a tf, ss, or zpk model. G is a stable, minimum-phase system that satisfies:

- H = G'\*S\*G, if you use the syntax [G,S] = spectralfact(H).
- G'\*S\*G = F'\*R\*F, if you use the syntax [G,S] = spectralfact(F,R).
- G'\*G = F'\*F, if you use the syntax G = spectralfact(F,[]).

#### S — Numeric factor

matrix

Numeric factor, returned as a symmetric matrix that satisfies:

- H = G'\*S\*G, if you use the syntax [G,S] = spectralfact(H). The dimensions of S match the I/O dimensions of H and G.
- G'\*S\*G = F'\*R\*F, if you use the syntax [G,S] = spectralfact(F,R). The size of S along each dimension matches the number of outputs of F.

## **Tips**

• spectralfact assumes that H is self-conjugate. In some cases when H is not self-conjugate, spectralfact returns G and S that do not satisfy H = G'\*S\*G.

Therefore, verify that your input model is in fact self-conjugate before using spectralfact. One way to verify H is to compare H to H - H' on a singular value plot.

```
sigmaplot(H,H-H')
```

If H is self-conjugate, the H - H' line on the plot lies far below the H line.

### See Also

modsep | stabsep

## **Topics**

"Arithmetic Operations"

Introduced in R2016a

### SS

Create state-space model, convert to state-space model

## **Syntax**

```
sys = ss(A,B,C,D)
sys = ss(A,B,C,D,Ts)
sys = ss(D)
sys = ss(A,B,C,D,ltisys)
sys_ss = ss(sys)
sys_ss = ss(sys,'minimal')
sys_ss = ss(sys,'explicit')
sys_ss = ss(sys,'measured')
sys_ss = ss(sys,'noise')
sys_ss = ss(sys,'augmented')
```

## **Description**

Use ss to create state-space models (ss model objects) with real- or complex-valued matrices or to convert dynamic system models to state-space model form. You can also use ss to create Generalized state-space (genss) models.

### **Creation of State-Space Models**

sys = ss(A,B,C,D) creates a state-space model object representing the continuous-time state-space model

```
\dot{x} = Ax + Buy = Cx + Du
```

For a model with Nx states, Ny outputs, and Nu inputs:

- A is an Nx-by-Nx real- or complex-valued matrix.
- $\bullet\ \ B$  is an Nx-by-Nu real- or complex-valued matrix.

- C is an Ny-by-Nx real- or complex-valued matrix.
- D is an Ny-by-Nu real- or complex-valued matrix.

To set D = 0, set D to the scalar 0 (zero), regardless of the dimension.

sys = ss(A,B,C,D,Ts) creates the discrete-time model x[n+1] = Ax[n] + Bu[n]

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time Ts (in seconds). Set Ts = -1 or Ts = [] to leave the sample time unspecified.

sys = ss(D) specifies a static gain matrix D and is equivalent to

$$sys = ss([],[],[],D)$$

sys = ss(A,B,C,D,ltisys) creates a state-space model with properties inherited from the model ltisys (including the sample time).

Any of the previous syntaxes can be followed by property name/property value pairs.

'PropertyName', PropertyValue

Each pair specifies a particular property of the model, for example, the input names or some notes on the model history. See "Properties" on page 2-1157 for more information about available ss model object properties.

The following expression:

```
sys = ss(A,B,C,D,'Property1',Value1,...,'PropertyN',ValueN)
```

is equivalent to the sequence of commands:

```
sys = ss(A,B,C,D)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

### **Conversion to State Space**

sys\_ss = ss(sys) converts a dynamic system model sys to state-space form. The output sys\_ss is an equivalent state-space model (ss model object). This operation is known as *state-space realization*.

 $sys\_ss = ss(sys, 'minimal')$  produces a state-space realization with no uncontrollable or unobservable states. This state-space realization is equivalent to  $sys\_ss = minreal(ss(sys))$ .

 $sys\_ss = ss(sys, 'explicit')$  computes an explicit realization (E = I) of the dynamic system model sys. If sys is improper, ss returns an error.

**Note** Conversions to state space are not uniquely defined in the SISO case. They are also not guaranteed to produce a minimal realization in the MIMO case. For more information, see "Recommended Working Representation".

### **Conversion of Identified Models**

An identified model is represented by an input-output equation of the form y(t) = Gu(t) + He(t), where u(t) is the set of measured input channels and e(t) represents the noise channels. If  $\Lambda = LL'$  represents the covariance of noise e(t), this equation can also be written as y(t) = Gu(t) + HLv(t), where cov(v(t)) = I.

 $sys\_ss = ss(sys)$  or  $sys\_ss = ss(sys)$ , 'measured') converts the measured component of an identified linear model into the state-space form. sys is a model of type idss, idproc, idtf, idpoly, or idgrey.  $sys\_ss$  represents the relationship between u and y.

sys\_ss = ss(sys, 'noise') converts the noise component of an identified linear model into the state space form. It represents the relationship between the noise input v(t) and output  $y\_noise = HL\ v(t)$ . The noise input channels belong to the InputGroup 'Noise'. The names of the noise input channels are v@yname, where yname is the name of the corresponding output channel. sys\_ss has as many inputs as outputs.

sys\_ss = ss(sys, 'augmented') converts both the measured and noise dynamics into a state-space model. sys\_ss has ny+nu inputs such that the first nu inputs represent the channels u(t) while the remaining by channels represent the noise channels v(t). sys\_ss.InputGroup contains 2 input groups- 'measured' and 'noise'. sys\_ss.InputGroup.Measured is set to 1:nu while sys\_ss.InputGroup.Noise is set to nu+1:nu+ny. sys ss represents the equation y(t) = [G HL][u; v]

**Tip** An identified nonlinear model cannot be converted into a state-space form. Use linear approximation functions such as linearize and linapp.

### **Creation of Generalized State-Space Models**

You can use the syntax:

```
gensys = ss(A,B,C,D)
```

to create a Generalized state-space (genss) model when one or more of the matrices A, B, C, D is a tunable realp or genmat model. For more information about Generalized state-space models, see "Models with Tunable Coefficients".

## **Properties**

ss objects have the following properties:

#### A,B,C,D,E

State-space matrices.

- A State matrix A. Square real- or complex-valued matrix with as many rows as states.
- B Input-to-state matrix *B*. Real- or complex-valued matrix with as many rows as states and as many columns as inputs.
- C State-to-output matrix *C*. Real- or complex-valued matrix with as many rows as outputs and as many columns as states.
- D Feedthrough matrix *D*. Real- or complex-valued matrix with as many rows as outputs and as many columns as inputs.
- E E matrix for implicit (descriptor) state-space models. By default e = [], meaning that the state equation is explicit. To specify an implicit state equation E dx/dt = Ax + Bu, set this property to a square matrix of the same size as A. See dss for more information about creating descriptor state-space models.

#### Scaled

Logical value indicating whether scaling is enabled or disabled.

When Scaled = 0 (false), most numerical algorithms acting on the state-space model automatically rescale the state vector to improve numerical accuracy. You can disable such auto-scaling by setting Scaled = 1 (true). For more information about scaling, see prescale.

**Default:** 0 (false)

#### **StateName**

State names, specified as one of the following:

- Character vector For first-order models, for example, 'velocity'.
- Cell array of character vectors For models with two or more states
- '' For unnamed states.

**Default:** '' for all states

#### StateUnit

State units, specified as one of the following:

- Character vector For first-order models, for example, 'velocity'
- Cell array of character vectors For models with two or more states
- '' For states without specified units

Use StateUnit to keep track of the units each state is expressed in. StateUnit has no effect on system behavior.

Default: ' ' for all states

### **InternalDelay**

Vector storing internal delays.

Internal delays arise, for example, when closing feedback loops on systems with delays, or when connecting delayed systems in series or parallel. For more information about internal delays, see "Closing Feedback Loops with Time Delays".

For continuous-time models, internal delays are expressed in the time unit specified by the TimeUnit property of the model. For discrete-time models, internal delays are expressed as integer multiples of the sample time Ts. For example, InternalDelay = 3 means a delay of three sampling periods.

You can modify the values of internal delays. However, the number of entries in sys.InternalDelay cannot change, because it is a structural property of the model.

#### **InputDelay**

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

#### Default: 0

### **OutputDelay**

Output delays. OutputDelay is a numeric vector specifying a time delay for each output channel. For continuous-time systems, specify output delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time Ts. For example, OutputDelay = 3 means a delay of three sampling periods.

For a system with Ny outputs, set OutputDelay to an Ny-by-1 vector, where each entry is a numerical value representing the output delay for the corresponding output channel. You can also set OutputDelay to a scalar value to apply the same delay to all channels.

**Default:** 0 for all output channels

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

#### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

#### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

#### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: '' for all output channels

### OutputUnit

Output channel units, specified as one of the following:

- $\bullet \quad \text{Character vector} \text{For single-output models. For example, 'seconds'}.$
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ' '

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

### **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

#### Default: []

### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

```
М
```

. . .

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

## **Examples**

### **Create Discrete-Time State-Space Model**

Create a state-space model with a sample time of 0.25 seconds and the following state-space matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

Specify the state-space matrices.

```
A = [0 1;-5 -2];
B = [0;3];
C = [0 1];
D = 0:
```

Specify the sample time.

```
Ts = 0.25:
```

Create the state-space model.

$$sys = ss(A,B,C,D,Ts);$$

# **Specify State and Input Names for Discrete-Time State-Space Model**

Create state-space matrices and specify sample time.

```
A = [0 1; -5 -2];
B = [0;3];
C = [0 1];
D = 0;
Ts = 0.05:
```

Create state-space model, specifying the state and input names.

```
sys = ss(A,B,C,D,Ts,'StateName',{'Position' 'Velocity'},...
'InputName','Force');
```

The number of state and input names must be consistent with the dimensions of A, B, C, and D.

### **Convert Transfer Function to State-Space Model**

Compute the state-space model of the following transfer function:

$$H(s) = \begin{bmatrix} \frac{s+1}{s^3 + 3s^2 + 3s + 2} \\ \frac{s^2 + 3}{s^2 + s + 1} \end{bmatrix}$$

Create the transfer function model.

```
H = [tf([1 1],[1 3 3 2]); tf([1 0 3],[1 1 1])];
```

Convert this model to a state-space model.

```
sys = ss(H);
```

Examine the size of the state-space model.

```
size(sys)
```

```
State-space model with 2 outputs, 1 inputs, and 5 states.
```

The number of states is equal to the cumulative order of the SISO entries in H(s).

To obtain a minimal realization of H(s), enter

```
sys = ss(H, 'minimal');
size(sys)
```

```
State-space model with 2 outputs, 1 inputs, and 3 states.
```

The resulting model has an order of three, which is the minimum number of states needed to represent H(s). To see this number of states, refactor H(s) as the product of a first-order system and a second-order system.

$$H(s) = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+1}{s^2+s+1} \\ \frac{s^2+3}{s^2+s+1} \end{bmatrix}$$

### **Explicit Realization of Descriptor State-Space Model**

Create a descriptor state-space model  $(E \neq I)$ .

```
a = [2 -4; 4 2];
b = [-1; 0.5];
c = [-0.5, -2];
d = [-1];
e = [1 0; -3 0.5];
sysd = dss(a,b,c,d,e);
```

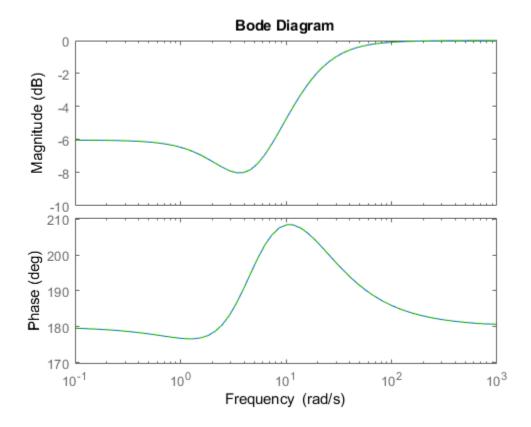
Compute an explicit realization of the system (E = I).

$$\begin{array}{ccc} D & = & & \\ & & u1 \\ y1 & -1 & \end{array}$$

Continuous-time state-space model.

Confirm that the descriptor and explicit realizations have equivalent dynamics.

bodeplot(sysd,syse,'g--')



# Create State-Space Model with Both Fixed and Tunable Parameters

This example shows how to create a state-space genss model having both fixed and tunable parameters.

$$A = \begin{bmatrix} 1 & a + b \\ 0 & ab \end{bmatrix}, \quad B = \begin{bmatrix} -3.0 \\ 1.5 \end{bmatrix}, \quad C = [0.3 \ 0], \quad D = 0,$$

where a and b are tunable parameters, whose initial values are -1 and 3, respectively.

Create the tunable parameters using realp.

```
a = realp('a',-1);
b = realp('b',3);
```

Define a generalized matrix using algebraic expressions of a and b.

```
A = [1 a+b;0 a*b];
```

A is a generalized matrix whose Blocks property contains a and b. The initial value of A is  $[1\ 2;0\ -3]$ , from the initial values of a and b.

Create the fixed-value state-space matrices.

```
B = [-3.0;1.5];
C = [0.3 0];
D = 0;
```

Use ss to create the state-space model.

```
sys = ss(A,B,C,D)
sys =

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 2 states, and
    a: Scalar parameter, 2 occurrences.
    b: Scalar parameter, 2 occurrences.
```

Type "ss(sys)" to see the current value, "get(sys)" to see all properties, and "sys.Blo

sys is a generalized LTI model (genss) with tunable parameters a and b. Confirm that the A property of sys is stored as a generalized matrix.

sys.A

```
ans =
   Generalized matrix with 2 rows, 2 columns, and the following blocks:
    a: Scalar parameter, 2 occurrences.
    b: Scalar parameter, 2 occurrences.

Type "double(ans)" to see the current value, "get(ans)" to see all properties, and "ans"
```

### **Extract Components from Identified State-Space Model**

Extract the measured and noise components of an identified polynomial model into two separate state-space models. The former (measured component) can serve as a plant model while the latter can serve as a disturbance model for control system design.

```
load icEngine
z = iddata(y,u,0.04);
sys = ssest(z,3);

sysMeas = ss(sys,'measured')
sysNoise = ss(sys,'noise')
```

Alternatively, use ss(sys) to extract the measured component.

# **Algorithms**

For TF to SS model conversion, ss(sys\_tf) returns a modified version of the controllable canonical form. It uses an algorithm similar to tf2ss, but further rescales the state vector to compress the numerical range in state matrix A and to improve numerics in subsequent computations.

For ZPK to SS conversion, ss(sys\_zpk) uses direct form II structures, as defined in signal processing texts. See *Discrete-Time Signal Processing* by Oppenheim and Schafer for details.

For example, in the following code, A and sys. A differ by a diagonal state transformation:

```
n=[1 1];
d=[1 1 10];
[A,B,C,D]=tf2ss(n,d);
```

For details, see balance.

### **See Also**

 $\verb"dss| frd| \verb"get| set| \verb"ssdata| tf| \verb"zpk"$ 

### **Topics**

"What Are Model Objects?"

### **Introduced before R2006a**

<sup>&</sup>quot;State-Space Models"

<sup>&</sup>quot;MIMO State-Space Models"

### ss2ss

State coordinate transformation for state-space model

# **Syntax**

$$sysT = ss2ss(sys,T)$$

# **Description**

Given a state-space model sys with equations

$$\dot{x} = Ax + Bu$$
$$v = Cx + Du$$

or the innovations form used by the identified state-space (IDSS) models:

$$\frac{dx}{dt} = Ax + Bu + Ke$$
$$y = Cx + Du + e$$

(or their discrete-time counterpart), ss2ss performs the similarity transformation  $\bar{x} = Tx$  on the state vector x and produces the equivalent state-space model sysT with equations.

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$

$$v = CT^{-1}\bar{x} + Du$$

or, in the case of an IDSS model:

$$\dot{\overline{x}} = TAT^{-1}\overline{x} + TBu + TKe$$
$$y = CT^{-1}\overline{x} + Du + e$$

(IDSS models require System Identification Toolbox software.)

sysT = ss2ss(sys,T) returns the transformed state-space model sysT given sys and the state coordinate transformation T. The model sys must be in state-space form and the

matrix  $\mathsf{T}$  must be invertible.  $\mathsf{ss2ss}$  is applicable to both continuous- and discrete-time models.

# **Examples**

Perform a similarity transform to improve the conditioning of the A matrix.

```
T = balance(sys.A)
sysb = ss2ss(sys,inv(T))
```

### See Also

balreal | canon

**Introduced before R2006a** 

### ssdata

Access state-space model data

# **Syntax**

```
[a,b,c,d] = ssdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
```

# **Description**

[a,b,c,d] = ssdata(sys) extracts the matrix (or multidimensional array) data A, B, C, D from the state-space model (LTI array) sys. If sys is a transfer function or zero-polegain model (LTI array), it is first converted to state space. See ss for more information on the format of state-space model data.

If sys appears in descriptor form (nonempty E matrix), an equivalent explicit form is first derived.

If sys has internal delays, A, B, C, D are obtained by first setting all internal delays to zero (creating a zero-order Padé approximation). For some systems, setting delays to zero creates singular algebraic loops, which result in either improper or ill-defined, zero-delay approximations. For these systems, ssdata cannot display the matrices and returns an error. This error does not imply a problem with the model sys itself.

For generalized state-space (genss) models, ssdata returns the state-space models evaluated at the current, nominal value of all control design blocks. To access the dependency of a genss model on its static control design blocks, use the A, B, C, and D properties of the model.

```
[a,b,c,d,Ts] = ssdata(sys) also returns the sample time Ts.
```

You can access the remaining LTI properties of sys with get or by direct referencing. For example:

```
sys.statename
```

For arrays of state-space models with variable numbers of states, use the syntax:

```
[a,b,c,d] = ssdata(sys,'cell')
```

to extract the state-space matrices of each model as separate cells in the cell arrays  ${\tt a}$ ,  ${\tt b}$ ,  ${\tt c}$ , and  ${\tt d}$ .

### **See Also**

dssdata | get | getDelayModel | set | ss | tfdata | zpkdata

**Introduced before R2006a** 

# stabsep

Stable-unstable decomposition

# **Syntax**

```
[GS,GNS]=stabsep(G)
[G1,GNS] = stabsep(G, 'abstol',ATOL, 'reltol',RTOL)
[G1,G2]=stabsep(G, ..., 'Mode', MODE, 'Offset', ALPHA)
[G1,G2] = stabsep(G, opts)
```

# **Description**

[GS, GNS] = stabsep(G) decomposes the LTI model G into its stable and unstable parts

```
G = GS + GNS
```

where GS contains all stable modes that can be separated from the unstable modes in a numerically stable way, and GNS contains the remaining modes. GNS is always strictly proper.

[G1,GNS] = stabsep(G, 'abstol', ATOL, 'reltol', RTOL) specifies absolute and relative error tolerances for the stable/unstable decomposition. The frequency responses of G and GS + GNS should differ by no more than ATOL+RTOL\*abs(G). Increasing these tolerances helps separate nearby stable and unstable modes at the expense of accuracy. The default values are ATOL=0 and RTOL=1e-8.

[G1,G2]=stabsep(G, ..., 'Mode', MODE, 'Offset', ALPHA) produces a more general stable/unstable decomposition where G1 includes all separable poles lying in the regions defined using offset ALPHA. This can be useful when there are numerical accuracy issues. For example, if you have a pair of poles close to, but slightly to the left of the  $j\omega$ -axis, you can decide not to include them in the stable part of the decomposition if numerical considerations lead you to believe that the poles may be in fact unstable

This table lists the stable/unstable boundaries as defined by the offset ALPHA.

Mode	Continuous Time Region	Discrete Time Region
1	Re(s) < -ALPHA*max(1,  Im(s) )	1  z  < 1-ALPHA
2	Re(s) > ALPHA*max(1, Im(s) )	2  z  > 1 + ALPHA

The default values are MODE=1 and ALPHA=0.

[G1,G2] = stabsep(G, opts) computes the stable/unstable decomposition of G using the options specified in the stabsepOptions object opts.

# **Examples**

Compute a stable/unstable decomposition with absolute error no larger than 1e-5 and an offset of 0.1:

```
h = zpk(1,[-2 -1 1 -0.001],0.1)
[hs,hns] = stabsep(h,stabsepOptions('AbsTol',1e-5,'Offset',0.1));
```

The stable part of the decomposition has poles at -1 and -2.

hs

The unstable part of the decomposition has poles at +1 and -.001 (which is nominally stable).

hns

```
Zero/pole/gain:
0.050075 (s-1)
------
(s+0.001) (s-1)
```

### See Also

modsep|stabsepOptions

### **Introduced before R2006a**

# **stabsepOptions**

Options for stable-unstable decomposition

# **Syntax**

```
opts = stabsepOptions
opts = stabsepOptions('OptionName', OptionValue)
```

# **Description**

opts = stabsepOptions returns the default options for the stabsep command.

opts = stabsepOptions('OptionName', OptionValue) accepts one or more comma-separated name/value pairs. Specify OptionName inside single quotes.

# **Input Arguments**

### **Name-Value Pair Arguments**

#### **Focus**

Focus of decomposition. Specified as one of the following values:

```
'stable' First output of stabsep contains only stable dynamics.

'unstable' First output of stabsep contains only unstable dynamics.
```

Default: 'stable'

#### AbsTol, RelTol

Absolute and relative error tolerance for stable/unstable decomposition. Positive scalar values. When decomposing a model G, stabsep ensures that the frequency responses of G and GS + GU differ by no more than AbsTol + RelTol\*abs(G). Increasing these

tolerances helps separate nearby stable and unstable modes at the expense of accuracy. See stabsep for more information.

Default: AbsTol = 0; RelTol = 1e-8

#### Offset

Offset for the stable/unstable boundary. Positive scalar value. The first output of stabsepincludes only poles satisfying:

Continuous time:

- Re(s) < -Offset \* max(1,|Im(s)|) (Focus = 'stable')
- Re(s) > Offset \* max(1,|Im(s)|) (Focus = 'unstable')

Discrete time:

- |z| < 1 Offset (Focus = 'stable')
- |z| >1 + Offset (Focus = 'unstable')

Increase the value of Offset to treat poles close to the stability boundary as unstable.

#### Default: 0

For additional information on the options and how to use them, see the stabsep reference page.

# **Examples**

Compute the stable/unstable decomposition of the system given by:

$$G(s) = \frac{10(s+0.5)}{(s+10^{-6})(s+2-5i)(s+2+5i)}$$

Use the Offset option to force stabsep to exclude the pole at  $s=10^{-6}$  from the stable term of the stable/unstable decomposition.

```
G = zpk(-.5,[-1e-6 -2+5i -2-5i],10);
opts = stabsepOptions('Offset',.001); % Create option set
[G1,G2] = stabsep(G,opts) % treats -1e-6 as unstable
```

These commands return the result:

The pole at  $s = 10^{-6}$  is in the second (unstable) output.

### See Also

stabsep

Introduced in R2010a

### stack

Build model array by stacking models or model arrays along array dimensions

### **Syntax**

```
sys = stack(arraydim,sys1,sys2,...)
```

# **Description**

sys = stack(arraydim,sys1,sys2,...) produces an array of dynamic system models sys by stacking (concatenating) the models (or arrays) sys1,sys2,... along the array dimension arraydim. All models must have the same number of inputs and outputs (the same I/O dimensions), but the number of states can vary. The I/O dimensions are not counted in the array dimensions. For more information about model arrays and array dimensions, see "Model Arrays".

For arrays of state-space models with variable order, you cannot use the dot operator (e.g., sys.A) to access arrays. Use the syntax

```
[A,B,C,D] = ssdata(sys,'cell')
```

to extract the state-space matrices of each model as separate cells in the cell arrays A, B, C, and D.

# **Examples**

### **Example 1**

If sys1 and sys2 are two models:

- stack(1,sys1,sys2) produces a 2-by-1 model array.
- stack(2,sys1,sys2) produces a 1-by-2 model array.
- stack(3, sys1, sys2) produces a 1-by-1-by-2 model array.

### **Example 2**

Stack identified state-space models derived from the same estimation data and compare their bode responses.

```
load iddata1 z1
sysc = cell(1,5);
opt = ssestOptions('Focus','simulation');
for i = 1:5
sysc{i} = ssest(z1,i-1,opt);
end
sysArray = stack(1, sysc{:});
bode(sysArray);
```

#### **Introduced before R2006a**

# step

Step response plot of dynamic system; step response data

# **Syntax**

```
step(sys)
step(sys,Tfinal)
step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,Tfinal)
step(sys1,sys2,...,sysN,t)
y = step(sys1,sys2,...,sysN,t)
[y,t] = step(sys,t)
[y,t] = step(sys,Tfinal)
[y,t,x] = step(sys)
[y,t,x,ysd] = step(sys)
[y,...] = step(sys,...,options)
```

# **Description**

step calculates the step response of a dynamic system. For the state-space case, zero initial state is assumed. When it is invoked with no output arguments, this function plots the step response on the screen.

step(sys) plots the step response of an arbitrary dynamic system model, sys. This model can be continuous- or discrete-time, and SISO or MIMO. The step response of multi-input systems is the collection of step responses for each input channel. The duration of simulation is determined automatically, based on the system poles and zeros.

step(sys, Tfinal) simulates the step response from t = 0 to the final time t = Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), step interprets Tfinal as the number of sampling periods to simulate.

step(sys,t) uses the user-supplied time vector t for simulation. Express t in the
system time units, specified in the TimeUnit property of sys. For discrete-time models, t

should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time models, t should be of the form Ti:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see "Algorithms" on page 2-1194). The step command always applies the step input at t=0, regardless of Ti.

To plot the step response of several models sys1,..., sysN on a single figure, use

```
step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,Tfinal)
step(sys1,sys2,...,sysN,t)
```

All of the systems plotted on a single plot must have the same number of inputs and outputs. You can, however, plot a mix of continuous- and discrete-time systems on a single plot. This syntax is useful to compare the step responses of multiple systems.

You can also specify a distinctive color, linestyle, marker, or all three for each system. For example,

```
step(sys1,'y:',sys2,'g--')
```

plots the step response of sys1 with a dotted yellow line and the step response of sys2 with a green dashed line.

When invoked with output arguments:

```
y = step(sys,t)
[y,t] = step(sys)
[y,t] = step(sys,Tfinal)
[y,t,x] = step(sys)
```

step returns the output response y, the time vector t used for simulation (if not supplied as an input argument), and the state trajectories x (for state-space models only). No plot generates on the screen. For single-input systems, y has as many rows as time samples (length of t), and as many columns as outputs. In the multi-input case, the step responses of each input channel are stacked up along the third dimension of y. The dimensions of y are then

 $(length \ of \ t) \times (number \ of \ outputs) \times (number \ of \ inputs)$ 

and y(:,:,j) gives the response to a unit step command injected in the jth input channel. Similarly, the dimensions of x are

```
(length \ of \ t) \times (number \ of \ states) \times (number \ of \ inputs)
```

For identified models (see idlti and idnlmodlel) [y,t,x,ysd] = step(sys) also computes the standard deviation ysd of the response y (ysd is empty if sys does not contain parameter covariance information).

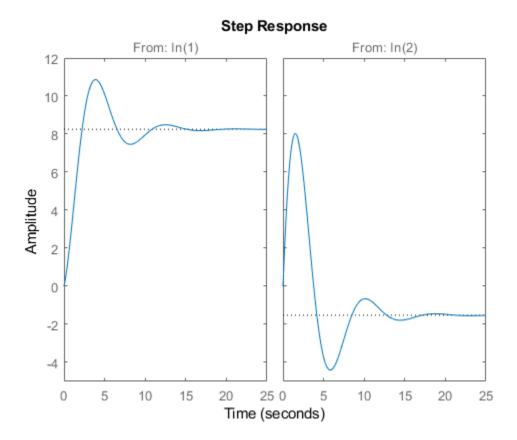
[y,...] = step(sys,...,options) specifies additional options for computing the step response, such as the step amplitude or input offset. Use stepDataOptions to create the option set options.

# **Examples**

#### **Step Response Plot of Dynamic System**

Plot the step response of the following second-order state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
 
$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
 
$$a = \begin{bmatrix} -0.5572, -0.7814; 0.7814, 0 \end{bmatrix};$$
 
$$b = \begin{bmatrix} 1, -1; 0, 2 \end{bmatrix};$$
 
$$c = \begin{bmatrix} 1.9691, 6.4493 \end{bmatrix};$$
 
$$sys = ss(a, b, c, 0);$$
 
$$step(sys)$$

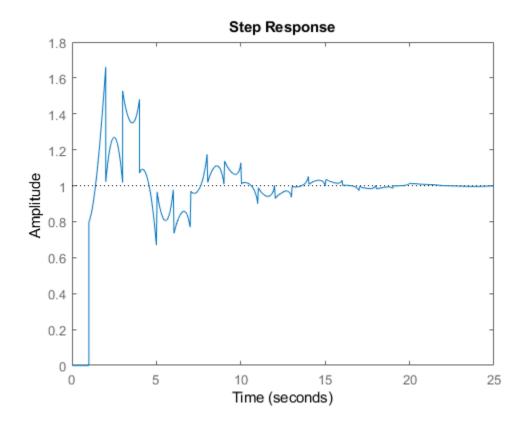


The left plot shows the step response of the first input channel, and the right plot shows the step response of the second input channel.

### **Step Response Plot of Feedback Loop with Delay**

Create a feedback loop with delay and plot its step response.

```
s = tf('s');
G = exp(-s) * (0.8*s^2+s+2)/(s^2+s);
T = feedback(ss(G),1);
step(T)
```



The system step response displayed is chaotic. The step response of systems with internal delays may exhibit odd behavior, such as recurring jumps. Such behavior is a feature of the system and not software anomalies.

### Step Responses of Identified Models with Confidence Regions

Compare the step response of a parametric identified model to a non-parametric (empirical) model. Also view their 3  $\sigma$  confidence regions.

Load the data.

```
load iddata1 z1
```

Estimate a parametric model.

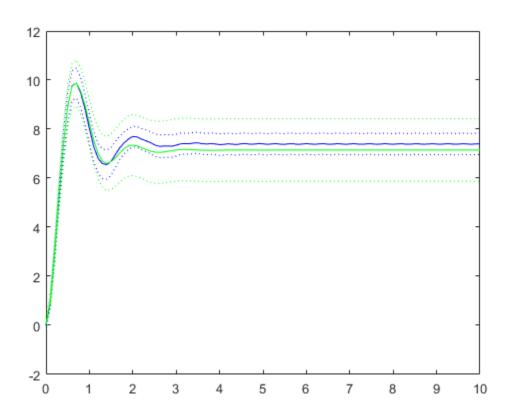
```
sys1 = ssest(z1,4);
```

Estimate a non-parametric model.

```
sys2 = impulseest(z1);
```

Plot the step responses for comparison.

```
t = (0:0.1:10)';
[y1, ~, ~, ysd1] = step(sys1,t);
[y2, ~, ~, ysd2] = step(sys2,t);
plot(t, y1, 'b', t, y1+3*ysd1, 'b:', t, y1-3*ysd1, 'b:')
hold on
plot(t, y2, 'g', t, y2+3*ysd2, 'g:', t, y2-3*ysd2, 'g:')
```



#### Validate Linearization of Identified Nonlinear ARX Model

Validate the linearization of a nonlinear ARX model by comparing the small amplitude step responses of the linear and nonlinear models.

Load the data.

load iddata2 z2;

Estimate a nonlinear ARX model.

```
nlsys = nlarx(z2, [4\ 3\ 10], 'tree', 'custom', \{'sin(y1(t-2)*u1(t)) + y1(t-2)*u1(t) + u1(t) .*u1(t) .*u1(t)
```

Determine an equilibrium operating point for nlsys corresponding to a steady-state input value of 1.

```
u0 = 1;
[X,~,r] = findop(nlsys, 'steady', 1);
y0 = r.SignalLevels.Output;
```

Obtain a linear approximation of nlsys at this operating point.

```
sys = linearize(nlsys,u0,X);
```

Validate the usefulness of sys by comparing its small-amplitude step response to that of nlsys.

The nonlinear system nlsys is operating at an equilibrium level dictated by (u0, y0). Introduce a step perturbation of size 0.1 about this steady-state and compute the corresponding response.

```
opt = stepDataOptions;
opt.InputOffset = u0;
opt.StepAmplitude = 0.1;
t = (0:0.1:10)';
ynl = step(nlsys, t, opt);
```

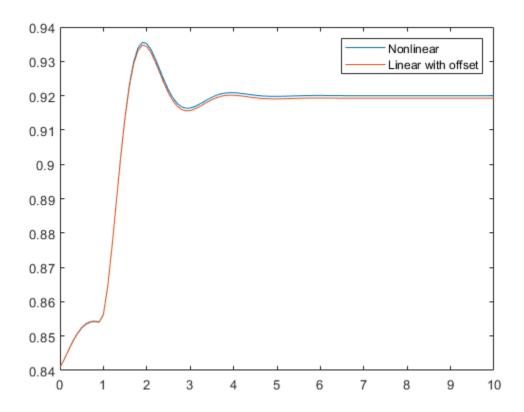
The linear system sys expresses the relationship between the perturbations in input to the corresponding perturbation in output. It is unaware of the nonlinear system's equilibrium values.

Plot the step response of the linear system.

```
opt = stepDataOptions;
opt.StepAmplitude = 0.1;
yl = step(sys, t, opt);
```

Add the steady-state offset,  $y\theta$  , to the response of the linear system and plot the responses.

```
plot(t, ynl, t, yl+y0)
legend('Nonlinear', 'Linear with offset')
```



#### **Step Response of Identified Time-Series Model**

Compute the step response of an identified time-series model.

A time-series model, also called a signal model, is one without measured input signals. The step plot of this model uses its (unmeasured) noise channel as the input channel to which the step signal is applied.

Load the data.

load iddata9;

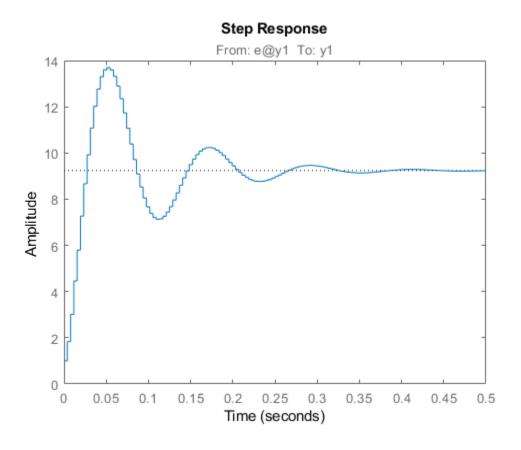
Estimate a time-series model.

$$sys = ar(z9, 4);$$

ys is a model of the form A y(t) = e(t), where e(t) represents the noise channel. For computation of step response, e(t) is treated as an input channel, and is named e@y1.

Plot the step response.

step(sys)



# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

# **Algorithms**

Continuous-time models without internal delays are converted to state space and discretized using zero-order hold on the inputs. The sample time is chosen automatically based on the system dynamics, except when a time vector t = 0:dt:Tf is supplied (dt is then used as sampling period). The resulting simulation time steps t are equisampled with spacing dt.

For systems with internal delays, Control System Toolbox software uses variable step solvers. As a result, the time steps t are not equisampled.

### References

[1] L.F. Shampine and P. Gahinet, "Delay-differential-algebraic equations in control theory," *Applied Numerical Mathematics*, Vol. 56, Issues 3-4, pp. 574–588.

### See Also

Linear System Analyzer | impulse | initial | lsim | stepDataOptions

Introduced before R2006a

# **stepDataOptions**

Options set for step

### **Syntax**

```
opt = stepDataOptions
opt = stepDataOptions(Name, Value)
```

# **Description**

opt = stepDataOptions creates the default options for step.

opt = stepDataOptions(Name, Value) creates an options set with the options
specified by one or more Name, Value pair arguments.

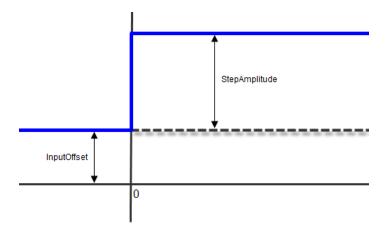
# **Input Arguments**

### Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

### InputOffset

Input signal level for all time t < 0, as shown in the next figure.



Default: 0

#### **StepAmplitude**

Change of input signal level which occurs at time t = 0, as shown in the previous figure.

Default: 1

# **Output Arguments**

#### opt

Option set containing the specified options for step.

# **Examples**

### **Specify Input Offset and Step Amplitude Level for Step Response**

Create a transfer function model.

$$sys = tf(1,[1,1]);$$

Create an option set for step to specify input offset and step amplitude level.

```
opt = stepDataOptions('InputOffset',-1,'StepAmplitude',2);
Calculate the step response using the specified options.
[y,t] = step(sys,opt);
```

# **See Also**

step

Introduced in R2012a

# stepinfo

Rise time, settling time, and other step-response characteristics

### **Syntax**

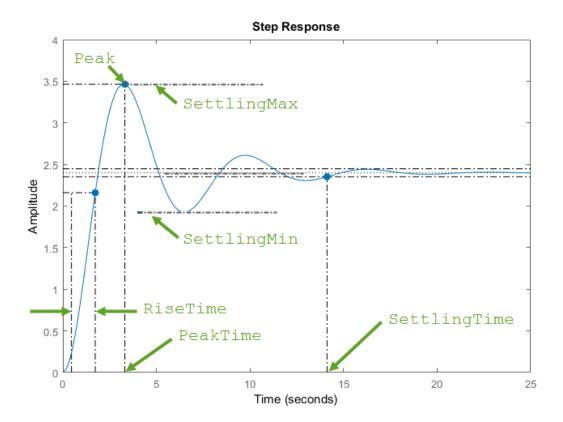
```
S = stepinfo(sys)
S = stepinfo(y,t)
S = stepinfo(y,t,yfinal)
S = stepinfo(____,'SettlingTimeThreshold',ST)
S = stepinfo(____,'RiseTimeLimits',RT)
```

# **Description**

S = stepinfo(sys)computes the step-response characteristics for a dynamic system model sys. The function returns the characteristics in a structure containing the fields:

- RiseTime Time it takes for the response to rise from 10% to 90% of the steadystate response.
- SettlingTime Time it takes for the error  $|y(t) y_{final}|$  between the response y(t) and the steady-state response  $y_{final}$  to fall to within 2% of  $y_{final}$ .
- SettlingMin Minimum value of y(t) once the response has risen.
- SettlingMax Maximum value of y(t) once the response has risen.
- Overshoot Percentage overshoot, relative to  $y_{final}$ ).
- Undershoot Percentage undershoot.
- Peak Peak absolute value of y(t)
- PeakTime Time at which the peak value occurs.

The following figure illustrates some of these quantities on a typical second-order response.



Using this syntax requires a Control System Toolbox license.

S = stepinfo(y,t) computes step-response characteristics from an array of step-response data y and corresponding time vector t. For SISO system responses, y is a vector with the same number of entries as t. For MIMO response data, y is an array containing the responses of each I/O channel. This syntax uses the last value in y (or the last value in each channel's corresponding response data) as the steady-state value for computing characteristics that depend on that value.

S = stepinfo(y,t,yfinal) computes step-response characteristics relative to the steady-state value yfinal. This syntax is useful when you know that the expected steady-state system response differs from the last value in y for reasons such as measurement noise.

For SISO responses, t and y are vectors with the same length NS. For systems with NU inputs and NY outputs, you can specify y as an NS-by-NY-by-NU array (see step) and yfinal as an NY-by-NU array. stepinfo then returns a NY-by-NU structure array S of performance metrics for each I/O pair.

 $S = stepinfo(\underline{\hspace{0.5cm}}$ , 'SettlingTimeThreshold', ST) lets you specify the threshold ST used in the definition of settling time. The response has settled when the error |y(t) - yfinal| becomes smaller than a fraction ST of its peak value. The default value is ST = 0.02 (2%). You can use this syntax with any of the previous input-argument combinations.

S = stepinfo(\_\_\_\_, 'RiseTimeLimits',RT) lets you specify the lower and upper thresholds used in the definition of rise time. By default, the rise time is defined as the time the response takes to rise from 10 to 90% of the steady-state value (RT = [0.1 0.9]). The upper threshold RT(2) is also used to calculate SettlingMin and SettlingMax. These values are the minimum and maximum values of the response occurring after the response has reached the upper threshold. You can use this syntax with any of the previous input-argument combinations.

# **Examples**

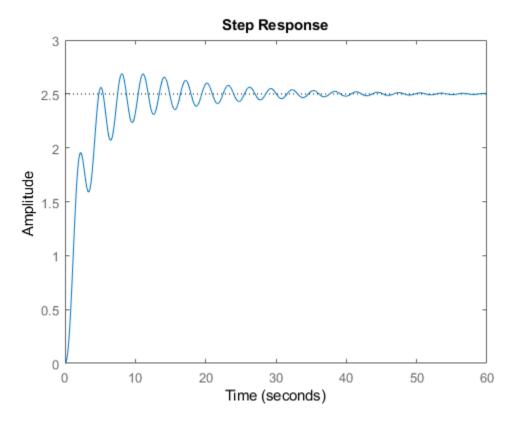
### **Step-Response Characteristics of Dynamic System**

Compute step-response characteristics such as rise time, settling time, and overshoot for a dynamic system model. For this example, use the continuous-time transfer function:

$$sys = \frac{s^2 + 5s + 5}{s^4 + 1.65s^3 + 5s^2 + 6.5s + 2}$$
.

Create the transfer function and examine its step response.

$$sys = tf([1 5 5],[1 1.65 5 6.5 2]);$$
  
 $step(sys)$ 



The plot shows that the response rises in a few seconds, and then rings down to a steady-state value of about 2.5. Compute the characteristics of this response using stepinfo.

### S = stepinfo(sys)

S = struct with fields:
 RiseTime: 3.8456
 SettlingTime: 27.9762
 SettlingMin: 2.0689
 SettlingMax: 2.6873
 Overshoot: 7.4915
 Undershoot: 0
 Peak: 2.6873

PeakTime: 8.0530

By default, the settling time is the time it takes for  $y(t) - y_{final}$  to fall below 2% of its peak value, where y(t) is the system response at time t and  $y_{final}$  is the steady-state response. The result  ${\tt S.SettlingTime}$  shows that for  ${\tt sys}$ , this condition occurs after about 28 seconds. The default definition of rise time is the time it takes for the response to go from 10% of its steady-state value to 90% of that value.  ${\tt S.RiseTime}$  shows that for  ${\tt sys}$ , this rise occurs in less than 4 seconds. The maximum overshoot is returned in  ${\tt S.Overshoot}$ . For this system, the peak value  ${\tt S.Peak}$ , which occurs at the time  ${\tt S.PeakTime}$ , overshoots the steady-state value by about 7.5% of the steady-state value.

#### **Step-Response Characteristics of MIMO System**

For a MIMO system, stepinfo returns a structure array in which each entry contains the response characteristics of the corresponding I/O channel of the system. For this example, use a two-output, two-input discrete-time system. Compute the step-response characteristics.

```
A = [0.68 -0.34; 0.34 0.68];
B = [0.18 -0.05; 0.04 0.11];
C = [0 -1.53; -1.12 -1.10];
D = [0 0; 0.06 -0.37];
sys = ss(A,B,C,D,0.2);

S = stepinfo(sys)

S=2×2 struct
   RiseTime
   SettlingTime
   SettlingMin
   SettlingMax
   Overshoot
   Undershoot
   Peak
   PeakTime
```

Access the response characteristics for a particular I/0 channel by indexing into S. For instance, examine the response characteristics for the response from the first input to the second output of sys, corresponding to S(2,1).

#### S(2,1)

```
ans = struct with fields:
    RiseTime: 0.4000
SettlingTime: 2.8000
SettlingMin: -0.6724
SettlingMax: -0.5188
    Overshoot: 24.6476
Undershoot: 11.1224
    Peak: 0.6724
PeakTime: 1
```

To access a particular value, use dot notation. For instance, extract the rise time of the (2,1) channel.

```
rt21 = S(2,1).RiseTime

rt21 = 0.4000
```

### **Specify Definition of Settling Time or Rise Time**

By default, stepinfo defines settling time as the time it takes for the error  $|y(t) - y_{final}|$  between the response y(t) and the steady-state response  $y_{final}$  to come within 2% of  $y_{final}$ . Also, stepinfo defines the rise time as the time it takes for the response to rise from 10% of  $y_{final}$  to 90% of  $y_{final}$ . You can change these definitions using SettlingTimeThreshold and RiseTimeThreshold. For this example, use the system given by:

$$sys = \frac{s^2 + 5s + 5}{s^4 + 1.65s^3 + 6.5s + 2}$$
.

Create the transfer function.

```
sys = tf([1 5 5],[1 1.65 5 6.5 2]);
```

Compute the time it takes for the error in the response of sys to to reach 0.5% of the steady-state response. To do so, set SettlingTimeThreshold to 0.5%, or 0.005.

```
S1 = stepinfo(sys,'SettlingTimeThreshold',0.005);
st1 = S1.SettlingTime
```

```
st1 = 46.1325
```

Compute the time it takes the response of sys to rise from 5% to 95% of the steady-state value. To do so, set RiseTimeThreshold to a vector containing those bounds.

```
S2 = stepinfo(sys,'RiseTimeThreshold',[0.05 0.95]);
rt2 = S2.RiseTime
rt2 = 4.1690
```

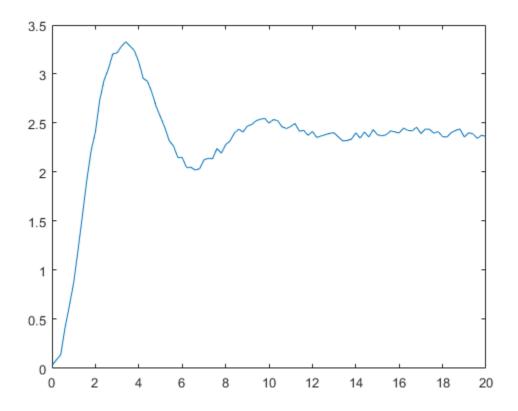
You can define both settling time and rise time in the same computation.

```
S3 = stepinfo(sys, 'SettlingTimeThreshold', 0.005, 'RiseTimeThreshold', [0.05 0.95])
S3 = struct with fields:
    RiseTime: 4.1690
SettlingTime: 46.1325
SettlingMin: 2.0689
SettlingMax: 2.6873
    Overshoot: 7.4915
Undershoot: 0
    Peak: 2.6873
PeakTime: 8.0530
```

### **Step-Response Characteristics from Response Data**

You can extract step-response characteristics from step-response data even if you do not have a model of your system. For instance, suppose you have measured the response of your system to a step input, and saved the resulting response data in a vector y of response values at the times stored in another vector, t. Load the response data and examine it.

```
load StepInfoData t y
plot(t,y)
```



Compute step-response characteristics from this response data using stepinfo. If you do not specify the steady-state response value yfinal, then stepinfo assumes that the last value in the response vector y is the steady-state response. Because there is some noise in the data, the last value in y is likely not the true steady-state response value. When you know what the steady-state value should be, you can provide it to stepinfo. For this example, suppose that the steady-state response is 2.4.

### S1 = stepinfo(y,t,2.4)

S1 = struct with fields:
 RiseTime: 1.2713
 SettlingTime: 19.6478
 SettlingMin: 2.0219
 SettlingMax: 3.3302

Overshoot: 38.7575 Undershoot: 0 Peak: 3.3302 PeakTime: 3.4000

Because of the noise in the data, the default definition of the settling time is too stringent, resulting in an arbitrary value of almost 20 seconds. To allow for the noise, increase the settling-time threshold from the default 2% to 5%.

## **Input Arguments**

Undershoot: 0

Peak: 3.3302 PeakTime: 3.4000

### sys — Dynamic system

dynamic system model

Dynamic system, specified as a SISO or MIMO dynamic system model. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.) For generalized models, stepinfo computes the step-response characteristics using the current value of tunable blocks and the nominal value of uncertain blocks.
- Identified LTI models, such as idtf, idss, or idproc models. (Using identified models requires System Identification Toolbox software.)

#### y — Step-response data

vector | array

Step-response data, specified as:

- For SISO response data, a vector of length Ns, where Ns is the number of samples in the response data.
- For MIMO response data, an Ns-by-Ny-by-Nu array, where Ny is the number of system outputs, and Nu is the number of system inputs.

#### t - Time vector

vector

Time vector corresponding to the response data in y, specified as a vector of length Ns.

### yfinal — Steady-state response

scalar | array

Steady-state response, specified as:

- For SISO response data, a scalar value.
- For MIMO response data, an Ny-by-Nu array, where each entry provides the steadystate response value for the corresponding system channel.

If you do not provide yfinal, then stepinfo uses the last value in the corresponding channel of y as the steady-state response value.

### ST — Threshold for defining settling time

0.02 (default) | scalar between 0 and 1

Threshold for defining settling time, specified as a scalar value between 0 and 1. By default, stepinfo defines settling time as the time it takes for the error  $|y(t) - y_{final}|$  between the response y(t) and the steady-state response  $y_{final}$  to fall to within 2% of  $y_{final}$ . To change this definition, set ST to a different value. For instance, to set a threshold of 5%, set ST to 0.05.

### RT — Thresholds for defining rise time

[0.1 0.9] (default) | 2-element row vector

Threshold for defining rise time, specified as a 2-element row vector of nondescending values between 0 and 1. By default, stepinfo defines rise time as the time it takes for the response to rise from 10% to 90% of the steady-state value  $y_{final}$ . To change this

definition, set RT to a different value. For instance, to define the rise time as the time it takes for the response to rise from 5% to 95% of the steady-state value, set RT to [0.05 0.95].

## **Output Arguments**

### **S — Step-response characteristics**

structure

Step-response characteristics of sys, returned as a structure containing the fields:

- RiseTime Time it takes for the response to rise from 10% to 90% of the steadystate response.
- SettlingTime Time it takes for the error  $|y(t) y_{final}|$  between the response y(t) and the steady-state response  $y_{final}$  to fall to within 2% of  $y_{final}$ .
- SettlingMin Minimum value of y(t) once the response has risen.
- SettlingMax Maximum value of y(t) once the response has risen.
- Overshoot Percentage overshoot, relative to  $y_{final}$ ).
- Undershoot Percentage undershoot.
- Peak Peak absolute value of y(t)
- PeakTime Time at which the peak value occurs.

For MIMO models or responses data, S is a structure array in which each entry contains the step-response characteristics of the corresponding I/O channel. For instance, if you provide a 3-input, 3-output model or array of response data, then S(2,3) contains the characteristics of the response from the third input to the second output. For an example, see "Step-Response Characteristics of MIMO System" on page 2-1202.

If sys is unstable, then all step-response characteristics are NaN, except for Peak and PeakTime, which are Inf.

### See Also

lsiminfo|step

Introduced in R2006a

# stepplot

Plot step response and return plot handle

## **Syntax**

```
h = stepplot(sys)
stepplot(sys,Tfinal)
stepplot(sys,t)
stepplot(sys1,sys2,...,sysN)
stepplot(sys1,sys2,...,sysN,Tfinal)
stepplot(sys1,sys2,...,sysN,t)
stepplot(AX,...)
stepplot(..., plotoptions)
stepplot(..., dataoptions)
```

## **Description**

h = stepplot(sys) plots the step response of the dynamic system model sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type

help timeoptions

for a list of available plot options.

For multiinput models, independent step commands are applied to each input channel. The time range and number of points are chosen automatically.

stepplot(sys, Tfinal) simulates the step response from t = 0 to the final time t = Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time (Ts = -1), stepplot interprets Tfinal as the number of sampling intervals to simulate.

stepplot(sys,t) uses the user-supplied time vector t for simulation. Express t in the
system time units, specified in the TimeUnit property of sys. For discrete-time models, t
should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time

models, t should be of the form Ti:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see step). The stepplot command always applies the step input at t=0, regardless of Ti.

To plot the step responses of multiple models sys1, sys2,... on a single plot, use:

```
stepplot(sys1,sys2,...,sysN)
stepplot(sys1,sys2,...,sysN,Tfinal)
stepplot(sys1,sys2,...,sysN,t)
You can also specify a color, line style, and marker for each system, as in
```

```
stepplot(sys1,'r',sys2,'y--',sys3,'qx')
stepplot(AX,...) plots into the axes with handle AX.
```

stepplot(..., plotoptions) customizes the plot appearance using the options set, plotoptions. Use timeoptions to create the options set.

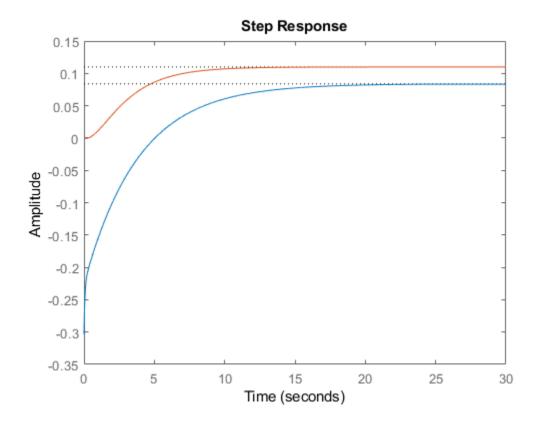
stepplot(..., dataoptions) specifies options such as the step amplitude and input offset using the options set, dataoptions. Use stepDataOptions to create the options set.

## **Examples**

### **Display Normalized Response on Step Plot**

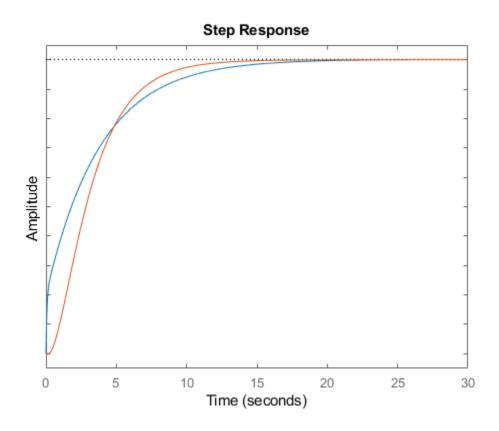
Generate a step response plot for two dynamic systems.

```
svs1 = rss(3);
sys2 = rss(3);
h = stepplot(sys1,sys2);
```



Each step response settles at a different steady-state value. Use the plot handle to normalize the plotted response.

```
setoptions(h,'Normalize','on')
```



Now, the responses settle at the same value expressed in arbitrary units.

### Plot Step Responses of Identified Models with Confidence Region

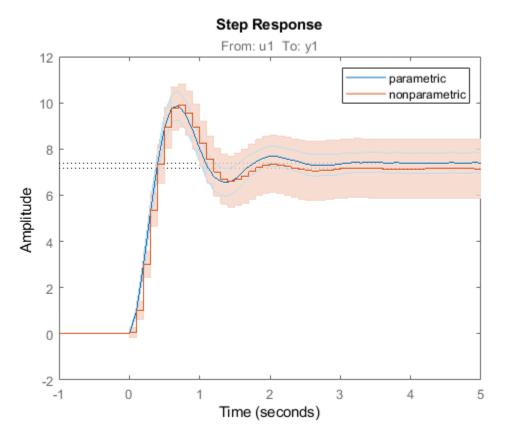
Compare the step response of a parametric identified model to a nonparametric (empirical) model, and view their 3- $\sigma$  confidence regions. (Identified models require System Identification Toolbox<sup>TM</sup> software.)

Identify a parametric and a nonparametric model from sample data.

```
load iddata1 z1
sys1 = ssest(z1,4);
sys2 = impulseest(z1);
```

Plot the step responses of both identified models. Use the plot handle to display the  $3\text{-}\sigma$  confidence regions.

```
t = -1:0.1:5;
h = stepplot(sys1,sys2,t);
showConfidence(h,3)
legend('parametric','nonparametric')
```



The nonparametric model sys2 shows higher uncertainty.

#### **Plot Step Response of Nonlinear Identified Model**

Load data for estimating a nonlinear Hammerstein-Wiener model.

```
load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'twotankdata'));
z = iddata(y,u,0.2, 'Name', 'Two tank system');
```

z is an iddata object that stores the input-output estimation data.

Estimate a Hammerstein-Wiener Model of order [1 5 3] using the estimation data. Specify the input nonlinearity as piecewise linear and output nonlinearity as one-dimensional polynomial.

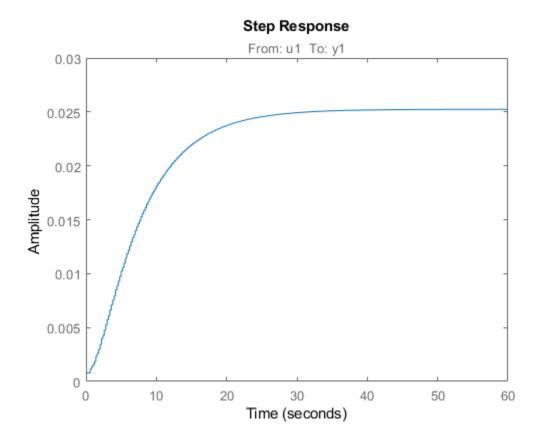
```
sys = nlhw(z,[1 5 3],pwlinear,poly1d);
```

Create an option set to specify input offset and step amplitude level.

```
opt = stepDataOptions('InputOffset',2,'StepAmplitude',0.5);
```

Plot the step response until 60 seconds using the specified options.

```
stepplot(sys,60,opt);
```



# **Tips**

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots".

## **See Also**

getoptions | setoptions | step

### **Introduced before R2006a**

# strseq

Create sequence of indexed character vectors

## **Syntax**

```
txtarray = strseq(TXT,INDICES)
```

## **Description**

txtarray = strseq(TXT,INDICES) creates a sequence of indexed character vectors
in the cell array txtarray by appending the integer values INDICES to the character
vector TXT.

**Note** You can use strvec to aid in system interconnection. For an example, see the sumblk reference page.

## **Examples**

### **Create a Cell Array of Indexed Text**

Index the text 'e' with the numbers 1, 2, and 4.

# **See Also**

connect | strcat

Introduced in R2008b

## sumblk

Summing junction for name-based interconnections

## **Syntax**

```
S = sumblk(formula)
S = sumblk(formula, signalsize)
S = sumblk(formula, signames1, signames2,...)
```

## **Description**

S = sumblk(formula) creates the transfer function, S, of the summing junction described by formula. The character vector formula specifies an equation that relates the scalar input and output signals of S.

S = sumblk(formula, signalsize) returns a vector-valued summing junction. The input and output signals are vectors with signalsize elements.

S = sumblk(formula, signames1, signames2,...) replaces aliases (signal names beginning with %) in formula by the signal names signames. The number of signames arguments must match the number of aliases in formula. The first alias in formula is replaced by signames1, the second by signames2, and so on.

## **Input Arguments**

#### formula

Equation that relates the input and output signals of the summing junction transfer function S, specified as a character vector. For example, the following command:

```
S = sumblk('e = r - y + d')
```

creates a summing junction with input names 'r', 'y', and 'd', output name 'e' and equation e = r-y+d.

If you specify a signalsize greater than 1, the inputs and outputs of S are vector-valued signals. sumblk automatically performs vector expansion of the signal names of S. For example, the following command:

```
S = sumblk('v = u + d',2)
```

specifies a summing junction with input names  $\{'u(1)'; 'u(2)'; 'd(1)'; 'd(2)'\}$  and output names  $\{'v(1)'; 'v(2)'\}$ . The formulas of this summing junction are v(1) = u(1) + d(1); v(2) = u(2) + d(2).

You can use one or more aliases in formula to refer to signal names defined in a variable. An alias is a signal name that begins with %. When formula contains aliases, sumblk replaces each alias with the corresponding signames argument.

Aliases are useful when you want to name individual entries in a vector-valued signal. Aliases also allow you to use input or output names of existing models. For example, if C and G are dynamic system models with nonempty InputName and OutputName properties, respectively, you can create a summing junction using the following expression.

```
S = sumblk('%e = r - %y',C.InputName,G.OutputName)
```

sumblk uses the values of C.InputName and G.OutputName in place of %e and %y, respectively. The vector dimension of C.InputName and G.OutputName must match. sumblk assigns the signal r the same dimension.

### signalsize

Number of elements in each input and output signal of S. Setting signalsize greater than 1 lets you specify a summing junction that operates on vector-valued signals.

#### Default: 1

#### signames

Signal names to replace one alias (signal name beginning with %) in the argument formula. You must provide one signames argument for each alias in formula.

Specify signames as:

- A cell array of signal names.
- The InputName or OutputName property of a model in the MATLAB workspace. For example:

```
S = sumblk('\%e = r - y', C.InputName)
```

This command creates a summing junction whose outputs have the same name as the inputs of the model C in the MATLAB workspace.

## **Output Arguments**

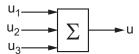
S

Transfer function for the summing junction, represented as a MIMO tf model object.

## **Examples**

#### **Summing Junction with Scalar-Valued Signals**

Create the summing junction of the following illustration. All signals are scalar-valued.



This summing junction has the formula u = u1 + u2 + u3.

```
S = sumblk('u = u1+u2+u3');
```

S is the transfer function (tf) representation of the sum u = u1 + u2 + u3. The transfer function S gets its input and output names from the formula.

S.OutputName, S. Inputname

```
ans =
'u'
ans =
'u1'
'u2'
'u3'
```

### **Summing Junction with Vector-Valued Signals**

Create the summing junction v = u - d where u,d,v are vector-valued signals of length 2.

```
S = sumblk('v = u-d',2);
```

sumblk automatically performs vector expansion of the signal names of S.

S.OutputName, S. Inputname

```
ans =

'v(1)'
'v(2)'

ans =

'u(1)'
'u(2)'
'd(1)'
'd(2)'
```

### **Summing Junction with Vector-Valued Signals That Have Specified Signal Names**

Create the summing junction

```
e(1) = setpoint(1) - alpha + d(1)e(2) = setpoint(2) - q + d(2)
```

The signals alpha and q have custom names that are not merely the vector expansion of a single signal name. Therefore, use an alias in the formula specifying the summing junction.

```
S = sumblk('e = setpoint - %y + d', {'alpha';'q'});
sumblk replaces the alias %y with the cell array {'alpha';'q'}.
S.OutputName, S.Inputname
ans =
    'e(1)'
    'e(2)'
```

```
ans =
    'setpoint(1)'
    'setpoint(2)'
    'alpha'
    'q'
    'd(1)'
    'd(2)'
```

# **Tips**

• Use sumblk in conjunction with connect to interconnect dynamic system models and derive aggregate models for block diagrams.

### See Also

connect|parallel|series|strseq

### **Topics**

"Multi-Loop Control System"
"MIMO Control System"

#### Introduced in R2008a

## systune

Tune fixed-structure control systems modeled in MATLAB

## **Syntax**

```
[CL,fSoft] = systune(CL0,SoftReqs)
[CL,fSoft,gHard] = systune(CL0,SoftReqs,HardReqs)
[CL,fSoft,gHard] = systune(CL0,SoftReqs,HardReqs,options)
[CL,fSoft,gHard,info] = systune( )
```

## **Description**

systune tunes fixed-structure control systems subject to both soft and hard design goals. systune can tune multiple fixed-order, fixed-structure control elements distributed over one or more feedback loops. For an overview of the tuning workflow, see "Automated Tuning Workflow".

This command tunes control systems modeled in MATLAB. For tuning Simulink models, use slTuner to create an interface to your Simulink model. You can then tune the control system with systune for slTuner (requires Simulink Control Design).

[CL,fSoft] = systune(CL0,SoftReqs) tunes the free parameters of the control system model, CL0, to best meet the soft tuning requirements. The best achieved soft constraint values are returned as fSoft. For robust tuning against real parameter uncertainty, use a control system model with uncertain real parameters. For robust tuning against a set of plant models, use an array of control system models CL0. (See "Input Arguments" on page 2-1234.)

[CL,fSoft,gHard] = systune(CL0,SoftReqs,HardReqs) tunes the control system to best meet the soft tuning requirements subject to satisfying the hard tuning requirements (constraints). It returns the best achieved values for the soft and hard constraints.

[CL,fSoft,gHard] = systume(CL0,SoftReqs,HardReqs,options) specifies options for the optimization.

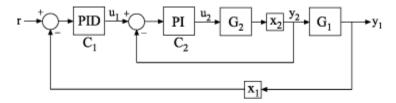
[CL,fSoft,gHard,info] = systume(\_\_\_\_) also returns detailed information about each optimization run. All input arguments described for the previous syntaxes also apply here.

## **Examples**

#### **Tune Control System to Soft Requirements**

Tune a cascaded control system to meet requirements of reference tracking and disturbance rejection.

The cascaded control system of the following illustration includes two tunable controllers, the PI controller for the inner loop,  $C_2$ , and the PID controller for the outer loop,  $C_1$ .



The blocks  $x_1$  and  $x_2$  mark analysis-point locations. These are locations at which loops can be opened or signals injected for the purpose of specifying requirements for tuning the system.

Tune the free parameters of this control system to meet the following requirements:

- The output signal,  $y_1$ , tracks the reference signal, r, with a response time of 10 seconds and a steady-state error of 1%.
- A disturbance injected at  $x_2$  is suppressed at  $y_1$  by a factor of 10.

Create tunable Control Design Blocks to represent the controllers, and numeric LTI models to represent the plants. Also, create AnalysisPoint blocks to mark the points of interest in each feedback loop.

```
G2 = zpk([],-2,3);
G1 = zpk([],[-1 -1 -1],10);
C20 = tunablePID('C2','pi');
```

```
C10 = tunablePID('C1','pid');
X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
```

Connect these components to build a model of the entire closed-loop control system.

```
InnerLoop = feedback(X2*G2*C20,1);
CL0 = feedback(G1*InnerLoop*C10,X1);
CL0.InputName = 'r';
CL0.OutputName = 'y';
```

CL0 is a tunable genss model. Specifying names for the input and output channels allows you to identify them when you specify tuning requirements for the system.

Specify tuning requirements for reference tracking and disturbance rejection.

```
Rtrack = TuningGoal.Tracking('r','y',10,0.01);
Rreject = TuningGoal.Gain('X2','y',0.1);
```

The TuningGoal.Tracking requirement specifies that the signal at 'y' track the signal at 'r' with a response time of 10 seconds and a tracking error of 1%.

The TuningGoal.Gain requirement limits the gain from the implicit input associated with the AnalysisPoint block, X2, to 'y'. (See AnalysisPoint.) Limiting this gain to a value less than 1 ensures that a disturbance injected at X2 is suppressed at the output.

Tune the control system.

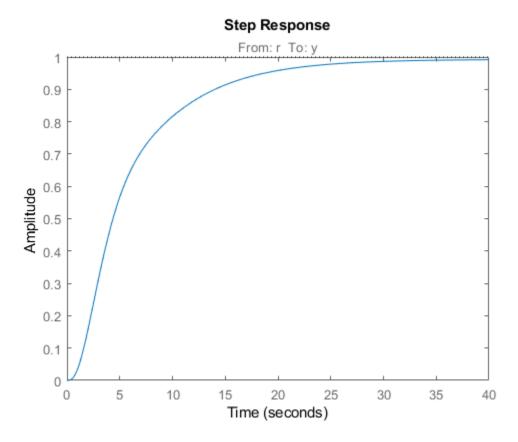
```
[CL,fSoft] = systume(CL0,[Rtrack,Rreject]);
Final: Soft = 1.24, Hard = -Inf, Iterations = 111
```

systune converts each tuning requirement into a normalized scalar value, f. The command adjusts the tunable parameters of CL0 to minimize the f values. For each requirement, the requirement is satisfied if f < 1 and violated if f > 1. fSoft is the vector of minimized f values. The largest of the minimized f values is displayed as f values.

The output model CL is the tuned version of CL0. CL contains the same Control Design Blocks as CL0, with current values equal to the tuned parameter values.

Validate that the tuned control system meets the tracking requirement by examining the step response from 'r' to 'y'.

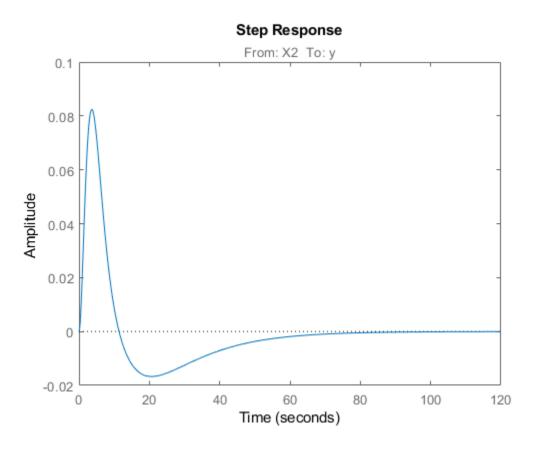
```
stepplot(CL)
```



The step plot shows that in the tuned control system, CL, the output tracks the input with approximately the desired response time.

Validate the tuned system against the disturbance rejection requirement by examining the closed-loop response to a signal injected at X2.

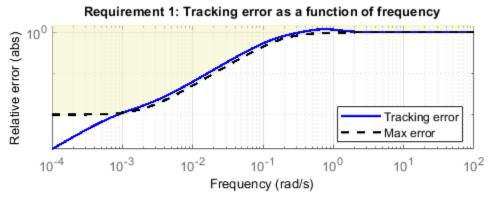
```
CLdist = getIOTransfer(CL, 'X2', 'y');
stepplot(CLdist);
```

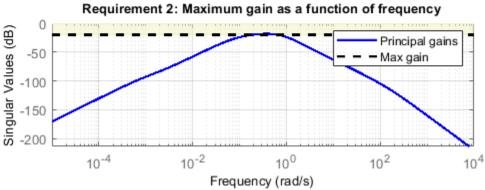


getIOTransfer extracts the closed-loop response from the specified inputs to outputs.
In general, getIOTransfer and getLoopTransfer are useful for validating a control
system tuned with systume.

You can also use viewGoal to compare the responses of the tuned control system directly against the tuning requirements, Rtrack and Rreject.

viewGoal([Rtrack,Rreject],CL)

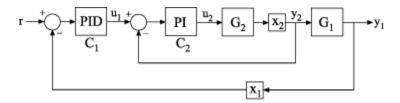




### **Tune Control System to Both Hard and Soft Requirements**

Tune a cascaded control system to meet requirements of reference tracking and disturbance rejection. These requirements are subject to a hard constraint on the stability margins of the inner and outer loops.

The cascaded control system of the following illustration includes two tunable controllers, the PI controller for the inner loop,  $C_2$ , and the PID controller for the outer loop,  $C_1$ .



The blocks  $x_1$  and  $x_2$  mark analysis-point locations. These are locations at which you can open loops or inject signals for the purpose of specifying requirements for tuning the system.

Tune the free parameters of this control system to meet the following requirements:

- The output signal,  $y_1$ , tracks the reference signal at r with a response time of 5 seconds and a steady-state error of 1%.
- A disturbance injected at  $x_2$  is suppressed at the output,  $y_1$ , by a factor of 10.

Impose these tuning requirements subject to hard constraints on the stability margins of both loops.

Create tunable Control Design Blocks to represent the controllers and numeric LTI models to represent the plants. Also, create AnalysisPoint blocks to mark the points of interest in each feedback loop.

```
G2 = zpk([],-2,3);
G1 = zpk([],[-1 -1 -1],10);

C20 = tunablePID('C2','pi');
C10 = tunablePID('C1','pid');

X1 = AnalysisPoint('X1');
X2 = AnalysisPoint('X2');
```

Connect these components to build a model of the entire closed-loop control system.

```
InnerLoop = feedback(X2*G2*C20,1);
CL0 = feedback(G1*InnerLoop*C10,X1);
CL0.InputName = 'r';
CL0.OutputName = 'y';
```

CL0 is a tunable genss model. Specifying names for the input and output channels allows you to identify them when you specify tuning requirements for the system.

Specify tuning requirements for reference tracking and disturbance rejection.

```
Rtrack = TuningGoal.Tracking('r','y',5,0.01);
Rreject = TuningGoal.Gain('X2','y',0.1);
```

The TuningGoal.Tracking requirement specifies that the signal at 'y' tracks the signal at 'r' with a response time of 5 seconds and a tracking error of 1%.

The TuningGoal.Gain requirement limits the gain from the implicit input associated with the AnalysisPoint block X2 to the output, 'y'. (See AnalysisPoint.) Limiting this gain to a value less than 1 ensures that a disturbance injected at X2 is suppressed at the output.

Specify tuning requirements for the gain and phase margins.

```
RmargOut = TuningGoal.Margins('X1',18,60);
RmargIn = TuningGoal.Margins('X2',18,60);
RmargIn.Openings = 'X1';
```

RmargOut imposes a minimum gain margin of 18 dB and a minimum phase margin of 60 degrees. Specifying X1 imposes that requirement on the outer loop. Similarly, RmargIn imposes the same requirements on the inner loop, identified by X2. To ensure that the inner-loop margins are evaluated with the outer loop open, include the outer-loop analysis-point location, X1, in RmargIn.Openings.

Tune the control system to meet the soft requirements of tracking and disturbance rejection, subject to the hard constraints of the stability margins.

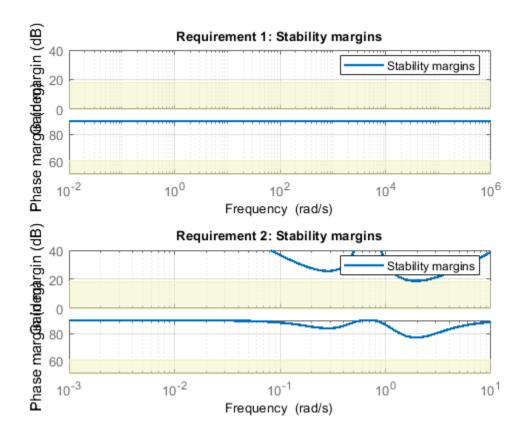
```
SoftReqs = [Rtrack,Rreject];
HardReqs = [RmargIn,RmargOut];
[CL,fSoft,gHard] = systune(CL0,SoftReqs,HardReqs);
Final: Soft = 1.13, Hard = 0.97742, Iterations = 112
```

systune converts each tuning requirement into a normalized scalar value, f for the soft constraints and g for the hard constraints. The command adjusts the tunable parameters of CL0 to minimize the f values, subject to the constraint that each g < 1.

The displayed value Hard is the largest of the minimized g values in gHard. This value is less than 1, indicating that both the hard constraints are satisfied.

Validate the tuned control system against the stability margin requirements.

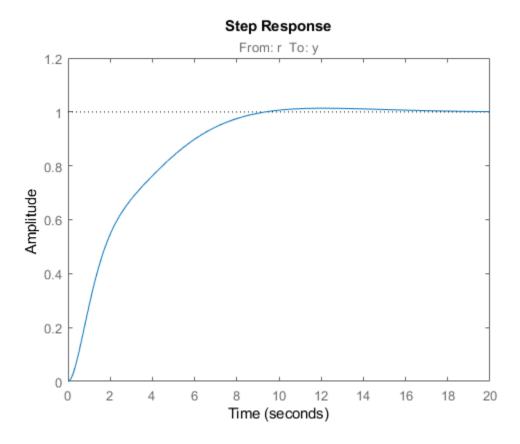




The viewGoal plot confirms that the stability margin requirements for both loops are satisfied by the tuned control system at all frequencies. The blue lines show the margin used in the optimization calculation, which is an upper bound on the actual margin of the tuned control system.

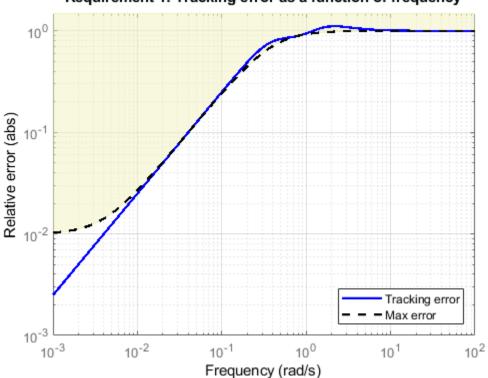
Examine whether the tuned control system meets the tracking requirement by examining the step response from 'r' to 'y'.

```
figure;
stepplot(CL,20)
```



The step plot shows that in the tuned control system, CL, the output tracks the input but the response is somewhat slower than desired and the tracking error may be larger than desired. For further information, examine the tracking requirement directly with viewGoal.

figure; viewGoal(Rtrack,CL)



Requirement 1: Tracking error as a function of frequency

The actual tracking error crosses into the shaded area between 1 and 10 rad/s, indicating that the requirement is not met in this regime. Thus, the tuned control system cannot meet the soft tracking requirement, time subject to the hard constraints of the stability margins. To achieve the desired performance, you may need to relax one of your requirements or convert one or more hard constraints to soft constraints.

## **Input Arguments**

### CL0 — Control system to tune

generalized state-space model | model array

Control system to tune, specified as a generalized state-space (genss) model or array of models with tunable parameters. To construct CL0:

- Parameterize the tunable elements of your control system. You can use predefined structures, such as tunablePID, tunableGain, and tunableTF. Alternatively, you can create your own structure from elementary tunable parameters (realp).
- 2 Build a closed-loop model of the overall control system as an interconnection of fixed and tunable components. To do so, use model interconnection commands such as feedback and connect. Use AnalysisPoint blocks to mark additional signals of interest for specifying and assessing tuning requirements.

For more information about creating models to tune, see "Setup for Tuning Control System Modeled in MATLAB".

For robust tuning of a control system against a set of plant models (requires Robust Control Toolbox), specify an array of tunable genss models that have the same tunable parameters. To make the controller robust against parameter uncertainty, use a model with uncertain real parameters defined with ureal or uss. In this case, CLO is a genss model that contains both tunable and uncertain control design blocks. For more information about robust tuning, see "Robust Tuning Approaches" (Robust Control Toolbox).

### SoftReqs — Soft tuning goals (objectives)

vector of TuningGoal objects

Soft tuning goals (objectives) for tuning the control system, specified as a vector of TuningGoal objects. These objects capture your design requirements, such as TuningGoal.Tracking, TuningGoal.StepTracking, or TuningGoal.Margins.

systune tunes the tunable parameters of the control system to minimize the soft tuning goals. This tuning is subject to satisfying the hard tuning goals (if any).

For more information about available tuning goals, see "Tuning Goals".

### HardReqs — Hard tuning goals (constraints)

[] (default) | vector of TuningGoal objects

Hard tuning goals (constraints) for tuning the control system, specified as a vector of TuningGoal objects. These objects capture your design requirements, such as TuningGoal.Tracking, TuningGoal.StepTracking, or TuningGoal.Margins.

systune converts each hard tuning goal to a normalized scalar value. systune then optimizes the free parameters to minimize those normalized values. A hard goal is satisfied if the normalized value is less than 1.

For more information about available tuning goals, see "Tuning Goals".

### options — Options for tuning algorithm

systuneOptions object

Options for the tuning algorithm, specified as an options set you create with systuneOptions. Available options include:

- Number of additional optimizations to run. Each optimization starts from random initial values of the free parameters.
- Tolerance for terminating the optimization.
- Flag for using parallel processing.

See the systumeOptions reference page for more details about all available options.

## **Output Arguments**

### **CL** — Tuned control system

generalized state-space model

Tuned control system, returned as a generalized state-space (genss) model. This model has the same number and type of tunable elements (Control Design Blocks) as CL0. The current values of these elements are the tuned parameters. Use getBlockValue or showTunable to access values of the tuned elements.

If you provide an array of control system models to tune as the input argument, CL0, systune tunes the parameters of all the models simultaneously. In this case, CL is an array of tuned genss models. For more information, see "Robust Tuning Approaches" (Robust Control Toolbox).

#### fSoft — Best achieved soft constraint values

vector

Best achieved soft constraint values, returned as a vector. systune converts the soft requirements to a function of the free parameters of the control system. The command then tunes the parameters to minimize that function subject to the hard constraints. (See

"Algorithms" on page 2-1240.) fSoft contains the best achieved value for each of the soft constraints. These values appear in fSoft in the same order that the constraints are specified in SoftReqs. fSoft values are meaningful only when the hard constraints are satisfied.

#### gHard — Best achieved hard constraint values

vector

Best achieved hard constraint values, returned as a vector. systune converts the hard requirements to a function of the free parameters of the control system. The command then tunes the parameters to drive those values below 1. (See "Algorithms" on page 2-1240.) gHard contains the best achieved value for each of the hard constraints. These values appear in gHard in the same order that the constraints are specified in HardReqs. If all values are less than 1, then the hard constraints are satisfied.

### info — Detailed information about optimization runs

structure

Detailed information about each optimization run, returned as a data structure. The fields of info are:

#### Run - Run number

scalar

Run number, returned as a scalar. If you use the RandomStart option of systuneOptions to perform multiple optimization runs, info is a struct array, and info.Run is the index.

#### Iterations — Total number of iterations

scalar

Total number of iterations performed during run, returned as a scalar. This value is the number of iterations performed in each run before the optimization terminates.

#### fBest — Best overall soft constraint value

scalar

Best overall soft constraint value, returned as a scalar. systume converts the soft requirements to a function of the free parameters of the control system. The command then tunes the parameters to minimize that function subject to the hard constraints. (See "Algorithms" on page 2-1240.) info.fBest is the maximum soft constraint value at the final iteration. This value is meaningful only when the hard constraints are satisfied.

### gBest — Best overall hard constraint value

scalar

Best overall hard constraint value, returned as a scalar. systune converts the hard requirements to a function of the free parameters of the control system. The command then tunes the parameters to drive those values below 1. (See "Algorithms" on page 2-1240.) info.gBest is the maximum hard constraint value at the final iteration. This value must be less than 1 for the hard constraints to be satisfied.

#### fSoft — Individual soft constraint values

vector

Individual soft constraint values, returned as a vector. systume converts each soft requirement to a normalized value that is a function of the free parameters of the control system. The command then tunes the parameters to minimize that value subject to the hard constraints. (See "Algorithms" on page 2-1240.) info.fSoft contains the individual values of the soft constraints at the end of each run. These values appear in fSoft in the same order that the constraints are specified in SoftReqs.

#### gHard — Individual hard constraint values

vector

Individual hard constraint values, returned as a vector. systume converts each hard requirement to a normalized value that is a function of the free parameters of the control system. The command then tunes the parameters to minimize those values. A hard requirement is satisfied if its value is less than 1. (See "Algorithms" on page 2-1240.) info.gHard contains the individual values of the hard constraints at the end of each run. These values appear in gHard in the same order that the constraints are specified in HardReqs.

### MinDecay — Minimum decay rate of closed-loop poles

vector

Minimum decay rate of closed-loop poles, returned as a vector.

By default, closed-loop pole locations of the tuned system are constrained to satisfy  $Re(p) < -10^{-7}$ . Use the MinDecay option of systuneOptions to change this constraint.

### Blocks — Tuned values of tunable blocks and parameters

structure

Tuned values of tunable blocks and parameters in the tuned control system, CL, returned as a structure. You can also use getBlockValue or showBlockValue to access the tuned parameter values.

# **LoopScaling — Optimal diagonal scaling for MIMO tuning requirements** state-space model

Optimal diagonal scaling for evaluating MIMO tuning requirements, returned as a state-space model.

When applied to multiloop control systems, TuningGoal.LoopShape and TuningGoal.Margins can be sensitive to the scaling of the loop transfer functions to which they apply. This sensitivity can lead to poor optimization results. systune automatically corrects scaling issues and returns the optimal diagonal scaling matrix d as a state-space model in info.LoopScaling.

The loop channels associated with each diagonal entry of D are listed in info.LoopScaling.InputName. The scaled loop transfer is D\L\*D, where L is the open-loop transfer measured at the locations info.LoopScaling.InputName.

### wcPert — Worst combinations of uncertain parameters

structure array

Worst combinations of uncertain parameters, returned as a structure array. (Applies for robust tuning of control systems with uncertainty only.) Each structure contains one set of uncertain parameter values. The perturbations with the worst performance are listed first.

### wcf — Worst objective value

positive scalar

Largest soft goal value over the uncertainty range when using the tuned controller. (Applies for robust tuning of control systems with uncertainty only.)

#### wcg — Worst constraint value

positive scalar

Largest hard goal value over the uncertainty range when using the tuned controller. (Applies for robust tuning of control systems with uncertainty only.)

#### wcDecay — Worst decay rate

scalar

Smallest closed-loop decay rate over the uncertainty range when using the tuned controller. (Applies for robust tuning of control systems with uncertainty only.) A positive value indicates robust stability. See MinDecay option in systumeOptions for details.

# **Algorithms**

x is the vector of tunable parameters in the control system to tune. systune converts each soft and hard tuning requirement SoftReqs(i) and HardReqs(j) into normalized values  $f_i(x)$  and  $g_j(x)$ , respectively. systune then solves the constrained minimization problem:

```
Minimize \max_{i} f_i(x) subject to \max_{j} g_j(x) < 1, for x_{\min} < x < x_{\max}.
```

 $x_{min}$  and  $x_{max}$  are the minimum and maximum values of the free parameters of the control system.

When you use both soft and hard tuning goals, the software approaches this optimization problem by solving a sequence of unconstrained subproblems of the form:

```
\min_{x} \max(\alpha f(x), g(x)).
```

The software adjusts the multiplier  $\alpha$  so that the solution of the subproblems converges to the solution of the original constrained optimization problem.

systume returns the control system with parameters tuned to the values that best solve the minimization problem. systume also returns the best achieved values of  $f_i(x)$  and  $g_i(x)$ , as fSoft and gHard respectively.

For information about the functions  $f_i(x)$  and  $g_j(x)$  for each type of constraint, see the reference pages for each TuningGoal requirement object.

systune uses the nonsmooth optimization algorithms described in [1],[2],[3],[4]

systune computes the  $H_{\infty}$  norm using the algorithm of [5]and structure-preserving eigensolvers from the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

# **Alternative Functionality**

## **App**

The **Control System Tuner** app provides a graphical interface to control system tuning.

# **Compatibility Considerations**

# **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

### References

- [1] Apkarian, P. and D. Noll, "Nonsmooth H-infinity Synthesis," *IEEE Transactions on Automatic Control*, Vol. 51, No. 1, (2006), pp. 71–86.
- [2] Apkarian, P. and D. Noll, "Nonsmooth Optimization for Multiband Frequency-Domain Control Design," *Automatica*, 43 (2007), pp. 724–731.
- [3] Apkarian, P., P. Gahinet, and C. Buhr, "Multi-model, multi-objective tuning of fixed-structure controllers," *Proceedings ECC* (2014), pp. 856–861.
- [4] Apkarian, P., M.-N. Dao, and D. Noll, "Parametric Robust Structured Control Design," *IEEE Transactions on Automatic Control*, 2015.
- [5] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the  $H_{\infty}$ -Norm of a Transfer Function Matrix," *System Control Letters*, Vol. 14, No, 4 (1990), pp. 287–293.

# **Extended Capabilities**

# **Automatic Parallel Support**

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox  $^{\text{\tiny TM}}$ .

To run in parallel, set 'UseParallel' to true using systumeOptions.

For more information, see "Speed Up Tuning with Parallel Computing Toolbox Software".

# See Also

AnalysisPoint | TuningGoal.Gain | TuningGoal.Margins | TuningGoal.Tracking | genss | looptune | looptune (for slTuner) | slTuner | systume (for slTuner) | systumeOptions | viewGoal

# **Topics**

"Tuning Control Systems with SYSTUNE"

"Building Tunable Models"

"Programmatic Tuning"

"Generalized Models"

#### Introduced in R2016a

<sup>&</sup>quot;Robust Tuning Approaches" (Robust Control Toolbox)

# systuneOptions

Set options for systune

# **Syntax**

```
options = systuneOptions
options = systuneOptions(Name, Value)
```

# **Description**

options = systumeOptions returns the default option set for the systume command.

options = systuneOptions(Name, Value) creates an option set with the options
specified by one or more Name, Value pair arguments.

# **Input Arguments**

## **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

systuneOptions takes the following Name arguments:

## Display

Amount of information to display during systune runs.

Display takes the following values:

'final' — Display a one-line summary at the end of each optimization run. The
display includes the best achieved values for the soft and hard constraints, fSoft and
gHard. The display also includes the number of iterations for each run.

### Example:

```
Final: Soft = 1.09, Hard = 0.68927, Iterations = 58
```

• 'sub' — Display the result of each optimization subproblem.

When you use both soft and hard tuning goals, the software solves the optimization as a sequence of subproblems of the form:

```
\min_{x} \max(\alpha f(x), g(x)).
```

Here, x is the vector of tunable parameters, f(x) is the largest normalized soft-constraint value, and g(x) is the largest normalized hard-constraint value. (See the "Algorithms" section of the systume reference page for more information.) The software adjusts the multiplier  $\alpha$  so that the solution of the subproblems converges to the solution of the original constrained optimization problem. When you select 'sub', the report includes the results of each of these subproblems.

### Example:

```
alpha=0.1: Soft = 3.97, Hard = 0.68927, Iterations = 8 alpha=0.5036: Soft = 1.36, Hard = 0.68927, Iterations = 8 alpha=1.47: Soft = 1.09, Hard = 0.68927, Iterations = 42 Final: Soft = 1.09, Hard = 0.68927, Iterations = 58
```

• 'iter' — Display optimization progress after each iteration. The display includes the value after each iteration of the objective parameter being minimized. The objective parameter is whichever is larger of  $\alpha f(x)$  and g(x). The display also includes a progress value that indicates the percent change in the constraints from the previous iteration.

### Example:

```
Iter 1: Objective = 4.664, Progress = 93%
Iter 2: Objective = 2.265, Progress = 51.4%
Iter 3: Objective = 0.7936, Progress = 65%
Iter 4: Objective = 0.7183, Progress = 9.48%
Iter 5: Objective = 0.6893, Progress = 4.04%
Iter 6: Objective = 0.6893, Progress = 0%
Iter 7: Objective = 0.6893, Progress = 0%
Iter 8: Objective = 0.6893, Progress = 0%
alpha=0.1: Soft = 3.97, Hard = 0.68927, Iterations = 8
Iter 1: Objective = 1.146, Progress = 42.7%
Iter 2: Objective = 1.01, Progress = 11.9%
...
```

```
alpha=1.47: Soft = 1.09, Hard = 0.68927, Iterations = 42 Final: Soft = 1.09, Hard = 0.68927, Iterations = 58
```

• 'off' — Run in silent mode, displaying no information during or after the run.

Default: 'final'

#### MaxIter

Maximum number of iterations in each optimization run, when the run does not converge to within tolerance.

Default: 300

### RandomStart

Number of additional optimizations starting from random values of the free parameters in the controller.

If RandomStart = 0, systune performs a single optimization run starting from the initial values of the tunable parameters. Setting RandomStart = N > 0 runs N additional optimizations starting from N randomly generated parameter values.

systune tunes by finding a local minimum of a gain minimization problem. To increase the likelihood of finding parameter values that meet your design requirements, set RandomStart > 0. You can then use the best design that results from the multiple optimization runs.

Use with UseParallel = true to distribute independent optimization runs among MATLAB workers (requires Parallel Computing Toolbox software).

**Default:** 0

#### UseParallel

Parallel processing flag.

Set to true to enable parallel processing by distributing randomized starts among workers in a parallel pool. If there is an available parallel pool, then the software performs independent optimization runs concurrently among workers in that pool. If no parallel pool is available, one of the following occurs:

- If **Automatically create a parallel pool** is selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), then the software starts a parallel pool using the settings in those preferences.
- If **Automatically create a parallel pool** is not selected in your preferences, then the software performs the optimization runs successively, without parallel processing.

If **Automatically create a parallel pool** is not selected in your preferences, you can manually start a parallel pool using parpool before running the tuning command.

Using parallel processing requires Parallel Computing Toolbox software.

### Default: false

### **SkipModels**

Models or design points to ignore, specified as an array of linear indices.

Use this option to skip specific models or ignore portions of the design space when tuning gain-scheduled control systems. For example, you might want to skip grid points outside the flight envelope of an airplane model, or points outside the operating range for tuning. Identify the models to skip by absolute index in the array of models to tune. Using SkipModels lets you narrow the scope of tuning without reconfiguring each tuning goal. For more information, see "Change Requirements with Operating Condition".

#### Default: []

### SoftTarget

Target value for soft constraints.

The optimization stops when the largest soft constraint value falls below the specified SoftTarget value. The default value SoftTarget = 0 minimizes the soft constrains subject to satisfying the hard constraints.

#### **Default:** 0

#### SoftTol

Relative tolerance for termination.

The optimization terminates when the relative decrease in the soft constraint value decreases by less than SoftTol over 10 consecutive iterations. Increasing SoftTol speeds up termination, and decreasing SoftTol yields tighter final values.

#### **Default:** 0.001

#### SoftScale

A priori estimate of best soft constraint value.

For problems that mix soft and hard constraints, providing a rough estimate of the optimal value of the soft constraints (subject to the hard constraints) helps to speed up the optimization.

#### **Default:** 1

### MinDecay

Minimum decay rate for stabilized dynamics.

Most tuning goals carry an implicit closed-loop stability or minimum-phase constraint. Stabilized dynamics refers to the poles and zeros affected by these constraints. The MinDecay option constrains all stabilized poles and zeros to satisfy:

- Re(s) < -MinDecay (continuous time).
- log(|z|) < -MinDecay (discrete time).

Adjust the minimum value if the optimization fails to meet the default value, or if the default value conflicts with other requirements. Alternatively, use TuningGoal.Poles to control the decay rate of a specific feedback loop.

For more information about implicit constraints for a particular tuning goal, see the reference page for that tuning goal.

#### Default: 1e-7

#### MaxRadius

Maximum spectral radius for stabilized dynamics.

This option constrains all stabilized poles and zeros to satisfy <code>|s| < MaxRadius</code>. Stabilized dynamics are those poles and zeros affected by implicit stability or minimumphase constraints of the tuning goals. The <code>MaxRadius</code> constraint is useful to prevent these poles and zeros from going to infinity as a result of algebraic loops becoming singular or control effort growing unbounded. Adjust the maximum radius if the optimization fails to meet the default value, or if the default value conflicts with other requirements.

MaxRadius is ignored for discrete-time tuning, where stability constraints already impose |z| < 1.

For more information about implicit constraints for a particular tuning goal, see the reference page for that tuning goal.

Default: 1e8

# **Output Arguments**

### options

Option set containing the specified options for the systune command.

# **Examples**

### **Create Options Set for systune**

Create an options set for a systune run using five random restarts. Also, set the display level to show the progress of each iteration, and increase the relative tolerance of the soft constraint value to 0.01.

Alternatively, use dot notation to set the values of options.

```
options = systuneOptions;
options.RandomStart = 5;
options.Display = 'iter';
options.SoftTol = 0.01;
```

## **Configure Option Set for Parallel Optimization Runs**

Configure an option set for a systune run using 20 random restarts. Execute these independent optimization runs concurrently on multiple workers in a parallel pool.

If you have the Parallel Computing Toolbox software installed, you can use parallel computing to speed up systune tuning of fixed-structure control systems. When you run multiple randomized systune optimization starts, parallel computing speeds up tuning by distributing the optimization runs among workers.

If **Automatically create a parallel pool** is not selected in your Parallel Computing Toolbox preferences (Parallel Computing Toolbox), manually start a parallel pool using parpool. For example:

```
parpool;
```

If **Automatically create a parallel pool** is selected in your preferences, you do not need to manually start a pool.

Create a systuneOptions set that specifies 20 random restarts to run in parallel.

```
options = systumeOptions('RandomStart', 20, 'UseParallel', true);
```

Setting UseParallel to true enables parallel processing by distributing the randomized starts among available workers in the parallel pool.

Use the systuneOptions set when you call systune. For example, suppose you have already created a tunable control system model, CLO. For tuning this system, you have created vectors SoftReqs and HardReqs of TuningGoal requirements objects. These vectors represent your soft and hard constraints, respectively. In that case, the following command uses parallel computing to tune the control system of CLO.

```
[CL,fSoft,gHard] = systume(CL0,SoftReqs,HardReqs,options);
```

# **Compatibility Considerations**

## **Functionality moved from Robust Control Toolbox**

Behavior changed in R2016a

Prior to R2016a, this functionality required a Robust Control Toolbox license.

## See Also

systune | systune (for slTuner)

### Introduced in R2016a

# tf

Transfer function model

# **Description**

Use tf to create real-valued or complex-valued transfer function models, or to convert dynamic system models to transfer function form.

Transfer functions are a frequency-domain representation of linear time-invariant systems. For instance, consider a continuous-time SISO dynamic system represented by the transfer function sys(s) = N(s)/D(s), where s = jw and N(s) and D(s) are called the numerator and denominator polynomials, respectively. The tf model object can represent SISO or MIMO transfer functions in continuous time or discrete time.

You can create a transfer function model object either by specifying its coefficients directly, or by converting a model of another type (such as a state-space model ss) to transfer-function form. For more information, see "Transfer Functions".

You can also use tf to create generalized state-space (genss) models or uncertain state-space (uss) models.

## Creation

# **Syntax**

```
sys = tf(numerator,denominator)
sys = tf(numerator,denominator,ts)
sys = tf(numerator,denominator,ltiSys)
sys = tf(m)
sys = tf(____,Name,Value)
sys = tf(ltiSys)
sys = tf(ltiSys,component)
```

```
s = tf('s')
z = tf('z',ts)
```

## **Description**

sys = tf(numerator, denominator) creates a continuous-time transfer function model, setting the Numerator and Denominator properties. For instance, consider a continuous-time SISO dynamic system represented by the transfer function sys(s) = N(s)/D(s), the input arguments numerator and denominator are the coefficients of N(s) and D(s), respectively.

sys = tf(numerator,denominator,ts) creates a discrete-time transfer function model, setting the Numerator, Denominator, and Ts properties. For instance, consider a discrete-time SISO dynamic system represented by the transfer function sys(z) = N(z)/D(z), the input arguments numerator and denominator are the coefficients of N(z) and D(z), respectively. To leave the sample time unspecified, set ts input argument to -1.

sys = tf(numerator, denominator, ltiSys) creates a transfer function model with properties inherited from the dynamic system model ltiSys, including the sample time.

sys = tf(m) creates a transfer function model that represents the static gain, m.

sys = tf(\_\_\_\_, Name, Value) sets properties of the transfer function model using one
or more Name, Value pair arguments for any of the previous input-argument
combinations.

sys = tf(ltiSys) converts the dynamic system model ltiSys to a transfer function model.

sys = tf(ltiSys,component) converts the specified component of ltiSys to transfer function form. Use this syntax only when ltiSys is an identified linear time-invariant (LTI) model.

s = tf('s') creates special variable s that you can use in a rational expression to create a continuous-time transfer function model. Using a rational expression can sometimes be easier and more intuitive than specifying polynomial coefficients.

z = tf('z',ts) creates special variable z that you can use in a rational expression to create a discrete-time transfer function model. To leave the sample time unspecified, set ts input argument to -1.

## **Input Arguments**

#### numerator — Numerator coefficients of the transfer function

row vector | Ny-by-Nu cell array of row vectors

Numerator coefficients of the transfer function, specified as:

- A row vector of polynomial coefficients.
- An Ny-by-Nu cell array of row vectors to specify a MIMO transfer function, where Ny is the number of outputs, and Nu is the number of inputs.

When you create the transfer function, specify the numerator coefficients in order of descending power. For instance, if the transfer function numerator is 3s^2-4s+5, then specify numerator as [3 -4 5]. For a discrete-time transfer function with numerator 2z-1, set numerator to [2 -1].

Also a property of the tf object. For more information, see Numerator.

#### denominator — Denominator coefficients of the transfer function

row vector | Ny-by-Nu cell array of row vectors

Denominator coefficients, specified as:

- A row vector of polynomial coefficients.
- An Ny-by-Nu cell array of row vectors to specify a MIMO transfer function, where Ny is the number of outputs and Nu is the number of inputs.

When you create the transfer function, specify the denominator coefficients in order of descending power. For instance, if the transfer function denominator is  $7s^2+8s-9$ , then specify denominator as  $[7\ 8\ -9]$ . For a discrete-time transfer function with denominator  $2z^2+1$ , set denominator to  $[2\ 0\ 1]$ .

Also a property of the tf object. For more information, see Denominator.

## ts — Sample time

scalar

Sample time, specified as a scalar. Also a property of the tf object. For more information, see Ts.

### ltiSys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, ss, or pid models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

The resulting transfer function assumes

- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.
- Frequency-response data models such as frd models.
- Identified LTI models, such as idtf, idss, idproc, idpoly, and idgrey models. To
  select the component of the identified model to convert, specify component. If you do
  not specify component, tf converts the measured component of the identified model
  by default. (Using identified models requires System Identification Toolbox software.)

### m — Static gain

scalar | matrix

Static gain, specified as a scalar or matrix. Static gain or steady state gain of a system represents the ratio of the output to the input under steady state condition.

## component — Component of identified model

```
'measured' (default) | 'noise' | 'augmented'
```

Component of identified model to convert, specified as one of the following:

- 'measured' Convert the measured component of sys.
- ullet 'noise' Convert the noise component of sys
- 'augmented' Convert both the measured and noise components of  ${\tt sys}.$

component only applies when sys is an identified LTI model.

For more information on identified LTI models and their measured and noise components, see "Identified LTI Models".

## **Output Arguments**

### sys — Output system model

tf model object | genss model object | uss model object

Output system model, returned as:

- A transfer function (tf) model object, when numerator and denominator input arguments are numeric arrays.
- A generalized state-space model (genss) object, when the numerator or denominator input arguments includes tunable parameters, such as realp parameters or generalized matrices (genmat). For an example, see "Tunable Low-Pass Filter" on page 2-1277.
- An uncertain state-space model (uss) object, when the numerator or denominator input arguments includes uncertain parameters. Using uncertain models requires Robust Control Toolbox software. For an example, see "Transfer Function with Uncertain Coefficients" (Robust Control Toolbox).

# **Properties**

#### Numerator — Numerator coefficients

row vector | Ny-by-Nu cell array of row vectors

Numerator coefficients, specified as:

- A row vector of polynomial coefficients in order of descending power (for Variable values 's', 'z', 'p', or 'q') or in order of ascending power (for Variable values 'z^-1' or 'q^-1').
- An Ny-by-Nu cell array of row vectors to specify a MIMO transfer function, where Ny is
  the number of outputs and Nu is the number of inputs. Each element of the cell array
  specifies the numerator coefficients for a given input/output pair. If you specify both
  Numerator and Denominator as cell arrays, they must have the same dimensions.

The coefficients of Numerator can be either real-valued or complex-valued.

#### **Denominator** — **Denominator coefficients**

row vector | Ny-by-Nu cell array of row vectors

Denominator coefficients, specified as:

- A row vector of polynomial coefficients in order of descending power (for values Variable values 's', 'z', 'p', or 'q') or in order of ascending power (for Variable values 'z^-1' or 'q^-1').
- An Ny-by-Nu cell array of row vectors to specify a MIMO transfer function, where Ny is
  the number of outputs and Nu is the number of inputs. Each element of the cell array
  specifies the numerator coefficients for a given input/ output pair. If you specify both
  Numerator and Denominator as cell arrays, they must have the same dimensions.

If all SISO entries of a MIMO transfer function have the same denominator, you can specify **Denominator** as the row vector while specifying **Numerator** as a cell array.

The coefficients of Denominator can be either real-valued or complex-valued.

# Variable — Transfer function display variable

```
's' (default) | 'z' | 'p' | 'q' | 'z^-1' | 'q^-1'
```

Transfer function display variable, specified as one of the following:

- 's' Default for continuous-time models
- 'z' Default for discrete-time models
- 'p' Equivalent to 's'
- 'q' Equivalent to 'z'
- 'z^-1' Inverse of 'z'
- 'q^-1' Equivalent to 'z^-1'

The value of Variable is reflected in the display, and also affects the interpretation of the Numerator and Denominator coefficient vectors for discrete-time models.

- For Variable values 's', 'z', 'p', or 'q', the coefficients are ordered in descending powers of the variable. For example, consider the row vector [ak ... al a0]. The polynomial order is specified as  $a_k z^k + ... + a_1 z + a_0$ .
- For Variable values 'z^-1' or 'q^-1', the coefficients are ordered in ascending powers of the variable. For example, consider the row vector [b0 b1 ... bk]. The polynomial order is specified as  $b_0 + b_1 z^{-1} + ... + b_k z^{-k}$ .

For examples, see "Specify Polynomial Ordering in Discrete-Time Transfer Function" on page 2-1275, "Transfer Function Model Using Rational Expression" on page 2-1269, and "Discrete-Time Transfer Function Model Using Rational Expression" on page 2-1269.

### **IODelay — Transport delay**

0 (default) | scalar | Ny-by-Nu array

Transport delay, specified as one of the following:

- Scalar Specify the transport delay for a SISO system or the same transport delay for all input/output pairs of a MIMO system.
- Ny-by-Nu array Specify separate transport delays for each input/output pair of a MIMO system. Here, Ny is the number of outputs and Nu is the number of inputs.

For continuous-time systems, specify transport delays in the time unit specified by the TimeUnit property. For discrete-time systems, specify transport delays in integer multiples of the sample time, Ts.

### InputDelay — Input delay

0 (default) | scalar | Nu-by-1 vector

Input delay for each input channel, specified as one of the following:

- Scalar Specify the input delay for a SISO system or the same delay for all inputs of a multi-input system.
- Nu-by-1 vector Specify separate input delays for input of a multi-input system, where Nu is the number of inputs.

For continuous-time systems, specify input delays in the time unit specified by the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time, Ts.

## OutputDelay — Output delay

0 (default) | scalar | Ny-by-1 vector

Output delay for each output channel, specified as one of the following:

- Scalar Specify the output delay for a SISO system or the same delay for all outputs of a multi-output system.
- $\bullet\,$  Ny-by-1 vector Specify separate output delays for output of a multi-output system, where Ny is the number of outputs.

For continuous-time systems, specify output delays in the time unit specified by the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time, Ts.

### Ts — Sample time

0 (default) | positive scalar | -1

Sample time, specified as:

- 0 for continuous-time systems.
- A positive scalar representing the sampling period of a discrete-time system. Specify
  Ts in the time unit specified by the TimeUnit property.
- -1 for a discrete-time system with an unspecified sample time.

Changing Ts does not discretize or resample the model. To convert between continuoustime and discrete-time representations, use c2d and d2c. To change the sample time of a discrete-time system, use d2d.

#### TimeUnit — Time variable units

```
'seconds' (default) | 'nanoseconds' | 'microseconds' | 'milliseconds' |
'minutes' | 'hours' | 'days' | 'weeks' | 'months' | 'years' | ...
```

Time variable units, specified as one of the following:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'davs'
- 'weeks'
- 'months'
- 'years'

Changing TimeUnit has no effect on other properties, but changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

### InputName — Input channel names

' ' (default) | character vector | cell array of character vectors

Input channel names, specified as one of the following:

- A character vector, for single-input models.
- A cell array of character vectors, for multi-input models.
- '', no names specified for any input channels.

Alternatively, you can assign input names for multi-input models using automatic vector expansion. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

### Use InputName to:

- Identify channels on model display and plots.
- Extract subsystems of MIMO systems.
- Specify connection points when interconnecting models.

### InputUnit — Input channel units

' ' (default) | character vector | cell array of character vectors

Input channel units, specified as one of the following:

- A character vector, for single-input models.
- A cell array of character vectors, for multi-input models.
- '', no units specified for any input channels.

Use InputUnit to specify input signal units. InputUnit has no effect on system behavior.

## InputGroup — Input channel groups

structure

Input channel groups, specified as a structure. Use InputGroup to assign the input channels of MIMO systems into groups and refer to each group by name. The field names of InputGroup are the group names and the field values are the input channels of each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1 and 2, and 3 and 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

By default, InputGroup is a structure with no fields.

### OutputName — Output channel names

```
' ' (default) | character vector | cell array of character vectors
```

Output channel names, specified as one of the following:

- A character vector, for single-output models.
- A cell array of character vectors, for multi-output models.
- '', no names specified for any output channels.

Alternatively, you can assign output names for multi-output models using automatic vector expansion. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can also use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Use OutputName to:

- · Identify channels on model display and plots.
- Extract subsystems of MIMO systems.
- Specify connection points when interconnecting models.

## OutputUnit — Output channel units

```
' ' (default) | character vector | cell array of character vectors
```

Output channel units, specified as one of the following:

- A character vector, for single-output models.
- A cell array of character vectors, for multi-output models.
- '', no units specified for any output channels.

Use OutputUnit to specify output signal units. OutputUnit has no effect on system behavior.

### OutputGroup — Output channel groups

structure

Output channel groups, specified as a structure. Use OutputGroupto assign the output channels of MIMO systems into groups and refer to each group by name. The field names of OutputGroup are the group names and the field values are the output channels of each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3 and 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

By default, OutputGroup is a structure with no fields.

### Name — System name

```
' ' (default) | character vector
```

System name, specified as a character vector. For example,  $'system_1'$ .

## Notes — User-specified text

```
{} (default) | character vector | cell array of character vectors
```

User-specified text that you want to associate with the system, specified as a character vector or cell array of character vectors. For example, 'System is MIMO'.

## UserData — User-specified data

```
[] (default) | any MATLAB data type
```

User-specified data that you want to associate with the system, specified as any MATLAB data type.

### SamplingGrid — Sampling grid for model arrays

structure array

Sampling grid for model arrays, specified as a structure array.

Use SamplingGrid to track the variable values associated with each model in a model array, including identified linear time-invariant (IDLTI) model arrays.

Set the field names of the structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables must be numeric scalars, and all arrays of sampled values must match the dimensions of the model array.

For example, you can create an 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, you can create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code maps the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

М

```
M(:,:,1,1) [zeta=0.3, w=5] =

25

s^2 + 3 s + 25

M(:,:,2,1) [zeta=0.35, w=5] =

25

s^2 + 3.5 s + 25
```

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For instance, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid automatically.

By default, SamplingGrid is a structure with no fields.

# **Object Functions**

The following lists contain a representative subset of the functions you can use with tf models. In general, any function applicable to "Dynamic System Models" is applicable to a tf object.

# **Linear Analysis**

step Step response plot of dynamic system; step response data impulse Impulse response plot of dynamic system; impulse response data sim Simulate time response of dynamic system to arbitrary inputs bode Bode plot of frequency response, or magnitude and phase data nyquist Nyquist plot of frequency response

nichols Nichols chart of frequency response bandwidth Frequency response bandwidth

# **Stability Analysis**

pole Poles of dynamic system

zero Zeros and gain of SISO dynamic system

pzplot Pole-zero plot of dynamic system model with plot customization options

margin Gain margin, phase margin, and crossover frequencies

### **Model Transformation**

zpk Create zero-pole-gain model; convert to zero-pole-gain model

ss Create state-space model, convert to state-space model

c2d Convert model from continuous to discrete time

d2c Convert model from discrete to continuous time

d2d Resample discrete-time model

### **Model Interconnection**

feedback Feedback connection of multiple models

connect Block diagram interconnections of dynamic systems

series Series connection of two models parallel Parallel connection of two models

# **Controller Design**

pidtune PID tuning algorithm for linear plant model

rlocus Root locus plot of dynamic system

lqr Linear-Quadratic Regulator (LQR) design lqg Linear-Quadratic-Gaussian (LQG) design

lgi Linear-Quadratic-Integral control

kalman Kalman filter design, Kalman estimator

# **Examples**

#### **SISO Transfer Function Model**

For this example, consider the following SISO transfer function model:

$$sys(s) = \frac{1}{2s^2 + 3s + 4}.$$

Specify the numerator and denominator coefficients ordered in descending powers of s, and create the transfer function model.

Continuous-time transfer function.

#### **Discrete-Time SISO Transfer Function Model**

For this example, consider the following discrete-time SISO transfer function model:

$$sys(z) = \frac{2z}{4z^3 + 3z - 1}.$$

Specify the numerator and denominator coefficients ordered in descending powers of z and the sample time of 0.1 seconds. Create the discrete-time transfer function model.

### **Second-Order Transfer Function from Damping Ratio and Natural Frequency**

For this example, consider a transfer function model that represents a second-order system with known natural frequency and damping ratio.

The transfer function of a second-order system, expressed in terms of its damping ratio  $\zeta$  and natural frequency  $\omega_0$ , is:

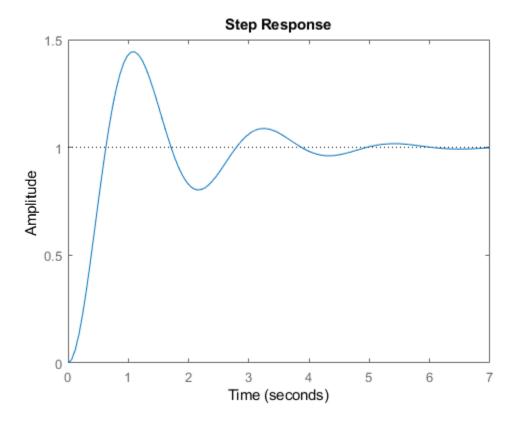
$$sys(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}.$$

Assuming a damping ratio,  $\zeta = 0.25$  and natural frequency,  $\omega_0 = 3$  rad/s, create the second order transfer function.

```
zeta = 0.25;
w0 = 3;
```

Continuous-time transfer function.

Examine the response of this transfer function to a step input. stepplot(sys)



The plot shows the ringdown expected of a second-order system with a low damping ratio.

#### **Discrete-Time MIMO Transfer Function Model**

Create a transfer function for the discrete-time, multi-input, multi-output model:

$$sys(z) = \begin{bmatrix} \frac{1}{z+0.3} & \frac{z}{z+0.3} \\ \frac{-z+2}{z+0.3} & \frac{3}{z+0.3} \end{bmatrix}$$

with sample time ts = 0.2 seconds.

Specify the numerator coefficients as a 2-by-2 matrix.

```
numerators = \{1 [1 0]; [-1 2] 3\};
```

Specify the coefficients of the common denominator as a row vector.

```
denominator = [1 \ 0.3];
```

Create the discrete-time MIMO transfer function model.

Sample time: 0.2 seconds Discrete-time transfer function.

For more information on creating MIMO transfer functions, see "MIMO Transfer Functions".

#### Concatenate SISO Transfer Functions into MIMO Transfer Function Model

In this example, you create a MIMO transfer function model by concatenating SISO transfer function models. Consider the following single-input, two-output transfer function:

$$sys(s) = \begin{bmatrix} \frac{s-1}{s+1} \\ \frac{s+2}{s^2+4s+5} \end{bmatrix}.$$

Specify the MIMO transfer function model by concatenating the SISO entries.

Continuous-time transfer function.

For more information on creating MIMO transfer functions, see "MIMO Transfer Functions".

### **Transfer Function Model Using Rational Expression**

For this example, create a continuous-time transfer function model using rational expressions. Using a rational expression can sometimes be easier and more intuitive than specifying polynomial coefficients of the numerator and denominator.

Consider the following system:

$$sys(s) = \frac{s}{s^2 + 2s + 10}.$$

To create the transfer function model, first specify s as a tf object.

```
s = tf('s')
s =
s
```

Continuous-time transfer function.

Create the transfer function model using s in the rational expression.

Continuous-time transfer function.

### **Discrete-Time Transfer Function Model Using Rational Expression**

For this example, create a discrete-time transfer function model using a rational expression. Using a rational expression can sometimes be easier and more intuitive than specifying polynomial coefficients.

Consider the following system:

$$sys(z) = \frac{z-1}{z^2 - 1.85z + 0.9}$$
.

To create the transfer function model, first specify z as a tf object and the sample time Ts.

```
ts = 0.1;
z = tf('z',ts)
z =
   z
Sample time: 0.1 seconds
Discrete-time transfer function.
```

Create the transfer function model using  $\boldsymbol{z}$  in the rational expression.

# **Transfer Function Model with Inherited Properties**

For this example, create a transfer function model with properties inherited from another transfer function model. Consider the following two transfer functions:

$$sys1(s) = \frac{2s}{s^2 + 8s}$$
 and  $sys2(s) = \frac{s - 1}{7s^4 + 2s^3 + 9}$ .

For this example, create sys1 with the TimeUnit and InputDelay property set to 'minutes'.

Create the second transfer function model with properties inherited from sys1.

Observe that the transfer function model sys2 has that same properties as sys1.

### **Array of Transfer Function Models**

You can use a for loop to specify an array of transfer function models.

First, pre-allocate the transfer function array with zeros.

```
sys = tf(zeros(1,1,3));
```

The first two indices represent the number of outputs and inputs for the models, while the third index is the number of models in the array.

Create the transfer function model array using a rational expression in the for loop.

### **Convert State-Space Model to Transfer Function**

For this example, compute the transfer function of the following state-space model:

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Create the state-space model using the state-space matrices.

```
A = [-2 -1;1 -2];
B = [1 1;2 -1];
C = [1 0];
D = [0 1];
ltiSys = ss(A,B,C,D);
```

Convert the state-space model ltiSys to a transfer function.

Continuous-time transfer function.

#### **Extract Transfer Functions from Identified Model**

For this example, extract the measured and noise components of an identified polynomial model into two separate transfer functions.

Load the Box-Jenkins polynomial model ltiSys in identifiedModel.mat.

```
load('identifiedModel.mat','ltiSys');
```

ltiSys is an identified discrete-time model of the form:  $y(t) = \frac{B}{F}u(t) + \frac{C}{D}e(t)$ , where  $\frac{B}{F}$  represents the measured component and  $\frac{C}{D}$  the noise component.

Extract the measured and noise components as transfer functions.

```
sysMeas = tf(ltiSys,'measured')
sysMeas =
  From input "u1" to output "y1":
            -0.1426 z^{-1} + 0.1958 z^{-2}
           1 - 1.575 z^{-1} + 0.6115 z^{-2}
Sample time: 0.04 seconds
Discrete-time transfer function.
sysNoise = tf(ltiSys, 'noise')
sysNoise =
  From input "v@y1" to output "y1":
          0.04556 + 0.03301 z^{-1}
  1 - 1.026 z^{-1} + 0.26 z^{-2} - 0.1949 z^{-3}
Input groups:
          Channels
    Name
    Noise
Sample time: 0.04 seconds
Discrete-time transfer function.
```

The measured component can serve as a plant model, while the noise component can be used as a disturbance model for control system design.

### **Specify Input and Output Names for MIMO Transfer Function Model**

Transfer function model objects include model data that helps you keep track of what the model represents. For instance, you can assign names to the inputs and outputs of your model.

Consider the following continuous-time MIMO transfer function model:

$$sys(s) = \begin{bmatrix} \frac{s+1}{s^2 + 2s + 2} \\ \frac{1}{s} \end{bmatrix}$$

The model has one input – Current, and two outputs – Torque and Angular velocity.

First, specify the numerator and denominator coefficients of the model.

```
numerators = {[1 1] ; 1};
denominators = {[1 2 2] ; [1 0]};
```

Create the transfer function model, specifying the input name and output names.

Continuous-time transfer function.

### Specify Polynomial Ordering in Discrete-Time Transfer Function

For this example, specify polynomial ordering in discrete-time transfer function models using the 'Variable' property.

Consider the following discrete-time transfer functions with sample time 0.1 seconds:

$$sys1(z) = \frac{z^2}{z^2 + 2z + 3}$$
  $sys2(z^{-1}) = \frac{1}{1 + 2z^{-1} + 3z^{-2}}.$ 

Create the first discrete-time transfer function by specifying the z coefficients.

The coefficients of sys1 are ordered in descending powers of z.

tf switches convention based on the value of the 'Variable' property. Since sys2 is the inverse transfer function model of sys1, specify 'Variable' as 'z^-1' and use the same numerator and denominator coefficients.

The coefficients of sys2 are now ordered in ascending powers of  $z^-1$ .

Based on different conventions, you can specify polynomial ordering in transfer function models using the 'Variable' property.

#### **Tunable Low-Pass Filter**

In this example, you will create a low-pass filter with one tunable parameter *a*:

$$F = \frac{a}{s+a}$$

Since the numerator and denominator coefficients of a tunableTF block are independent, you cannot use tunableTF to represent F. Instead, construct F using the tunable real parameter object realp.

Create a real tunable parameter with an initial value of 10.

Use tf to create the tunable low-pass filter F.

```
numerator = a;
denominator = [1,a];
F = tf(numerator, denominator)

F =

Generalized continuous-time state-space model with 1 outputs, 1 inputs, 1 states, and a: Scalar parameter, 2 occurrences.
```

Type "ss(F)" to see the current value, "get(F)" to see all properties, and "F.Blocks"

F is a genss object which has the tunable parameter a in its Blocks property. You can connect F with other tunable or numeric models to create more complex control system models. For an example, see "Control System with Tunable Components".

#### Static Gain MIMO Transfer Function Model

In this example, you will create a static gain MIMO transfer function model.

Consider the following two-input, two-output static gain matrix m:

$$m = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

Specify the gain matrix and create the static gain transfer function model.

```
m = [2,4;...
    3,5];
sys1 = tf(m)
sys1 =
    From input 1 to output...
    1: 2
    2: 3
    From input 2 to output...
    1: 4
    2: 5
Static gain.
```

You can use static gain transfer function model sys1 obtained above to cascade it with another transfer function model.

For this example, create another two-input, two-output discrete transfer function model and use the series function to connect the two models.

```
numerators = {1,[1,0];[-1,2],3};
denominator = [1,0.3];
```

```
ts = 0.2;
sys2 = tf(numerators,denominator,ts)
sys2 =
  From input 1 to output...
      1
   1: -----
      z + 0.3
      -z + 2
   2: -----
       z + 0.3
  From input 2 to output...
         Ζ
  1: -----
       z + 0.3
         3
   2: -----
       z + 0.3
Sample time: 0.2 seconds
Discrete-time transfer function.
sys = series(sys1, sys2)
sys =
  From input 1 to output...
      3 z^2 + 2.9 z + 0.6
   1: -----
      z^2 + 0.6 z + 0.09
      -2 z^2 + 12.4 z + 3.9
   2: ---
       z^2 + 0.6 z + 0.09
  From input 2 to output...
      5 z^2 + 5.5 z + 1.2
   1: --
      z^2 + 0.6 z + 0.09
       -4 z^2 + 21.8 z + 6.9
```

```
2: z^2 + 0.6z + 0.09
```

Sample time: 0.2 seconds
Discrete-time transfer function.

## Limitations

- Transfer function models are ill-suited for numerical computations. Once created, convert them to state-space form before combining them with other models or performing model transformations. You can then convert the resulting models back to transfer function form for inspection purposes
- An identified nonlinear model cannot be directly converted into a transfer function model using tf. To obtain a transfer function model:
  - Convert the nonlinear identified model to an identified LTI model using linapp, idnlarx/linearize, or idnlhw/linearize.
  - 2 Then, convert the resulting model to a transfer function model using tf.

## See Also

filt | frd | genmat | genss | get | realp | set | ss | tfdata | tunableTF | zpk

### **Topics**

"What Are Model Objects?"

"Transfer Functions"

"Discrete-Time Numeric Models"

"MIMO Transfer Functions"

"Using the Right Model Representation"

"Using FEEDBACK to Close Feedback Loops"

#### Introduced before R2006a

## tfdata

Access transfer function data

# **Syntax**

```
[num,den] = tfdata(sys)
[num,den,Ts] = tfdata(sys)
[num,den,Ts,sdnum,sdden]=tfdata(sys)
[num,den,Ts,...]=tfdata(sys,J1,...,Jn)
```

# **Description**

[num,den] = tfdata(sys) returns the numerator(s) and denominator(s) of the transfer function for the TF, SS or ZPK model (or LTI array of TF, SS or ZPK models) sys. For single LTI models, the outputs num and den of tfdata are cell arrays with the following characteristics:

- num and den have as many rows as outputs and as many columns as inputs.
- The (i,j) entries num{i,j} and den{i,j} are row vectors specifying the
  numerator and denominator coefficients of the transfer function from input j to output
  i. These coefficients are ordered in descending powers of s or z.

For arrays sys of LTI models, num and den are multidimensional cell arrays with the same sizes as sys.

If sys is a state-space or zero-pole-gain model, it is first converted to transfer function form using tf. For more information on the format of transfer function model data, see the tf reference page.

For SISO transfer functions, the syntax

```
[num,den] = tfdata(sys,'v')
```

forces tfdata to return the numerator and denominator directly as row vectors rather than as cell arrays (see example below).

[num,den,Ts] = tfdata(sys) also returns the sample time Ts.

[num,den,Ts,sdnum,sdden]=tfdata(sys) also returns the uncertainties in the numerator and denominator coefficients of identified system sys.  $sdnum\{i,j\}(k)$  is the 1 standard uncertainty in the value  $num\{i,j\}(k)$  and  $sdden\{i,j\}(k)$  is the 1 standard uncertainty in the value  $den\{i,j\}(k)$ . If sys does not contain uncertainty information, sdnum and sdden are empty ([]).

```
[num,den,Ts,...]=tfdata(sys,J1,...,Jn) extracts the data for the (J1,...,JN) entry in the model array sys.
```

You can access the remaining LTI properties of sys with get or by direct referencing, for example,

```
sys.Ts
sys.variable
```

# **Examples**

## **Example 1**

Given the SISO transfer function

```
h = tf([1 1],[1 2 5])
```

you can extract the numerator and denominator coefficients by typing

This syntax returns two row vectors.

If you turn h into a MIMO transfer function by typing

```
H = [h ; tf(1,[1 1])]
the command
```

```
[num,den] = tfdata(H)
```

now returns two cell arrays with the numerator/denominator data for each SISO entry. Use celldisp to visualize this data. Type

```
celldisp(num)
```

This command returns the numerator vectors of the entries of H.

```
num\{1\} = 0    1    1
num\{2\} = 0    1
```

Similarly, for the denominators, type

## **Example 2**

Extract the numerator, denominator and their standard deviations for a 2-input, 1 output identified transfer function.

```
load iddata7
```

transfer function model

```
sys1 = tfest(z7, 2, 1, 'InputDelay',[1 0]);
an equivalent process model
sys2 = procest(z7, {'P2UZ', 'P2UZ'}, 'InputDelay',[1 0]);
[num1, den1, ~, dnum1, dden1] = tfdata(sys1);
[num2, den2, ~, dnum2, dden2] = tfdata(sys2);
```

## See Also

```
get | ssdata | tf | zpkdata
```

### **Introduced before R2006a**

## thiran

Generate fractional delay filter based on Thiran approximation

# **Syntax**

```
sys = thiran(tau, Ts)
```

# **Description**

sys = thiran(tau, Ts) discretizes the continuous-time delay tau using a Thiran filter to approximate the fractional part of the delay. Ts specifies the sample time.

# **Input Arguments**

#### tau

Time delay to discretize.

#### Ts

Sample time.

# **Output Arguments**

### sys

Discrete-time tf object.

# **Examples**

Approximate and discretize a time delay that is a noninteger multiple of the target sample time.

The time delay is 2.4 s, and the sample time is 1 s. Therefore, sys1 is a discrete-time transfer function of order 3.

Discretize a time delay that is an integer multiple of the target sample time.

```
sys2 = thiran(10, 1)
Transfer function:
    1
----
z^10
Sample time: 1
```

# **Tips**

- If tau is an integer multiple of Ts, then sys represents the pure discrete delay  $z^{-N}$ , with N = tau/Ts. Otherwise, sys is a discrete-time, all-pass, infinite impulse response (IIR) filter of order ceil(tau/Ts).
- thiran approximates and discretizes a pure time delay. To approximate a pure continuous-time time delay without discretizing, use pade. To discretize continuous-time models having time delays, use c2d.

# **Algorithms**

The Thiran fractional delay filter has the following form:

$$H(z) = \frac{a_N z^N + a_{N-1} z^{N-1} + \cdots + a_1}{a_0 z^N + a_1 z^{N-1} + \cdots + a_N} \,.$$

The coefficients  $a_0$ , ...,  $a_N$  are given by:

$$\begin{aligned} a_k &= (-1)^k \binom{N}{k}_i \prod_{i=0}^N \frac{D-N+i}{D-N+k+i}, & \forall k \colon 1, 2, ..., N \\ a_0 &= 1 \end{aligned}$$

where  $D = \tau/T_s$  and N = ceil(D) is the filter order. See [1].

## References

[1] T. Laakso, V. Valimaki, "Splitting the Unit Delay", *IEEE Signal Processing Magazine*, Vol. 13, No. 1, p.30-60, 1996.

## See Also

c2d | pade | tf

Introduced in R2010a

# timeoptions

Create list of time plot options

# **Syntax**

```
P = timeoptions
P = timeoptions('cstprefs')
```

# **Description**

P = timeoptions returns a list of available options for time plots with default values set. You can use these options to customize the time value plot appearance from the command line.

P = timeoptions('cstprefs') initializes the plot options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the available time plot options.

Option	Description
Title, XLabel, YLabel	Label text and style
TickLabel	Tick label style
Grid	Show or hide the grid, specified as one of the following values: 'off'   'on'  Default: 'off'
GridColor	Color of the grid lines, specified as one of the following: Vector of RGB values in the range [0,1]  character vector of color name   'none'. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'.  Default: [0.15,0.15,0.15]

Option	Description
XlimMode, YlimMode	Limit modes
Xlim, Ylim	Axes limits
IOGrouping	Grouping of input-output pairs, specified as one of the following values: 'none'  'inputs' 'outputs' 'all' Default: 'none'
InputLabels, OutputLabels	Input and output label styles
InputVisible, OutputVisible	Visibility of input and output channels
Normalize	Normalize responses, specified as one of the following values: 'on'  'off'  Default: 'off'
SettleTimeThreshold	Settling time threshold
RiseTimeLimits	Rise time limits

Option	Description
TimeUnits	Time units, specified as one of the following values:
	• 'nanoseconds'
	• 'microseconds'
	• 'milliseconds'
	• 'seconds'
	• 'minutes'
	• 'hours'
	• 'days'
	• 'weeks'
	• 'months'
	• 'years'
	Default: 'seconds'
	You can also specify 'auto' which uses time units specified in the TimeUnit property of the input system. For multiple systems with different time units, the units of the first system is used.

# **Examples**

### **Plot Normalized Step Response**

Create a default time options set.

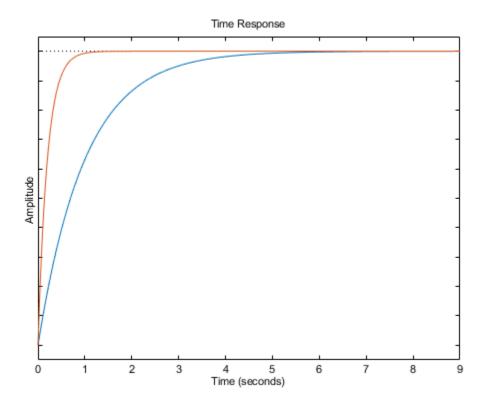
```
opt = timeoptions;
```

Enable plotting of normalized responses.

```
opt.Normalize = 'on';
```

Plot the step response of two transfer function models using the specified options.

```
sys1 = tf(10,[1,1]);
sys2 = tf(5,[1,5]);
stepplot(sys1,sys2,opt);
```



The plot shows the normalized step response for the two transfer function models.

## See Also

getoptions | impulseplot | initialplot | lsimplot | setoptions | stepplot

### Introduced in R2008a

# totaldelay

Total combined I/O delays for LTI model

# **Syntax**

```
td = totaldelay(sys)
```

# **Description**

td = totaldelay(sys) returns the total combined I/O delays for an LTI model sys.
The matrix td combines contributions from the InputDelay, OutputDelay, and
ioDelayMatrix properties.

Delays are expressed in seconds for continuous-time models, and as integer multiples of the sample period for discrete-time models. To obtain the delay times in seconds, multiply td by the sample time sys.Ts.

# **Examples**

### **Compute Combined Input-Output Delay for Transfer Function**

```
Create the transfer function model, 1/s.

sys = tf(1,[1 0]);
```

Specify a 2 second input delay, and a 1.5 second output delay.

```
sys.InputDelay = 2;
sys.Outputdelay = 1.5;
```

Compute the combined input-output delay for sys.

```
td = totaldelay(sys)
td = 3,5000
```

The resulting transfer function has the following form:

$$e^{-2s}x\frac{1}{s}e^{-1.5s} = e^{-3.5s}\frac{1}{s}$$

This result is equivalent to specifying an input-output delay of 3.5 seconds for the original transfer function, 1/s.

## **See Also**

absorbDelay | hasdelay

Introduced before R2006a

## tunableGain

Tunable static gain block

# **Syntax**

```
blk = tunableGain(name,Ny,Nu)
blk = tunableGain(name,G)
```

# **Description**

Model object for creating tunable static gains. tunableGain lets you parametrize tunable static gains for parameter studies or for automatic tuning with tuning commands such as systume or looptune.

tunableGain is part of the Control Design Block family of parametric models. Other Control Design Blocks include tunablePID, tunableSS, and tunableTF.

## Construction

blk = tunableGain(name, Ny, Nu) creates a parametric static gain block named name. This block has Ny outputs and Nu inputs. The tunable parameters are the gains across each of the Ny-by-Nu I/O channels.

blk = tunableGain(name,G) uses the double array G to dimension the block and initialize the tunable parameters.

### **Input Arguments**

#### name

Block Name, specified as a character vector such as 'K' or 'gain1'. (See "Properties" on page 2-1295.)

### Ny

Non-negative integer specifying the number of outputs of the parametric static gain block blk.

#### Nu

Non-negative integer specifying the number of inputs of the parametric static gain block blk.

#### G

Double array of static gain values. The number of rows and columns of G determine the number of inputs and outputs of blk. The entries G are the initial values of the parametric gain block parameters.

# **Properties**

### Gain

Parametrization of the tunable gain.

blk.Gain is a param.Continuous object. For general information about the properties of the param.Continuous object blk.Gain, see the param.Continuous object reference page.

The following fields of blk. Gain are used when you tune blk using hinfstruct:

Field	Description
Value	Current value of the gain matrix. For a block that has Ny outputs and Nu inputs, blk.Gain.Value is a Ny-by-Nu matrix.
	If you use the G input argument to create blk, blk.Gain.Value initializes to the values of G. Otherwise, all entries of blk.Gain.Value initialize to zero.
	hinfstruct tunes all entries in blk.Gain.Value except those whose values are fixed by blk.Gain.Free.
	Default: Array of zero values
Free	Array of logical values determining whether the gain entries in blk.Gain.Value are fixed or free parameters.
	• If blk.Gain.Free(i,j) = 1, then blk.Gain.Value(i,j) is a tunable parameter.
	• If blk.Gain.Free(i,j) = 0, then blk.Gain.Value(i,j) is fixed.
	Default: Array of 1 (true) values
Minimum	Minimum value of the parameter. This property places a lower bound on the tuned value of the parameter. For example, setting blk.Gain.Minimum = 1 ensures that all entries in the gain matrix have gain greater than 1.
	Default: -Inf

Field	Description
Maximum	Maximum value of the parameter. This property places an upper bound on the tuned value of the parameter. For example, setting blk.Gain.Maximum = 100 ensures that all entries in the gain matrix have gain less than 100.  Default: Inf

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

Default: 0 (continuous time)

### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- $\bullet \quad \text{Cell array of character vectors} \quad \text{For multi-output models}.$

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

### OutputUnit

Output channel units, specified as one of the following:

- Character vector For single-output models. For example, 'seconds'.
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ''

#### Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

Default: [0×1 string]

### **UserData**

Any type of data you want to associate with system, specified as any MATLAB data type.

Default: []

# **Examples**

Create a 2-by-2 parametric gain block of the form

$$\begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}$$

where  $g_1$  and  $g_2$  are tunable parameters, and the off-diagonal elements are fixed to zero.

```
blk = tunableGain('gainblock',2,2); % 2 outputs, 2 inputs
blk.Gain.Free = [1 0; 0 1]; % fix off-diagonal entries to zero
```

All entries in blk.Gain.Value initialize to zero. Initialize the diagonal values to 1 as follows.

```
blk.Gain.Value = eye(2); % set diagonals to 1
```

Create a two-input, three-output parametric gain block and initialize all the parameter values to 1.

To do so, create a matrix to dimension the parametric gain block and initialize the parameter values.

```
G = ones(3,2);
blk = tunableGain('gainblock',G);
Create a 2-by-2 parametric gain block and assign names to the inputs.
blk = tunableGain('gainblock',2,2) % 2 outputs, 2 inputs
blk.InputName = {'Xerror','Yerror'} % assign input names
```

# **Tips**

- Use the blk.Gain.Free field of blk to specify additional structure or fix the values of specific entries in the block. To fix the gain value from input i to output j, set blk.Gain.Free(i,j) = 0. To allow hinfstruct to tune this gain value, set blk.Gain.Free(i,j) = 1.
- To convert a tunableGain parametric model to a numeric (non-tunable) model object, use model commands such as tf, zpk, or ss.

# **Compatibility Considerations**

## Name changed from Itiblock.gain

Behavior changed in R2016a

Prior to R2016a, tunableGain was called ltiblock.gain.

## See Also

genss | hinfstruct | looptune | systume | tunablePID | tunablePID2 | tunableSS | tunableTF

## **Topics**

"Control Design Blocks"

"Models with Tunable Coefficients"

### Introduced in R2016a

## tunablePID

Tunable PID controller

# **Syntax**

blk = tunablePID(name,type)
blk = tunablePID(name,type,Ts)
blk = tunablePID(name,sys)

# **Description**

Model object for creating tunable one-degree-of-freedom PID controllers. tunablePID lets you parametrize a tunable SISO PID controller for parameter studies or for automatic tuning with tuning commands such as systume, looptume, or the Robust Control Toolbox command, hinfstruct.

tunablePID is part of the family of parametric Control Design Blocks. Other parametric Control Design Blocks include tunableGain, tunableSS, and tunableTF.

## Construction

blk = tunablePID(name, type) creates the one-degree-of-freedom continuous-time PID controller:

$$blk = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + T_f s'}$$

with tunable parameters Kp, Ki, Kd, and Tf. The type argument sets the controller type by fixing some of these values to zero (see "Input Arguments" on page 2-1305).

blk = tunablePID(name, type, Ts) creates a discrete-time PID controller with sample
time Ts:

$$blk = K_p + K_i IF(z) + \frac{K_d}{T_f + DF(z)},$$

where IF(z) and DF(z) are the discrete integrator formulas for the integral and derivative terms, respectively. The values of the IFormula and DFormula properties set the discrete integrator formulas (see "Properties" on page 2-1306).

blk = tunablePID(name, sys) uses the dynamic system model, sys, to set the sample time, Ts, and the initial values of the parameters Kp, Ki, Kd, and Tf.

## **Input Arguments**

#### name

PID controller Name, specified as a character vector such as 'C' or 'PI1'. (See "Properties" on page 2-1306.)

### type

Controller type, specified as one of the values in the following table. Specifying a controller type fixes up to three of the PID controller parameters.

Value for type	Controller Type	Effect on PID Parameters
'P'	Proportional only	Ki and Kd are fixed to zero; Tf is fixed to 1; Kp is free
'PI'	Proportional-integral	Kd is fixed to zero; Tf is fixed to 1; Kp and Ki are free
' PD '	Proportional-derivative with first-order filter on derivative action	Ki is fixed to zero; Kp, Kd, and Tf are free
'PID'	Proportional-integral- derivative with first-order filter on derivative action	Kp, Ki, Kd, and Tf are free

#### Ts

Sample time, specified as a scalar.

#### sys

Dynamic system model representing a PID controller.

# **Properties**

Kp, Ki, Kd, Tf

Parametrization of the PID gains Kp, Ki, Kd, and filter time constant Tf of the tunable PID controller blk.

The following fields of blk.Kp, blk.Ki, blk.Kd, and blk.Tf are used when you tune blk using a tuning command such as systune:

Field	Description
Value	Current value of the parameter.
Free	Logical value determining whether the parameter is fixed or tunable. For example,  • If blk.Kp.Free = 1, then
	blk.Kp.Value is tunable.
	• If blk.Kp.Free = 0, then blk.Kp.Value is fixed.
Minimum	Minimum value of the parameter. This property places a lower bound on the tuned value of the parameter. For example, setting blk.Kp.Minimum = 0 ensures that Kp remains positive.
	blk.Tf.Minimum must always be positive.
Maximum	Maximum value of the parameter. This property places an upper bound on the tuned value of the parameter. For example, setting blk.Tf.Maximum = 100 ensures that the filter time constant does not exceed 100.

blk.Kp, blk.Ki, blk.Kd, and blk.Tf are param.Continuous objects. For general information about the properties of these param.Continuous objects, see the param.Continuous object reference page.

### IFormula, DFormula

Discrete integrator formulas IF(z) and DF(z) for the integral and derivative terms, respectively, specified as one of the values in the following table.

Value	IF(z) or DF(z) Formula
'ForwardEuler'	$\frac{T_s}{z-1}$
'BackwardEuler'	$\frac{T_{S}z}{z-1}$
'Trapezoidal'	$\frac{T_s}{2} \frac{z+1}{z-1}$

Default: 'ForwardEuler'

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. Unspecified sample time (Ts = -1) is not supported for PID blocks.

Changing this property does not discretize or resample the model.

Default: 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'

- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### InputName

Input channel name, specified as a character vector. Use this property to name the input channel of the controller model. For example, assign the name error to the input of a controller model C as follows.

```
C.InputName = 'error';
```

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- · Specifying connection points when interconnecting models

Default: Empty character vector, ''

### InputUnit

Input channel units, specified as a character vector. Use this property to track input signal units. For example, assign the concentration units mol/m^3 to the input of a controller model C as follows.

```
C.InputUnit = 'mol/m^3';
```

InputUnit has no effect on system behavior.

Default: Empty character vector, ''

### InputGroup

Input channel groups. This property is not needed for PID controller models.

#### **Default:** struct with no fields

### **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the OutputName property. For example, C.y is equivalent to C.OutputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

**Default:** Empty character vector, ''

### **OutputUnit**

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

Default: Empty character vector, ''

### OutputGroup

Output channel groups. This property is not needed for PID controller models.

Default: struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ' '

#### Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

### Default: [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

### Default: []

# **Examples**

#### **Tunable Controller with a Fixed Parameter**

Create a tunable PD controller. Then, initialize the parameter values, and fix the filter time constant.

```
blk = tunablePID('pdblock','PD');
blk.Kp.Value = 4;    % initialize Kp to 4
blk.Kd.Value = 0.7;    % initialize Kd to 0.7
blk.Tf.Value = 0.01;    % set parameter Tf to 0.01
```

### **Controller Initialized by Dynamic System Model**

Create a tunable discrete-time PI controller. Use a pid object to initialize the parameters and other properties.

blk takes the value of properties, such as Ts and IFormula, from C.

## **Controller with Named Input and Output**

Create a tunable PID controller, and assign names to the input and output.

```
blk = tunablePID('pidblock','pid')
blk.InputName = {'error'} % assign input name
blk.OutputName = {'control'} % assign output name
```

# **Tips**

- You can modify the PID structure by fixing or freeing any of the parameters Kp, Ki, Kd, and Tf. For example, blk.Tf.Free = false fixes Tf to its current value.
- To convert a tunablePID parametric model to a numeric (nontunable) model object, use model commands such as pid, pidstd, tf, or ss. You can also use getValue to obtain the current value of a tunable model.

# **Compatibility Considerations**

## Name changed from Itiblock.pid

Behavior changed in R2016a

Prior to R2016a, tunablePID was called ltiblock.pid.

## See Also

genss | hinfstruct | looptune | systune | tunableGain | tunablePID2 |
tunableSS | tunableTF

# **Topics**

"Control Design Blocks"
"Models with Tunable Coefficients"

### Introduced in R2016a

# tunablePID2

Tunable two-degree-of-freedom PID controller

# **Syntax**

blk = tunablePID2(name,type)
blk = tunablePID2(name,type,Ts)
blk = tunablePID2(name,sys)

# **Description**

Model object for creating tunable two-degree-of-freedom PID controllers. tunablePID2 lets you parametrize a tunable SISO two-degree-of-freedom PID controller. You can use this parametrized controller for parameter studies or for automatic tuning with tuning commands such as systume, looptume, or the Robust Control Toolbox command hinfstruct.

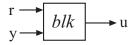
tunablePID2 is part of the family of parametric Control Design Blocks. Other parametric Control Design Blocks include tunableGain, tunableSS, and tunableTF.

# Construction

blk = tunablePID2(name, type) creates the two-degree-of-freedom continuous-time PID controller described by the equation:

$$u=K_p(br-y)+\frac{K_i}{s}(r-y)+\frac{K_ds}{1+T_fs}(cr-y)\,.$$

r is the setpoint command, y is the measured response to that setpoint, and u is the control signal, as shown in the following illustration.



The tunable parameters of the block are:

- Scalar gains Kp, Ki, and Kd
- Filter time constant Tf
- Scalar weights b and c

The type argument sets the controller type by fixing some of these values to zero (see "Input Arguments" on page 2-1314).

blk = tunablePID2(name, type, Ts) creates a discrete-time PID controller with sample time Ts. The equation describing this controller is:

$$u=K_p(br-y)+K_iIF(z)(r-y)+\frac{K_d}{T_f+DF(z)}(cr-y)\,.$$

IF(z) and DF(z) are the discrete integrator formulas for the integral and derivative terms, respectively. The values of the IFormula and DFormula properties set the discrete integrator formulas (see "Properties" on page 2-1315).

blk = tunablePID2(name, sys) uses the dynamic system model, sys, to set the sample time, Ts, and the initial values of all the tunable parameters. The model sys must be compatible with the equation of a two-degree-of-freedom PID controller.

## **Input Arguments**

#### name

PID controller Name, specified as a character vector such as 'C' or '2D0FPID1'. (See "Properties" on page 2-1315.)

### type

Controller type, specified as one of the values in the following table. Specifying a controller type fixes up to three of the PID controller parameters.

Value for type	Controller Type	Effect on PID Parameters
'P'	1 -	Ki and Kd are fixed to zero; Tf is fixed to 1; Kp is free

Value for type	Controller Type	Effect on PID Parameters
'PI'	Proportional-integral	Kd is fixed to zero; Tf is fixed to 1; Kp and Ki are free
' PD '	Proportional-derivative with first-order filter on derivative action	Ki is fixed to zero; Kp, Kd, and Tf are free
'PID'	Proportional-integral- derivative with first-order filter on derivative action	Kp, Ki, Kd, and Tf are free

### Ts

Sample time, specified as a scalar.

### sys

Dynamic system model representing a two-degree-of-freedom PID controller.

# **Properties**

Kp,Ki,Kd,Tf,b,c

Parametrization of the PID gains Kp, Ki, Kd, the filter time constant, Tf, and the scalar gains, b and c.

The following fields of blk.Kp, blk.Ki, blk.Kd, blk.Tf, blk.b, and blk.c are used when you tune blk using a tuning command such as systune:

Field	Description
	Current value of the parameter. blk.b.Value, and blk.c.Value are always nonnegative.

Field	Description
Free	Logical value determining whether the parameter is fixed or tunable. For example:
	• If blk.Kp.Free = 1, then blk.Kp.Value is tunable.
	• If blk.Kp.Free = 0, then blk.Kp.Value is fixed.
Minimum	Minimum value of the parameter. This property places a lower bound on the tuned value of the parameter. For example, setting blk.Kp.Minimum = 0 ensures that Kp remains positive.
	blk.Tf.Minimum must always be positive.
Maximum	Maximum value of the parameter. This property places an upper bound on the tuned value of the parameter. For example, setting blk.c.Maximum = 1 ensures that c does not exceed unity.

blk.Kp, blk.Ki, blk.Kd, blk.Tf, blk.b, and blk.c are param.Continuous objects. For more information about the properties of these param.Continuous objects, see the param.Continuous object reference page.

## IFormula, DFormula

Discrete integrator formulas IF(z) and DF(z) for the integral and derivative terms, respectively, specified as one of the values in the following table.

Value	IF(z) or DF(z) Formula
'ForwardEuler'	$\frac{T_s}{z-1}$
'BackwardEuler'	$\frac{T_{S}z}{z-1}$
'Trapezoidal'	$\frac{T_s}{2} \frac{z+1}{z-1}$

### Default: 'ForwardEuler'

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. Unspecified sample time (Ts = -1) is not supported for PID blocks.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### **InputName**

Input channel name, specified as a character vector or a 2-by-1 cell array of character vectors. Use this property to name the input channels of the controller model. For

example, assign the names setpoint and measurement to the inputs of a 2-DOF PID controller model C as follows.

```
C.InputName = {'setpoint';'measurement'};
```

Alternatively, use automatic vector expansion to assign both input names. For example:

```
C.InputName = 'C-input';
```

The input names automatically expand to {'C-input(1)'; 'C-input(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, C.u is equivalent to C.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Specifying connection points when interconnecting models

**Default:** { ' ' ; ' ' }

## InputUnit

Input channel units, specified as a 2-by-1 cell array of character vectors. Use this property to track input signal units. For example, assign the units Volts to the reference input and the concentration units mol/m^3 to the measurement input of a 2-DOF PID controller model C as follows.

```
C.InputUnit = {'Volts';'mol/m^3'};
```

InputUnit has no effect on system behavior.

**Default:** { ' ' ; ' ' }

## InputGroup

Input channel groups. This property is not needed for PID controller models.

Default: struct with no fields

### **OutputName**

Output channel name, specified as a character vector. Use this property to name the output channel of the controller model. For example, assign the name control to the output of a controller model C as follows.

```
C.OutputName = 'control';
```

You can use the shorthand notation y to refer to the OutputName property. For example, C.y is equivalent to C.OutputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Specifying connection points when interconnecting models

Default: Empty character vector, ''

### OutputUnit

Output channel units, specified as a character vector. Use this property to track output signal units. For example, assign the unit Volts to the output of a controller model C as follows.

```
C.OutputUnit = 'Volts';
```

OutputUnit has no effect on system behavior.

Default: Empty character vector, ''

## **OutputGroup**

Output channel groups. This property is not needed for PID controller models.

**Default:** struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ' '

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

## Default: [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

## Default: []

# **Examples**

## Tunable Two-Degree-of-Freedom Controller with a Fixed Parameter

Create a tunable two-degree-of-freedom PD controller. Then, initialize the parameter values, and fix the filter time constant.

```
blk = tunablePID2('pdblock','PD');
blk.b.Value = 1;
blk.c.Value = 0.5;
blk.Tf.Value = 0.01;
```

```
blk.Tf.Free = false;
blk

blk =

Parametric continuous-time 2-DOF PID controller "pdblock" with equation:

u = Kp (b*r-y) + Kd -------- (c*r-y)
    Tf*s+1

where r,y are the controller inputs and Kp, Kd, b, c are tunable gains.

Type "showBlockValue(blk)" to see the current value and "get(blk)" to see all properties.
```

### **Controller Initialized by Dynamic System Model**

Create a tunable two-degree-of-freedom PI controller. Use a two-input, one-output tf model to initialize the parameters and other properties.

```
s = tf('s');
Kp = 10;
Ki = 0.1;
b = 0.7;
sys = [(b*Kp + Ki/s), (-Kp - Ki/s)];
blk = tunablePID2('PI2dof',sys)

blk =

Parametric continuous-time 2-DOF PID controller "PI2dof" with equation:

u = Kp (b*r-y) + Ki --- (r-y)
s

where r,y are the controller inputs and Kp, Ki, b are tunable gains.

Type "showBlockValue(blk)" to see the current value and "get(blk)" to see all properties.
```

blk takes initial parameter values from sys.

If sys is a discrete-time system, blk takes the value of properties, such as Ts and IFormula, from sys.

## **Controller with Named Inputs and Output**

Create a tunable PID controller, and assign names to the inputs and output.

```
blk = tunablePID2('pidblock','pid');
blk.InputName = {'reference','measurement'};
blk.OutputName = {'control'};
```

blk. InputName is a cell array containing two names, because a two-degree-of-freedom PID controller has two inputs.

# **Tips**

- You can modify the PID structure by fixing or freeing any of the parameters. For example, blk.Tf.Free = false fixes Tf to its current value.
- To convert a tunablePID2 parametric model to a numeric (nontunable) model object, use model commands such as tf or ss. You can also use getValue to obtain the current value of a tunable model.

# **Compatibility Considerations**

# Name changed from Itiblock.pid2

Behavior changed in R2016a

Prior to R2016a, tunablePID2 was called ltiblock.pid2.

## See Also

genss | hinfstruct | looptune | systume | tunableGain | tunablePID | tunableSS | tunableTF

# **Topics**

"Control Design Blocks"

"Models with Tunable Coefficients"

### Introduced in R2016a

# tunableSS

Tunable fixed-order state-space model

# **Syntax**

```
blk = tunableSS(name,Nx,Ny,Nu)
blk = tunableSS(name,Nx,Ny,Nu,Ts)
blk = tunableSS(name,sys)
blk = tunableSS(...,Astruct)
```

# **Description**

Model object for creating tunable fixed-order state-space models. tunableSS lets you parametrize a state-space model of a given order for parameter studies or for automatic tuning with tuning commands such as systume or looptune.

tunableSS is part of the Control Design Block family of parametric models. Other Control Design Blocks include tunablePID, tunableGain, and tunableTF.

## Construction

blk = tunableSS(name, Nx, Ny, Nu) creates the continuous-time parametric state-space model named name. The state-space model blk has Nx states, Ny outputs, and Nu inputs. The tunable parameters are the entries in the A, B, C, and D matrices of the state-space model.

blk = tunableSS(name, Nx, Ny, Nu, Ts) creates a discrete-time parametric state-space model with sample time Ts.

blk = tunableSS(name, sys) uses the dynamic system sys to dimension the parametric state-space model, set its sample time, and initialize the tunable parameters.

blk = tunableSS(..., Astruct) creates a parametric state-space model whose A matrix is restricted to the structure specified in Astruct.

# **Input Arguments**

#### name

Parametric state-space model Name, specified as a character vector such as 'C0'. (See "Properties" on page 2-1325.)

#### Nx

Nonnegative integer specifying the number of states (order) of the parametric state-space model blk.

## Ny

Nonnegative integer specifying the number of outputs of the parametric state-space model blk.

#### Nu

Nonnegative integer specifying the number of inputs of the parametric state-space model blk.

#### Ts

Scalar sample time.

#### **Astruct**

Constraints on the form of the A matrix of the parametric state-space model blk, specified as one of the following values:

Value for Astruct	Structure of A matrix
'tridiag'	A is tridiagonal. In tridiagonal form, A has free elements only in the main diagonal, the first diagonal below the main diagonal, and the first diagonal above the main diagonal. The remaining elements of A are fixed to zero.
'full'	A is full (every entry in A is a free parameter).

Value for Astruct	Structure of A matrix
·	A is in companion form. In companion form, the characteristic polynomial of the system appears explicitly in the rightmost column of the A matrix. See canon for more information.

If you do not specify Astruct, blk defaults to 'tridiag' form.

### sys

Dynamic system model providing number of states, number of inputs and outputs, sample time, and initial values of the parameters of blk. To obtain the dimensions and initial parameter values, tunableSS converts sys to a state-space model with the structure specified in Astruct. If you omit Astruct, tunableSS converts sys into tridiagonal state-space form.

# **Properties**

### A, B, C, D

Parametrization of the state-space matrices A, B, C, and D of the tunable state-space model blk.

blk.A, blk.B, blk.C, and blk.D are param.Continuous objects. For general information about the properties of these param.Continuous objects, see the param.Continuous object reference page.

The following fields of blk.A, blk.B, blk.C, and blk.D are used when you tune blk using hinfstruct:

Field	Description
Value	Current values of the entries in the parametrized state-space matrix. For example, blk.A.Value contains the values of the A matrix of blk.
	hinfstruct tunes all entries in blk.A.Value, blk.B.Value, blk.C.Value, and blk.D.Value except those whose values are fixed by blk.Gain.Free.
Free	2-D array of logical values determining whether the corresponding state-space matrix parameters are fixed or free parameters. For example:
	<ul> <li>If blk.A.Free(i,j) = 1, then blk.A.Value(i,j) is a tunable parameter.</li> <li>If blk.A.Free(i,j) = 0, then blk.A.Value(i,j) is fixed.</li> </ul>
	Defaults: By default, all entries in B, C, and D are tunable. The default free entries in A depend upon the value of Astruct:
	• 'tridiag' — entries on the three diagonals of blk.A.Free are 1; the rest are 0.
	• 'full' — all entries in blk.A.Free are 0.
	• 'companion' — blk.A.Free(1,:) = 1 and blk.A.Free(j,j-1) = 1; all other entries are 0.

Field	Description
Minimum	Minimum value of the parameter. This property places a lower bound on the tuned value of the parameter. For example, setting blk.A.Minimum(1,1) = 0 ensures that the first entry in the A matrix remains positive.  Default: -Inf
Maximum	Maximum value of the parameter. This property places an upper bound on the tuned value of the parameter. For example, setting blk.A.Maximum(1,1) = 0 ensures that the first entry in the A matrix remains negative.  Default: Inf

#### **StateName**

State names, specified as one of the following:

- Character vector For first-order models, for example, 'velocity'.
- Cell array of character vectors For models with two or more states
- '' For unnamed states.

Default: ' ' for all states

### StateUnit

State units, specified as one of the following:

- Character vector For first-order models, for example, 'velocity'
- Cell array of character vectors For models with two or more states
- '' For states without specified units

Use StateUnit to keep track of the units each state is expressed in. StateUnit has no effect on system behavior.

**Default:** '' for all states

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### **InputName**

Input channel names, specified as one of the following:

 $\bullet \quad \hbox{Character vector} - \hbox{For single-input models, for example, '} \verb|controls'|.$ 

• Cell array of character vectors — For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- · Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- · Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

## InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

## **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector For single-output models. For example, 'seconds'.
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

### **Default:** ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

#### Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sys1 has a string."
```

```
ans =
   'sys2 has a character vector.'
```

Default: [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

Default: []

# **Examples**

Create a parametrized 5th-order SISO model with zero D matrix.

```
blk = tunableSS('ssblock',5,1,1);
blk.D.Value = 0;  % set D = 0
blk.D.Free = false;  % fix D to zero
```

By default, the A matrix is in tridiagonal form. To parametrize the model in companion form, use the 'companion' input argument:

```
blk = tunableSS('ssblock',5,1,1,'companion');
blk.D.Value = 0; % set D = 0
blk.D.Free = false; % fix D to zero
```

Create a parametric state-space model, and assign names to the inputs.

```
blk = tunableSS('ssblock',5,2,2) % 5 states, 2 outputs, 2 inputs
blk.InputName = {'Xerror', 'Yerror'} % assign input names
```

# **Tips**

• Use the Astruct input argument to constrain the structure of the A matrix of the parametric state-space model. To impose additional structure constrains on the state-space matrices, use the fields blk.A.Free, blk.B.Free, blk.C.Free, and blk.D.Free to fix the values of specific entries in the parameter matrices.

For example, to fix the value of blk.B(i,j), set blk.B.Free(i,j) = 0. To allow hinfstruct to tune blk.B(i,j), set blk.B.Free(i,j) = 1.

• To convert a tunableSS parametric model to a numeric (non-tunable) model object, use model commands such as ss, tf, or zpk.

# **Compatibility Considerations**

# Name changed from Itiblock.ss

Behavior changed in R2016a

Prior to R2016a, tunableSS was called ltiblock.ss.

## See Also

genss | hinfstruct | looptune | systume | tunableGain | tunablePID |
tunablePID2 | tunableTF

# **Topics**

"Control Design Blocks"
"Models with Tunable Coefficients"

Introduced in R2016a

# tunableSurface

Create tunable gain surface for gain scheduling

# **Syntax**

```
K = tunableSurface(name,K0init,domain,shapefcn)
K = tunableSurface(name,K0init,domain)
```

# **Description**

tunableSurface lets you parameterize and tune gain schedules, which are gains that vary as a function of one or more scheduling variables.

For tuning purposes, it is convenient to parameterize a variable gain as a smooth gain surface of the form:

$$K(n(\sigma)) = \gamma [K_0 + K_1 F_1(n(\sigma)) + ... + K_M F_M(n(\sigma))],$$

#### where

- $\sigma$  is a vector of scheduling variables.
- $n(\sigma)$  is a normalization function (see the Normalization property of the output argument K).
- $\gamma$  is a scaling factor (see the Normalization property of the output argument K).
- $F_1,...,F_M$  are user-selected basis functions.
- $K_0,...,K_M$  are the coefficients to be tuned

You can use terms in a generic polynomial expansion as basis functions. Or, when the expected shape of  $K(\sigma)$  is known, you can use more specific functions. You can then use systune to tune the coefficients  $K_0,...,K_M$ , subject to your design requirements, over the range of scheduling-variable values.

K = tunableSurface(name, K0init, domain, shapefcn) creates the tunable gain surface:

$$K(n(\sigma)) = \gamma [K_0 + K_1 F_1(n(\sigma)) + ... + K_M F_M(n(\sigma))].$$

The tunable surface K stores the basis functions specified by shapefcn and a discrete set of  $\sigma$  values (the design points) given by domain. The tunable gain surface has tunable coefficients  $K_0,...,K_M$ . The gain value is initialized to the constant gain K0init. You can combine K with other static or dynamic elements to construct a closed-loop model of your gain-scheduled control system. Or, use K to parameterize a lookup table in an slTuner interface to a Simulink model. Then, use systume to tune  $K_0,...,K_M$  so that the closed-loop system meets your design requirements at the selected design points.

K = tunableSurface(name, K0init, domain) creates a flat surface with constant, tunable gain. This syntax is equivalent to tunableGain(name, K0init).

# **Examples**

### **Tunable Gain With One Scheduling Variable**

Create a scalar gain *K* that varies as a quadratic function of *t*:

$$K(t) = K_0 + K_1 n(t) + K_2 (n(t))^2$$
.

This gain surface can represent a gain that varies with time. The coefficients  $K_0$ ,  $K_1$ , and  $K_2$  are the tunable parameters of this time-varying gain. For this example, suppose that t varies from 0 to 40. In that case, the normalization function is n(t) = (t - 20)/20.

To represent the tunable gain surface K(t) in MATLAB®, first choose a vector of t values that are the design points of your system. For example, if your design points are snapshots of a time-varying system every 5 seconds from times t = 0 to t = 40, use the following sampling grid:

```
t = 0:5:40;
domain = struct('t',t);
```

Specify a quadratic function for the variable gain.

```
shapefcn = @(x)[x,x^2];
```

shapefcn is the handle to an anonymous vector function. Each entry in the vector gives a term in the polynomial expansion that describes the variable gain. tunableSurface

implicitly assumes the constant function  $f_0(t) = 1$ , so it need not be included in shapefon.

Create the tunable gain surface K(t).

```
K = tunableSurface('K',1,domain,shapefcn)

K =
   Tunable surface "K" of scalar gains with:
    * Scheduling variables: t
    * Basis functions: t,t^2
   * Design points: 1x9 grid of t values
   * Normalization: default (from design points)
```

The display summarizes the characteristics of the gain surface, including the design points and the basis functions. Examine the properties of K.

```
get(K)
```

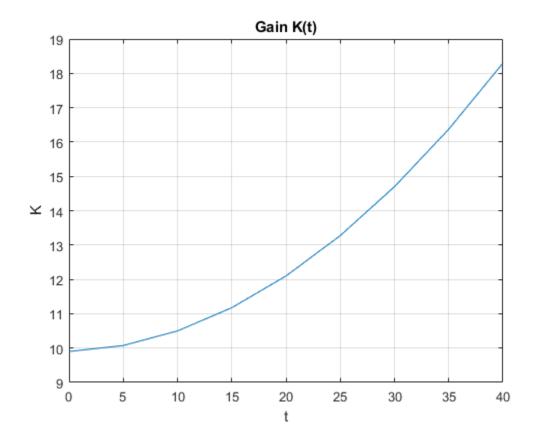
```
BasisFunctions: @(x)[x,x^2]
Coefficients: [1x3 realp]
SamplingGrid: [1x1 struct]
Normalization: [1x1 struct]
Name: 'K'
```

The Coefficients property of the tunable surface is the array of tunable coefficients,  $[K_0, K_1, K_2]$ , stored as an array-valued realp block.

You can now use the tunable surface in a control system model. For tuning in MATLAB, interconnect K with other control system elements just as you would use a Control Design Block to create a tunable control system model. For tuning in Simulink®, use setBlockParam to make K the parameterization of a tunable block in an slTuner interface. When you tune the model or slTuner interface using systune, the resulting model or interface contains tuned values for the coefficients  $K_0$ ,  $K_1$ , and  $K_2$ .

After you tune the coefficients, you can view the shape of the resulting gain curve using the viewSurf command. For this example, instead of tuning, manually set the coefficients to non-zero values. View the resulting gain as a function of time.

```
Ktuned = setData(K,[12.1,4.2,2]);
viewSurf(Ktuned)
```



viewSurf displays the gain as a function of the scheduling variable, for the range of scheduling-variable values specified by domain and stored in the SamplingGrid property of the gain surface.

## **Tunable Gain with Two Independent Scheduling Variables**

This example shows how to model a scalar gain K with a bilinear dependence on two scheduling variables. You do so by creating a grid of design points representing the independent dependence of the two variables.

Suppose that the first variable  $\alpha$  is an angle of incidence that ranges from 0 to 15 degrees, and the second variable V is a speed that ranges from 300 to 600 m/s. By default, the normalized variables are:

$$x = \frac{\alpha - 7.5}{7.5}$$
,  $y = \frac{V - 450}{150}$ .

The gain surface is modeled as:

$$K(\alpha, V) = K_0 + K_1 x + K_2 y + K_3 x y,$$

where  $K_0, \ldots, K_3$  are the tunable parameters.

Create a grid of design points,  $(\alpha, V)$ , that are linearly spaced in  $\alpha$  and V. These design points are the scheduling-variable values used for tuning the gain-surface coefficients. They must correspond to parameter values at which you have sampled the plant.

```
[alpha,V] = ndgrid(0:3:15,300:50:600);
```

These arrays, alpha and V, represent the independent variation of the two scheduling variables, each across its full range. Put them into a structure to define the design points for the tunable surface.

```
domain = struct('alpha',alpha,'V',V);
```

Create the basis functions that describe the bilinear expansion.

```
shapefcn = Q(x,y) [x,y,x*y]; % or use polyBasis('canonical',1,2)
```

In the array returned by shapefon, the basis functions are:

$$F_1(x,y)=x$$

$$F_2(x,y)=y$$

$$F_3(x,y)=xy.$$

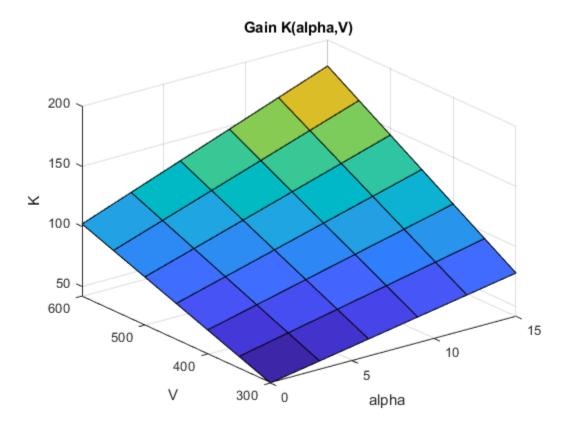
Create the tunable gain surface.

```
K = tunableSurface('K',1,domain,shapefcn);
```

You can use the tunable surface as the parameterization for a lookup table block or a MATLAB Function block in a Simulink model. Or, use model interconnection commands to incorporate it as a tunable element in a control system modeled in MATLAB. After you

tune the coefficients, you can examine the resulting gain surface using the viewSurf command. For this example, instead of tuning, manually set the coefficients to non-zero values and view the resulting gain.

Ktuned = setData(K,[100,28,40,10]);
viewSurf(Ktuned)



viewSurf displays the gain surface as a function of the scheduling variables, for the ranges of values specified by domain and stored in the SamplingGrid property of the gain surface.

### **Gain Surface Over Nonregular Grid**

Create a gain surface using design points that do not form a regular grid in the operating domain. The gain surface varies as a bilinear function of normalized scheduling variables  $\alpha_N$  and  $\beta_N$ :

$$K(\alpha_N, \beta_N) = K_0 + K_1 \alpha_N + K_2 \beta_N + K_3 \alpha_N \beta_N.$$

Suppose that the values of interest of the scheduling variables are the following  $(\alpha, \beta)$  pairs.

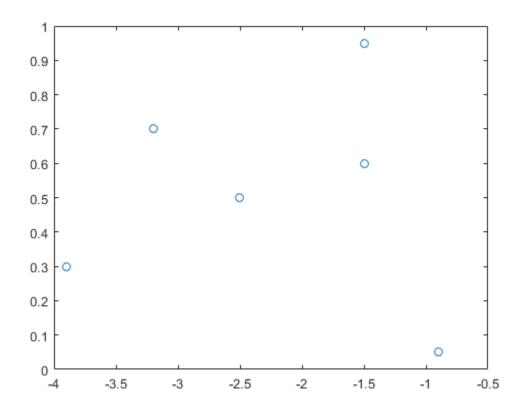
$$(\alpha, \beta) = \begin{cases} (-0.9, 0.05) \\ (-1.5, 0.6) \\ (-1.5, 0.95) \\ (-2.5, 0.5) \\ (-3.2, 0.7) \\ (-3.9, 0.3) \end{cases}$$

Specify the  $(\alpha, \beta)$  sample values as vectors.

```
alpha = [-0.9;-1.5;-1.5;-2.5;-3.2;-3.9];
beta = [0.05;0.6;0.95;0.5;0.7;0.3];
domain = struct('alpha',alpha,'beta',beta);
```

Instead of a regular grid of  $(\alpha, \beta)$  values, here the system is sampled at irregularly spaced points on  $(\alpha, \beta)$ -space.

```
plot(alpha,beta,'o')
```



Specify the basis functions.

shapefcn = @(x,y)[x,y,x\*y];

Create the tunable model of the gain surface using these sampled function values.

K = tunableSurface('K',1,domain,shapefcn)

K =

Tunable surface "K" of scalar gains with:

- \* Scheduling variables: alpha, beta
- \* Basis functions: alpha, beta, alpha\*beta
- \* Design points: 6x1 grid of (alpha, beta) values
- \* Normalization: default (from design points)

The domain is the list of six  $(\alpha, \beta)$  pairs. The normalization, by default, shifts  $\alpha$  and  $\beta$  so that the center of the range of each variable is zero, and scales them so that they range from -1 to 1.

#### K.Normalization

#### **Matrix-Valued Tunable Surface**

Create a tunable gain surface that takes two scheduling variables and returns a 3-by-3 gain matrix. Each entry in the gain matrix is an independent function of the two scheduling variables.

```
Create a grid of design points (alpha, V).
```

```
[alpha,V] = ndgrid(0:3:15,300:50:600);
domain = struct('alpha',alpha,'V',V);
```

Create the basis function that describes how the surface varies with the scheduling variables. Use a basis that describes a bilinear expansion in alpha and V.

```
shapefcn = polyBasis('canonical',1,2);
```

To create the tunable surface, specify the initial value of the matrix-valued gain surface. This value sets the value of the gain surface when the normalized scheduling variables are both zero. tunableSurface takes the dimensions of the gain surface from the initial value you specify. Thus, to create a 3-by-3 gain matrix, use a 3-by-3 initial value.

```
K0init = diag([0.05 0.05 -0.05]);
K0 = tunableSurface('K',K0init,domain,shapefcn)

K0 =
   Tunable surface "K" of 3x3 gain matrices with:
     * Scheduling variables: alpha,V
     * Basis functions: @(x1,x2)utFcnBasisOuterProduct(FDATA_,x1,x2)
     * Design points: 6x7 grid of (alpha,V) values
```

\* Normalization: default (from design points)

# **Input Arguments**

### name — Identifying label for the tunable gain

character vector

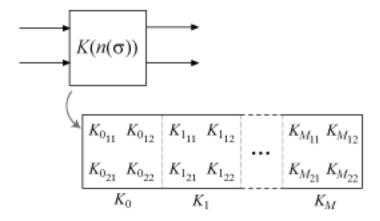
Identifying label for the tunable gain surface, specified as a character vector. tunableSurface uses this name for the realp block that represents the tunable coefficients of the surface. Therefore, you can use this name to refer to the tunable gain coefficients within a genss model of a control system or an slTuner interface.

### **K0init** — **Initial value of constant term**

scalar | array

Initial value of the constant term in the tunable gain surface, specified as a scalar or an array. The dimensions of K0init determine the I/O dimensions of the gain surface. For example, if the gain surface represents a two-input, two-output gain, you can set K0init = ones(2). The remaining coefficients K1, K2,... always have the same size as K0. The tunable coefficients automatically expand so that the gains in each I/O channel are tuned independently.

For example, for a two-input, two-output surface, there is a set of expansion coefficients for each entry in the gain matrix.



Each entry  $K_{ii}$  in the tunable gain matrix  $K(n(\sigma))$  is given by:

$$K_{ij}(n(\sigma)) = K_{ij_0} + K_{ij_1}F_1(n(\sigma)) + ... + K_{ij_M}F_M(n(\sigma)).$$

## domain — Design points

structure

Design points at which the gain surface is tuned, specified as a structure. The structure has fields containing the scheduling variables values at which you sample the plant for gain-scheduled tuning. For example, suppose that you want to tune a gain that varies as a function of two scheduling variables,  $\alpha$  and V. You linearize the plant at a grid of  $\alpha$  and V values, with  $\alpha = [0.5, 0.10, 0.15]$  and V = [700, 800, 900, 1000]

. Specify the design points as follows:

```
[alpha,V] = ndgrid([0.5,0.10,0.15],[700,800,900,1000]);

domain = struct('alpha',alpha,'V',V);
```

The design points do not have to lie on a rectangular or regularly spaced grid (see "Gain Surface Over Nonregular Grid" on page 2-1339). However, for best results use design points that cover the full range of operating conditions. Since tuning only considers these design points, the validity of the tuned gain schedule is questionable at operating conditions far from the design points.

## shapefcn — Basis functions

function handle

Basis functions used to model the gain surface in terms of the scheduling variables, specified as a function handle. The function associated with the handle takes normalized values of the scheduling variables as inputs and returns a vector of basis-function values. The basis functions always operate on the normalized range [-1,1]. tunableSurface implicitly normalizes the scheduling variables to this interval.

For example, consider the scheduling-variable values  $\alpha = [0.5, 0.10, 0.15]$  and V = [700, 800, 900, 1000]. The following expression creates basis functions for a gain surface that is bilinear in these variables:

```
shapefcn = @(x,y) [x y x*y];
```

shapefcn is an anonymous function (MATLAB) of two variables. The basis functions describe a parameterized gain  $G(\alpha, V) = G_0 + G_1\alpha_N + G_2V_N + G_3\alpha_NV_N$  where  $\alpha_N$  and  $V_N$  are the normalized scheduling variables (see the Normalization property of K).

You can use anonymous functions to specify any set of basis functions that you need to describe the variable gain. Alternatively, you can use helper functions to generate basis functions automatically for commonly used expansions:

- polyBasis Power series expansion and Chebyshev expansion.
- fourierBasis Periodic Fourier series expansion. The basis functions generated by fourierBasis are periodic such that a gain surface K defined by those functions satisfies K(-1) = K(1). When you create a gain surface using tunableSurface, the software normalizes the scheduling-variable range that you specify with domain to the interval [-1,1]. Therefore, if you use periodic basis functions, then the sampled range of the corresponding scheduling variable must be exactly one period. This restriction ensures that the periodicity of the basis function matches that of the scheduling variable. For example, if the periodically varying scheduling variable is an angle that ranges from 0 to  $2\pi$ , then the corresponding values in domain must also range from 0 to  $2\pi$ .
- ndBasis Build multidimensional expansions from lower-dimensional expansions.
   This function is useful when you want to use different basis functions for different scheduling variables.

See the reference pages for those functions for more information about the basis functions they generate.

# **Output Arguments**

## **K** — Tunable gain surface

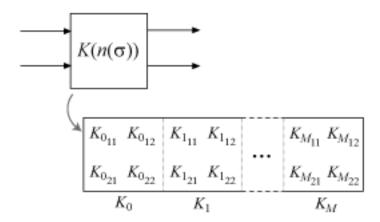
tunableSurface object

Tunable gain surface, returned as a tunableSurface object. This object has the following properties that store the coefficients, basis functions, and other information about the gain surface:

- BasisFunctions Basis functions, specified as a function handle. When you create the gain surface, the shapefcn input argument sets the initial value of this property.
- Coefficients Tunable coefficients of the gain surface, specified as an arrayvalued realp tunable parameter. The dimensions of K0init and the number of basis functions in shapefcn determine the dimensions of K.Coefficients.

For scalar gains, K.Coefficients has dimensions [1,M+1], where M is the number of basis functions. The entries in K.Coefficients correspond to the tunable coefficients  $K_0,...,K_M$ .

For array-valued gains, each coefficient expands to the dimension of K0init. These expanded coefficients are concatenated horizontally in K.Coefficients. Therefore, for example, for a two-input, two-output gain surface, K.Coefficients has dimensions [2,2(M+1)].



Each entry  $K_{ij}$  in the tunable gain matrix  $K(n(\sigma))$  is given by:

$$K_{ij}(n(\sigma)) = K_{ij_0} + K_{ij_1}F_1(n(\sigma)) + \dots + K_{ij_M}F_M(n(\sigma)).$$

- SamplingGrid Grid of design points, specified as a data structure. When you create the gain surface, the domain input argument sets the initial value of this property.
- Normalization Normalization offset and scaling, specified as a structure with fields:
  - InputOffset Vector of offsets for each scheduling variable.
  - InputScaling Vector of scaling factors for each scheduling variable.
  - OutputScaling Scaling factor for overall gain.

In general, the tunableSurface parameterization takes the form:

$$K(\sigma) = \text{OutputScaling}[K_0 + K_1 F_1(n(\sigma)) + \dots + K_m F_m(n(\sigma))],$$

where  $n(\sigma)$  is the normalized scheduling variable, given by:

$$n(\sigma) = \frac{\sigma - \text{InputOffset}}{\text{InputScaling}}.$$

tunableSurface normalizes the scheduling variables to compress their numerical range and improve the numerical stability of the optimization process. By default, OutputScaling = 1, and tunableSurface computes values for InputOffset and InputScaling that map the SamplingGrid domain of each scheduling variable to [-1,1]. Thus, n=0 at the center of the design-point range.

You can change the default normalization by adjusting the values of these fields. For example:

- If you have a known gain value for a particular design point, you can set Normalization. InputOffset so that n=0 at that design point. You can then set K0init to the known gain value.
- If you want to restrict a scheduling variable to nonnegative values, set Normalization.InputOffset to the minimum value of that variable in your design grid. This restriction is useful, for example, when your basis function includes  $\sqrt{\sigma}$ .
- Name Name of the gain surface, specified as a character vector. When you create the gain surface, the name input argument sets the initial value of this property.

## **Tips**

• To tune a gain surface in a control system modeled in MATLAB: Connect the gain surface with an array of plant models corresponding to the design points in domain. For example, suppose G is such an array, and K represents a variable integration time. The following command builds a closed-loop model that you can tune with the systume command.

```
C0 = tf(K,[1 0]);
T0 = feedback(C0*G,1);
```

• To tune a gain surface in a control system modeled in Simulink: Use the gain surface to parameterize a lookup-table, matrix interpolation, or MATLAB function block in the Simulink model. For example, suppose ST0 is an slTuner interface to a Simulink model, and GainTable is the name of a tuned block in the interface. The following command sets the parameterization of GainTable to the tunable gain surface.

```
ST0 = setBlockParam(ST0, 'GainTable', K);
```

See "Parameterize Gain Schedules" for more information.

 When you use writeBlockValue to write a tuned gain surface back to a Simulink model, the software uses codegen to generate MATLAB code for the gain surface. You can use codegen yourself to examine this code.

### See Also

#### **Functions**

codegen | evalSurf | fourierBasis | ndBasis | ndgrid | polyBasis | systume |
viewSurf

### **Topics**

"Parameterize Gain Schedules"

"Gain Scheduling Basics"

"Tuning of Gain-Scheduled Three-Loop Autopilot"

#### Introduced in R2015b

### tunableTF

Tunable transfer function with fixed number of poles and zeros

## **Syntax**

```
blk = tunableTF(name,Nz,Np)
blk = tunableTF(name,Nz,Np,Ts)
blk = tunableTF(name,sys)
```

## **Description**

Model object for creating tunable SISO transfer function models of fixed order. tunableTF lets you parameterize a transfer function of a given order for parameter studies or for automatic tuning with tuning commands such as systume or looptune.

tunableTF is part of the Control Design Block family of parametric models. Other Control Design Blocks include tunablePID, tunableSS, and tunableGain.

### Construction

blk = tunableTF(name,Nz,Np) creates the parametric SISO transfer function:

$$blk = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}.$$

n = Np is the maximum number of poles of blk, and m = Nz is the maximum number of zeros. The tunable parameters are the numerator and denominator coefficients  $a_0, ..., a_m$  and  $b_0, ..., b_{n-1}$ . The leading coefficient of the denominator is fixed to 1.

blk = tunableTF(name,Nz,Np,Ts) creates a discrete-time parametric transfer function with sample time Ts.

blk = tunableTF(name, sys) uses the tf model sys to set the number of poles, number of zeros, sample time, and initial parameter values.

### **Input Arguments**

#### name

Parametric transfer function Name, specified as a character vector such as 'filt' or 'DM'. (See "Properties" on page 2-1350.)

#### Νz

Nonnegative integer specifying the number of zeros of the parametric transfer function blk.

### Np

Nonnegative integer specifying the number of poles of the parametric transfer function blk.

#### Ts

Scalar sample time.

#### sys

tf model providing number of poles, number of zeros, sample time, and initial values of the parameters of blk.

## **Properties**

#### Numerator, Denominator

Parameterization of the numerator coefficients  $a_m$ , ...,  $a_0$  and the denominator coefficients  $1,b_{n-1}$ , ...,  $b_0$  of the tunable transfer function blk.

blk.Numerator and blk.Denominator are param.Continuous objects. For general information about the properties of these param.Continuous objects, see the param.Continuous object reference page.

The following fields of blk.Numerator and blk.Denominator are used when you tune blk using hinfstruct:

Field	Description
Value	Array of current values of the numerator $a_m$ ,, $a_0$ or the denominator coefficients $1,b_{n-1}$ ,, $b_0$ . blk.Numerator.Value has length Nz + 1. blk.Denominator.Value has length Np + 1. The leading coefficient of the denominator (blk.Denominator.Value(1)) is always fixed to 1.
	By default, the coefficients initialize to values that yield a stable, strictly proper transfer function. Use the input sys to initialize the coefficients to different values.  hinfstruct tunes all values except those
	whose Free field is zero.
Free	Array of logical values determining whether the coefficients are fixed or tunable. For example:
	• If blk.Numerator.Free(j) = 1, then blk.Numerator.Value(j) is tunable.
	• If blk.Numerator.Free(j) = 0, then blk.Numerator.Value(j) is fixed.
	Default: blk.Denominator.Free(1) = 0; all other entries are 1.
Minimum	Minimum value of the parameter. This property places a lower bound on the tuned value of the parameter. For example, setting blk.Numerator.Minimum(1) = 0 ensures that the leading coefficient of the numerator remains positive.
	Default: -Inf

Field	Description
Maximum	Maximum value of the parameter. This property places an upper bound on the tuned value of the parameter. For example, setting blk.Numerator.Maximum(1) = 1 ensures that the leading coefficient of the numerator does not exceed 1.  Default: Inf

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

### OutputUnit

Output channel units, specified as one of the following:

- Character vector For single-output models. For example, 'seconds'.
- Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

**Default:** ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

**Default:** Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ''

#### Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

**Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

Default: []

## **Examples**

Create a parametric SISO transfer function with two zeros, four poles, and at least one integrator.

A transfer function with an integrator includes a factor of 1/s. Therefore, to ensure that a parameterized transfer function has at least one integrator regardless of the parameter values, fix the lowest-order coefficient of the denominator to zero.

```
blk = tunableTF('tfblock',2,4); % two zeros, four poles
blk.Denominator.Value(end) = 0; % set last denominator entry to zero
blk.Denominator.Free(end) = 0; % fix it to zero
```

Create a parametric transfer function, and assign names to the input and output.

## **Tips**

• To convert a tunableTF parametric model to a numeric (non-tunable) model object, use model commands such as tf, zpk, or ss.

## **Compatibility Considerations**

### Name changed from ltiblock.tf

Behavior changed in R2016a

Prior to R2016a, tunableTF was called ltiblock.tf.

### See Also

genss | hinfstruct | looptune | systune | tunableGain | tunablePID |
tunablePID2 | tunableSS

### **Topics**

"Control Design Blocks"
"Models with Tunable Coefficients"

Introduced in R2016a

### tzero

Invariant zeros of linear system

## **Syntax**

```
z = tzero(sys)
z = tzero(A,B,C,D,E)
z = tzero(____,tol)
[z,nrank] = tzero(____)
```

## **Description**

z = tzero(sys) returns the invariant zeros on page 2-1362 of the multi-input, multi-output (MIMO) dynamic system, sys. If sys is a minimal realization, the invariant zeros coincide with the transmission zeros on page 2-1362 of sys.

z = tzero(A,B,C,D,E) returns the invariant zeros on page 2-1362 of the state-space model

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du.$$

Omit E for an explicit state-space model (E = I).

z = tzero(\_\_\_\_, tol) specifies the relative tolerance, tol, controlling rank decisions.

[z,nrank] = tzero(\_\_\_\_) also returns the normal rank of the transfer function of sys or of the transfer function  $H(s) = D + C(sE - A)^{-1}B$ .

## **Input Arguments**

### sys

MIMO dynamic system model. If sys is not a state-space model, then tzero computes tzero(ss(sys)).

### A,B,C,D,E

State-space matrices describing the linear system

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du.$$

tzero does not scale the state-space matrices when you use the syntax z = tzero(A,B,C,D,E). Use prescale if you want to scale the matrices before using tzero.

Omit E to use E = I.

#### tol

Relative tolerance controlling rank decisions. Increasing tolerance helps detect nonminimal modes and eliminate very large zeros (near infinity). However, increased tolerance might artificially inflate the number of transmission zeros.

**Default:** eps^(3/4)

## **Output Arguments**

#### Z

Column vector containing the invariant zeros on page 2-1362 of sys or the state-space model described by A,B,C,D,E.

#### nrank

Normal rank of the transfer function of sys or of the transfer function  $H(s) = D + C(sE - A)^{-1}B$ . The normal rank is the rank for values of s other than the transmission zeros on page 2-1362.

To obtain a meaningful result for nrank, the matrix s\*E-A must be regular (invertible for most values of s). In other words, sys or the system described by A,B,C,D,E must have a finite number of poles.

## **Examples**

#### Find Transmission Zeros of MIMO Transfer Function

Create a MIMO transfer function, and locate its invariant zeros.

```
s = tf('s');
H = [1/(s+1) 1/(s+2);1/(s+3) 2/(s+4)];
z = tzero(H)

z = 2×1 complex
    -2.5000 + 1.3229i
    -2.5000 - 1.3229i
```

The output is a column vector listing the locations of the invariant zeros of H. This output shows that H a has complex pair of invariant zeros. Confirm that the invariant zeros coincide with the transmission zeros.

Check whether the first invariant zero is a transmission zero of H.

If z(1) is a transmission zero of H, then H drops rank at s = z(1).

```
H1 = evalfr(H,z(1));

svd(H1)

ans = 2×1

1.5000

0.0000
```

H1 is the transfer function, H, evaluated at s = z(1). H1 has a zero singular value, indicating that H drops rank at that value of s. Therefore, z(1) is a transmission zero of H.

A similar analysis shows that z(2) is also a transmission zero.

### **Identify Unobservable and Uncontrollable Modes of MIMO Model**

Obtain a MIMO model.

```
load ltiexamples gasf
size(gasf)
State-space model with 4 outputs, 6 inputs, and 25 states.
```

gasf is a MIMO model that might contain uncontrollable or unobservable states.

To identify the unobservable and uncontrollable modes of gasf, you need the state-space matrices A, B, C, and D of the model. tzero does not scale state-space matrices. Therefore, use prescale with ssdata to scale the state-space matrices of gasf.

```
[A,B,C,D] = ssdata(prescale(gasf));
```

Identify the uncontrollable states of gasf.

```
uncon = tzero(A,B,[],[])
uncon = 6 \times 1

-0.0568
-0.0568
-0.0568
-0.0568
-0.0568
-0.0568
```

When you provide A and B matrices to tzero, but no C and D matrices, the command returns the eigenvalues of the uncontrollable modes of gasf. The output shows that there are six degenerate uncontrollable modes.

Identify the unobservable states of gasf.

```
unobs = tzero(A,[],C,[])
unobs =
0x1 empty double column vector
```

When you provide A and C matrices, but no B and D matrices, the command returns the eigenvalues of the unobservable modes. The empty result shows that gasf contains no unobservable states.

### **More About**

### **Invariant zeros**

For a MIMO state-space model

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du,$$

the invariant zeros are the complex values of s for which the rank of the system matrix

$$\begin{bmatrix} A - sE & B \\ C & D \end{bmatrix}$$

drops from its normal value. (For explicit state-space models, E = I).

### **Transmission zeros**

For a MIMO state-space model

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du,$$

the transmission zeros are the complex values of s for which the rank of the equivalent transfer function  $H(s) = D + C(sE - A)^{-1}B$  drops from its normal value. (For explicit statespace models, E = I.)

Transmission zeros are a subset of the invariant zeros on page 2-1362. For minimal realizations, the transmission zeros and invariant zeros are identical.

## **Tips**

• You can use the syntax z = tzero(A,B,C,D,E) to find the uncontrollable or unobservable modes of a state-space model. When C and D are empty or zero, tzero returns the uncontrollable modes of (A-sE,B). Similarly, when B and D are empty or zero, tzero returns the unobservable modes of (C,A-sE). See "Identify Unobservable and Uncontrollable Modes of MIMO Model" on page 2-1361 for an example.

## **Algorithms**

tzero is based on SLICOT routines AB08ND, AG08BD, and AB8NXZ. tzero implements the algorithms in [1] and [2].

### **Alternatives**

To calculate the zeros and gain of a single-input, single-output (SISO) system, use zero.

### References

- [1] Emami-Naeini, A. and P. Van Dooren, "Computation of Zeros of Linear Multivariable Systems," *Automatica*, 18 (1982), pp. 415–430.
- [2] Misra, P, P. Van Dooren, and A. Varga, "Computation of Structural Invariants of Generalized State-Space Systems," *Automatica*, 30 (1994), pp. 1921-1936.

### See Also

pole|pzmap|zero

Introduced in R2012a

## unscentedKalmanFilter

Create unscented Kalman filter object for online state estimation

## **Syntax**

```
obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn,
InitialState)
obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn,
InitialState, Name, Value)

obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn)
obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn,
Name, Value)
obj = unscentedKalmanFilter(Name, Value)
```

## **Description**

obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn, InitialState) creates an unscented Kalman filter object for online state estimation of a discrete-time nonlinear system. StateTransitionFcn is a function that calculates the state of the system at time k, given the state vector at time k-1. MeasurementFcn is a function that calculates the output measurement of the system at time k, given the state at time k. InitialState specifies the initial value of the state estimates.

After creating the object, use the correct and predict commands to update state estimates and state estimation error covariance values using a discrete-time unscented Kalman filter algorithm and real-time data.

obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn, InitialState, Name, Value) specifies additional attributes of the unscented Kalman filter object using one or more Name, Value pair arguments.

obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn) creates an unscented Kalman filter object using the specified state transition and measurement functions. Before using the predict and correct commands, specify the initial state

values using dot notation. For example, for a two-state system with initial state values [1;0], specify obj.State = [1;0].

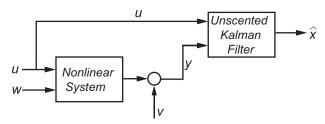
obj = unscentedKalmanFilter(StateTransitionFcn, MeasurementFcn, Name, Value) specifies additional attributes of the unscented Kalman filter object using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the initial state values using Name, Value pair arguments or dot notation.

obj = unscentedKalmanFilter(Name, Value) creates an unscented Kalman filter object with properties specified using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the state transition function, measurement function, and initial state values using Name, Value pair arguments or dot notation.

## **Object Description**

unscentedKalmanFilter creates an object for online state estimation of a discrete-time nonlinear system using the discrete-time unscented Kalman filter algorithm.

Consider a plant with states x, input u, output y, process noise w, and measurement noise v. Assume that you can represent the plant as a nonlinear system.



The algorithm computes the state estimates  $\hat{x}$  of the nonlinear system using state transition and measurement functions specified by you. The software lets you specify the noise in these functions as additive or nonadditive:

• **Additive Noise Terms** — The state transition and measurements equations have the following form:

$$x[k] = f(x[k-1], u_s[k-1]) + w[k-1]$$
  
$$y[k] = h(x[k], u_m[k]) + v[k]$$

Here f is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function h relates x to the measurements y at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional input arguments that are denoted by  $u_s$  and  $u_m$  in the equations. For example, the additional arguments could be time step k or the inputs u to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, x(k) is linearly related to the process noise w(k-1), and y(k) is linearly related to the measurement noise v(k).

• Nonadditive Noise Terms — The software also supports more complex state transition and measurement functions where the state x[k] and measurement y[k] are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$x[k] = f(x[k-1], w[k-1], u_s[k-1])$$
  
$$y[k] = h(x[k], v[k], u_m[k])$$

When you perform online state estimation, you first create the nonlinear state transition function f and measurement function h. You then construct the unscentedKalmanFilter object using these nonlinear functions, and specify whether the noise terms are additive or nonadditive. After you create the object, you use the predict command to predict state estimates at the next time step, and correct to correct state estimates using the unscented Kalman filter algorithm and real-time data. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You can use the following commands with unscentedKalmanFilter objects:

Command	Description
	Correct the state and state estimation error covariance at time step $k$ using measured data at time step $k$ .
predict	Predict the state and state estimation error covariance at time the next time step.

Command	Description
residual	Return the difference between the actual and predicted measurements.
	Create another object with the same object property values.  Do not create additional objects using syntax obj2 = obj. Any changes made to the properties of the new object created in
	this way (obj2) also change the properties of the original object (obj).

For unscentedKalmanFilter object properties, see "Properties" on page 2-1373.

## **Examples**

### **Create Unscented Kalman Filter Object for Online State Estimation**

To define an unscented Kalman filter object for estimating the states of your system, you write and save the state transition function and measurement function for the system.

In this example, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1. The oscillator has two states.

Specify an initial guess for the two states. You specify the initial state guess as an Melement row or column vector, where M is the number of states.

```
initialStateGuess = [1;0];
```

Create the unscented Kalman filter object. Use function handles to provide the state transition and measurement functions to the object.

```
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,initialStateGuess);
```

The object has a default structure where the process and measurement noise are additive.

To estimate the states and state estimation error covariance from the constructed object, use the correct and predict commands and real-time data.

# Specify Process and Measurement Noise Covariances in Unscented Kalman Filter Object

Create an unscented Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions are written for additive process and measurement noise terms. Specify the initial state values for the two states as [2;0].

Since the system has two states and the process noise is additive, the process noise is a 2-element vector and the process noise covariance is a 2-by-2 matrix. Assume there is no cross-correlation between process noise terms, and both the terms have the same variance 0.01. You can specify the process noise covariance as a scalar. The software uses the scalar value to create a 2-by-2 diagonal matrix with 0.01 on the diagonals.

Specify the process noise covariance during object construction.

Alternatively, you can specify noise covariances after object construction using dot notation. For example, specify the measurement noise covariance as 0.2.

```
obj.MeasurementNoise = 0.2;
```

Since the system has only one output, the measurement noise is a 1-element vector and the MeasurementNoise property denotes the variance of the measurement noise.

### Specify Nonadditive Measurement Noise in Unscented Kalman Filter Object

Create an unscented Kalman filter object for a van der Pol oscillator with two states and one output. Assume that the process noise terms in the state transition function are additive. That is, there is a linear relation between the state and process noise. Also assume that the measurement noise terms are nonadditive. That is, there is a nonlinear relation between the measurement and measurement noise.

```
obj = unscentedKalmanFilter('HasAdditiveMeasurementNoise',false);
```

Specify the state transition function and measurement functions. Use the previously written and saved functions, vdpStateFcn.m and vdpMeasurementNonAdditiveNoiseFcn.m.

The state transition function is written assuming the process noise is additive. The measurement function is written assuming the measurement noise is nonadditive.

```
obj.StateTransitionFcn = @vdpStateFcn;
obj.MeasurementFcn = @vdpMeasurementNonAdditiveNoiseFcn;
```

Specify the initial state values for the two states as [2;0].

```
obj.State = [2;0];
```

You can now use the correct and predict commands to estimate the state and state estimation error covariance values from the constructed object.

### Specify Additional Inputs in State Transition and Measurement Functions

Consider a nonlinear system with input u whose state x and measurement y evolve according to the following state transition and measurement equations:

$$x[k] = \sqrt{x[k-1] + u[k-1]} + w[k-1]$$
$$v[k] = x[k] + 2 * u[k] + v[k]^{2}$$

The process noise  $\boldsymbol{v}$  of the system is additive while the measurement noise  $\boldsymbol{v}$  is nonadditive.

Create the state transition function and measurement function for the system. Specify the functions with an additional input u.

```
f = @(x,u)(sqrt(x+u));

h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an unscented Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1, and the measurement noise as nonadditive.

```
obj = unscentedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
```

Specify the measurement noise covariance.

```
obj.MeasurementNoise = 0.01;
```

You can now estimate the state of the system using the predict and correct commands. You pass the values of u to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement y[k]=0.8 and input u[k]=0.2 at time step k.

```
correct(obj, 0.8, 0.2)
```

Predict the state at next time step, given u[k]=0.2.

```
predict(obj,0.2)
```

## **Input Arguments**

#### StateTransitionFcn — State transition function

function handle

State transition function f, specified as a function handle. The function calculates the Ns-element state vector of the system at time step k, given the state vector at time step k-1. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system, and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

HasAdditiveProcessNoise is true — The process noise w is additive, and the state
transition function specifies how the states evolve as a function of state values at the
previous time step:

$$x(k) = f(x(k-1), Us1, \dots, Usn)$$

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.

 HasAdditiveProcessNoise is false — The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$x(k) = f(x(k-1), w(k-1), Us1, ..., Usn)$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

### MeasurementFcn — Measurement function

function handle

Measurement function h, specified as a function handle. The function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. N is the number of measurements of the system. You write and save the measurement function, and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn as qvdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

 HasAdditiveMeasurementNoise is true — The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:

$$y(k) = h(x(k),Um1,...,Umn)$$

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an

additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command, which in turn passes them to the measurement function.

 HasAdditiveMeasurementNoise is false — The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

```
y(k) = h(x(k), v(k), Um1, ..., Umn)
```

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

#### InitialState — Initial state estimates

vector

Initial state estimates, specified as an Ns-element vector, where Ns is the number of states in the system. Specify the initial state values based on your knowledge of the system.

The specified value is stored in the State property of the object. If you specify InitialState as a column vector then State is also a column vector, and predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned.

If you want a filter with single-precision floating-point variables, specify InitialState as a single-precision vector variable. For example, for a two-state system with state transition and measurement functions vdpStateFcn.m and vdpMeasurementFcn.m, create the unscented Kalman filter object with initial states [1;2] as follows:

```
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,single([1;2]))
Data Types: double | single
```

### **Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify properties on page 2-1373 of unscentedKalmanFilter object during object creation. For example, to create an unscented Kalman filter object and specify the process noise covariance as 0.01:

```
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState,'ProcessNoise',0.01);
```

## **Properties**

unscentedKalmanFilter object properties are of three types:

 Tunable properties that you can specify multiple times, either during object construction using Name, Value arguments, or any time afterwards during state estimation. After object creation, use dot notation to modify the tunable properties.

```
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState);
obj.ProcessNoise = 0.01;
```

The tunable properties are State, StateCovariance, ProcessNoise, MeasurementNoise, Alpha, Beta, and Kappa.

- Nontunable properties that you can specify once, either during object construction or afterward using dot notion. Specify these properties before state estimation using correct and predict. The StateTransitionFcn and MeasurementFcn properties belong to this category.
- Nontunable properties that you must specify during object construction. The HasAdditiveProcessNoise and HasAdditiveMeasurementNoise properties belong to this category.

### Alpha — Spread of sigma points

1e-3 (default) | scalar value between 0 and 1

Spread of sigma points around mean state value, specified as a scalar value between 0 and 1 (0 < Alpha <= 1).

The unscented Kalman filter algorithm treats the state of the system as a random variable with mean value State and variance StateCovariance. To compute the state and its statistical properties at the next time step, the algorithm first generates a set of state values distributed around the mean State value by using the unscented transformation. These generated state values are called sigma points. The algorithm uses each of the sigma points as an input to the state transition and measurement functions to get a new set of transformed state points and measurements. The transformed points are used to compute the state and state estimation error covariance value at the next time step.

The spread of the sigma points around the mean state value is controlled by two parameters Alpha and Kappa. A third parameter, Beta, impacts the weights of the transformed points during state and measurement covariance calculations:

- Alpha Determines the spread of the sigma points around the mean state value. It is usually a small positive value. The spread of sigma points is proportional to Alpha. Smaller values correspond to sigma points closer to the mean state.
- Kappa A second scaling parameter that is usually set to 0. Smaller values
  correspond to sigma points closer to the mean state. The spread is proportional to the
  square-root of Kappa.
- Beta Incorporates prior knowledge of the distribution of the state. For Gaussian distributions, Beta = 2 is optimal.

If you know the distribution of state and state covariance, you can adjust these parameters to capture the transformation of higher-order moments of the distribution. The algorithm can track only a single peak in the probability distribution of the state. If there are multiple peaks in the state distribution of your system, you can adjust these parameters so that the sigma points stay around a single peak. For example, choose a small Alpha to generate sigma points close to the mean state value.

For more information, see "Unscented Kalman Filter Algorithm".

Alpha is a tunable property. You can change it using dot notation.

#### Beta — Characterization of state distribution

2 (default) | scalar value greater than or equal to 0

Characterization of the state distribution that is used to adjust weights of transformed sigma points, specified as a scalar value greater than or equal to 0. For Gaussian distributions, Beta = 2 is an optimal choice.

For more information, see the Alpha property description.

Beta is a tunable property. You can change it using dot notation.

# HasAdditiveMeasurementNoise — Measurement noise characteristics true (default) | false

Measurement noise characteristics, specified as one of the following values:

• true — Measurement noise v is additive. The measurement function h that is specified in MeasurementFcn has the following form:

$$y(k) = h(x(k), Um1, ..., Umn)$$

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function.

• false — Measurement noise is nonadditive. The measurement function specifies how the output measurement evolves as a function of the state *and* measurement noise:

$$y(k) = h(x(k), v(k), Um1, ..., Umn)$$

HasAdditiveMeasurementNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

### HasAdditiveProcessNoise — Process noise characteristics

true (default) | false

Process noise characteristics, specified as one of the following values:

• true — Process noise w is additive. The state transition function f specified in StateTransitionFcn has the following form:

$$x(k) = f(x(k-1), Us1, \dots, Usn)$$

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function.

• false — Process noise is nonadditive. The state transition function specifies how the states evolve as a function of the state *and* process noise at the previous time step:

$$x(k) = f(x(k-1),w(k-1),Us1,...,Usn)$$

HasAdditiveProcessNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

### Kappa — Spread of sigma points

0 (default) | scalar value between 0 and 3

Spread of sigma points around mean state value, specified as a scalar value between 0 and 3 ( $\theta \le \text{Kappa} \le 3$ ). Kappa is typically specified as  $\theta$ . Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square-root of Kappa. For more information, see the Alpha property description.

Kappa is a tunable property. You can change it using dot notation.

#### MeasurementFcn — Measurement function

function handle

Measurement function h, specified as a function handle. The function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. N is the number of measurements of the system. You write and save the measurement function and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn as QvdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

 HasAdditiveMeasurementNoise is true — The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:

```
y(k) = h(x(k), Um1, ..., Umn)
```

Where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command which in turn passes them to the measurement function.

• HasAdditiveMeasurementNoise is false — The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

$$y(k) = h(x(k), v(k), Um1, ..., Umn)$$

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

MeasurementFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

#### MeasurementNoise — Measurement noise covariance

1 (default) | scalar | matrix

Measurement noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true Specify the covariance as a scalar or an *N*-by-*N* matrix, where *N* is the number of measurements of the system. Specify a scalar if there is no cross-correlation between measurement noise terms and all the terms have the same variance. The software uses the scalar value to create an *N*-by-*N* diagonal matrix.
- HasAdditiveMeasurementNoise is false Specify the covariance as a V-by-V matrix, where V is the number of measurement noise terms. MeasurementNoise must be specified before using correct. After you specify MeasurementNoise as a matrix for the first time, to then change MeasurementNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the measurement noise terms and all the terms have the same variance. The software extends the scalar to a V-by-V diagonal matrix with the scalar on the diagonals.

MeasurementNoise is a tunable property. You can change it using dot notation.

#### ProcessNoise — Process noise covariance

1 (default) | scalar | matrix

Process noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveProcessNoise property:

- HasAdditiveProcessNoise is true Specify the covariance as a scalar or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. The software uses the scalar value to create an *Ns*-by-*Ns* diagonal matrix.
- HasAdditiveProcessNoise is false Specify the covariance as a W-by-W matrix, where W is the number of process noise terms. ProcessNoise must be specified before using predict. After you specify ProcessNoise as a matrix for the first time, to then change ProcessNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the process noise terms and all the terms have the same variance. The software extends the scalar to a W-by-W diagonal matrix.

ProcessNoise is a tunable property. You can change it using dot notation.

### State — State of nonlinear system

[] (default) | vector

State of the nonlinear system, specified as a vector of size Ns, where Ns is the number of states of the system.

When you use the predict command, State is updated with the predicted value at time step k using the state value at time step k-1. When you use the correct command, State is updated with the estimated value at time step k using measured data at time step k.

The initial value of State is the value you specify in the InitialState input argument during object creation. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned. If you want a filter with single-precision floating-point variables, you must specify State as a single-precision variable during object construction using the InitialState input argument.

**State** is a tunable property. You can change it using dot notation.

#### StateCovariance — State estimation error covariance

1 (default) | scalar | matrix

State estimation error covariance, specified as a scalar or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. If you specify a scalar, the software uses the scalar value to create an *Ns*-by-*Ns* diagonal matrix.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in the InitialState input argument.

When you use the predict command, StateCovariance is updated with the predicted value at time step k using the state value at time step k-1. When you use the correct command, StateCovariance is updated with the estimated value at time step k using measured data at time step k.

StateCovariance is a tunable property. You can change it using dot notation after using the correct or predict commands.

### StateTransitionFcn — State transition function

function handle

State transition function f, specified as a function handle. The function calculates the Ns-element state vector of the system at time step k, given the state vector at time step k-1. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

• HasAdditiveProcessNoise is true — The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at previous time step:

$$x(k) = f(x(k-1), Us1, \dots, Usn)$$

Where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.

 HasAdditiveProcessNoise is false — The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$x(k) = f(x(k-1), w(k-1), Us1, ..., Usn)$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

StateTransitionFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## **Output Arguments**

### obj — unscented Kalman filter object for online state estimation

unscentedKalmanFilter object

Unscented Kalman filter object for online state estimation, returned as an unscentedKalmanFilter object. This object is created using the specified properties on page 2-1373. Use the correct and predict commands to estimate the state and state estimation error covariance using the unscented Kalman filter algorithm.

When you use predict, obj.State and obj.StateCovariance are updated with the predicted value at time step k using the state value at time step k-1. When you use correct, obj.State and obj.StateCovariance are updated with the estimated values at time step k using measured data at time step k.

## **Extended Capabilities**

### **C/C++ Code Generation**

Generate C and C++ code using MATLAB® Coder™.

Usage notes and limitations:

For more information, see "Generate Code for Online State Estimation in MATLAB".

Supports MATLAB Function block: No

### See Also

#### **Functions**

clone | correct | extendedKalmanFilter | kalman | kalmd | predict | residual

#### **Blocks**

Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

### **Topics**

- "Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
- "Generate Code for Online State Estimation in MATLAB"
- "Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
- "Validate Online State Estimation at the Command Line"

#### Introduced in R2016b

<sup>&</sup>quot;Troubleshoot Online State Estimation"

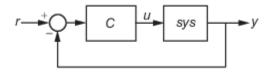
### **Tune PID Controller**

Tune PID Controller for LTI plant in the Live Editor

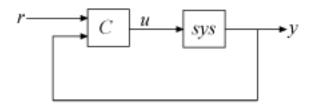
## **Description**

The **Tune PID Controller** Live Editor task lets you interactively tune a PID controller for a plant. The task automatically generates MATLAB code for your live script. For more information about Live Editor tasks generally, see "Add Interactive Tasks to a Live Script" (MATLAB).

**Tune PID Controller** automatically tunes the gains of a PID controller for a SISO plant to achieve a balance between performance and robustness. You can specify the controller type, such as PI, PD, or PID with or without a derivative filter. By default, **Tune PID Controller** assumes the following standard unit-feedback control configuration.



You can also use **Tune PID Controller** to design a 2-DOF PID controller for the feedback configuration of this illustration:



**Tune PID Controller** generates analysis plots that let you examine controller performance in the time and frequency domains. You can interactively refine the performance of the controller to adjust response time, loop bandwidth, or phase margin, or to favor setpoint tracking or disturbance rejection.

To get started with the **Tune PID Controller** task, select the plant model and specify the type of controller you want to design. Experiment with the sliders and observe their effect on the closed-loop system response. For an example, see "PID Controller Design in the Live Editor".

### **Related Functionality**

- Tune PID Controller generates code using pidtune and pidtuneOptions.
- To perform interactive PID tuning in a standalone app, use **PID Tuner**.

## **Description (collapsed portion)**

### **Related Functionality**

- Tune PID Controller generates code using pidtune and pidtuneOptions.
- To perform interactive PID tuning in a standalone app, use **PID Tuner**.

## Open the Task

To add the Tune PID Controller task to a live script in the MATLAB Editor:

- On the Live Editor tab, select Task > Tune PID Controller.
- In a code block in your script, type a relevant keyword, such as tune or PID. Select Tune PID Controller from the suggested command completions.

### **Parameters**

**Plant and Controller Settings** 

### Plant — Current plant

LTI model

Choose a plant for which to design a controller. The list contains continuous-time or discrete-time SISO LTI models present in the MATLAB workspace, such as:

- State-space (ss), transfer function (tf), and zero-pole-gain (zpk) models.
- For frequency-response data (frd) models. For such plants, only frequency-domain design goals and response plots are available.
- Generalized state-space (genss) or uncertain state-space (uss) models. For such models, Tune PID Controllers uses the current, nominal value of the tunable and uncertain components.
- Identified models, such as idss and idtf models.

#### Form — Controller form

'Parallel'|'Standard'

Specify the controller form. The two forms differ in the parameters used to express the proportional, integral, and derivative actions and the filter on the derivative term. For information about parallel and standard forms, see:

- "Proportional-Integral-Derivative (PID) Controllers"
- "Discrete-Time Proportional-Integral-Derivative (PID) Controllers"
- "Two-Degree-of-Freedom PID Controllers"

# **Degrees of Freedom — Specify 1-DOF or 2-DOF controller** 1D0F (default) | 2D0F

By default, **Tune PID Controller** designs a one-degree-of-freedom (1-DOF) controller. Such a controller has up to four coefficients (see "Proportional-Integral-Derivative (PID) Controllers").

You can instead design a two-degree-of-freedom (2-DOF) PID controller. Such controllers include setpoint weighting on the proportional and derivative terms. A 2-DOF PID controller is capable of fast disturbance rejection without significant increase of overshoot in setpoint tracking. You can also use 2-DOF PID controllers to mitigate the influence of changes in the reference signal on the control signal. For more information, see "Two-Degree-of-Freedom PID Controllers".

# Controller Type — Specify the terms the controller has 'PI' (default) | 'PIDF' | 'PID2' | ...

The controller type specifies which terms are present in the PID controller. For instance, a PI controller has a proportional and an integral term. A PDF controller has a proportional term and a filtered derivative term. For details on available controller types, see "PID Controller Types for Tuning".

#### **System Performance Goals**

#### Domain — Domain for specifying performance targets

'Time' (default) | 'Frequency'

Choose the domain in which the task displays the target performance parameters.

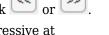
- Time Use the sliders to set performance goals in terms of response time and transient behavior. Time-domain tuning is not available for frequency-response data plants such as frd plants.
- Frequency Use the sliders to set performance goals in terms of loop bandwidth and phase margin.

The choice of domain does not affect the underlying controller design or the results. You can use whichever is more convenient for you or more appropriate for your application. For instance, if your design goals include a target rise time, you might find it convenient to work in the time domain. If you have a target loop bandwidth, you might prefer working in the frequency domain. In both domains, there is a trade-off between reference tracking and disturbance rejection performance.

# Response Time, Transient Behavior — Time-domain performance goals sliders

When you set **Domain** to Time, use these sliders to adjust the responsiveness and robustness of the controller.

- Use the **Response Time** slider to make the closed-loop response of the control system faster or slower. To change the limits of the slider, drag the slider to the left or right
  - end. To decrease or increase the response time by a factor of 10, click



 Use the Transient Behavior slider to make the controller more aggressive at disturbance rejection (smaller values) or more robust against plant uncertainty (larger values).

Time-domain tuning is not available for frequency-response data plants such as frd plants.

# **Bandwidth, Phase Margin — Frequency-domain performance goals** sliders

When you set **Domain** to Frequency, use these sliders to adjust the bandwidth and phase margin of the control system.

• Use the **Bandwidth** slider to make the closed-loop response of the control system faster or slower (the response time is  $2/w_c$ , where  $w_c$  is the bandwidth). To change the limits of the slider, drag the slider to the left or right end. To decrease or increase the

bandwidth by a factor of 10, click or

For discrete-time controllers, **Tune PID Controller** limits the maximum bandwidth to pi/Ts, where Ts is the sample time of the selected plant.

• Use the **Phase Margin** slider to make the controller more aggressive at disturbance rejection (smaller values) or more robust against plant uncertainty (larger values).

#### **Optional Parameters**

#### Design focus — Closed-loop performance objective to favor

Balanced (default) | Reference tracking | Input disturbance rejection

For a given target phase margin, **Tune PID Controller** chooses a controller design that balances the two measures of performance, reference tracking and disturbance rejection. When you change the **Design focus** option, the tuning algorithm attempts to adjust the PID gains to favor either reference tracking or disturbance rejection while achieving the same target phase margin.

#### The **Design focus** options follow:

- Balanced For a given robustness, tune the controller to balance reference tracking and disturbance rejection.
- ullet Reference tracking Tune the controller to favor reference tracking, if possible.
- Input disturbance rejection Tune the controller to favor disturbance rejection, if possible.

The more tunable parameters there are in the system, the more likely it is that the PID algorithm can achieve the desired design focus without sacrificing robustness. For example, setting the design focus is more likely to be effective for PID controllers than for P or PI controllers.

In all cases, how much you can fine-tune the performance of the system depends strongly on the properties of your plant. For some plants, changing the **Design Focus** option might have little or no effect.

# Integral formula, Filter formula — Formula for discrete integral and derivative terms

Forward Euler (default) | Backward Euler | Trapezoidal

For discrete-time PID controllers, there are different ways to implement the integrator and filter terms. For instance, for a parallel-form discrete-time PID controller, the controller transfer function is

$$C = K_p + K_i IF(z) + \frac{K_d}{T_f + DF(z)} \,.$$

IF(z) and DF(z) are the discrete integrator formulas for the integrator and derivative filter. (To see how IF(z) and DF(z) affect other controller forms, including standard form and 2-DOF controllers, see "Discrete-Time Proportional-Integral-Derivative (PID) Controllers".)

Use **Integral formula** and **Filter formula** to select the values of IF(z) and DF(z), respectively.

• Forward Euler — IF(z) or  $DF(z) = \frac{T_s}{z-1}$ .

This formula is best for small sample times, where the Nyquist limit is large compared to the bandwidth of the controller. For larger sample times, the Forward Euler formula can result in instability, even when you discretize a system that is stable in continuous time.

• Backward Euler — IF(z) or  $DF(z) = \frac{T_S z}{z-1}$ .

An advantage of the Backward Euler formula is that discretizing a stable continuous-time system using this formula always yields a stable discrete-time result.

Trapezoidal — IF(z) or  $DF(z) = \frac{T_s}{2} \frac{z+1}{z-1}$ .

An advantage of the Trapezoidal formula is that discretizing a stable continuous-time system using this method always yields a stable discrete-time result. Of all available discrete integrator formulas, Trapezoidal yields the closest match between the frequency-domain properties of the discretized system and the corresponding continuous-time system.

#### **Display**

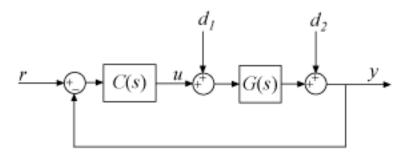
#### **Output Plot — System response plot to generate**

Step Plot: Reference tracking (default) | Step Plot: Input disturbance rejection | Bode Plot: Open-loop | ...

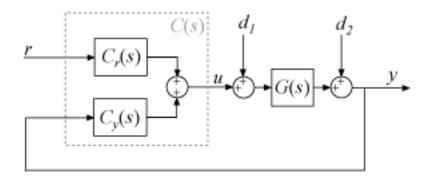
Specify a response plot for observing the effect of the PID controller on system performance. You can specify a time-domain step response plot or a frequency-domain Bode plot for different system responses. The code that **Tune PID Controller** generates in your live script includes code for generating the plot you select.

#### **Available System Responses**

For 1-DOF PID controller types such as PI, PIDF, and PDF, the software computes system responses based upon the following single-loop control architecture, where G is your specified plant and C is the PID controller:



For 2-DOF PID controller types such as PI2, PIDF2, and I-PD, the software computes responses based upon the following architecture:



The system responses are based on the decomposition of the 2-DOF PID controller, C, into a setpoint component Cr and a feedback component Cy, as described in "Two-Degree-of-Freedom PID Controllers".

The following table summarizes the available responses for analysis plots. (For frequency-response-data plants such as frd models, time-domain response plots are not available.)

Response	Plotted System (1- DOF)	Plotted System (2- DOF)	Description
Plant	G	G	Plant response. Use to examine plant dynamics.
Open-loop	GC	$-GC_y$	Response of the open-loop controller-plant system. Use for frequency-domain design. Use when your design specifications include robustness criteria such as open-loop gain margin and phase margin.

Response	Plotted System (1- DOF)	Plotted System (2- DOF)	Description
Reference tracking	$\frac{GC}{1+GC} \text{ (from } r \text{ to } y\text{)}$	$\frac{GC_r}{1 - GC_y} \text{ (from } r \text{ to } y\text{)}$	Closed-loop system response to a step change in setpoint. Use when your design specifications include setpoint tracking.
Controller effort	$\frac{C}{1 + GC} \text{ (from } r \text{ to } u\text{)}$	$\frac{C_r}{1 - GC_y} \text{ (from } r \text{ to } u\text{)}$	Closed-loop controller output response to a step change in setpoint. Use when your design is limited by practical constraints, such as controller saturation.
Input disturbance rejection	$\frac{G}{1+GC} \text{ (from } d_1 \text{ to } y)$	$\frac{G}{1 - GC_y} \text{ (from } d_1 \text{ to } y\text{)}$	Closed-loop system response to load disturbance (a step disturbance at the plant input). Use when your design specifications include input disturbance rejection.
Output disturbance rejection	$\frac{1}{1+GC} (\text{from } d_2 \text{ to } y)$	$\frac{1}{1 - GC_y} \text{ (from } d_2 \text{ to } y\text{)}$	Closed-loop system response to a step disturbance at plant output. Use when you want to analyze sensitivity to modeling errors.

# $\label{eq:constraints} \textbf{System response characteristics} \ - \ \textbf{Display numeric characteristics of closed-loop response}$

off (default) | on

Select this option to generate a display of numeric characteristics of the closed-loop or open-loop response.

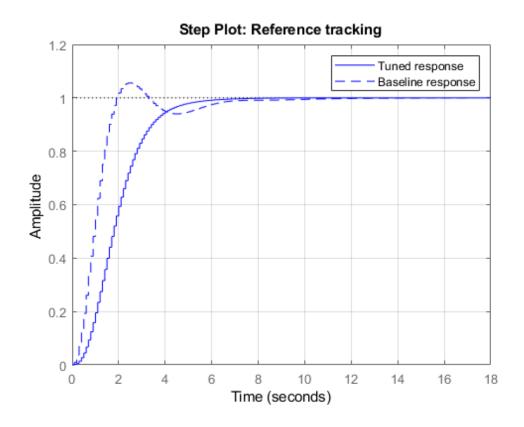
- When Output Plot is a step plot, the display includes characteristics such as rise
  time, settling time, and percent overshoot. These values are always those of the
  closed-loop step response from the control system input r to output y, regardless of
  which specific step response you choose for the plot. Tune PID Controller uses
  stepinfo to compute the step-response characteristics. For details about how to
  interpret these values, see the stepinfo reference page.
- When **Output Plot** is a Bode plot, the display includes characteristics such as gain margin and phase margin. These values are always those of the open-loop system response *GC*, regardless of which specific Bode plot you choose. **Tune PID Controller** uses allmargin to compute the frequency-response characteristics. For details about how to interpret these values, see the allmargin reference page.

# Baseline controller — Controller for performance comparison None (default) | Select from workspace

Use this option when you want to compare the performance of the tuned controller to another PID controller in the MATLAB workspace. To do so, choose Select from workspace. Another menu appears containing PID model objects that are currently in the workspace. The list includes PID model objects of all types (pid, pidstd, pid2, or pidstd2) that are of the same time domain as the currently specified plant. For instance, if the plant is a discrete-time state-space model, then any discrete-time PID model object in the workspace is available as a baseline controller.

When you specify a baseline controller, the response plot updates to include a dotted-line plot of the system response with the baseline controller.

→ Display results		
Output Plot	Step Plot: Reference tracking ▼	System response characteristics
Baseline controller	Select from workspace ▼	C ▼
	*	



#### **Store Controller for Comparison**

You can store a design to use as a baseline while you experiment further with controller types, performance goals, and other settings. To do so, when you find the design that you want to use as a baseline:

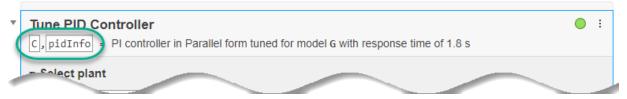
- Note the name of the controller workspace variable in the task summary line (see "Tips" on page 2-1392). For instance, if the name is C, then the current controller is in the MATLAB workspace as C.
- 2 Change the name of the controller variable in the task summary line. For instance, change it to Cnew.
- **3** Select **Baseline Controller** and specify the stored controller C as the baseline.

When you experiment further with the controller design, the task stores changes to the controller in the workspace as Cnew. The plot shows you the baseline response using C and the tuned response using Cnew.

## **Tips**

After you select a plant, the task creates the controller and stores it in the MATLAB workspace. The stored controller is a pid, pidstd, pid2, or pidstd2 model object, as specified by your selections for Controller Form and Degrees of Freedom.

The default variable name for the stored controller is C. You can change the variable name by typing a new name into the task summary line.



• The task also provides information about the performance and robustness of the closed-loop system in a structure called pidInfo by default. For information about this structure, see the info output argument on the pidtune reference page.

# **Algorithms**

Tune PID Controller uses the algorithm discussed in "PID Tuning Algorithm".

### See Also

**Apps** PID Tuner

Functions
pidtune|pidtuneOptions

**Objects** pid | pid2 | pidstd | pidstd2

### **Topics**

"PID Controller Design in the Live Editor" "Choosing a PID Controller Design Tool"

Introduced in R2019b

# updateSystem

Update dynamic system data in a response plot

## **Syntax**

```
updateSystem(h,sys)
updateSystem(h,sys,N)
```

# **Description**

updateSystem(h,sys) replaces the dynamic system used to compute a response plot with the dynamic system model or model array sys, and updates the plot. If the plot with handle h contains more than one system response, this syntax replaces the first response in the plot. updateSystem is useful, for example, to cause a plot in a GUI to update in response to interactive input. See "Build GUI With Interactive Response-Plot Updates".

updateSystem(h,sys,N) replaces the data used to compute the Nth response in the
plot.

# **Examples**

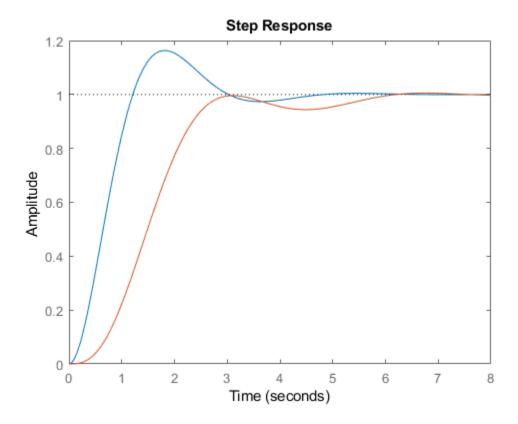
### **Update System Data in Response Plot**

Replace step response data in an existing plot with data computed from a different dynamic system model.

Suppose you have a plant model and pure integrator controller that you designed for that plant. Plot the step responses of the plant and the closed-loop system.

```
w = 2;
zeta = 0.5;
G = tf(w^2,[1,2*zeta*w,w^2]);
C1 = pid(0,0.621);
```

```
CL1 = feedback(G*C1,1);
h = stepplot(G,CL1);
```



h is the plot handle that identifies the plot created by stepplot. In this figure, G is used to compute the first response, and CL1 is used to compute the second response. This ordering corresponds to the order of inputs to stepplot.

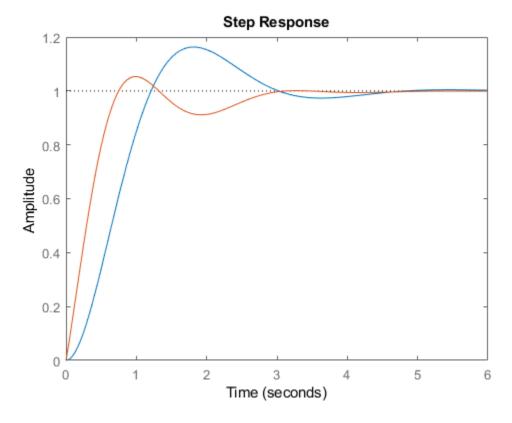
Suppose you also have a PID controller design that you want to analyze. Create a model of the closed-loop system using this alternate controller.

```
C2 = pid(2,2.6,0.4,0.002);

CL2 = feedback(G*C2,1);
```

Update the step plot to display the second closed-loop system instead of the first. The closed-loop system is the second response in the plot, so specify the index value 2.

updateSystem(h,CL2,2);



The updateSystem command replaces the system used to compute the second response displayed in the plot. Instead of displaying response data derived from CL1, the plot now shows data derived from CL2.

When you build a GUI that displays a response plot, use updateSystem in GUI control callbacks to cause those GUI controls to update the response plot. For an example showing how to implement such a GUI control, see "Build GUI With Interactive Response-Plot Updates".

## **Input Arguments**

#### h — Plot to update

plot handle

Plot to update with new system data, specified as a plot handle. Typically, you obtain the plot handle as an output argument of a response plotting command such as stepplot or bodeplot. For example, the command h = bodeplot(G) returns a handle to a plot containing the Bode response of a dynamic system, G.

#### sys — System for new response data

dynamic system model | model array

System from which to compute new response data for the response plot, specified as a dynamic system model or model array.

sys must match the plotted system that it replaces in both I/O dimensions and array dimensions. For example, suppose h refers to a plot that displays the step responses of a 5-element vector of 2-input, 2-output systems. In this case, sys must also be a 5-element vector of 2-input, 2-output systems. The number of states in the elements of sys need not match the number of states in the plotted systems.

### N — Index of system to replace

1 (default) | positive integer

Index of system to replace in the plot, specified as a positive integer. For example, suppose you create a plot using the following command.

```
h = impulseplot(G1,G2,G3,G4);
```

To replace the impulse data of G3 with data from a new system, sys, use the following command.

```
updateSystem(h,sys,3);
```

### See Also

### **Topics**

"Build GUI With Interactive Response-Plot Updates"

### Introduced in R2013b

# upsample

Upsample discrete-time models

## **Syntax**

```
sysl = upsample(sys,L)
```

# **Description**

sysl = upsample(sys,L) resamples the discrete-time dynamic system model sys at a sampling rate that is L-times faster than the sample time of sys  $(Ts_0)$ . L must be a positive integer. When sys is a TF model, H(z), upsample returns sysl as  $H(z^L)$  with the sample time  $Ts_0/L$ .

The responses of models sys and sysl have the following similarities:

- The time responses of sys and sysl match at multiples of  $Ts_0$ .
- The frequency responses of sys and sysl match up to the Nyquist frequency  $\pi / Ts_0$ .

Note sysl has L times as many states as sys.

# **Examples**

### **Upsample Discrete-Time Transfer Function**

Create a transfer function with sample time 2.25 seconds.

```
z^2 + 10 z + 2
Sample time: 2.25 seconds
Discrete-time transfer function.
```

Create a transfer function with a sample time that is 14 times faster than sys.

The sample time of sys1 is 0.16071 seconds, which is 14 times faster than the sample time of sys.

### See Also

c2d | d2c | d2d

Introduced in R2008b

# varyingGoal

Variable tuning goal for gain-scheduled controllers

## **Syntax**

```
VG = varyingGoal(FH,par1,par2,...)
VG = varyingGoal(____,Name,Value)
```

# **Description**

When tuning fixed or gain-scheduled controllers at multiple design points (operating conditions), you might need to adjust the tuning goals as a function of operating condition, for example, to relax performance in some regions of the operating range. Use <code>varyingGoal</code> to construct tuning goals that depend implicitly or explicitly on the design point.

VG = varyingGoal(FH,par1,par2,...) specifies a varying goal using a template and sets of goal-parameter values. The template FH is a function handle that specifies a function, TG = FH(p1,p2,...), that evaluates to one of the TuningGoal objects. The arrays par1,par2,... specify the values of the tuning-goal parameters p1,p2,... at each design point. Use VG as you would use any TuningGoal object in an input to systume.

 $VG = varyingGoal(\_\__, Name, Value)$  configures additional properties of the tuning goal.

# **Examples**

### **Varying Tuning Goal**

Create a tuning goal that specifies variable gain and phase margins across a grid of design points.

Suppose you use the following 5-by-5 grid of design points to tune your controller.

```
[alpha, V] = ndgrid(linspace(0, 20, 5), linspace(700, 1300, 5));
```

Suppose further that you have 5-by-5 arrays of target gain margins and target phase margins corresponding to each of the design points, such as the following.

```
[GM,PM] = ndgrid(linspace(7,20,5),linspace(45,70,5));
```

To enforce the specified margins at each design point, first create a template for the margins goal. The template is a function that takes gain and phase margin values and returns a TuningGoal.Margins object with those margins.

```
FH = @(gm,pm) TuningGoal.Margins('u',gm,pm);
```

Use the template and the margin arrays to create the varying goal.

```
VG = varyingGoal(FH,GM,PM);
```

To make it easier to trace which goal applies to which design point, use the SamplingGrid property to attach the design-point information to VG.

```
VG.SamplingGrid = struct('alpha',alpha,'V',V);
```

Use VG with systune as you would use any other tuning goal. Use viewGoal to visualize the tuning goal and identify design points that fail to meet the target margins.

### **Varying Tuning Goal With LTI Parameter**

Create a tuning goal that specifies a loop shape that varies with one scheduling variable, a.

Suppose you want to specify a loop shape with a crossover frequency that varies as 2\*a over three design points. To enforce this requirement, first create a template for the loop-shape goal. The template is a function that takes a numeric scalar input parameter and returns a TuningGoal.LoopShape object. The function input must be scalar, so the function constructs the LTI models representing the loop shapes.

```
a = [5;10;15];
s = tf('s');
FH = @(A) TuningGoal.LoopShape('u',2*A/s);
```

Here, 'u' is an Analysis Point in the system, the location at which you want to impose the loop-shape requirements.

Use the template and the array to create the varying goal.

```
VG = varyingGoal(FH,a);
```

Attach the design-point information to VG.

```
VG.SamplingGrid = struct('a',a);
```

Now each value of a is associated with a tuning goal that enforces the corresponding loop shape. For example, confirm that the third entry in a, a = 15, is associated with the third loop shape, 30/s.

```
LS3 = getGoal(VG,'index',3);
tf(LS3.LoopGain)
ans =
   30
   --
   s
Continuous-time transfer function.
```

### Varying Tuning Goal With Frequency Focus and Loop Opening

Create a tuning goal that specifies variable gain and phase margins across a grid of design points. Configure the tuning goal to be evaluated with a loop opening at a location 'LO' and to apply only in the frequency range between 1 and 100 rad/s.

Specify the grid of design points and the corresponding grid of target gain margins and phase margins. Also, create the template function for the varying margins goal.

```
[alpha,V] = ndgrid(linspace(0,20,5),linspace(700,1300,5));
[GM,PM] = ndgrid(linspace(7,20,5),linspace(45,70,5));
FH = @(gm,pm) TuningGoal.Margins('u',gm,pm);
```

Use the template function and the margin arrays to create the varying goal. Also, use Name, Value pairs to specify:

• The location at which the loop is opened for evaluating the tuning goal (Openings property).

• The frequency range in which the varying goal applies (Focus property).

The property names and values that you specify for the underlying tuning goal are stored in the Settings property of the varying goal.

To make it easier to trace which goal applies to which design point, use the SamplingGrid property to attach the design-point information to VG.

```
VG.SamplingGrid = struct('alpha',alpha,'V',V);
```

## **Input Arguments**

#### FH — Template for varying goal

function handle

Template for varying goal, specified as a function handle. FH specifies a function of one or more parameters that evaluates to one of the TuningGoal objects. For example, suppose you want to constrain the overshoot in the step response from an input r to an output y in your system, and you want to allow the constraint to vary across different design points. Specify the template as a function that returns a TuningGoal.Overshoot object. For example, you can specify FH as an anonymous function (MATLAB).

```
FH = @(os) TuningGoal.Overshoot('r','y',os);
```

Because TuningGoal.Overshoot has only one parameter besides the input and output signals, FH is a handle to a function of one argument. For other tuning goals, use more arguments. For example, TuningGoal.Margins has two parameters, the gain margin and phase margin. Therefore, for a variable margin goal, FH has two arguments.

```
FH = @(gm,pm) TuningGoal.Margins('u',gm,pm);
```

The template function allows great flexibility in constructing the design goals. For example, you can write a function, goalspec(a,b), that constructs the tuning goal specification as a nontrivial function of the parameters (a,b), and save the function in a MATLAB file. Your template function then calls goalspec as follows.

```
FH = @(a,b) TuningGoal.Margins('u',goalspec(a,b));
```

Similarly, if the tuning-goal parameters do not fit in numeric arrays, you can use the design-point index as input to FH. For example, suppose the gain and phase margin data is stored in a struct array, S, with fields GM and PM, you can use the following.

```
FH = @(idx) TuningGoal.Margins('u',S(idx).GM,S(idx).PM);
```

idx is an absolute index into the grid of design points.

#### par — Tuning goal parameters

numerical array

Tuning goal parameters, specified as a numerical array with the same dimensions as the model array used for gain-scheduled tuning. Provide an array for each parameter in the tuning goal, where each entry in the array is the parameter value you want to apply to the corresponding design point. For example, the TuningGoal.Overshoot goal has only one parameter, the maximum overshoot. Therefore, specify the parameters as follows.

```
par = osvals;
```

osvals is an array of overshoot values to enforce at each design point.

The TuningGoal.Margins goal has two parameters, the gain margin and phase margin. Therefore, for a variable margin goal, specify the parameters as follows:

```
par1 = GM;
par2 = PM;
```

Here, GM is an array of gain-margin values and PM is an array of phase-margin values to enforce at each design point.

To make the variable tuning goal inactive at a particular design point, set the corresponding entry of the par array (or arrays) to NaN.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value pairs to specify properties of the underlying tuning goal. For instance, suppose you want to create a varying gain goal that specifies a variable gain profile between points 'L' and 'V'. You also want to enforce the gain goal only in the frequency band [0 pi/Ts], with a loop opening at an analysis point labeled OuterLoop. You use Name, Value pairs to specify these properties for the gain goal.

```
FH = @(w) TuningGoal.Gain('F','V',tf(w,[1 w]));
VG = varyingGoal(FH,wdata,'Focus',[0 pi/Ts],'Openings','OuterLoop');
```

Which properties you can set depends on which of the TuningGoal objects your function FH evaluates to. For instance, for most varying tuning goals, you can set properties such as Openings, Models, and Focus. For a variable TuningGoal.Gain goal, you can also use Name, Value pairs to set properties such as Stabilize, InputScaling, and OutputScaling. See the individual TuningGoal object reference pages for a list of the properties of each tuning goal.

## **Output Arguments**

### VG — Variable tuning goal

varyingGoal object

Variable tuning goal, returned as a varyingGoal object. This object captures the tuning goal and its variation across design points in a MATLAB variable. Use VG in an input argument to systune just as you would use any TuningGoal object.

VG has the following properties.

Property	Description	
Template	Template for the varying goal, stored as a function handle to a function of one or more parameters whose values change over the operating range for tuning. The initial value of this property is set by the FH input argument.	
Parameters	Tuning goal parameters at each design point, stored as a cell array. Each entry in the cell array is a numeric array containing the parameter values at each design point. For example, for a variable margins goal with template	
	FG = FH = @(gm,pm) TuningGoal.Margins( the value of this property is {GM,PM}, where GM and PM are arrays containing the desired gain and phase margins at each design point.  The initial value of this property is set by the input arguments par1,par2,	'u',gm,pm)
Settings	Property names and values to be applied to each goal instance in the varying goal, stored as a cell array.  Default: {}	

Property	Description
SamplingGrid	Design points, stored as a structure containing an array of values for each sampling variable. The design points need not lie on a rectangular grid and can be scattered throughout the operating range. The sizes of the arrays in the SamplingGrid and Parameters properties must match.  For more information about sampling grids, see the SamplingGrid property description on the ss reference page.  Default: struct with no fields
Name	Name for the variable goal. Set Name to a string to label the goal.  Default: ' '

# **Tips**

• Use viewGoal to visualize tuning goals. For varying tuning goals, the tuning-goal plot generated by viewGoal lets you examine the tuning goal at each design point. For more information, see "Validate Gain-Scheduled Control Systems".

### See Also

getGoal | systume | tunableSurface | viewGoal

### **Topics**

"Change Requirements with Operating Condition"

 $\hbox{``Validate Gain-Scheduled Control Systems''}$ 

#### Introduced in R2017b

### viewGoal

View tuning goals; validate design against tuning goals

### **Syntax**

viewGoal(Req)
viewGoal(Req,T)

# **Description**

viewGoal(Req) displays a graphical view of a tuning goal or vector of tuning goals, specified as TuningGoal objects. The form of the tuning-goal plot depends on the specific tuning goals you use. Plots for time-domain tuning goals typically show the target time-domain response specified in the tuning goal. Plots for frequency-domain tuning goals typically show a shaded area that represents the region in which the tuning goal is violated.

When you provide a vector of tuning goals, viewGoal plots each tuning goal on separate axes in a single figure window.

viewGoal (Req,T) plots the performance of a tuned control system against the tuning goal or goals. The form of the tuning-goal plot depends on the specific tuning goals you use. Typically, the plot shows both the target response specified in the tuning goal and the corresponding response of the control system represented by T. For more information about interpreting tuning-goal plots, see "Visualize Tuning Goals".

# **Examples**

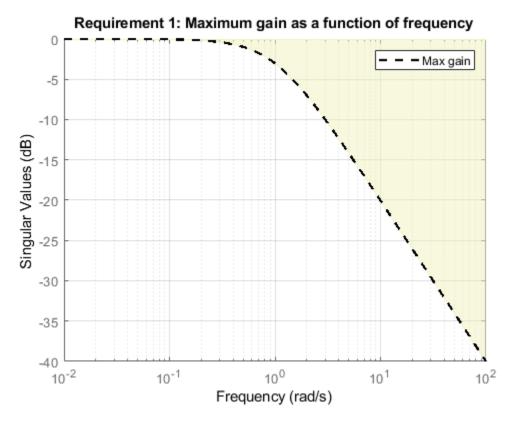
### **Visualize Tuning Goal as Function of Frequency**

Create a tuning goal that constrains the response from a signal 'd' to another signal 'y' to roll off at 20 dB/decade at frequencies greater than 1. The tuning goal also imposes disturbance rejection (maximum gain of 1) in the frequency range [0,1].

```
gmax = frd([1 1 0.01],[0 1 100]);
Req = TuningGoal.MaxGain('du','u',gmax);
```

When you use a frequency response data (frd) model to sketch the bounds of a gain constraint or loop shape, the tuning goal interpolates the constraint. This interpolation converts the constraint to a smooth function of frequency. Examine the interpolated gain constraint using viewGoal.

viewGoal(Reg)



The dotted line shows the gain profile specified in the tuning goal. The shaded region represents gain values that violate the tuning requirement. For more information about interpreting tuning-goal plots, see "Visualize Tuning Goals".

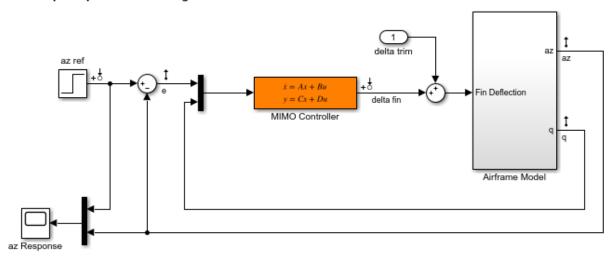
#### **Visualize Tuned Responses Against Tuning Goals**

Examine the tuned response of a control system against tuning goals, to determine where and by how much the tuning goals are violated. This visualization helps you determine whether the tuned control system comes satisfactorily close to meeting your soft requirements.

Open a Simulink® model of a control system to tune.

```
open_system('rct_airframe2')
```

#### Two-loop autopilot for controlling the vertical acceleration of an airframe

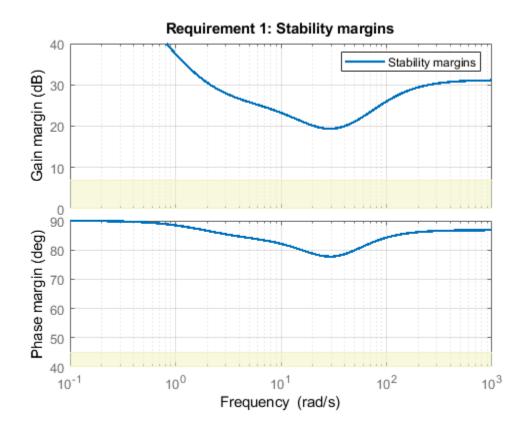


Create tuning goals. For this example, use tracking, roll-off, stability margin, and disturbance-rejection tuning goals.

```
Req1 = TuningGoal.Tracking('az ref','az',1);
Req2 = TuningGoal.Gain('delta fin','delta fin',tf(25,[1 0]));
Req3 = TuningGoal.Margins('delta fin',7,45);
MaxGain = frd([2 200 200],[0.02 2 200]);
Req4 = TuningGoal.Gain('delta fin','az',MaxGain);
```

Create an slTuner interface, and tune the model with these tuning goals designated as soft goals.

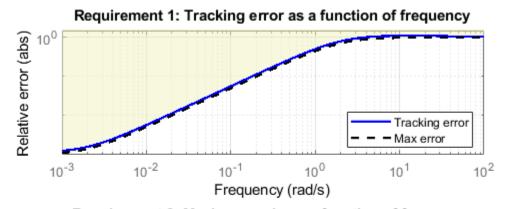
```
ST0 = slTuner('rct_airframe2','MIMO Controller');
addPoint(ST0,'delta fin');
rng('default');
[ST1,fSoft] = systune(ST0,[Req1,Req2,Req3,Req4]);
Final: Soft = 1.13, Hard = -Inf, Iterations = 96
Verify that the tuned system satisfies the margin requirement.
figure;
viewGoal(Req3,ST1)
```

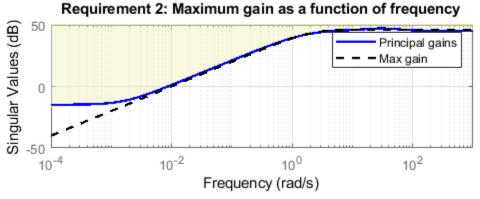


The shaded region corresponds to margins falling short of the target of 7 dB gain margin and 45 degrees phase margin. The solid line shows that the margin requirement is satisfied at all frequencies.

Examine system responses compared to the tracking and disturbance-rejection tuning goals. When you provide a vector of tuning goals, viewGoal plots them on separate axes in a single figure.

figure
viewGoal([Req1,Req4],ST1)





The first plot shows that the tuned system response very nearly satisfies the tracking requirement. The slight violation suggests that setpoint tracking will perform close to expectations.

The second plot shows that the gain requirement is satisfied except at low frequency. For this tuning goal, the shaded region, which represents the effective tuning constraint, diverges from the specified maximum gain profile at low frequency. This modification to the gain profile is to avoid a pole at s=0 in the weighting function used to normalize the goal (see Tips on this page). While the tuned gain exceeds the specified gain below 0.001 rad/s, it is still about 60 dB less than the peak value, which is typically enough in practice.

To further examine the responses of the tuned system, use getIOTransfer to extract the relevant transfer functions for analysis with time-domain commands such as step.

## **Input Arguments**

#### Req — Tuning goal to view or validate

TuningGoal object | vector of TuningGoal objects

Tuning goal to view or validate, specified as a TuningGoal object or vector of TuningGoal objects. For a list of all TuningGoal objects, see "Tuning Goals".

#### T — Tuned control system

generalized state-space model | slTuner interface object

Tuned control system, specified as a generalized state-space (genss) model or an slTuner interface to a Simulink model. T is typically the result of using the tuning goal to tune control system parameters with systune.

Example: [T,fSoft,gHard] = systume(T0,SoftReq,HardReq), where T0 is a tunable genss model

Example: [T,fSoft,gHard] = systume(ST0,SoftReq,HardReq), where ST0 is a slTuner interface object

## **Tips**

- For general information about how to interpret tuning-goal plots, see "Visualize Tuning Goals". For information about interpreting margin-goal plots in particular, see "Stability Margins in Control System Tuning".
- For varying tuning goals that you create with varyingGoal, the tuning-goal plot generated by viewGoal lets you examine the tuning goal at each design point. For more information, see "Validate Gain-Scheduled Control Systems".

- With some frequency-domain tuning goals, there might be a difference between the gain profile you specify in the tuning goal (dashed line), and the profile the software uses for tuning (shaded region). In this case, the shaded region of the plot reflects the profile that the software uses for tuning. The gain profile you specify and the gain profile used for tuning might differ if:
  - You tune a control system in discrete time, but specify the gain profile in continuous time.
  - The software modifies the asymptotes of the specified gain profile to improve numeric stability.

For more information about how an enforced tuning goal might differ from the goal, see "Visualize Tuning Goals".

 For MIMO feedback loops, the LoopShape, MinLoopGain, MaxLoopGain, Margins, Sensitivity, and Rejection goals are sensitive to the relative scaling of each SISO loop. systune tries to balance the overall loop-transfer matrix while enforcing such goals. The optimal loop scaling is stored in the tuned closed-loop model or slTuner interface T returned by systune. For consistency, viewGoal(R,T) takes this scaling into account, and plots the scaled open-loop response or sensitivity. To omit this scaling, use viewGoal(R,clearTuningInfo(T)).

Modifying T might compromise the validity of the stored scaling. Therefore, if you make significant modifications to T, retuning is recommended to update the scaling data.

### See Also

evalGoal|genss|systume|systume (for slTumer)

### **Topics**

"Visualize Tuning Goals"

"Stability Margins in Control System Tuning"

"Tuning Goals"

#### Introduced in R2012b

# viewSpec

(Not recommended) View tuning goals; validate design against tuning goals

**Note** viewSpec is not recommended. Use viewGoal instead.

# **Syntax**

```
viewSpec(Req)
viewSpec(Req,T)
viewSpec(Req,T,[])
```

# **Description**

viewSpec(Req) displays a graphical view of a tuning goal or vector of tuning goals, specified as TuningGoal objects. The form of the tuning-goal plot depends on the specific tuning goals you use. Plots for time-domain tuning goals typically show the target time-domain response specified in the tuning goal. Plots for frequency-domain tuning goals typically show a shaded area that represents the region in which the tuning goal is violated.

When you provide a vector of tuning goals, viewSpec plots each tuning goal on separate axes in a single figure window.

viewSpec(Req,T) plots the performance of a tuned control system against the tuning goal or goals. The form of the tuning-goal plot depends on the specific tuning goals you use. Typically, the plot shows both the target response specified in the tuning goal and the corresponding response of the control system represented by T. For more information about interpreting tuning-goal plots, see "Visualize Tuning Goals".

viewSpec(Req,T,[]) disregards scaling information stored with the tuned control system T when computing system responses. For more information, see "Tips" on page 2-1422.

## **Examples**

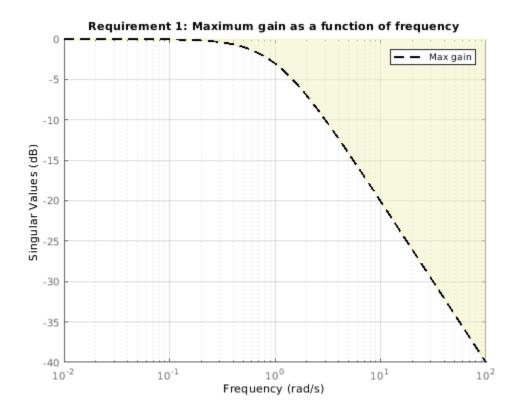
#### **Visualize Tuning Goal as Function of Frequency**

Create a tuning goal that constrains the response from a signal 'd' to another signal 'y' to roll off at 20 dB/decade at frequencies greater than 1. The tuning goal also imposes disturbance rejection (maximum gain of 1) in the frequency range [0,1].

```
gmax = frd([1 1 0.01],[0 1 100]);
Req = TuningGoal.MaxGain('du','u',gmax);
```

When you use a frequency response data (frd) model to sketch the bounds of a gain constraint or loop shape, the tuning goal interpolates the constraint. This interpolation converts the constraint to a smooth function of frequency. Examine the interpolated gain constraint using viewSpec.

```
viewSpec(Req)
```



The dotted line shows the gain profile specified in the tuning goal. The shaded region represents gain values that violate the tuning requirement. For more information about interpreting tuning-goal plots, see "Visualize Tuning Goals".

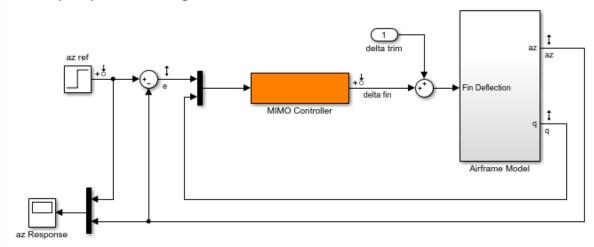
### **Visualize Tuned Responses Against Tuning Goals**

Examine the tuned response of a control system against tuning goals, to determine where and by how much the tuning goals are violated. This visualization helps you determine whether the tuned control system comes satisfactorily close to meeting your soft requirements.

Open a Simulink® model of a control system to tune.

```
open_system('rct_airframe2')
```

#### Two-loop autopilot for controlling the vertical acceleration of an airframe



Create tuning goals. For this example, use tracking, roll-off, stability margin, and disturbance-rejection tuning goals.

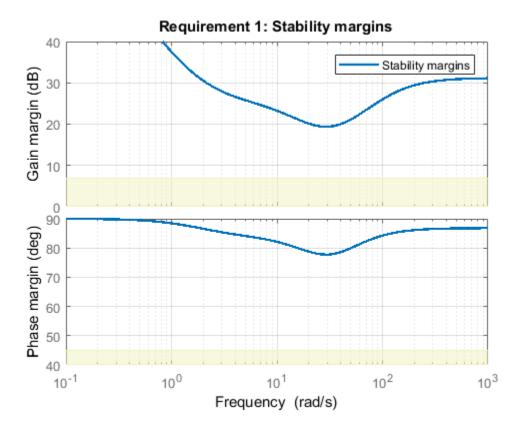
```
Req1 = TuningGoal.Tracking('az ref','az',1);
Req2 = TuningGoal.Gain('delta fin','delta fin',tf(25,[1 0]));
Req3 = TuningGoal.Margins('delta fin',7,45);
MaxGain = frd([2 200 200],[0.02 2 200]);
Req4 = TuningGoal.Gain('delta fin','az',MaxGain);
```

Create an slTuner interface, and tune the model with these tuning goals designated as soft goals.

```
ST0 = slTuner('rct_airframe2','MIMO Controller');
addPoint(ST0,'delta fin');
rng('default');
[ST1,fSoft] = systune(ST0,[Req1,Req2,Req3,Req4]);
Final: Soft = 1.13, Hard = -Inf, Iterations = 68
```

Verify that the tuned system satisfies the margin requirement.

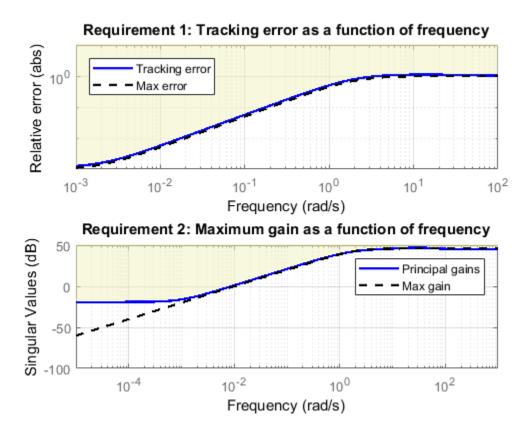




The shaded region corresponds to margins falling short of the target of 7 dB gain margin and 45 degrees phase margin. The solid line shows that the margin requirement is satisfied at all frequencies.

Examine system responses compared to the tracking and disturbance-rejection tuning goals. When you provide a vector of tuning goals, viewSpec plots them on separate axes in a single figure.

```
figure
viewSpec([Req1,Req4],ST1)
```



The first plot shows that the tuned system response very nearly satisfies the tracking requirement. The slight violation suggests that setpoint tracking will perform close to expectations.

The second plot shows that the gain requirement is satisfied except at low frequency. For this tuning goal, the shaded region, which represents the effective tuning constraint, diverges from the specified maximum gain profile at low frequency. This modification to the gain profile is to avoid a pole at s=0 in the weighting function used to normalize the goal (see "Tips" on page 2-1422). While the tuned gain exceeds the specified gain below 0.001 rad/s, it is still about 60 dB less than the peak value, which is typically enough in practice.

To further examine the responses of the tuned system, use getIOTransfer to extract the relevant transfer functions for analysis with time-domain commands such as step.

## **Input Arguments**

### Req — Tuning goal to view or validate

TuningGoal object | vector of TuningGoal objects

Tuning goal to view or validate, specified as a TuningGoal object or vector of TuningGoal objects. For a list of all TuningGoal objects, see "Tuning Goals".

#### T — Tuned control system

generalized state-space model | slTuner interface object

Tuned control system, specified as a generalized state-space (genss) model or an slTuner interface to a Simulink model. T is typically the result of using the tuning goal to tune control system parameters with systune.

```
Example: [T,fSoft,gHard] = systume(T0,SoftReq,HardReq), where T0 is a tunable genss model
```

```
Example: [T,fSoft,gHard] = systume(ST0,SoftReq,HardReq), where ST0 is a slTuner interface object
```

# Tips

- With some frequency-domain tuning goals, there might be a difference between the
  gain profile you specify in the tuning goal (dashed line), and the profile the software
  uses for tuning (shaded region). In this case, the shaded region of the plot reflects the
  profile that the software uses for tuning. The gain profile you specify and the gain
  profile used for tuning might differ if:
  - You tune a control system in discrete time, but specify the gain profile in continuous time.
  - The software modifies the asymptotes of the specified gain profile to improve numeric stability.

For more information about how an enforced tuning goal might differ from the goal, see "Visualize Tuning Goals".

 For MIMO feedback loops, the LoopShape, MinLoopGain, MaxLoopGain, Margins, Sensitivity, and Rejection goals are sensitive to the relative scaling of each SISO loop. systune tries to balance the overall loop-transfer matrix while enforcing such goals. The optimal loop scaling is stored in the tuned closed-loop model CL returned by systune. For consistency, viewSpec(R,CL) takes this scaling into account, and plots the scaled open-loop response or sensitivity. To omit this scaling, use viewSpec(R,CL,[]).

Modifying CL might compromise the validity of the stored scaling. Therefore, if you make significant modifications to CL, retuning is recommended to update the scaling data.

## **Compatibility Considerations**

### viewSpec is not recommended

Not recommended starting in R2017b

Beginning in R2017b, viewSpec is not recommended. Use viewGoal instead.

### See Also

evalGoal|genss|systune|systune (for slTuner)|viewGoal

### **Topics**

"Visualize Tuning Goals"
"Tuning Goals"

Introduced in R2012b

## viewSurf

Visualize gain surface as a function of scheduling variables

## **Syntax**

```
viewSurf(GS)
view(GS,xvar,xdata)
view(GS,xvar,xdata,yvar,ydata)
```

## **Description**

viewSurf(GS) plots the values of a 1-D or 2-D gain surface as a function of the scheduling variables. GS is a tunable gain surface that you create with tunableSurface. The plot uses the independent variable values specified in GS.SamplingGrid. For 2-D gain surfaces, the design points in GS.SamplingGrid must lie on a rectangular grid.

view(GS,xvar,xdata) plots the gain surface GS at the scheduling-variable values listed
in xdata. The variable name xvar must match a scheduling variable name in
GS.SamplingGrid. However, the values in xdata need not match design points in
GS.SamplingGrid.

For a 2-D gain surface, the plot shows a parametric family of curves with one curve per value of the other scheduling variable. In the 2-D case, the design points in GS.SamplingGrid must lie on a rectangular grid.

view(GS,xvar,xdata,yvar,ydata) creates a surface plot of a 2-D gain surface
evaluated over a grid of scheduling variable values given by ndgrid(xdata,ydata). In
this case, the design points of GS do not need to lie on a rectangular grid, and xdata and
ydata do not need to match the design points.

# **Examples**

#### **View Gain Surface**

Display a tunable gain surface that depends on two independent variables.

Model a scalar gain K with a bilinear dependence on two scheduling variables,  $\alpha$  and V, as follows:

$$K(\alpha, V) = K_0 + K_1 x + K_2 y + K_3 x y$$
.

Here, x and y are the normalized scheduling variables. Suppose that  $\alpha$  is an angle of incidence that ranges from 0 degrees to 15 degrees, and V is a speed that ranges from 300 m/s to 600 m/s. Then, x and y are given by:

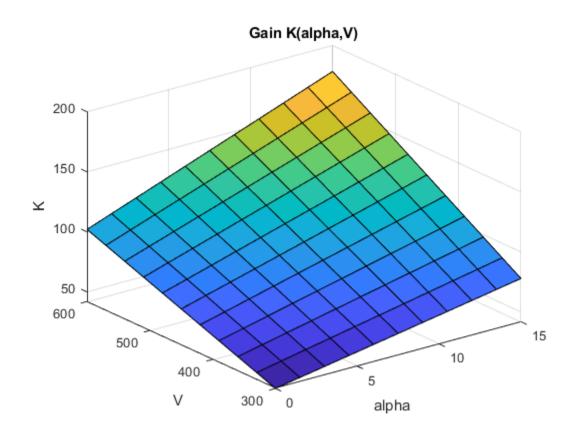
$$x = \frac{\alpha - 7.5}{7.5}$$
,  $y = \frac{V - 450}{150}$ .

The coefficients  $K_0$ , . . . ,  $K_3$  are the tunable parameters of this variable gain. Use tunableSurface to model this variable gain.

```
[alpha,V] = ndgrid(0:1.5:15,300:30:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
K = tunableSurface('K',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. You would then use setBlockValue or setData to write the tuned coefficients back to K, and view the tuned gain surface. For this example, instead of tuning, manually set the coefficients to non-zero values and view the resulting gain.

```
Ktuned = setData(K,[100,28,40,10]);
viewSurf(Ktuned)
```



viewSurf displays the gain surface as a function of the scheduling variables, for the ranges of values specified by domain and stored in Ktuned.SamplingGrid.

### **Plot Gain Surface for Specified Breakpoints**

View a 1-D gain surface evaluated at different design points from the points specified in the gain surface.

When you create a gain surface using tunableSurface, you specify design points at which the gain coefficients are tuned. These points are the typically the scheduling-variable values at which you have sampled or linearized the plant. However, you might

want to implement the gain surface as a lookup table with breakpoints that are different from the specified design points. In this example, you create a gain surface with a set of design points and then view the surface using a different set of scheduling variable values.

Create a scalar gain that varies as a quadratic function of one scheduling variable, t. Suppose that you have linearized your plant every five seconds from t = 0 to t = 40.

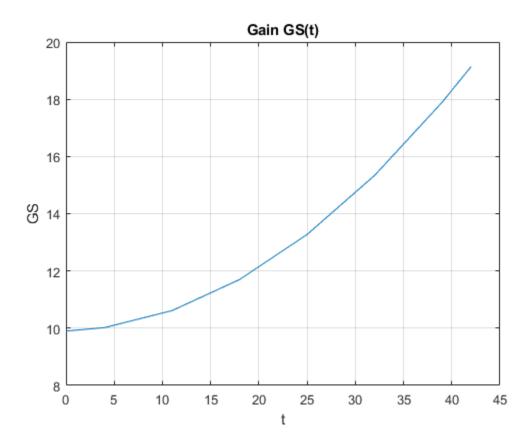
```
t = 0:5:40;
domain = struct('t',t);
shapefcn = @(x) [x,x^2];
GS = tunableSurface('GS',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS,[12.1,4.2,2]);
```

Plot the gain surface evaluated at a different set of time values.

```
tvals = [0,4,11,18,25,32,39,42];
viewSurf(GS,'t',tvals)
```



The plot shows that the gain curve bends at the points specified in tvals, rather than the design points specified in domain. Also, tvals includes values outside of the scheduling-variable range of domain. If you attempt to extrapolate too far out of the range of values used for tuning, the software issues a warning.

## View 1-Dimensional Projections of 2-D Gain Surface

Plot gain surface values as a function of one independent variable, for a gain surface that depends on two independent variables.

Create a gain surface that is a bilinear function of two independent variables,  $\alpha$  and V.

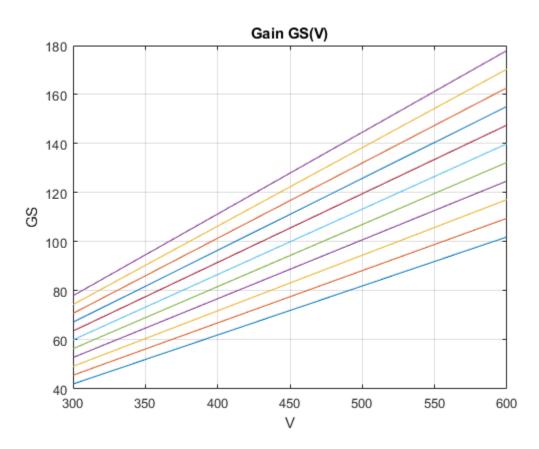
```
[alpha,V] = ndgrid(0:1.5:15,300:30:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
GS = tunableSurface('GS',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS, [100, 28, 40, 10]);
```

Plot the gain at selected values of V.

```
Vplot = [300:50:600];
viewSurf(GS,'V',Vplot);
```



viewSurf evaluates the gain surface at the specified values of V, and plots the dependence on V for all values of  $\alpha$  in domain. Clicking any of the lines in the plot displays the corresponding  $\alpha$  value. This plot is useful to visualize the full range of gain variation due to one independent variable.

#### Plot 2-D Gain Surface for Specified Breakpoints

View a 2-D gain surface evaluated at different scheduling-variable values from the design points specified in the gain surface.

When you create a gain surface using tunableSurface, you specify design points at which the gain coefficients are tuned. These points are the typically the scheduling-variable values at which you have sampled or linearized the plant. However, you might want to implement the gain surface as a lookup table with breakpoints that are different from the specified design points. In this example, you create a gain surface with a set of design points and then view the surface using a different set of scheduling-variable values.

Create a gain surface that is a bilinear function of two independent variables,  $\alpha$  and V.

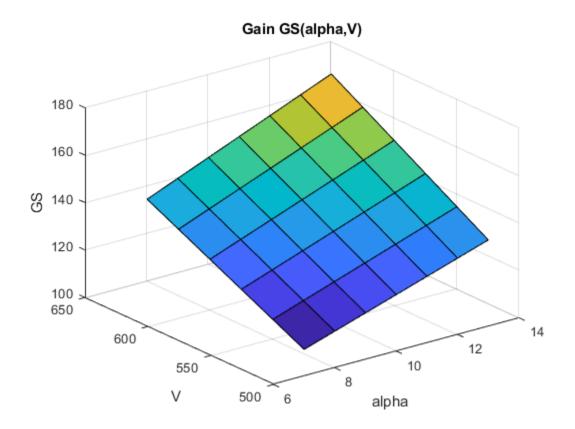
```
[alpha,V] = ndgrid(0:1.5:15,300:30:600);
domain = struct('alpha',alpha,'V',V);
shapefcn = @(x,y) [x,y,x*y];
GS = tunableSurface('GS',1,domain,shapefcn);
```

Typically, you would tune the coefficients as part of a control system. For this example, instead of tuning, manually set the coefficients to non-zero values.

```
GS = setData(GS, [100, 28, 40, 10]);
```

Plot the gain at selected values of  $\alpha$  and V.

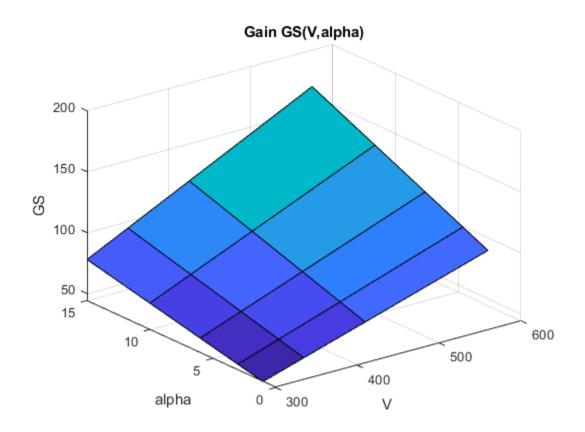
```
alpha_vec = [7:1:13];
V_vec = [500:25:625];
viewSurf(GS,'alpha',alpha_vec,'V',V_vec);
```



The breakpoints at which you evaluate the gain surface need not fall within the range specified by domain. However, if you attempt to evaluate the gain too far outside the range used for tuning, the software issues a warning.

The breakpoints also need not be regularly spaced. In addition, you can specify the scheduling variables in any order to get a different perspective on the shape of the surface. The variable that you specify first is used as the X-axis in the plot.

```
alpha_vec2 = [1,3,6,10,15];
V_vec2 = [300,350,425,575];
viewSurf(GS,'V',V_vec2,'alpha',alpha_vec2);
```



# **Input Arguments**

### **GS** — **Gain** surface

tunableSurface object

Gain surface to plot, specified as a tunableSurface object. GS can depend on one or two scheduling variables, and must be scalar-valued.

#### xvar — X-axis variable

character vector

X-axis variable in the plot, specified as a character vector. The variable name xvar must match a scheduling variable name in GS.SamplingGrid.

#### xdata — X-axis-variable values

numeric vector

X-axis-variable values at which to evaluate and plot the gain surface, specified as a numeric vector.

#### yvar — Y-axis variable

character vector

Y-axis variable in the plot, specified as a character vector. The variable name yvar must match a scheduling variable name in GS.SamplingGrid.

#### ydata — Y-axis-variable values

numeric vector

Y-axis-variable values at which to evaluate and plot the gain surface, specified as a numeric vector.

### **See Also**

evalSurf | tunableSurface

Introduced in R2015b

## voidModel

Mark missing or irrelevant models in model array

## **Syntax**

```
Mout = voidModel(M, void)
```

## **Description**

Mout = voidModel(M, void) sets the models specified by void to NaN static gains. When working with model arrays defined on a multidimensional grid of design points, use voidModel to indicate that no model is available at specific grid points. For example, when using systume to tune controller parameters for a model array, remove models at points outside the design envelope or points to be ignored during analysis or design.

- If void is a vector of integers, then voidModel sets M(:,:,void) to NaN.
- If void is a logical array, then voidModel sets the models selected by void to NaN.

## **Examples**

### **Mark Unneeded Models in Array**

Generate an array of tunable genss models. To do so, first create an array of plant models by varying parameters in a second-order transfer function. Then, interconnect the resulting array of plant models with a tunable controller element.

```
G = tf(zeros(1,1,3,3));
zeta = [0.66,0.71,0.75];
w = [1.0,1.2,1.5];
for i = 1:length(zeta)
    for j = 1:length(w)
        G(:,:,i,j) = tf(w(j)^2,[1 2*zeta(i)*w(j) w(j)^2]);
    end
end
```

```
C = tunablePID('C', 'PID');
M = feedback(C*G,1)
M =
  3x3 array of generalized continuous-time state-space models.
  Each model has 1 outputs, 1 inputs, 3 states, and the following blocks:
    C: Parametric PID controller, 1 occurrences.
Type "ss(M)" to see the current value, "get(M)" to see all properties, and "M.Blocks"
Suppose that you want to tune the PID controller gains for all of the models in M, but that
the parameter combinations (zeta,w) = (0.66,1.0) and (zeta,w) = (0.75,1.5)
do not occur in your physical system. Void these models so that they do not contribute to
any tuning or analysis of the model array. These models are the first and last models in
the 3-by-3 array, with linear indices 1 and 9.
void = [1,9]
void = 1 \times 2
     1
            9
Mout = voidModel(M, void)
Mout =
  3x3 array of generalized continuous-time state-space models.
  Each model has 1 outputs, 1 inputs, between 0 and 3 states, and between 0 and 1 block
Type "ss(Mout)" to see the current value, "get(Mout)" to see all properties, and "Mout
The display indicates that the models in Mout have 0-3 states and 0-1 blocks. The 0-state,
0-block models are the voided entries in Mout. For instance, examine the first entry and
confirm that it is a NaN static gain.
tf(Mout(:,:,1,1))
ans =
  NaN
```

Static gain.

Instead of using linear indices to specify the models to void, you can use a logical array.

```
void = logical([1 0 0;0 0 0;0 0 1]);
Mout1 = voidModel(M, void);

Confirm that the first and last models in Mout1 are NaN.

tf(Mout1(:,:,1,1))
ans =
    NaN

Static gain.

tf(Mout1(:,:,3,3))
ans =
```

## **Input Arguments**

### M - Model array

LTI model array

NaN

Static gain.

Model array, specified as an LTI model array such as an array of genss models.

#### void - Models to void

vector | logical array

Models to void, specified as a vector of integer values or a logical array.

- If void is a vector of integers, then voidModel sets M(:,:,void) to NaN. For instance, using Void = [1,10] voids M(:,:,[1 10]), the 1st and 10th models in M determined by linear indexing, regardless of the array dimensions of M.
- If void is a logical array, then voidModel sets the models selected by void to NaN.
   For instance, if M is a 2-by-2 array of models, then using void = logical([0,1;0,0]) voids the second model in the first row of M.

# **Output Arguments**

### Mout — Array with voided models

LTI model array

Array with voided models set to NaN, returned as an LTI model array of the same type and size as M.

### See Also

Introduced in R2017b

## **xperm**

Reorder states in state-space models

## **Syntax**

```
sys = xperm(sys, P)
```

## **Description**

sys = xperm(sys, P) reorders the states of the state-space model sys according to the permutation P. The vector P is a permutation of 1:NX, where NX is the number of states in sys. For information about creating state-space models, see ss and dss.

# **Examples**

### **Alphabetically Order States of State-Space Model**

Load a previously saved state space model  ${\tt ssF8}$  with four states.

```
load ltiexamples
ssF8
ssF8 =
 A =
             PitchRate Velocity
                                    AOA PitchAngle
  PitchRate
              -0.7 -0.0458
                                   -12.2
                        -0.014
  Velocity
                  0
                                  -0.2904
                                             -0.562
                        -0.0057 -1.4
                   1
  A0A
                             (-)
                                      0
                                                 (-)
  PitchAngle
 B =
            Elevator Flaperon
  PitchRate -19.1
                        -3.1
  Velocity -0.0119 -0.0096
```

Continuous-time state-space model.

The states in  ${\tt ssF8}$  now appear in alphabetical order.

## **See Also**

dss | ss

Introduced in R2008b

### zero

Zeros and gain of SISO dynamic system

## **Syntax**

```
Z = zero(sys)
[Z,gain] = zero(sys)
[Z,gain] = zero(sys,J1,...,JN)
```

## **Description**

Z = zero(sys) returns the zeros of the single-input, single-output (SISO) dynamic system model, sys. The output is expressed as the reciprocal of the time units specified in sys.TimeUnit.

[Z,gain] = zero(sys) also returns the zero-pole-gain of sys.

[Z,gain] = zero(sys,J1,...,JN) returns the zeros and gain of the entries in the model array sys with subscripts J1,...,JN.

## **Examples**

#### **Zeros of Transfer Function**

Compute the zeros of the following transfer function:

$$sys(s) = \frac{4.2s^2 + 0.25s - 0.004}{s^2 + 9.6s + 17}$$

$$sys = tf([4.2,0.25,-0.004],[1,9.6,17]);$$

$$Z = zero(sys)$$

$$Z = 2 \times 1$$

```
-0.0726
0.0131
```

#### Zeros and Gain of Transfer Function

Calculate the zero locations and zero-pole gain of the following transfer function:

$$sys(s) = \frac{4.2s^2 + 0.25s - 0.004}{s^2 + 9.6s + 17}$$

$$sys = tf([4.2, 0.25, -0.004], [1, 9.6, 17]);$$

$$[z,gain] = zero(sys)$$

$$z = 2 \times 1$$

$$-0.0726$$

$$0.0131$$

$$gain = 4.2000$$

The zero locations are expressed in second<sup>-1</sup>, because the time unit of the transfer function (H.TimeUnit) is seconds.

### Zeros and Gain of Models in an Array

For this example, load a 3-by-1 array of transfer function models.

```
load('tfArray.mat','sys');
size(sys)
3x1 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Find the zeros and gain values of the models in the array.

```
[Z, gain] = zero(sys);
Z(:,:,1,1)
```

```
ans = 2 \times 1

-0.0726

0.0131

gain(:,:,1,1)

ans = 4.2000
```

zero returns an array each for the zeros and the gain values respectively. Here, Z(:,:,1,1) and gain(:,:,1,1) corresponds to the zero and the gain value of the first model in the array, that is, sys(:,:,1,1).

## **Input Arguments**

#### sys — Dynamic system

dynamic system model | model array

Dynamic system, specified as a SISO dynamic system model, or an array of SISO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, zero returns the zeros of the current or nominal value of sys. If sys is an array of models, zero returns the zeros of the model corresponding to its subscript J1,..., JN in sys. For more information on model arrays, see "Model Arrays".

# J1, . . . , JN — Indices of models in array whose zeros you want to extract positive integer

Indices of models in array whose zeros you want to extract, specified as a positive integer. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4-by-5 array of dynamic system models, the following command extracts the zeros for entry (2,3) in the array.

```
Z = zero(sys, 2, 3);
```

## **Output Arguments**

#### Z — Zeros of the dynamic system

column vector | array

Zeros of the dynamic system, returned as a column vector or an array. If sys is:

- A single model, then Z is a column vector of zeros of the dynamic system model sys
- A model array, then Z is an array containing the zeros of each model in sys

Z is expressed as the reciprocal of the time units specified in sys.TimeUnit. For example, zero is expressed in 1/minute if sys.TimeUnit = 'minutes'.

## $\label{eq:gain-def} \mathbf{gain} - \mathbf{Zero\text{-}pole\text{-}gain} \ \mathbf{of} \ \mathbf{the} \ \mathbf{dynamic} \ \mathbf{system}$

scalar

Zero-pole-gain of the dynamic system, returned as a scalar. In other words, gain is the value of K when the model is written in zpk form.

## **Tips**

- If sys has internal delays, zero sets all internal delays to zero, creating a zero-order Padé approximation. This approximation ensures that the system has a finite number of zeros. zero returns an error if setting internal delays to zero creates singular algebraic loops. To assess the stability of models with internal delays, use step or impulse.
- To calculate the transmission zeros of a multi-input, multi-output (MIMO) system, use tzero.

### See Also

impulse | pole | pzmap | pzplot | step | tzero

### **Topics**

"Pole and Zero Locations"

#### **Introduced before R2006a**

# zgrid

Generate z-plane grid of constant damping factors and natural frequencies

## **Syntax**

```
zgrid
zgrid(zeta,wn)
zgrid(___,'new')
zgrid(AX,___)
```

# **Description**

zgrid generates a grid of constant damping factors from 0 to 1 in steps of 0.1 and natural frequencies from 0 to  $\pi$  in steps of  $\pi/10$  for root locus and pole-zero maps. zgrid then plots the grid over the current axis. zgrid creates the grid over the plot without altering the current axis limits if the current axis contains a discrete z-plane root locus diagram or pole-zero map.

zgrid(zeta,wn) plots a grid of constant damping factor and natural frequency lines for
the damping factors and normalized natural frequencies in the vectors zeta and wn,
respectively. zgrid(zeta,wn) creates the grid over the plot if the current axis contains
a discrete z-plane root locus diagram or pole-zero map. You can also plot frequency lines
for non-normalized (true) frequencies using zgrid(zeta,wn/Ts), where Ts is the
sample time.

Alternatively, you can select **Grid** from the context menu to generate the same z-plane grid.

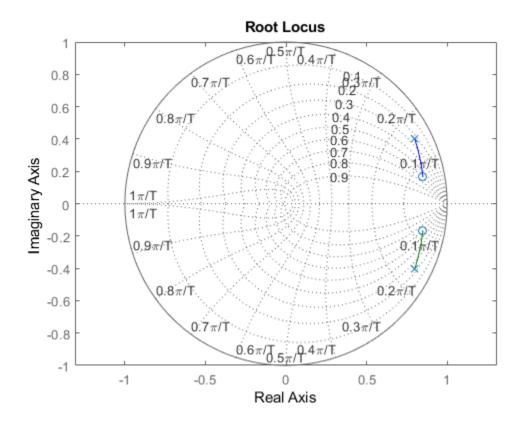
zgrid( ,'new') clears the current axes first and sets hold on.

zgrid(AX, \_\_\_\_) plots the z-plane grid on the Axes or UIAxes object in the current figure with the handle AX. Use this syntax when creating apps with zgrid in the App Designer.

# **Examples**

### Plot z-plane Grid Lines on the Root Locus

To see the z-plane grid on the root locus plot, type



# **Input Arguments**

### zeta — Damping ratio

vector

Damping ratio, specified as a vector in the same order as wn.

## ${\bf wn-Normalized\ natural\ frequency}$

vector

Normalized natural frequency, specified as a vector. You can also plot frequency lines for non-normalized (true) frequencies using  ${\tt zgrid(zeta,wn/Ts)}$ , where Ts is the sample time.

### AX — Object handle

Axes object | UIAxes object

Object handle, specified as an Axes or UIAxes object. Use AX to create apps with zgrid in the App Designer.

### See Also

pzmap | rlocus | sgrid

Introduced before R2006a

# zpk

Create zero-pole-gain model; convert to zero-pole-gain model

## **Syntax**

```
sys = zpk(Z,P,K)
sys = zpk(Z,p,k,Ts)
sys = zpk(M)
sys = zpk(Z,p,k,ltisys)
s = zpk('s')
z = zpk('z',Ts)
zsys = zpk(sys)
zsys = zpk(sys, 'measured')
zsys = zpk(sys, 'noise')
zsys = zpk(sys, 'augmented')
```

# **Description**

Used zpk to create zero-pole-gain models (zpk model objects), or to convert dynamic systems to zero-pole-gain form.

### **Creation of Zero-Pole-Gain Models**

sys = zpk(Z,P,K) creates a continuous-time zero-pole-gain model with zeros Z, poles P, and gain(s) K. The output sys is a zpk model object storing the model data.

In the SISO case, Z and P are the vectors of real- or complex-valued zeros and poles, and K is the real- or complex-valued scalar gain:

$$h(s) = k \frac{(s - z(1))(s - z(2))...(s - z(m))}{(s - p(1))(s - p(2))...(s - p(n))}$$

Set Z or p to [] for systems without zeros or poles. These two vectors need not have equal length and the model need not be proper (that is, have an excess of poles).

To create a MIMO zero-pole-gain model, specify the zeros, poles, and gain of each SISO entry of this model. In this case:

- Z and P are cell arrays of vectors with as many rows as outputs and as many columns as inputs, and K is a matrix with as many rows as outputs and as many columns as inputs.
- The vectors Z{i,j} and P{i,j} specify the zeros and poles of the transfer function from input j to output i.
- K(i,j) specifies the (scalar) gain of the transfer function from input j to output i.

See below for a MIMO example.

sys = zpk(Z,p,k,Ts) creates a discrete-time zero-pole-gain model with sample time Ts (in seconds). Set Ts = -1 or Ts = [] to leave the sample time unspecified. The input arguments Z, P, K are as in the continuous-time case.

```
sys = zpk(M) specifies a static gain M.
```

sys = zpk(Z,p,k,ltisys) creates a zero-pole-gain model with properties inherited
from the LTI model ltisys (including the sample time).

To create an array of zpk model objects, use a for loop, or use multidimensional cell arrays for Z and P, and a multidimensional array for K.

Any of the previous syntaxes can be followed by property name/property value pairs.

```
'PropertyName',PropertyValue
```

Each pair specifies a particular property of the model, for example, the input names or the input delay time. For more information about the properties of zpk model objects, see "Properties" on page 2-1452. Note that

```
sys = zpk(Z,P,K,'Property1',Value1,...,'PropertyN',ValueN)
```

is a shortcut for the following sequence of commands.

```
sys = zpk(Z,P,K)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

### Zero-Pole-Gain Models as Rational Expressions in s or z

You can also use rational expressions to create a ZPK model. To do so, first type either:

- s = zpk('s') to specify a ZPK model using a rational function in the Laplace variable, s.
- z = zpk('z',Ts) to specify a ZPK model with sample time Ts using a rational function in the discrete-time variable, z.

Once you specify either of these variables, you can specify ZPK models directly as rational expressions in the variable s or z by entering your transfer function as a rational expression in either s or z.

### **Conversion to Zero-Pole-Gain Form**

zsys = zpk(sys) converts an arbitrary LTI model sys to zero-pole-gain form. The output zsys is a ZPK object. By default, zpk uses zero to compute the zeros when converting from state-space to zero-pole-gain. Alternatively,

```
zsys = zpk(sys,'inv')
```

uses inversion formulas for state-space models to compute the zeros. This algorithm is faster but less accurate for high-order models with low gain at s = 0.

### **Conversion of Identified Models**

An identified model is represented by an input-output equation of the form y(t) = Gu(t) + He(t), where u(t) is the set of measured input channels and e(t) represents the noise channels. If  $\Lambda = LL'$  represents the covariance of noise e(t), this equation can also be written as y(t) = Gu(t) + HLv(t), where cov(v(t)) = I.

zsys = zpk(sys), or zsys = zpk(sys, 'measured') converts the measured
component of an identified linear model into the ZPK form. sys is a model of type idss,
idproc, idtf, idpoly, or idgrey. zsys represents the relationship between u and y.

zsys = zpk(sys, 'noise') converts the noise component of an identified linear model into the ZPK form. It represents the relationship between the noise input, v(t) and output,  $y_noise = HL v(t)$ . The noise input channels belong to the InputGroup 'Noise'. The names of the noise input channels are v@yname, where yname is the name of the corresponding output channel. zsys has as many inputs as outputs.

zsys = zpk(sys, 'augmented') converts both the measured and noise dynamics into a ZPK model. zsys has ny+nu inputs such that the first nu inputs represent the channels u(t) while the remaining by channels represent the noise channels v(t).

```
zsys.InputGroup contains 2 input groups, 'measured' and 'noise'.
zsys.InputGroup.Measured is set to 1:nu while zsys.InputGroup.Noise is set to nu+1:nu+ny. zsys represents the equation y(t) = [G HL] [u; v].
```

**Tip** An identified nonlinear model cannot be converted into a ZPK system. Use linear approximation functions such as linearize and linapp.

### **Variable Selection**

As for transfer functions, you can specify which variable to use in the display of zero-polegain models. Available choices include s (default) and p for continuous-time models, and z (default),  $z^1$ ,  $q^1$  (equivalent to  $z^1$ ), or q (equivalent to z) for discrete-time models. Reassign the 'Variable' property to override the defaults. Changing the variable affects only the display of zero-pole-gain models.

## **Properties**

zpk objects have the following properties:

#### Z

System zeros.

The Z property stores the transfer function zeros (the numerator roots). For SISO models, Z is a vector containing the zeros. For MIMO models with Ny outputs and Nu inputs, Z is a Ny-by-Nu cell array of vectors of the zeros for each input/output pair.

#### Ρ

System poles.

The P property stores the transfer function poles (the denominator roots). For SISO models, P is a vector containing the poles. For MIMO models with Ny outputs and Nu inputs, P is a Ny-by-Nu cell array of vectors of the poles for each input/output pair.

#### K

System gains.

The K property stores the transfer function gains. For SISO models, K is a scalar value. For MIMO models with Ny outputs and Nu inputs, K is a Ny-by-Nu matrix storing the gains for each input/output pair.

#### **DisplayFormat**

Specifies how the numerator and denominator polynomials are factorized for display purposes.

The numerator and denominator polynomials are each displayed as a product of first- and second-order factors. DisplayFormat controls the display of those factors. DisplayFormat can take the following values:

- 'roots' (default) Display factors in terms of the location of the polynomial roots.
- 'frequency' Display factors in terms of root natural frequencies  $\omega_0$  and damping ratios  $\zeta$ .

The 'frequency' display format is not available for discrete-time models with  $Variable\ value\ 'z^-1'$  or 'q^-1'.

• 'time constant' — Display factors in terms of root time constants  $\tau$  and damping ratios  $\zeta$ .

The 'time constant' display format is not available for discrete-time models with  $Variable\ value\ 'z^-1'$  or 'q^-1'.

For continuous-time models, the following table shows how the polynomial factors are written in each display format.

DisplayName Value	First-Order Factor (Real Root R)	Second-Order Factor (Complex Root pair $R = a \pm jb$ )
'roots'	(s-R)	$(s^2 - \alpha s + \beta)$ , where $\alpha = 2a$ , $\beta = a^2 + b^2$
'frequency'	$(1 - s/\omega_0)$ , where $\omega_0 = R$	1 - $2\zeta(s/\omega_0)$ + $(s/\omega_0)^2$ , where $\omega_0^2 = a^2 + b^2$ , $\zeta = a/\omega_0$
'time constant'	$(1 - \tau s)$ , where $\tau = 1/R$	1 - $2\zeta(\tau s)$ + $(\tau s)^2$ , where $\tau = 1/\omega_0$ , $\zeta = a\tau$

For discrete-time models, the polynomial factors are written as in continuous time, with the following variable substitutions:

$$s \to w = \frac{z-1}{T_s}; \quad R \to \frac{R-1}{T_s},$$

where  $T_s$  is the sample time. In discrete time,  $\tau$  and  $\omega_0$  closely match the time constant and natural frequency of the equivalent continuous-time root, provided  $|z-1| \ll T_s$  ( $\omega_0 \ll \pi/T_s = \text{Nyquist frequency}$ ).

Default: 'roots'

#### Variable

Transfer function display variable, specified as one of the following:

- 's' Default for continuous-time models
- 'z' Default for discrete-time models
- 'p' Equivalent to 's'
- 'q' Equivalent to 'z'
- 'z^-1' Inverse of 'z'
- 'q^-1' Equivalent to 'z^-1'

The value of Variable only affects the display of zpk models.

Default: 's'

### **IODelay**

Transport delays. IODelay is a numeric array specifying a separate transport delay for each input/output pair.

For continuous-time systems, specify transport delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify transport delays in integer multiples of the sample time, Ts.

For a MIMO system with Ny outputs and Nu inputs, set IODelay to a Ny-by-Nu array. Each entry of this array is a numerical value that represents the transport delay for the corresponding input/output pair. You can also set IODelay to a scalar value to apply the same delay to all input/output pairs.

Default: 0 for all input/output pairs

#### InputDelay

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

#### Default: 0

#### **OutputDelay**

Output delays. OutputDelay is a numeric vector specifying a time delay for each output channel. For continuous-time systems, specify output delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify output delays in integer multiples of the sample time Ts. For example, OutputDelay = 3 means a delay of three sampling periods.

For a system with Ny outputs, set OutputDelay to an Ny-by-1 vector, where each entry is a numerical value representing the output delay for the corresponding output channel. You can also set OutputDelay to a scalar value to apply the same delay to all channels.

**Default:** 0 for all output channels

#### Ts

Sample time. For continuous-time models, Ts = 0. For discrete-time models, Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts = -1.

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

**Default:** 0 (continuous time)

#### **TimeUnit**

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

#### InputName

Input channel names, specified as one of the following:

- Character vector For single-input models, for example, 'controls'.
- Cell array of character vectors For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to {'controls(1)';'controls(2)'}.

You can use the shorthand notation u to refer to the InputName property. For example, sys.u is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

**Default:** ' ' for all input channels

#### InputUnit

Input channel units, specified as one of the following:

- Character vector For single-input models, for example, 'seconds'.
- Cell array of character vectors For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

**Default:** ' ' for all input channels

#### **InputGroup**

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1, 2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:

```
sys(:,'controls')
```

**Default:** Struct with no fields

#### **OutputName**

Output channel names, specified as one of the following:

- Character vector For single-output models. For example, 'measurements'.
- Cell array of character vectors For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:

```
sys.OutputName = 'measurements';
The output names automatically expand to
{'measurements(1)';'measurements(2)'}.
```

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- · Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

#### OutputUnit

Output channel units, specified as one of the following:

- $\bullet \quad \text{Character vector} \text{For single-output models. For example, 'seconds'}.$
- ullet Cell array of character vectors For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

Default: ' ' for all output channels

### **OutputGroup**

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1, and 3, 5, respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

Default: Struct with no fields

#### Name

System name, specified as a character vector. For example, 'system\_1'.

Default: ''

#### **Notes**

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes

ans =
    "sys1 has a string."

ans =
    'sys2 has a character vector.'
```

## **Default:** [0×1 string]

#### UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

#### Default: []

#### **SamplingGrid**

Sampling grid for model arrays, specified as a data structure.

For model arrays that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model in the array. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, suppose you create a 11-by-1 array of linear models, sysarr, by taking snapshots of a linear time-varying system at times t = 0:10. The following code stores the time samples with the linear models.

```
sysarr.SamplingGrid = struct('time',0:10)
```

Similarly, suppose you create a 6-by-9 model array, M, by independently sampling two variables, zeta and w. The following code attaches the (zeta,w) values to M.

```
[zeta,w] = ndgrid(<6 values of zeta>,<9 values of w>)
M.SamplingGrid = struct('zeta',zeta,'w',w)
```

When you display M, each entry in the array includes the corresponding zeta and w values.

```
М
```

. . .

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

## **Examples**

## **Example 1**

Create the continuous-time SISO transfer function:

$$h(s) = \frac{-2s}{(s-1+j)(s-1-j)(s-2)}$$

Create h(s) as a zpk object using:

$$h = zpk(0, [1-i 1+i 2], -2);$$

### **Example 2**

Specify the following one-input, two-output zero-pole-gain model:

$$H(z) = \begin{bmatrix} \frac{1}{z - 0.3} \\ \frac{2(z + 0.5)}{(z - 0.1 + j)(z - 0.1 - j)} \end{bmatrix}.$$

To do this, enter:

```
Z = {[] ; -0.5};
P = {0.3 ; [0.1+i 0.1-i]};
K = [1 ; 2];
H = zpk(Z,P,K,-1); % unspecified sample time
```

## **Example 3**

Convert the transfer function

```
h = tf([-10 \ 20 \ 0],[1 \ 7 \ 20 \ 28 \ 19 \ 5]);
```

to zero-pole-gain form, using:

```
zpk(h)
```

This command returns the result:

## **Example 4**

Create a discrete-time ZPK model from a rational expression in the variable z.

```
z = zpk('z', 0.1);

H = (z+.1)*(z+.2)/(z^2+.6*z+.09)
```

This command returns the following result:

```
Zero/pole/gain:
(z+0.1) (z+0.2)
(z+0.3)^2
```

Sample time: 0.1

## **Example 5**

Create a MIMO zpk model using cell arrays of zeros and poles.

Create the two-input, two-output zero-pole-gain model

$$H(s) = \begin{bmatrix} \frac{-1}{s} & \frac{3(s+5)}{(s+1)^2} \\ \frac{2(s^2 - 2s + 2)}{(s-1)(s-2)(s-3)} & 0 \end{bmatrix}$$

by entering:

```
Z = {[],-5;[1-i 1+i] []};
P = {0,[-1 -1];[1 2 3],[]};
K = [-1 3;2 0];
H = zpk(Z,P,K);
```

Use [] as a place holder in Z or P when the corresponding entry of H(s) has no zeros or poles.

## **Example 6**

Extract the measured and noise components of an identified polynomial model into two separate ZPK models. The former (measured component) can serve as a plant model while the latter can serve as a disturbance model for control system design.

```
load icEngine
z = iddata(y,u,0.04);
nb = 2; nf = 2; nc = 1; nd = 3; nk = 3;
sys = bj(z, [nb nc nd nf nk]);

sys is a model of the form, y(t) = B/F u(t) + C/D e(t), where B/F represents the measured component and C/D the noise component.

sysMeas = zpk(sys, 'measured')

Alternatively, use can simply use zpk(sys) to extract the measured component.

sysNoise = zpk(sys, 'noise')
```

## **Algorithms**

zpk uses the MATLAB function roots to convert transfer functions and the functions
zero and pole to convert state-space models.

## See Also

frd|get|set|ss|tf|zpkdata

### **Introduced before R2006a**

## zpkdata

Access zero-pole-gain data

## **Syntax**

```
[z,p,k] = zpkdata(sys)
[z,p,k,Ts] = zpkdata(sys)
[z,p,k,Ts,covz,covp,covk] = zpkdata(sys)
```

## **Description**

[z,p,k] = zpkdata(sys) returns the zeros z, poles p, and gain(s) k of the zero-polegain model sys. The outputs z and p are cell arrays with the following characteristics:

- z and p have as many rows as outputs and as many columns as inputs.
- The (i,j) entries z{i,j} and p{i,j} are the (column) vectors of zeros and poles of the transfer function from input j to output i.

The output k is a matrix with as many rows as outputs and as many columns as inputs such that k(i,j) is the gain of the transfer function from input j to output i. If sys is a transfer function or state-space model, it is first converted to zero-pole-gain form using zpk.

For SISO zero-pole-gain models, the syntax

```
[z,p,k] = zpkdata(sys,'v')
```

forces zpkdata to return the zeros and poles directly as column vectors rather than as cell arrays (see example below).

```
[z,p,k,Ts] = zpkdata(sys) also returns the sample time Ts.
```

[z,p,k,Ts,covz,covp,covk] = zpkdata(sys) also returns the covariances of the zeros, poles and gain of the identified model sys. covz is a cell array such that  $covz\{ky,ku\}$  contains the covariance information about the zeros in the vector  $z\{ky,ku\}$ .  $covz\{ky,ku\}$  is a 3-D array of dimension 2-by-2-by-Nz, where Nz is the

length of  $z\{ky, ku\}$ , so that the (1,1) element is the variance of the real part, the (2,2) element is the variance of the imaginary part, and the (1,2) and (2,1) elements contain the covariance between the real and imaginary parts. covp has a similar relationship to p.covk is a matrix containing the variances of the elements of k.

You can access the remaining LTI properties of sys with get or by direct referencing, for example,

```
sys.Ts
sys.inputname
```

## **Examples**

## **Example 1**

Given a zero-pole-gain model with two outputs and one input

Sample time: unspecified

you can extract the zero/pole/gain data embedded in H with

To access the zeros and poles of the second output channel of H, get the content of the second cell in z and p by typing

```
z{2,1}

ans =

-0.5000

p{2,1}

ans =

0.1000+ 1.0000i

0.1000- 1.0000i
```

## **Example 2**

Extract the ZPK matrices and their standard deviations for a 2-input, 1 output identified transfer function.

```
load iddata7

transfer function model

sys1 = tfest(z7, 2, 1, 'InputDelay',[1 0]);
an equivalent process model

sys2 = procest(z7, {'P2UZ', 'P2UZ'}, 'InputDelay',[1 0]);

1, p1, k1, ~, dz1, dp1, dk1] = zpkdata(sys1);
[z2, p2, k2, ~, dz2, dp2, dk2] = zpkdata(sys2);

Use iopzplot to visualize the pole-zero locations and their covariances

h = iopzplot(sys1, sys2);
showConfidence(h)
```

## **See Also**

get | ssdata | tfdata | zpk

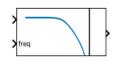
Introduced before R2006a

# **Block Reference**

## **Discrete Varying Lowpass**

Discrete Butterworth filter with varying coefficients

Library: Control System Toolbox / Linear Parameter Varying



Discrete Varying Lowpass

## **Description**

The block implements the Tustin discretization of a continuous-time  $N^{th}$ -order Butterworth filter. The result is a digital filter with unit DC gain and varying cutoff frequency.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

### u — Filter input

scalar

Lowpass filter input signal.

### freq — Cutoff frequency

scalar

Continuous-time value of the cutoff frequency, specified in rad/s.

## Output

#### y — Filter output

scalar

Lowpass filter output signal.

## **Parameters**

#### Filter order - Lowpass filter order

1 (default) | positive integer

Lowpass filter order, specified as a positive integer.

## Pre-warping frequency w0 (rad/s) — Pre-warping frequency

0 (default) | positive scalar

Pre-warping frequency, specified as a positive scalar. Discretization of the continuous-time Butterworth filter can shift the cutoff frequency when it is close to the Nyquist frequency. To ensure that the analog and digital filters have matching frequency response near a particular frequency  $w\theta$ , set this parameter to  $w\theta$ . The default value  $w\theta = \theta$  corresponds to the bilinear (Tustin) transformation without pre-warp:

$$s = \frac{2}{T_s} \left( \frac{z - 1}{z + 1} \right),$$

where  $T_s$  is the block sample time, specified with the **Sample time Ts** parameter.

### Sample time Ts — Sample time

1 (default) | positive scalar

Block sample time, specified as a positive scalar. This block does not support inherited sample time, because it requires a specified sample time to compute the discretization of the Butterworth filter.

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

## **See Also**

Varying Lowpass Filter

## **Topics**

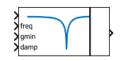
"Model Gain-Scheduled Control Systems in Simulink"

Introduced in R2017b

## **Discrete Varying Notch**

Discrete-time notch filter with varying coefficients

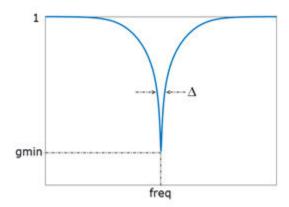
Library: Control System Toolbox / Linear Parameter Varying



Discrete Varying Notch

## **Description**

The block implements the Tustin discretization of a continuous-time notch filter with varying coefficients. Feed the continuous-time values of the notch frequency, minimum gain, and damping ratio to the **freq**, **gmin**, and **damp** input ports, respectively. These parameters control the notch depth and frequency of the continuous-time notch frequency as shown in the following illustration. The damping ratio **damp** controls the notch width  $\Delta$ ; larger **damp** means larger  $\Delta$ .



Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

#### u - Filter input

scalar

Notch filter input signal

#### freq - Notch frequency

scalar

Continuous-time value of the notch frequency, specified in rad/s.

#### gmin — Gain at notch frequency

scalar

Continuous-time value of the gain at notch frequency, in absolute units. This value controls the notch depth. The notch filter has unit gain at low and high frequency. The gain is lowest at the notch frequency.

### damp — Damping ratio of the filter poles

scalar

Continuous-time value of the damping ratio, specified as a positive scalar value. The damping ratio controls the notch width; the closer to 0, the steeper the notch.

## **Output**

### y — Filter output

scalar

Notch filter output signal.

## **Parameters**

Pre-warping frequency w0 (rad/s) — Pre-warping frequency 0 (default) | positive scalar

Pre-warping frequency, specified as a positive scalar. Discretization of the continuous-time notch-filter transfer function can shift the notch frequency when it is close to the Nyquist frequency. To ensure that the continuous and discrete filters have matching frequency response near a particular frequency  $w\theta$ , set this parameter to  $w\theta$ . The default value  $w\theta = \theta$  corresponds to the bilinear (Tustin) transformation without pre-warp:

$$s = \frac{2}{T_s} \left( \frac{z - 1}{z + 1} \right),$$

where  $T_s$  is the block sample time, specified with the **Sample time Ts** parameter.

#### Sample time Ts — Sample time

1 (default) | positive scalar

Block sample time, specified as a positive scalar. This block does not support inherited sample time, because it requires a specified sample time to compute the discretization of the notch filter.

## **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

## See Also

Varying Notch Filter

## **Topics**

"Model Gain-Scheduled Control Systems in Simulink"

#### Introduced in R2017b

## **Discrete Varying Observer Form**

Discrete-time observer-form state-space model with varying matrix values

Library: Control System Toolbox / Linear Parameter Varying

A
B
X<sub>k+1</sub> = A X<sub>k</sub> + B Q + LE<sub>k</sub>
C
U<sub>k</sub> = -K X<sub>k</sub>
D
E<sub>k</sub> = -C X<sub>k</sub> - D Q
K
L
Discrete Varying Observer Form

## **Description**

Use this block to implement a discrete-time varying state-space model in observer form. The system matrices A, B, C, and D describe the plant dynamics, and the matrices K and L specify the state-feedback and state-observer gains, respectively. Feed the instantaneous values of these matrices to the corresponding input ports. The observer form is given by:

$$x_{k+1} = Ax_k + Bu_k + L\varepsilon_k$$
  

$$u_k = -Kx_k$$
  

$$\varepsilon_k = y_k - Cx_k - Du_k,$$

where  $u_k$  is the plant input,  $y_k$  is the plant output,  $x_k$  is the estimated state, and  $\varepsilon_k$  is the innovation, the difference between the predicted and measured plant output. The observer form works well for gain scheduling of state-space controllers. In particular, the state  $x_k$  tracks the plant state, and all controllers are expressed with the same state coordinates.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

### Input

#### y<sub>k</sub> — Measurement signal

scalar | vector

Measured plant output signal.

#### A — Plant state matrix

matrix

Plant state matrix of dimensions  $N_x$ -by- $N_x$ , where  $N_x$  is the number of plant states.

#### **B** — Plant input matrix

matrix

Plant input matrix of dimensions  $N_x$ -by- $N_u$ , where  $N_u$  is the number of plant inputs.

#### C — Plant output matrix

matrix

Plant output matrix of dimensions  $N_v$ -by- $N_x$ , where  $N_v$  is the number of plant outputs.

#### D — Plant feedforward matrix

matrix

Plant feedforward matrix of dimensions  $N_y$ -by- $N_u$ .

#### K — State-feedback matrix

matrix

State-feedback matrix of dimensions  $N_u$ -by- $N_x$ .

#### L — State-observer matrix

matrix

State-observer matrix of dimensions  $N_x$ -by- $N_v$ .

## Output

#### u<sub>k</sub> — Control signal

scalar | vector

Control signal (plant input).

#### $x_k$ — Estimated plant state vector

vector

Vector of estimated plant states.

#### **Dependencies**

To enable this port, select the **Output states** parameter.

#### $x_{k+1}$ — Estimated next state vector

vector

Estimated state values at next time step.

#### **Dependencies**

To enable this port, select the **Output state updates** parameter.

## **Parameters**

## Initial conditions — System initial conditions

0 (default) | scalar | vector

Initial state values, specified as a scalar or a vector whose length is the number of plant states.

## Sample time (-1 for inherited) — Block sample time

-1 (default) | positive scalar

Block sample time, specified as either -1 (inherited sample time) or a positive scalar value.

## **Output states** — **Provide state output**

on (default) | off

Select to enable the estimated states output port,  $x_e$ .

**Output state derivatives — Provide state updates** on (default) | off

Select to enable the estimated state updates output port,  $\mathbf{x}_{k+1}$ .

## **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

## See Also

Discrete Varying State Space | Varying Observer Form

## **Topics**

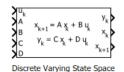
"Model Gain-Scheduled Control Systems in Simulink"

Introduced in R2017b

## **Discrete Varying State Space**

Discrete-time state-space model with varying matrix values

Library: Control System Toolbox / Linear Parameter Varying



## **Description**

Use this block to implement a discrete-time state-space model with varying matrices. Feed the instantaneous values of the state matrix A, input matrix B, output matrix C, and feedforward matrix D to the corresponding input ports. The system response is given by:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k,$$

where  $u_k$  is the system input,  $y_k$  is the system output,  $x_k$  is the current system state, and  $x_k$  is the system state at the next time step.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

 $u_k$  — System input scalar | vector

1

System input signal.

A — State matrix

matrix

State matrix of dimensions  $N_x$ -by- $N_x$ , where  $N_x$  is the number of system states.

#### **B** — Input matrix

matrix

Input matrix of dimensions  $N_x$ -by- $N_u$ , where  $N_u$  is the number of system inputs.

#### C — Output matrix

matrix

Output matrix  $N_{v}$ -by- $N_{x}$ , where  $N_{v}$  is the number of system outputs.

#### D — Feedforward matrix

matrix

Feedforward matrix of dimensions  $N_v$ -by- $N_u$ .

## **Output**

#### $y_k$ — System output

scalar | vector

System output signal.

#### $x_k$ — Current state vector

vector

Current state values.

#### **Dependencies**

To enable this port, select the **Output states** parameter.

#### $x_{k+1}$ — Next state vector

vector

State values at next time step.

#### **Dependencies**

To enable this port, select the **Output state updates** parameter.

## **Parameters**

#### Initial conditions — System initial conditions

0 (default) | scalar | vector

Initial state values, specified as a scalar or a vector whose length is the number of system states.

### Sample time (-1 for inherited) — Block sample time

-1 (default) | positive scalar

Block sample time, specified as either -1 (inherited sample time) or a positive scalar value.

#### Output states — Provide state output

on (default) | off

Select to enable the state values output port,  $\mathbf{x}$ .

#### Output state derivatives — Provide state updates

on (default) | off

Select to enable the state updates output port,  $x_{k+1}$ .

## **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

## **See Also**

Varying State Space

## **Topics**

"Model Gain-Scheduled Control Systems in Simulink"

## Introduced in R2017b

## **Discrete Varying Transfer Function**

Discrete-time transfer function with varying coefficients

Library: Control System Toolbox / Linear Parameter Varying

$$\begin{cases} b0 & b_0 + b_1 z^{-1} \\ a1 & 1 + a_1 z^{-1} \end{cases}$$

Discrete Varying Transfer Function

## **Description**

This block implements a discrete-time transfer function with varying coefficients. The instantaneous transfer function is given by:

$$H(z) = \frac{b_0 + b_1/z + \dots + b_N/z^N}{1 + a_1/z + \dots + a_N/z^N},$$

where N is number of poles, specified with the **Transfer function order** parameter. Feed the values of the coefficients  $a_1, ..., a_N$  and  $b_0, b_1, ..., b_N$  to the corresponding block input ports.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

u — Transfer function input

scalar

Transfer function input signal.

**b0, b1, ... — Numerator coefficients** scalar

Transfer function numerator coefficients. The number of coefficient ports is determined by the **Transfer function order** parameter.

# **a1**, **a2**, ... — Denominator coefficients scalar

Transfer function denominator coefficients. The number of coefficient ports is determined by the **Transfer function order** parameter.

## Output

### y — Transfer function output

scalar

Transfer function output signal.

### **Parameters**

#### Transfer function order — Number of poles

1 (default) | positive integer

Transfer function (number of poles), specified as a positive integer. This parameter determines the number of coefficient input ports on the block.

# Sample time (-1 for inherited) — Block sample time -1 (default) | positive scalar

Block sample time, specified as either -1 (inherited sample time) or a positive scalar value.

## **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

## **See Also**

Varying Transfer Function

## **Topics**

"Model Gain-Scheduled Control Systems in Simulink"

**Introduced in R2017b** 

## **Kalman Filter**

Estimate states of discrete-time or continuous-time linear system



## **Description**

Use the Kalman Filter block to estimate states of a state-space plant model given process and measurement noise covariance data. The state-space model can be time-varying. A steady-state Kalman filter implementation is used if the state-space model and the noise covariance matrices are all time-invariant. A time-varying Kalman filter is used otherwise.

Kalman filter provides the optimal solution to the following continuous or discrete estimation problems:

#### **Continuous-Time Estimation**

Given the continuous plant

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + G(t)w(t) \text{ (state equation)}$$

$$y(t) = C(t)x(t) + D(t)u(t) + H(t)w(t) + v(t) \text{ (measurement equation)}$$

with known inputs u, white process noise w, and white measurement noise v satisfying:

$$E[w(t)] = E[v(t)] = 0$$

$$E[w(t)w^{T}(t)] = Q(t)$$

$$E[w(t)v^{T}(t)] = N(t)$$

$$E[v(t)v^{t}] = R(t)$$

construct a state estimate  $\widehat{x}$  that minimizes the state estimation error covariance  $P(t) = E[(x - \widehat{x})(x - \widehat{x})^T].$ 

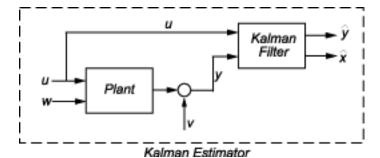
The optimal solution is the Kalman filter with equations

$$\begin{split} L(t) &= (P(t)C^T(t) + \overline{N}), \\ \dot{P}(t) &= A(t)P(t) + P(t)A^T(t) + \overline{Q}(t) - L(t)\overline{R}(t)L^T(t), \\ \dot{\widehat{x}}(t) &= A(t)\widehat{x}(t) + B(t)u(t) + L(t)(y(t) - C(t)\widehat{x}(t) - D(t)u(t)), \end{split}$$

where

$$\begin{split} \overline{Q}(t) &= G(t)Q(t)G^T(t), \\ \overline{R}(t) &= R(t) + H(t)N(t) + N^T(t)H^T(t) + H(t)Q(t)H^T(t), \\ \overline{N}(t) &= G(t)(O(t)H^T(t) + N(t)). \end{split}$$

The Kalman filter uses the known inputs u and the measurements y to generate the state estimates  $\widehat{x}$ . If you want, the block can also output the estimates of the true plant output  $\widehat{y}$ .



The block implements the steady-state Kalman filter when the system matrices (A(t), B(t), C(t), D(t), G(t), H(t)) and noise covariance matrices (Q(t), R(t), N(t)) are constant (specified in the Block Parameters dialog box). The steady-state Kalman filter uses a constant matrix P that minimizes the steady-state estimation error covariance and solves the associated continuous-time algebraic Riccati equation:

$$P = \lim_{t \to \infty} E[(x - \widehat{x})(x - \widehat{x})^T].$$

#### **Discrete-Time Estimation**

Given the discrete plant

$$x[n+1] = A[n]x[n] + B[n]u[n] + G[n]w[n],$$
  
$$y[n] = C[n]x[n] + D[n]u[n] + H[n]w[n] + v[n],$$

with known inputs u, white process noise w and white measurement noise v satisfying

$$E[w[n]] = E[v[n]] = 0,$$
  
 $E[w[n]w^{T}[n]] = Q[n],$   
 $E[v[n]v^{T}[n]] = R[n],$   
 $E[w[n]v^{T}[n]] = N[n].$ 

The estimator has the following state equation

$$\widehat{x}[n+1|n] = A[n]\widehat{x}[n|n-1] + B[n]u[n] + L[n](y[n] - C[n]\widehat{x}[n|n-1] - D[n]u[n]),$$

where the gain L[n] is calculated through the discrete Riccati equation:

$$\begin{split} L[n] &= (A[n]P[n]C^T[n] + \overline{N}[n])(^C, \\ M[n] &= P[n]C^T[n](^C, \\ Z[n] &= (I - M[n]C[n])P[n](^I + M[n]\overline{R}[n]M^T[n], \\ P[n+1] &= (A[n] - \overline{N}[n]\overline{R}^{-1}[n]C[n])Z(^A + \overline{Q}[n] - N[n]\overline{R}^{-1}[n]N^T[n], \end{split}$$

where I is the identity matrix of appropriate size and

$$\begin{split} & \overline{Q}[\mathbf{n}] = G[\mathbf{n}]Q[\mathbf{n}]G^T[\mathbf{n}], \\ & \overline{R}[\mathbf{n}] = R[\mathbf{n}] + H[\mathbf{n}]N[\mathbf{n}] + N^T[\mathbf{n}]H^T[\mathbf{n}] + H[\mathbf{n}]Q[\mathbf{n}]H^T[\mathbf{n}], \\ & \overline{N}[\mathbf{n}] = G[\mathbf{n}](Q[\mathbf{n}]H^T[\mathbf{n}] + N[\mathbf{n}]), \\ & \text{and} \\ & P[\mathbf{n}] = E[(x - \widehat{x}[n \mid n - 1])(^x], \\ & Z[\mathbf{n}] = E[(x - \widehat{x}[n \mid n])(^x], \end{split}$$

The steady-state Kalman filter uses a constant matrix P that minimizes the steady-state estimation error covariance and solves the associated discrete-time algebraic Riccati equation.

There are two variants of discrete-time Kalman filters:

• The current estimator generates the state estimates  $\widehat{x}[n|n]$  using all measurement available, including y[n]. The filter updates  $\widehat{x}[n|n-1]$  with y[n] and outputs:

$$\widehat{x}[n|n] = \widehat{x}[n|n-1] + M[n](y[n] - C[n]\widehat{x}[n|n-1] - D[n]u[n]),$$

$$\widehat{y}[n|n] = C[n]\widehat{x}[n|n] + D[n]u[n].$$

• The delayed estimator generates the state estimates  $\widehat{x}[n|n-1]$  using measurements up to y[n-1]. The filter outputs  $\widehat{x}[n|n-1]$  as defined previously, along with the optional output  $\widehat{y}[n|n-1]$ 

$$\widehat{y}[n|n-1] = C[n]\widehat{x}[n|n-1] + D[n]u[n]$$

The current estimator has better estimation accuracy compared to the delayed estimator, which is important for slow sample times. However, it has higher computational cost, making it harder to implement inside control loops. More specifically, it has direct feedthrough. This leads to an algebraic loop if the Kalman filter is used in a feedback loop that does not contain any delays (the feedback loop itself also has direct feedthrough). The algebraic loop can impact the speed of simulation. You cannot generate code if your model contains algebraic loops.

The Kalman Filter block differs from the kalman command in the following ways:

- When calling kalman(sys,...), sys includes the G and H matrices. Specifically, sys.B has [B G] and sys.D has [D H]. When you provide a LTI variable to the Kalman Filter block, it does not assume that the LTI variable provided contains G and H. They are optional and separate.
- The kalman command outputs [yhat;xhat] by default. The block only outputs xhat by default.

## **Parameters**

The following table summarizes the Kalman Filter block parameters, accessible via the Block Parameter dialog box.

Task	Parameters	
Specify filter settings	Time domain on page 3-23	
	• Use the current measurement y[n] to improve xhat[n] on page 3-24	

Task	Parameters
Specify the system model	Model source on page 3-24 in Model Parameters tab
Specify initial state estimates	Source on page 3-25 in Model Parameters tab
Specify noise characteristics	In Model Parameters tab:
	• Use G and H matrices (default G=I and H=0) on page 3-26
	• <b>Q</b> on page 3-26, <b>Time-invariant Q</b> on page 3-27
	• <b>R</b> on page 3-27, <b>Time-invariant R</b> on page 3-27
	• N on page 3-27, Time-invariant N on page 3-27
Specify additional inports	In <b>Options</b> tab:
	Add input port u on page 3-27
	Add input port Enable to control measurement updates on page 3-28
	• External reset on page 3-28
Specify additional outports	In <b>Options</b> tab:
	Output estimated model output y on page 3-29
	Output state estimation error covariance Z on page 3-29

## **Time domain**

Specify whether to estimate continuous-time or discrete-time states:

- Discrete-Time (**Default**) Block estimates discrete-time states
- Continuous-Time Block estimates continuous-time states

When the Kalman Filter block is in a model with synchronous state control (see the State Control block), you cannot select Continuous-time.

# Use the current measurement y[n] to improve xhat[n]

Use the current estimator variant of the discrete-time Kalman filter. When not selected, the delayed estimator (variant) is used.

This option is available only when **Time Domain** is **Discrete-Time**.

## Model source

Specify how the A, B, C, D matrices are provided to the block. Must be one of the following:

- Dialog: LTI State-Space Variable Use the values specified in the LTI state-space variable. You must also specify the variable name in **Variable**. The sample time of the model must match the setting in the **Time domain** option, i.e. the model must be discrete-time if the **Time domain** is discrete-time.
- Dialog: Individual A, B, C, D matrices Specify values in the following block parameters:
  - A Specify the A matrix. It must be real and square.
  - B Specify the B matrix. It must be real and have as many rows as the A matrix.
     This option is available only when Add input port u is selected in the Options tab.
  - C Specify the C matrix. It must be real and have as many columns as the A matrix.
  - D Specify the D matrix. It must be real. It must have as many rows as the C matrix and as many columns as the B matrix. This option is available only when Add input port u is selected in the Options tab.
- External Specify the A, B, C, D matrices as input signals to the Kalman Filter block. If you select this option, the block includes additional input ports A, B, C and D. You must also specify the following in the block parameters:
  - Number of states Number of states to be estimated, specified as a positive integer. The default value is 2.
  - Number of inputs Number of known inputs in the model, specified as a
    positive integer. The default value is 2. This option is only available when Add
    input port u is selected.
  - Number of outputs Number of measured outputs in the model, specified as a positive integer. The default value is 2.

# **Sample Time**

Block sample time, specified as -1 or a positive scalar.

This option is available only when **Time Domain** is Discrete Time and **Model Source** is Dialog: Individual A, B, C, D matrices or External. The sample time is obtained from the LTI state-space variable if the Model Source is Dialog: LTI State-Space Variable.

The default value is -1, which implies that the block inherits its sample time based on the context of the block within the model. All block input ports must have the same sample time.

## Source

Specify how to enter the initial state estimates and initial state estimation error covariance:

- Dialog Specify the values directly in the dialog box. You must also specify the following parameters:
  - **Initial states** x[0] Specify the initial state estimate as a real scalar or vector. If you specify a scalar, all initial state estimates are set to this scalar. If you specify a vector, the length of the vector must match with the number of states in the model.
  - State estimation error covariance P[0] (only when time-varying Kalman filter is used) Specify the initial state estimation error covariance P[0] for discrete-time Kalman filter or P(0) for continuous-time Kalman filter. Must be specified as one of the following:
    - Real nonnegative scalar. P is an Ns-by-Ns diagonal matrix with the scalar on the diagonals. Ns is the number of states in the model.
    - Vector of real nonnegative scalars. P is an Ns-by-Ns diagonal matrix with the elements of the vector on the diagonals of P.
    - Ns-by-Ns positive semi-definite matrix.
- External Inherit the values from input ports. The block includes an additional input port X0. A second additional input port P0 is added when time-varying Kalman filter is used. X0 and P0 must satisfy the same conditions described previously when you specify them in the dialog box.

## Use the Kalman Gain K from the model variable

Specify whether to use the pre-identified Kalman Gain contained in the state-space plant model. This option is available only when:

- **Model Source** is Dialog: LTI State-Space Variable and **Variable** is an identified state-space model (idss) with a nonzero K matrix.
- Time Invariant Q, Time Invariant R and Time Invariant N options are selected.

If the Use G and H matrices (default G=I and H=0) option is selected, Time Invariant G and Time Invariant H options must also be selected.

## Use G and H matrices (default G=I and H=0)

Specify whether to use non-default values for the G and H matrices. If you select this option, you must specify:

- **G** Specify the G matrix. It must be a real matrix with as many rows as the A matrix. The default value is 1.
- **Time-invariant G** Specify if the G matrix is time invariant. If you unselect this option, the block includes an additional input port G.
- $\mathbf{H}$  Specify the H matrix. It must be a real matrix with as many rows as the C matrix and as many columns as the G matrix. The default value is 0.
- **Time-invariant H** Specify if the H matrix is time invariant. If you unselect this option, the block includes an additional input port G.
- **Number of process noise inputs** Specify the number of process noise inputs in the model. The default value is 1.

This option is available only when  $Time-invariant\ G$  and  $Time-invariant\ H$  are cleared. Otherwise, this information is inferred from the G or H matrix.

## Q

Process noise covariance matrix, specified as one of the following:

- Real nonnegative scalar. Q is an Nw-by-Nw diagonal matrix with the scalar on the diagonals. Nw is the number of process noise inputs in the model.
- Vector of real nonnegative scalars. Q is an Nw-by-Nw diagonal matrix with the elements of the vector on the diagonals of Q.

Nw-by-Nw positive semi-definite matrix.

## **Time Invariant Q**

Specify if the Q matrix is time invariant. If you unselect this option, the block includes an additional input port Q.

### R

Measurement noise covariance matrix, specified as one of the following:

- Real positive scalar. R is an Ny-by-Ny diagonal matrix with the scalar on the diagonals. Ny is the number of measured outputs in the model.
- Vector of real positive scalars. R is an Ny-by-Ny diagonal matrix with the elements of the vector on the diagonals of R.
- Ny-by-Ny positive-definite matrix.

## **Time Invariant R**

Specify if the R matrix is time invariant. If you unselect this option, the block includes an additional input port R.

## N

Process and measurement noise cross-covariance matrix. Specify it as a Nw-by-Ny matrix. The matrix  $[Q \ N; \ N^T \ R]$  must be positive definite.

## Time Invariant N

Specify if the N matrix is time invariant. If you unselect this option, the block includes an additional input port N.

## Add input port u

Select this option if your model contains known inputs u(t) or u[k]. The option is selected by default. Unselecting this option removes the input port u from the block and removes the  $\mathbf{B}$ ,  $\mathbf{D}$  and  $\mathbf{Number\ of\ inputs}$  parameters from the block dialog box.

# Add input port Enable to control measurement updates

Select this option if you want to control the measurement updates. The block includes an additional inport Enable. The Enable input port takes a scalar signal. This option is cleared by default.

By default the block does measurement updates at each time step to improve the state and output estimates  $\widehat{x}$  and  $\widehat{y}$  based on measured outputs. The measurement update is skipped for the current sample time when the signal in the Enable port is 0. Concretely, the equation for state estimates become  $\widehat{x}(t) = A(t)\widehat{x}(t) + B(t)u(t)$  for continuous-time Kalman filter and  $\widehat{x}[n+1|n] = A[n]\widehat{x}[n|n-1] + B[n]u[n]$  for discrete-time.

## **External Reset**

Option to reset estimated states and parameter covariance matrix using specified initial values.

Suppose you reset the block at a time step, t. If the block is enabled at t, the software uses the initial parameter values specified either in the block dialog or the input ports P0 and X0 to estimate the states. In other words, at t, the block performs a time update and if it is enabled, a measurement update after the reset. The block outputs these updated estimates.

Specify one of the following:

- None (Default) Estimated states  $\hat{x}$  and state estimation error covariance matrix P values are not reset.
- Rising Triggers a reset when the control signal rises from a negative or zero value to a positive value. If the initial value is negative, rising to zero triggers a reset.
- Falling Triggers a reset when the control signal falls from a positive or a zero
  value to a negative value. If the initial value is positive, falling to zero triggers a reset.
- Either Triggers a reset when the control signal is either rising or falling.
- ullet Level Triggers a reset in either of these cases:
  - The control signal is nonzero at the current time step.
  - The control signal changes from nonzero at the previous time step to zero at the current time step.
- Level hold Triggers reset when the control signal is nonzero at the current time step.

When you choose an option other than None, a Reset input port is added to the block to provide the reset control input signal.

# Output estimated model output y

Add  $\widehat{y}$  output port to the block to output the estimated model outputs. The option is cleared by default.

# Output state estimation error covariance P or Z

Add P output port or Z output port to the block. The Z matrix is provided only when **Time Domain** is **Discrete** Time and the **Use the current measurement y[n] to improve xhat[n]** is selected. Otherwise, the P matrix, as described in the "Description" on page 3-19 section previously, is provided.

The option is cleared by default.

# **Ports**

Port Name	Port Type	Description
	(In/ Out)	
u (Optional)	In	Known inputs, specified as a real scalar or vector.
У	In	Measured outputs, specified as a real scalar or vector.
xhat	Out	Estimated states, returned as a real scalar or vector.
yhat (Optional)	Out	Estimated outputs, returned as a real scalar or vector.
P or Z (Optional)	Out	State estimation error covariance, returned as a matrix.
A (Optional)	In	A matrix, specified as a real matrix.
B (Optional)	In	B matrix, specified as a real matrix.
C (Optional)	In	C matrix, specified as a real matrix.
D (Optional)	In	D matrix, specified as a real matrix.

Port Name	Port Type	Description
	(In/ Out)	
G (Optional)	In	G matrix, specified as a real matrix.
H (Optional)	In	H matrix, specified as a real matrix.
Q (Optional)	In	Q matrix, specified as a real scalar, vector or matrix.
R (Optional)	In	R matrix, specified as a real scalar, vector or matrix.
N (Optional)	In	N matrix, specified as a real matrix.
P0 (Optional)	In	P matrix at initial time, specified as a real scalar, vector, or matrix.
X0 (Optional)	In	Initial state estimates, specified as a real scalar or vector.
Enable (Optional)	In	Control signal to enable measurement updates, specified as a real scalar.
Reset (Optional)	In	Control signal to reset state estimates, specified as a real scalar.

# **Supported Data Types**

- Double-precision floating point
- Single-precision floating point (for discrete-time Kalman filter only)

### Note

- All input ports except **Enable** and **Reset** must have the same data type (single or double).
- Enable and Reset ports support single, double, int8, uint8, int16, uint16, int32, uint32, and boolean data types.

# Limitations

- The plant and noise data must satisfy:
  - (C,A) detectable
  - $\overline{R} > 0$  and  $\overline{Q} \overline{N}\overline{R}^{-1}\overline{N}^T \ge 0$
  - $(A \overline{N}\overline{R}^{-1}C, \overline{Q} \overline{N}\overline{R}^{-1}\overline{N}^T)$  has no uncontrollable mode on the imaginary axis (or unit circle in discrete time) with the notation

$$\overline{Q} = GQG^{T}$$

$$\overline{R} = R + HN + N^{T}H^{T} + HQH^{T}$$

$$\overline{N} = G(OH^{T} + N)$$

• The continuous-time Kalman filter cannot be used in Function-Call Subsystems or Triggered Subsystems.

## References

- [1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.
- [2] Lewis, F., Optimal Estimation, John Wiley & Sons, Inc, 1986.

# **Extended Capabilities**

# C/C++ Code Generation

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

# **PLC Code Generation**

Generate Structured Text code using Simulink  $\! \! \! \! \! \! \! ^{\mathtt{B}}$  PLC Coder  $\! \! \! \! \! \! ^{\mathtt{TM}}.$ 

# **See Also**

### **Functions**

extendedKalmanFilter|kalman|kalmd|particleFilter|
unscentedKalmanFilter

### **Blocks**

Extended Kalman Filter | Particle Filter | Unscented Kalman Filter

## **Topics**

"State Estimation Using Time-Varying Kalman Filter" "Validate Online State Estimation in Simulink"

"Troubleshoot Online State Estimation"

## Introduced in R2014b

# **Extended Kalman Filter**

Estimate states of discrete-time nonlinear system using extended Kalman filter

**Library:** Control System Toolbox / State Estimation System Identification Toolbox / Estimators

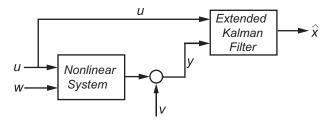
y1 xhat

Extended Raiman Filter

# **Description**

The Extended Kalman Filter block estimates the states of a discrete-time nonlinear system using the first-order discrete-time extended Kalman filter algorithm.

Consider a plant with states x, input u, output y, process noise w, and measurement noise v. Assume that you can represent the plant as a nonlinear system.



Using the state transition and measurement functions of the system and the extended Kalman filter algorithm, the block produces state estimates  $\widehat{x}$  for the current time step. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You create the nonlinear state transition function and measurement functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement functions, each corresponding to a sensor in the system. You can also specify the Jacobians of the state transition and measurement functions. If you do not specify them, the software numerically computes the Jacobians. For more information, see "State Transition and Measurement Functions" on page 3-50.

# **Ports**

## Input

y1, y2, y3, y4, y5 — Measured system outputs

vector

Measured system outputs corresponding to each measurement function that you specify in the block. The number of ports equals the number of measurement functions in your system. You can specify up to five measurement functions. For example, if your system has two sensors, you specify two measurement functions in the block. The first port **y1** is available by default. When you click **Apply**, the software generates port **y2** corresponding to the second measurement function.

Specify the ports as N-dimensional vectors, where N is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and velocity of an object, then there is only one port y1. The port is specified as a 2-dimensional vector with values corresponding to position and velocity.

### **Dependencies**

The first port y1 is available by default. Ports y2 to y5 are generated when you click **Add Measurement**, and click **Apply**.

Data Types: single | double

# StateTransitionFcnInputs — Additional optional input argument to state transition function

scalar | vector | matrix

Additional optional input argument to the state transition function f other than the state x and process noise w. For information about state transition functions see, "State Transition and Measurement Functions" on page 3-50.

Suppose that your system has nonadditive process noise, and the state transition function f has the following form:

x(k+1) = f(x(k), w(k), StateTransitionFcnInputs)

Here k is the time step, and StateTransitionFcnInputs is an additional input argument other than x and w.

If you create f using a MATLAB function (.m file), the software generates the port **StateTransitionFcnInputs** when you click **Apply**. You can specify the inputs to this port as a scalar, vector, or matrix.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Extended Kalman Filter block.

#### **Dependencies**

This port is generated only if both of the following conditions are satisfied:

- You specify f in **Function** using a MATLAB function, and f is on the MATLAB path.
- f requires only one additional input argument apart from x and w.

Data Types: single | double

MeasurementFcn1Inputs, MeasurementFcn2Inputs, MeasurementFcn3Inputs, MeasurementFcn4Inputs, MeasurementFcn5Inputs — Additional optional input argument to each measurement function

```
scalar | vector | matrix
```

Additional optional inputs to the measurement functions other than the state x and measurement noise v. For information about measurement functions see, "State Transition and Measurement Functions" on page 3-50.

**MeasurementFcn1Inputs** corresponds to the first measurement function that you specify, and so on. For example, suppose that your system has three sensors and nonadditive measurement noise, and the three measurement functions h1, h2, and h3 have the following form:

```
y1[k] = h1(x[k],v1[k],MeasurementFcn1Inputs)
y2[k] = h2(x[k],v2[k],MeasurementFcn2Inputs)
y3[k] = h3(x[k],v3[k])
```

Here k is the time step, and MeasurementFcnlInputs and MeasurementFcnlInputs are the additional input arguments to h1 and h2.

If you specify h1, h2, and h3 using MATLAB functions (.m files) in **Function**, the software generates ports **MeasurementFcn1Inputs** and **MeasurementFcn2Inputs** when you click **Apply**. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Extended Kalman Filter block.

#### **Dependencies**

A port corresponding to a measurement function h is generated only if both of the following conditions are satisfied:

- You specify h in Function using a MATLAB function, and h is on the MATLAB path.
- h requires only one additional input argument apart from x and v.

Data Types: single | double

## Q — Time-varying process noise covariance

scalar | vector | matrix

Time-varying process noise covariance, specified as a scalar, vector, or matrix depending on the value of the **Process noise** parameter:

- **Process noise** is Additive Specify the covariance as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. Specify a vector of length *Ns*, if there is no cross-correlation between process noise terms, but all the terms have different variances.
- **Process noise** is **Nonadditive** Specify the covariance as a *W*-by-*W* matrix, where *W* is the number of process noise terms in the state transition function.

#### **Dependencies**

This port is generated if you specify the process noise covariance as **Time-Varying**. The port appears when you click **Apply**.

Data Types: single | double

# R1,R2,R3,R4,R5 — Time-varying measurement noise covariance matrix

Time-varying measurement noise covariances for up to five measurement functions of the system, specified as matrices. The sizes of the matrices depend on the value of the **Measurement noise** parameter for the corresponding measurement function:

- **Measurement noise** is Additive Specify the covariance as an *N*-by-*N* matrix, where *N* is the number of measurements of the system.
- Measurement noise is Nonadditive Specify the covariance as a V-by-V matrix, where V is the number of measurement noise terms in the corresponding measurement function.

#### **Dependencies**

A port is generated if you specify the measurement noise covariance as **Time-Varying** for the corresponding measurement function. The port appears when you click **Apply**.

Data Types: single | double

# Enable1, Enable2, Enable3, Enable4, Enable5 — Enable correction of estimated states when measured data is available

scalar

Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement function. Use a signal value other than 0 at the **Enable1** port to enable the correction of estimated states when measured data is available. Specify the port value as 0 when measured data is not available. Similarly, if measured output data is not available at all time points at the port yi for the i<sup>th</sup> measurement function, specify the corresponding port **Enable**i as a value other than 0.

### **Dependencies**

A port corresponding to a measurement function is generated if you select **Add Enable port** for that measurement function. The port appears when you click **Apply**.

Data Types: single | double | Boolean

## Output

#### xhat — Estimated states

vector

Estimated states, returned as a vector of size Ns, where Ns is the number of states of the system. To access the individual states, use the Selector block.

When the **Use the current measurements to improve state estimates** parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

Data Types: single | double

#### P — State estimation error covariance

matrix

State estimation error covariance, returned as an Ns-by-Ns matrix, where Ns is the number of states of the system. To access the individual covariances, use the Selector block.

#### **Dependencies**

This port is generated if you select **Output state estimation error covariance** in the **System Model** tab, and click **Apply**.

Data Types: single | double

# **Parameters**

# **System Model Tab**

#### **State Transition**

#### Function — State transition function name

myStateTransitionFcn (default) | function name

The state transition function calculates the Ns-element state vector of the system at time step k+1, given the state vector at time step k. Ns is the number of states of the nonlinear system. You create the state transition function and specify the function name in **Function**. For example, if vdpStateFcn.m is the state transition function that you created and saved, specify **Function** as vdpStateFcn.

The inputs to the function you create depend on whether you specify the process noise as additive or nonadditive in **Process noise**.

• **Process noise** is Additive — The state transition function *f* specifies how the states evolve as a function of state values at previous time step:

$$x(k+1) = f(x(k), Us1(k), \dots, Usn(k)),$$

where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

• **Process noise** is **Nonadditive** — The state transition function also specifies how the states evolve as a function of the process noise w:

$$x(k+1) = f(x(k),w(k),Us1(k),...,Usn(k)).$$

For more information, see "State Transition and Measurement Functions" on page 3-50.

You can create f using a Simulink Function block or as a MATLAB function (.m file).

 You can use a MATLAB function only if f has one additional input argument Us1 other than x and w.

$$x(k+1) = f(x(k),w(k),Us1(k))$$

The software generates an additional input port **StateTransitionFcnInputs** to specify this argument.

• If you are using a Simulink Function block, specify x and w using Argument Inport blocks and the additional inputs Us1,...,Usn using Inport blocks in the Simulink Function block. You do not provide Us1,...,Usn to the Extended Kalman Filter block.

#### **Programmatic Use**

**Block Parameter:** StateTransitionFcn

**Type:** character vector, string

**Default:** 'myStateTransitionFcn'

# Jacobian — Jacobian of state transition function

off (default) | on

Jacobian of state transition function f, specified as one of the following:

• off — The software computes the Jacobian numerically. This computation may increase processing time and numerical inaccuracy of the state estimation.

• on — You create a function to compute the Jacobian, and specify the name of the function in **Jacobian**. For example, if vdpStateJacobianFcn.m is the Jacobian function, specify **Jacobian** as vdpStateJacobianFcn. If you create the state transition function f using a Simulink Function block, then create the Jacobian using a Simulink Function block. If you create f using a MATLAB function (.m file), then create the Jacobian using a MATLAB function.

The function calculates the partial derivatives of the state transition function with respect to the states and process noise. The number of inputs to the Jacobian function must equal the number of inputs of the state transition function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the **Process noise** parameter:

• **Process noise** is Additive — The function calculates the partial derivative of the state transition function f with respect to the states  $(\partial f/\partial x)$ . The output is an Ns-by-Ns Jacobian matrix, where Ns is the number of states.

To see an example of a Jacobian function for additive process noise, type edit vdpStateJacobianFcn at the command line.

• **Process noise** is Nonadditive — The function must also return a second output that is the partial derivative of the state transition function f with respect to the process noise terms  $(\partial f/\partial w)$ . The second output is returned as an Ns-by-W matrix, where W is the number of process noise terms in the state transition function.

#### **Programmatic Use**

Block Parameter: HasStateTransitionJacobianFcn

Type: character vector Values: 'off','on' Default: 'off'

Block Parameter: StateTransitionJacobianFcn

**Type:** character vector, string

Default: ' '

#### **Process noise — Process noise characteristics**

Additive (default) | Nonadditive

Process noise characteristics, specified as one of the following values:

Additive — Process noise w is additive, and the state transition function f that you specify in Function has the following form:

```
x(k+1) = f(x(k),Us1(k),...,Usn(k)),
```

where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function.

 Nonadditive — Process noise is nonadditive, and the state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:

```
x(k+1) = f(x(k),w(k),Us1(k),...,Usn(k)).
```

### **Programmatic Use**

Block Parameter: HasAdditiveProcessNoise

**Type:** character vector

Values: 'Additive', 'Nonadditive'

Default: 'Additive'

## **Covariance** — Time-invariant process noise covariance

1 (default) | scalar | vector | matrix

Time-invariant process noise covariance, specified as a scalar, vector, or matrix depending on the value of the **Process noise** parameter:

- **Process noise** is Additive Specify the covariance as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms and all the terms have the same variance. Specify a vector of length *Ns*, if there is no cross-correlation between process noise terms but all the terms have different variances.
- **Process noise** is **Nonadditive** Specify the covariance as a *W*-by-*W* matrix, where *W* is the number of process noise terms.

If the process noise covariance is time-varying, select  $\mathbf{Time\text{-}varying}$ . The block generates input port  $\mathbf{Q}$  to specify the time-varying covariance.

## **Dependencies**

This parameter is enabled if you do not specify the process noise as **Time-Varying**.

#### **Programmatic Use**

**Block Parameter:** ProcessNoise **Type:** character vector, string

Default: '1'

## Time-varying — Time-varying process noise covariance

'off' (default) | 'on'

If you select this parameter, the block includes an additional input port  $\mathbf{Q}$  to specify the time-varying process noise covariance.

#### **Programmatic Use**

**Block Parameter:** HasTimeVaryingProcessNoise

Type: character vector Values: 'off', 'on' Default: 'off'

#### Initialization

### Initial state — Initial state estimate

0 (default) | vector

Initial state estimate value, specified as an Ns-element vector, where Ns is the number of states in the system. Specify the initial state values based on your knowledge of the system.

#### **Programmatic Use**

**Block Parameter:** InitialState **Type:** character vector, string

Default: '0'

#### Initial covariance — State estimation error covariance

1 (default) | scalar | vector | matrix

State estimation error covariance, specified as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. If you specify a scalar or vector, the software creates an *Ns*-by-*Ns* diagonal matrix with the scalar or vector elements on the diagonal.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in **Initial state**.

#### **Programmatic Use**

Block Parameter: InitialStateCovariance

**Type:** character vector, string

Default: '1'

#### Measurement

#### Function — Measurement function name

myMeasurementFcn (default) | function name

The measurement function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. You create the measurement function and specify the function name in **Function**. For example, if vdpMeasurementFcn.m is the measurement function that you created and saved, specify **Function** as vdpMeasurementFcn.

The inputs to the function you create depend on whether you specify the measurement noise as additive or nonadditive in **Measurement noise**.

 Measurement noise is Additive — The measurement function h specifies how the measurements evolve as a function of state Values:

```
y(k) = h(x(k), Um1(k), ..., Umn(k)),
```

where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using a sensor for tracking an object, an additional input could be the sensor position.

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line.

• **Measurement noise** is **Nonadditive**— The measurement function also specifies how the output measurement evolves as a function of the measurement noise v:

```
y(k) = h(x(k),v(k),Um1(k),...,Umn(k)).
```

To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

For more information, see "State Transition and Measurement Functions" on page 3-50.

You can create h using a Simulink Function block or as a MATLAB function (.m file).

 You can use a MATLAB function only if h has one additional input argument Um1 other than x and v.

```
y[k] = h(x[k],v[k],Um1(k))
```

The software generates an additional input port **MeasurementFcniInputs** to specify this argument for the *i*th measurement function.

• If you are using a Simulink Function block, specify x and v using Argument Inport blocks and the additional inputs Um1,...,Umn using Inport blocks in the Simulink

Function block. You do not provide Um1,...,Umn to the Extended Kalman Filter block.

If you have multiple sensors in your system, you can specify multiple measurement functions. You can specify up to five measurement functions using the **Add Measurement** button. To remove measurement functions, use **Remove Measurement**.

#### **Programmatic Use**

Block Parameter: MeasurementFcn1, MeasurementFcn2, MeasurementFcn3,

MeasurementFcn4, MeasurementFcn5

Type: character vector, string
Default: 'myMeasurementFcn'

### Jacobian — Jacobian of measurement function

off (default) | on

Jacobian of measurement function h, specified as one of the following:

- off The software computes the Jacobian numerically. This computation may increase processing time and numerical inaccuracy of the state estimation.
- on You create a function to compute the Jacobian of the measurement function h, and specify the name of the function in Jacobian. For example, if vdpMeasurementJacobianFcn.m is the Jacobian function, specify MeasurementJacobianFcn as vdpMeasurementJacobianFcn. If you create h using a Simulink Function block, then create the Jacobian using a Simulink Function block. If you create h using a MATLAB function (.m file), then create the Jacobian using a MATLAB function.

The function calculates the partial derivatives of the measurement function h with respect to the states and measurement noise. The number of inputs to the Jacobian function must equal the number of inputs to the measurement function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the **Measurement noise** parameter:

• Measurement noise is Additive — The function calculates the partial derivatives of the measurement function with respect to the states  $(\partial h/\partial x)$ . The output is as an N-by-Ns Jacobian matrix, where N is the number of measurements of the system and Ns is the number of states.

To see an example of a Jacobian function for additive measurement noise, type edit vdpMeasurementJacobianFcn at the command line.

• **Measurement noise** is Nonadditive — The function also returns a second output that is the partial derivative of the measurement function with respect to the measurement noise terms  $(\partial h/\partial v)$ . The second output is returned as an *N*-by-*V* Jacobian matrix, where *V* is the number of measurement noise terms.

### **Programmatic Use**

**Block Parameter:** HasMeasurementJacobianFcn1, HasMeasurementJacobianFcn2, HasMeasurementJacobianFcn3,

HasMeasurementJacobianFcn4,HasMeasurementJacobianFcn5

Type: character vector Values: 'off','on' Default: 'off'

Block Parameter: MeasurementJacobianFcn1, MeasurementJacobianFcn2,

MeasurementJacobianFcn3, MeasurementJacobianFcn4,

MeasurementJacobianFcn5

**Type:** character vector

Default: ' '

## Measurement noise — Measurement noise characteristics

Additive (default) | Nonadditive

Measurement noise characteristics, specified as one of the following values:

• Additive — Measurement noise v is additive, and the measurement function h that you specify in **Function** has the following form:

$$y(k) = h(x(k), Um1(k), ..., Umn(k)),$$

where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function.

 Nonadditive — Measurement noise is nonadditive, and the measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

$$y(k) = h(x(k),v(k),Um1(k),...,Umn(k)).$$

### **Programmatic Use**

**Block Parameter:** HasAdditiveMeasurementNoise1, HasAdditiveMeasurementNoise2, HasAdditiveMeasurementNoise3, HasAdditiveMeasurementNoise4, HasAdditiveMeasurementNoise5 **Type:** character vector

Values: 'Additive', 'Nonadditive'

Default: 'Additive'

### **Covariance** — Time-invariant measurement noise covariance

1 (default) | matrix

Time-invariant measurement noise covariance, specified as a matrix. The size of the matrix depends on the value of the **Measurement noise** parameter:

- **Measurement noise** is Additive Specify the covariance as an *N*-by-*N* matrix, where *N* is the number of measurements of the system.
- **Measurement noise** is **Nonadditive** Specify the covariance as a *V*-by-*V* matrix, where *V* is the number of measurement noise terms.

If the measurement noise covariance is time-varying, select **Time-varying**. The block generates input port  $\mathbf{R}\mathbf{i}$  to specify the time-varying covariance for the  $i^{th}$  measurement function.

### **Dependencies**

This parameter is enabled if you do not specify the process noise as **Time-Varying**.

#### **Programmatic Use**

 $\textbf{Block Parameter:} \ \texttt{MeasurementNoise1}, \ \texttt{MeasurementNoise2}, \ \texttt{MeasurementNoise3}, \\$ 

MeasurementNoise4, MeasurementNoise5

Type: character vector, string

Default: '1'

# Time-varying — Time-varying measurement noise covariance

off (default) | on

If you select this parameter for the measurement noise covariance of the first measurement function, the block includes an additional input port  $\mathbf{R1}$ . You specify the time-varying measurement noise covariance in  $\mathbf{R1}$ . Similarly, if you select  $\mathbf{Time-varying}$  for the  $i^{th}$  measurement function, the block includes an additional input port  $\mathbf{Ri}$  to specify the time-varying measurement noise covariance for that function.

### **Programmatic Use**

**Block Parameter:** HasTimeVaryingMeasurementNoise1, HasTimeVaryingMeasurementNoise2, HasTimeVaryingMeasurementNoise3, HasTimeVaryingMeasurementNoise4, HasTimeVaryingMeasurementNoise5 Type: character vector Values: 'off', 'on' Default: 'off'

# Add Enable Port — Enable correction of estimated states only when measured data is available

off (default) | on

Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement function. Select **Add Enable port** to generate an input port **Enable1**. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the i<sup>th</sup> measurement function, select the corresponding **Add Enable port**.

## **Programmatic Use**

**Block Parameter:** HasMeasurementEnablePort1, HasMeasurementEnablePort2, HasMeasurementEnablePort3, HasMeasurementEnablePort4,

HasMeasurementEnablePort5

Type: character vector Values: 'off', 'on' Default: 'off'

### **Settings**

# Use the current measurements to improve state estimates — Choose between corrected or predicted state estimate

on (default) | off

When this parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

#### **Programmatic Use**

Block Parameter: UseCurrentEstimator

Type: character vector Values: 'off', 'on'

Default: 'on'

# Output state estimation error covariance — Output state estimation error covariance

off (default) | on

If you select this parameter, a state estimation error covariance output port  ${\bf P}$  is generated in the block.

#### **Programmatic Use**

**Block Parameter:** OutputStateCovariance

Type: character vector Values: 'off','on' Default: 'off'

#### Data type — Data type for block parameters

double (default) | single

Use this parameter to specify the data type for all block parameters.

#### **Programmatic Use**

**Block Parameter:** DataType

**Type:** character vector

Values: 'single', 'double'

Default: 'double'

## Sample time — Block sample time

1 (default) | positive scalar

Block sample time, specified as a positive scalar. If the sample times of your state transition and measurement functions are different, select **Enable multirate operation** in the **Multirate** tab, and specify the sample times in the **Multirate** tab instead.

#### **Dependencies**

This parameter is available if in the **Multirate** tab, the **Enable multirate operation** parameter is off.

## **Programmatic Use**

**Block Parameter:** SampleTime **Type:** character vector, string

Default: '1'

## **Multirate Tab**

# Enable multirate operation — Enable specification of different sample times for state transition and measurement functions

off (default) | on

Select this parameter if the sample times of the state transition and measurement functions are different. You specify the sample times in the **Multirate** tab, in **Sample time**.

### **Programmatic Use**

Block Parameter: EnableMultirate

Type: character vector Values: 'off', 'on' Default: 'off'

# **Sample time — State transition and measurement function sample times** positive scalar

If the sample times for state transition and measurement functions are different, specify **Sample time**. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function Additional input to state transition function StateTransitionFcnInputs and time-varying process noise covariance Q.
   The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement functions.
- Ports corresponding to i<sup>th</sup> measurement function Measured output yi, additional input to measurement function MeasurementFcniInputs, enable signal at port Enablei, and time-varying measurement noise covariance Ri. The sample times of these ports for the same measurement function must always be the same, but can differ from the sample time for the state transition function and other measurement functions.

### **Dependencies**

This parameter is available if in the **Multirate** tab, the **Enable multirate operation** parameter is on.

### **Programmatic Use**

**Block Parameter:** StateTransitionFcnSampleTime, MeasurementFcn1SampleTime1, MeasurementFcn1SampleTime2, MeasurementFcn1SampleTime3, MeasurementFcn1SampleTime4,

MeasurementFcn1SampleTime5 **Type:** character vector, string

Default: '1'

## **More About**

## **State Transition and Measurement Functions**

The algorithm computes the state estimates  $\widehat{x}$  of the nonlinear system using state transition and measurement functions specified by you. You can specify up to five measurement functions, each corresponding to a sensor in the system. The software lets you specify the noise in these functions as additive or nonadditive.

• **Additive Noise Terms** — The state transition and measurements equations have the following form:

$$x[k+1] = f(x[k], u_s[k]) + w[k]$$
  
 $y[k] = h(x[k], u_m[k]) + v[k]$ 

Here f is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function h relates x to the measurements y at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional optional input arguments that are denoted by  $u_s$  and  $u_m$  in the equations. For example, the additional arguments could be time step k or the inputs u to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, x(k+1) is linearly related to the process noise w(k), and y(k) is linearly related to the measurement noise v(k). For additive noise terms, you do not need to specify the noise terms in the state transition and measurement functions. The software adds the terms to the output of the functions.

• **Nonadditive Noise Terms** — The software also supports more complex state transition and measurement functions where the state x[k] and measurement y[k] are

nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

```
x[k+1] = f(x[k], w[k], u_s[k])

y[k] = h(x[k], v[k], u_m[k])
```

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

The state transition, measurement, and Jacobian functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

# See Also

#### **Blocks**

Kalman Filter | Particle Filter | Unscented Kalman Filter

#### **Functions**

extendedKalmanFilter|kalman|kalmd|particleFilter|
unscentedKalmanFilter

# **Topics**

"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

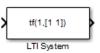
"Validate Online State Estimation in Simulink"

#### Introduced in R2017a

<sup>&</sup>quot;Troubleshoot Online State Estimation"

# LTI System

Use linear time invariant system model object in Simulink **Library:** Control System Toolbox



# **Description**

The LTI System block imports linear system model objects into the Simulink environment. You specify the LTI model to import in the **LTI system variable** parameter. You can import any type of proper linear time-invariant dynamic system model. If the imported system is a state-space (ss) model, you can specify initial state values in the **Initial states** parameter.

## **Ports**

## Input

Port\_1(In1) — Input signal

scalar | vector

For a single-input LTI system, the input signal is a scalar. For multiple-input systems, combine the system inputs into a vector signal, using blocks such as:

- Mux
- Vector Concatenate
- Bus Creator

# Output

Port\_1(0ut1) — Output signal

scalar | vector

For a single-output LTI system, the output signal is a scalar. For multiple-output systems, the output signal is a vector. To split system outputs into scalar signals, use blocks such as:

- Demux
- · Bus Selector

# **Parameters**

## LTI system variable — Linear system

dynamic system model

Specify the linear system for the block as a MATLAB expression or a variable in the MATLAB workspace, the model workspace, or a data dictionary. The model can be SISO or MIMO.

Most linear time-invariant dynamic system models are supported, except:

- Frequency-response data models, such as frd and genfrd models.
- Nonlinear identified models, such as idnlarx.
- Models with unmodeled dynamics, such as udyn.

The specified model must be proper (see isproper).

The model can be either continuous time or discrete time. When the LTI system block is in a Simulink model with synchronous state control (see the State Control block), you must specify a discrete-time model.

Simulink converts the model to its state-space equivalent prior to initializing the simulation.

# Initial states (state-space only) — Initial state values for state-space model

[] (default) | vector | scalar

If the linear system is in state-space form, specify the initial state values as a vector with as many entries as the system has states. If you specify a scalar value, the block applies that value to each state in the system. The default value, [], initializes all states to zero.

The concept of initial state is not well-defined for linear systems that are not in state-space form, such as transfer functions or zero-pole-gain models. For such models, the

initial state depends on the choice of state coordinates used by the realization algorithm. As a result, the block ignores this parameter for such models.

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

# See Also

LPV System

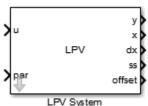
# **Topics**

"Import LTI Model Objects into Simulink"

Introduced before R2006a

# **LPV System**

Simulate Linear Parameter-Varying (LPV) systems



# **Description**

Represent and simulate Linear Parameter-Varying (LPV) systems in Simulink. The block also supports code generation.

A linear parameter-varying (LPV) system is a linear state-space model whose dynamics vary as a function of certain time-varying parameters called scheduling parameters. In MATLAB, an LPV model is represented in a state-space form using coefficients that are parameter dependent.

Mathematically, an LPV system is represented as:

$$dx(t) = A(p)x(t) + B(p)u(t)$$
  

$$y(t) = C(p)x(t) + D(p)u(t)$$
  

$$x(0) = x_0$$
(3-1)

#### where

- u(t) are the inputs
- y(t) the outputs
- x(t) are the model states with initial value x0
- dx(t) is the state derivative vector  $\dot{x}$  for continuous-time systems and the state update vector  $x(t + \Delta T)$  for discrete-time systems.  $\Delta T$  is the sample time.
- A(p), B(p), C(p) and D(p) are the state-space matrices parameterized by the scheduling parameter vector p.

 The parameters p = p(t) are measurable functions of the inputs and the states of the model. They can be a scalar quantity or a vector of several parameters. The set of scheduling parameters define the *scheduling space* over which the LPV model is defined.

The block implements a grid-based representation of the LPV system. You pick a grid of values for the scheduling parameters. At each value  $p = p^*$ , you specify the corresponding linear system as a state-space (ss or idss) model object. You use the generated array of state-space models to configure the LPV System block.

The block accepts an array of state-space models with operating point information. The information on the scheduling variables is extracted from the SamplingGrid property of the LTI array. The scheduling variables define the grid of the LPV models. They are scalar-valued quantities that can be functions of time, inputs and states, or constants. They are used to pick the local dynamics in the operating space. The software interpolates the values of these variables. The block uses this array with data interpolation and extrapolation techniques for simulation.

The LPV system representation can be extended to allow offsets in dx, x, u and y variables. This form is known as affine form of the LPV model. Mathematically, the following represents an LPV system:

$$dx(t) = A(p)x(t) + B(p)u(t) + \left(\overline{dx}(p) - A(p)\overline{x}(p) - B(p)\overline{u}(p)\right)$$

$$y(t) = C(p)x(t) + D(p)u(t) + (\overline{y}(p) - C(p)\overline{x}(p) - D(p)\overline{u}(p))$$

$$x(0) = x_0$$
(3-2)

 $\overline{dx}(p)$ ,  $\overline{x}(p)$ ,  $\overline{y}(p)$  are the offsets in the values of dx(t), x(t), u(t) and y(t) at a given parameter value p = p(t).

To obtain such representations of the linear system array, linearize a Simulink model over a batch of operating points (see "Batch Linearization" (Simulink Control Design).) The offsets correspond to the operating points at which you linearized the model.

You can obtain the offsets by returning additional linearization information when calling functions such as linearize or getIOTransfer. You can then extract the offsets using getOffsetsForLPV. For an example, see "LPV Approximation of a Boost Converter Model" (Simulink Control Design).

The following limitations apply to the LPV System block:

- Internal delays cannot be extrapolated to be less than their minimum value in the state-space model array.
- When using an irregular grid of linear models to define the LPV system, only the nearest neighbor interpolation scheme is used. This may reduce the accuracy of simulation results. It is recommended to work with regular grids. To learn more about regular and irregular grids, see "Regular vs. Irregular Grids".

# **Data Type Support**

Single and double data. You must convert any other data type for input signals or model properties to these data types.

## **Parameters**

The LPV System Block Parameter dialog box contains five tabs for specifying the system data, scheduling algorithm and output ports. The following table summarizes the block parameters.

Task	Parameters
Specify an array of state-space models and	In <b>LPV Model</b> tab:
initial states	• State-space array on page 3-59
	• Initial state on page 3-59
Specify operating point offsets	In <b>LPV Model</b> tab:
	• Input offset on page 3-59
	Output offset on page 3-59
	• State offset on page 3-60
Specify offsets in state derivative or update	In the LPV Model tab:
variable	State derivative/update offset on page 3-60

Task	Parameters
Specify which model matrices are fixed and their nominal values to override entries in model data.  In some situations, you may want to replace a parameter-dependent matrix such as A(p) with a fixed value A* for simulation. For example, A* may represent an average value over the scheduling range.	<ul> <li>In the Fixed Entries tab:</li> <li>Nominal Model on page 3-61</li> <li>Fixed Coefficient Indices on page 3-61</li> </ul>
Specify options for interpolation and extrapolation	<ul> <li>In the Scheduling tab:</li> <li>Interpolation method on page 3-62</li> <li>Extrapolation method on page 3-63</li> <li>Index search method on page 3-63</li> <li>Begin index search using previous index result on page 3-63</li> </ul>
Specify additional outputs for the block	<ul> <li>In the Outputs tab:</li> <li>Output states on page 3-64</li> <li>Output state derivatives (continuous-time) or updates (discrete-time) on page 3-64</li> <li>Output interpolated state-space data on page 3-64</li> <li>Output interpolated offsets on page 3-64</li> </ul>
Specify code generation settings	<ul> <li>In the Code Generation tab:</li> <li>Block data type (discrete-time case only) on page 3-64</li> <li>Initial buffer size for delays on page 3-65</li> <li>Use fixed buffer size on page 3-65</li> </ul>

# **State-space array**

An array of state-space (ss or idss) models. All the models in the array must use the same definition of states. Use the SamplingGrid property of the state-space object to specify scheduling parameters for the model. See the ss or idss model reference page for more information on the SamplingGrid property.

When the block is in a model with synchronous state control (see the State Control block), you must specify an array of discrete-time models.

#### **Initial state**

Initial conditions to use with the local model to start the simulation, specified one of the following:

- 0 (**Default**)
- Double vector of length equal to the number of model states

# Input offset

Offsets in input u(t), specified as one of the following:

- 0 (**Default**) Use when there are no input offsets ( $\overline{u}(p) = 0 \quad \forall \quad p$ ).
- Double vector of length equal to the number of inputs Use when input offset is the same across the scheduling space.
- Double array of size [nu 1 sysArraySize] Use when offsets are present and they vary across the scheduling space. Here, nu = number of inputs, sysArraySize = array size of state-space array. Use size to determine the array size.

You can obtain offsets during linearization and convert them to the format supported by the LPV System block. For more information, see "Approximating Nonlinear Behavior Using an Array of LTI Systems" (Simulink Control Design) and getOffsetsForLPV.

# **Output offset**

Offsets in output y(t), specified as one of the following:

• 0 (**Default**) — Use when there are no output offsets  $\overline{y}(p) = 0 \quad \forall \quad p$ .

- Double vector of length equal to the number of outputs. Use when output offsets are the same across the scheduling space.
- Double array of size [ny 1 sysArraySize]. Use when offsets are present and they vary across the scheduling space. Here, ny = number of outputs, sysArraySize = array size of state-space array. Use size to determine the array size.

You can obtain offsets during linearization and convert them to the format supported by the LPV System block. For more information, see "Approximating Nonlinear Behavior Using an Array of LTI Systems" (Simulink Control Design) and getOffsetsForLPV.

#### State offset

Offsets in states x(t), specified as one of the following:

- 0 (**Default**) Use when there are no state offsets  $\bar{x}(p) = 0 \quad \forall p$ .
- Double vector of length equal to the number of states. Use when the state offsets are the same across the scheduling space.
- Double array of size [nx 1 sysArraySize], where nx = number of states, sysArraySize = array size of state-space array. Use when offsets are present and they vary across the scheduling space. Here, nx = number of states, sysArraySize = array size of state-space array. Use size to determine the array size.

You can obtain offsets during linearization and convert them to the format supported by the LPV System block. For more information, see "Approximating Nonlinear Behavior Using an Array of LTI Systems" (Simulink Control Design) and getOffsetsForLPV.

# State derivative/update offset

Offsets in state derivative or update variable dx(t), specified as one of the following:

- If you obtained the linear system array by linearization under equilibrium conditions, select the **Assume equilibrium conditions** option. This option corresponds to an offset of  $\overline{dx}(p) = 0$  for a continuous-time system and  $\overline{dx}(p) = \overline{x}(p)$  for a discrete-time system. This option is selected by default.
- If the linear system contains at least one system that you obtained under nonequilibrium conditions, clear the **Assume equilibrium conditions** option. Specify one of the following in the **Offset value** field:

- If the dx offset values are the same across the scheduling space, specify as a double vector of length equal to the number of states.
- If the dx offsets are present and they vary across the scheduling space, specify as a
  double array of size [nx 1 sysArraySize], where nx = number of states, and
  sysArraySize = array size of state-space array.

You can obtain offsets during linearization and convert them to the format supported by the LPV System block. For more information, see "Approximating Nonlinear Behavior Using an Array of LTI Systems" (Simulink Control Design) and getOffsetsForLPV.

### **Nominal Model**

State-space model that provides the values of the fixed coefficients, specified as one of the following:

• Use the first model in state-space array (**Default:**) — The first model in the state-space array is used to represent the LPV model. In the following example, the state-space array is specified by object sys and the fixed coefficients are taken from model sys(:,:,1).

```
% Specify a 4-by-5 array of state-space models.
sys = rss(4,2,3,4,5);
a = 1:4;
b = 10:10:50;
[av,bv] = ndgrid(a,b);
% Use "alpha" and "beta" variables as scheduling parameters.
sys.SamplingGrid = struct('alpha',av,'beta',bv);
```

Fixed coefficients are taken from the model sysFixed = sys(:,:,1), which corresponds to [alpha=1, beta=10]. If the (2,1) entry of A matrix is forced to be fixed, its value used during the simulation is sysFixed.A(2,1).

• Custom value — Specify a different state-space model for fixed entries. Specify a variable for the fixed model in the **State space model** field. The fixed model must use the same state basis as the state-space array in the LPV model.

# **Fixed Coefficient Indices**

Specify which coefficients of the state-space matrices and delay vectors are fixed.

Specify one of the following:

 Scalar Boolean (true or false), if all entries of a matrix are to be treated the same way.

The default value is false for the state-space matrices and delay vectors, which means that they are treated as free.

Logical matrix of a size compatible with the size of the corresponding matrix:

State-space matrix	Size of fixed entry matrix
A matrix	nx-by-nx
B matrix	nx-by-nu
C matrix	ny-by-nx
D matrix	ny-by-nu
Input delay	nu-by-1
Output delay	ny-by-1
Internal delay	ni-by-1

where, nu = number of inputs, ny = number of outputs, nx = number of states, ni = length of internal delay vector.

Numerical indices to specify the location of fixed entries. See sub2ind reference page
for more information on how to generate numerical indices corresponding to a given
subscript (i,j) for an element of a matrix.

# Interpolation method

Interpolation method. Defines how the state-space data must be computed for scheduling parameter values that are located away from their grid locations.

Specify one of the following options:

- Flat Choose the state-space data at the grid point closest, but not larger than, the current point. The *current point* is the value of the scheduling parameters at current time.
- Nearest Choose the state-space data at the closest grid point in the scheduling space.
- Linear Obtain state-space data by linear interpolation of the nearest 2d neighbors in the scheduling space, where d = number of scheduling parameters.

The default interpolation scheme is Linear for regular grids of scheduling parameter values. For irregular grids, the Nearest interpolation scheme is always used regardless of the choice made. to learn more about regular and irregular grids, see "Regular vs. Irregular Grids".

The Linear method provides the highest accuracy but takes longer to compute. The Flat and Nearest methods are good for models that have mode-switching dynamics.

# **Extrapolation method**

Extrapolation method. Defines how to compute the state-space data for scheduling parameter values that fall outside the range over which the state-space array has been provided (as specified in the SamplingGrid property).

Specify one of the following options:

- Clip (**Default:**) Disables extrapolation and returns the data corresponding to the last available scheduling grid point that is closest to the current point.
- Linear Fits a line between the first or last pair of values for each scheduling
  parameter, depending upon whether the current value is less than the first or greater
  than the last grid point value, respectively. This method returns the point on that line
  corresponding to the current value. Linear extrapolation requires that the
  interpolation scheme be linear too.

### Index search method

The location of the current scheduling parameter values in the scheduling space is determined by a prelookup algorithm. Select Linear search or Binary search. Each search method has speed advantages in different situations. For more information on this parameter, see the Prelookup block reference page.

# Begin index search using previous index result

Select this check box when you want the block to start its search using the index found at the previous time step. For more information on this parameter, see the Prelookup block reference page.

# **Output states**

Add x port to the block to output state values. This option is selected by default.

# Output state derivatives (continuous-time) or updates (discrete-time)

Add dx port to the block to output state derivative values or update the values. This option is selected by default.

# **Output interpolated state-space data**

Add ss port to the block to output state-space data as a structure. This option is selected by default.

The fields of the generated structure are:

- State-space matrices A, B, C, D.
- Delays InputDelay, OutputDelay, and InternalDelay. The InternalDelay field is available only when the model has internal delay.

# **Output interpolated offsets**

Add offset port to the block to output LPV model offsets  $(\overline{u}(p), \overline{y}(p), \overline{x}(p), \overline{dx}(p))$ .

The fields of the structure are:

- InputOffset, OutputOffset, StateOffset, and StateDerivativeOffset in continuous-time.
- InputOffset, OutputOffset, StateOffset, and StateUpdateOffset in discretetime.

# Block data type (discrete-time case only)

Supported data type. Use this option only for discrete-time state-space models. Specify double or single.

# Initial buffer size for delays

Initial memory allocation for the number of input points to store for models that contain delays. If the number of input points exceeds the initial buffer size, the block allocates additional memory. The default size is 1024.

When you run the model in Accelerator mode or build the model, make sure the initial buffer size is large enough to handle maximum anticipated delay in the model.

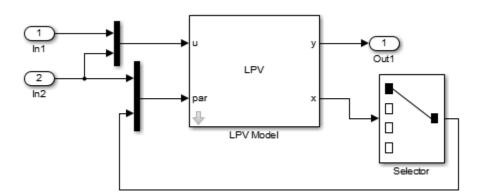
#### Use fixed buffer size

Specify whether to use a fixed buffer size to save delayed input and output data from previous time steps. Use this option for continuous-time LPV systems that contain input or output delays. If the buffer is full, new data replaces data already in the buffer. The software uses linear extrapolation to estimate output values that are not in the buffer.

# **Examples**

# Configure the Scheduling Parameter Input Port

Consider a 2-input, 3-output, 4-state LPV model. Use input u(2) and state x(1) as scheduling parameters. Configure the Simulink model as shown in the following figure.



# Simulate a Linear Parameter-Varying System

Consider a linear mass-spring-damper system whose mass changes as a function of an external load command. The governing equation is:

$$m(u)\ddot{y} + c\dot{y} + k(y)y = F(t)$$

where m(u) is the mass dependent upon the external command u, c is the damping ratio, k is the stiffness of the spring and F(t) is the forcing input. y(t) is position of the mass at a given time t. For a fixed value of u, the system is linear and expressed as:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$\dot{x} = Ax + Bu, \quad y = Cx$$

where  $x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$  is the state vector and **m** is the value of the mass for a given value of **u**.

In this example, you want to study the model behavior over a range of input values from 1 to 10 Volts. For each value of u, measure the mass and compute the linear representation of the system. Suppose, mass is related to the input by the relationship:

 $m(u) = 10u + 0.1u^2$ . For values of u ranging from 1:10 results in the following array of linear systems.

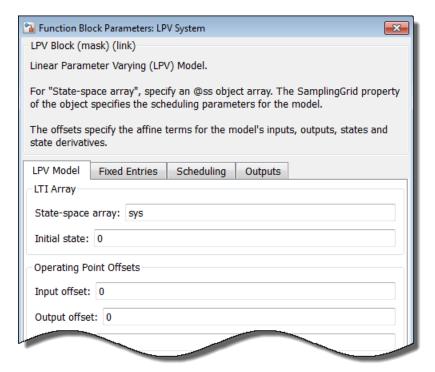
```
% Specify damping coefficient.
c = 5;
% Specify stiffness.
k = 300;
% Specify load command.
u = 1:10;
% Specify mass.
m = 10*u + 0.1*u.^2;
% Compute linear system at a given mass value.
for i = 1:length(u)
    A = [0 1; -k/m(i), -c/m(i)];
    B = [0; 1/m(i)];
    C = [1 0];
    sys(:,:,i) = ss(A,B,C,0);
end
```

The variable u is the scheduling input. Add this information to the model.

```
sys.SamplingGrid = struct('LoadCommand',u);
```

Configure the LPV System block:

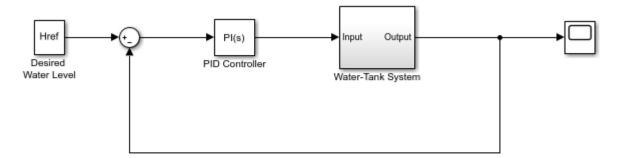
- Type sys in the State-space array field.
- Connect the input port par to a one-dimensional source signal that generates the
  values of the load command. If the source provides values between 1 and 10,
  interpolation is used to compute the linear model at a given time instance. Otherwise,
  extrapolation is used.



### **Extract LPV Offsets from Linearization Results**

Open the Simulink model.

```
model = 'watertank';
open_system(model)
```



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Specify linearization I/Os.

```
io(1) = linio('watertank/Desired Water Level',1,'input');
io(2) = linio('watertank/Water-Tank System',1,'output');
```

Vary plant parameters A and b, and create a 3-by-4 parameter grid.

```
 [A\_grid,b\_grid] = ndgrid(linspace(0.9*A,1.1*A,3),linspace(0.9*b,1.1*b,4)); \\ params(1).Name = 'A'; \\ params(1).Value = A\_grid; \\ params(2).Name = 'b'; \\ params(2).Value = b\_grid;
```

Create a linearization option set, setting the  ${\tt StoreOffsets}$  option to  ${\tt true}.$ 

```
opt = linearizeOptions('StoreOffsets',true);
```

Linearize the model using the specified parameter grid, and return the linearization offsets in the info structure.

```
[sys,op,info] = linearize('watertank',io,params,opt);
```

Extract the linearization offsets.

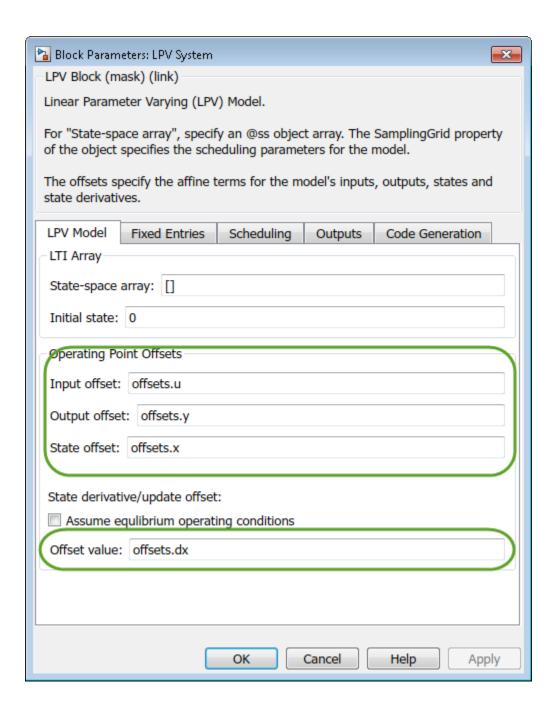
```
offsets = getOffsetsForLPV(info)
```

```
struct with fields:
```

offsets =

```
x: [2x1x3x4 double]
y: [1x1x3x4 double]
u: [1x1x3x4 double]
dx: [2x1x3x4 double]
```

To configure an LPV System block, use the fields from offsets directly.



# <sup>3-70</sup> **Ports**

Port Name	Port	Description
	Type	

Port Name	Port Type (In/	Description
	Out)	
par	In	Provides the signals for variables defining the scheduling space ("sampling grid" variables). The scheduling variables can be functions of time, inputs and states, or constants. The required dependence can be achieved by preparing a scheduling signal using clock input (for time), input signal (u), and the outputs signals (x, dx/dt, y) of the LPV block, as required.
у	Out	Model output
X	Out	Values of the model states
xdot	Out	Values of the state derivatives. The state derivatives are sometimes used to define the scheduling parameters.
ss	Out	Local state-space model at the major simulation time steps
offset	Out	LPV model offsets

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder  $^{\scriptscriptstyle{\text{TM}}}.$ 

# See Also

 ${\tt get0ffsetsForLPV}$ 

# **Topics**

<sup>&</sup>quot;Linear Parameter-Varying Models"

<sup>&</sup>quot;Using LTI Arrays for Simulating Multi-Mode Dynamics"

<sup>&</sup>quot;Approximating Nonlinear Behavior Using an Array of LTI Systems" (Simulink Control Design)

<sup>&</sup>quot;LPV Approximation of a Boost Converter Model" (Simulink Control Design)

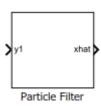
### Introduced in R2014b

# **Particle Filter**

Estimate states of discrete-time nonlinear system using particle filter

Library: Control System Toolbox / State Estimation

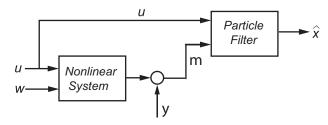
System Identification Toolbox / Estimators



# **Description**

The Particle Filter block estimates the states of a discrete-time nonlinear system using the discrete-time particle filter algorithm.

Consider a plant with states x, input u, output m, process noise w, and measurement y. Assume that you can represent the plant as a nonlinear system.



The algorithm computes the state estimates  $\hat{x}$  of the nonlinear system using the state transition and measurement likelihood functions you specify.

You create the nonlinear state transition function and measurement likelihood functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement likelihood functions, each corresponding to a sensor in the system.

### **Ports**

# Input

# y1,y2,y3,y4,y5 — Measured system outputs vector

Measured system outputs corresponding to each measurement likelihood function that you specify in the block. The number of ports equals the number of measurement likelihood functions in your system. You can specify up to five measurement likelihood functions. For example, if your system has two sensors, you specify two measurement likelihood functions in the block. The first port y1 is available by default. Click Add Measurement, to generate port y2 corresponding to the second measurement likelihood

Specify the ports as N-dimensional vectors, where N is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and velocity of an object, then there is only one port y1. The port is specified as a two-dimensional vector with values corresponding to position and velocity.

#### **Dependencies**

function.

The first port y1 is available by default. Ports y2 to y5 are generated when you click **Add Measurement**.

# StateTransitionFcnInputs — Optional input argument to state transition function

scalar | vector | matrix

Optional input argument to the state transition function f other than the state x.

If you create f using a MATLAB function (.m file), the software generates the port **StateTransitionFcnInputs** when you enter the name of your function, and click **Apply**.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Particle Filter block.

#### **Dependencies**

This port is generated only if both of the following conditions are satisfied:

- You specify f in Function using a MATLAB function, and f is on the MATLAB path.
- f requires only one additional input argument apart from particles.

# MeasurementLikelihoodFcn1Inputs,...,MeasurementLikelihoodFcn5Inputs — Optional input argument to each measurement likelihood function scalar | vector | matrix

Optional inputs to the measurement likelihood functions other than the state x and measurement y.

**MeasurementLikelihoodFcn1Inputs** corresponds to the first measurement likelihood function that you specify, and so on.

If you specify two measurement inputs using MATLAB functions (.m files) in **Function**, the software generates ports **MeasurementLikelihoodFcn1Inputs** and **MeasurementLikelihoodFcn2Inputs** when you click **Apply**. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement likelihood functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Particle Filter block.

#### **Dependencies**

A port corresponding to a measurement likelihood function h is generated only if both of the following conditions are satisfied:

- You specify measurement input h in Function using a MATLAB function, and h is on the MATLAB path.
- h requires only one additional input argument apart from particles and measurement.

# Enable1, Enable2, Enable3, Enable4, Enable5 — Enable correction of estimated states when measured data is available

scalar

Enable correction of estimated states when measured data is available.

For example, consider that measured output data is not available at all time points at the port y1 that corresponds to the first measurement likelihood function. Then, use a signal value other than 0 at the **Enable1** port to enable the correction of estimated states when

measured data is available. Specify the port value as  $\mathbf{0}$  when measured data is not available. Similarly, if measured output data is not available at all time points at the port  $\mathbf{y}\mathbf{i}$  for the  $i^{th}$  measurement likelihood function, specify the corresponding port **Enable** $\mathbf{i}$  as a value other than  $\mathbf{0}$ .

#### **Dependencies**

If you select **Add Enable port** for a measurement likelihood function, a port corresponding to that measurement likelihood function is generated. The port appears when you click **Apply**.

# **Output**

#### xhat — Estimated states

vector

Estimated states, returned as a vector of size *Ns*, where *Ns* is the number of states of the system. To access the individual states, use the Selector block.

When the **Use the current measurements to improve state estimates** parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

#### P — State estimation error covariance

matrix

State estimation error covariance, returned as an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. To access the individual covariances, use the Selector block.

You can output the error covariance only if you select **Output state estimation error covariance** in the **Block outputs, Multirate** tab, and click **Apply**.

#### **Dependencies**

This parameter is available if in the **Block outputs, Multirate** tab, the **State estimation method** parameter is set to 'Mean'.

#### Particles — Particle values used for state estimation

array

Particle values used for state estimation, returned as an *Ns*-by-*Np* or *Np*-by-*Ns* array. *Ns* is the number of states of the system, and *Np* is the number of particles.

- If the StateOrientation parameter is specified as 'column', then **Particles** is returned as an *Ns*-by-*Np* array.
- If the StateOrientation parameter is specified as 'row', then **Particles** is returned as an *Np*-by-*Ns* array.

#### **Dependencies**

This port is generated if you select **Output all particles** in the **Block outputs**, **Multirate** tab, and click **Apply**.

#### Weights — Particle weights used for state estimation

vector

Particle weights used for state estimation, returned as a 1-by-Np or Np-by-1 vector, where Np is the number of particles used for state estimation.

- If the StateOrientation parameter is specified as 'column', then **Weights** is returned as a 1-by-*Np* vector, where each weight is associated with the particle in the same column in the Particles array.
- If the StateOrientation parameter is specified as 'row', then **Weights** is returned as a *Np*-by-1 vector, where each weight is associated with the particle in the same row in the Particles array.

#### **Dependencies**

This port is generated if you select **Output weights** in the **Block outputs, Multirate** tab, and click **Apply**.

# **Parameters**

# **System Model Tab**

#### State Transition

#### Function — State transition function name

'vdpParticleFilterStateFcn' (default) | function name

The particle filter state transition function calculates the particles at time step k+1, given particles at time step k per the dynamics of your system and process noise. This function has the syntax:

```
particlesNext = f(particles, param1, param2, ...)
```

where, particles and particlesNext have dimensions Ns-by-Np if **State Orientation** is specified as 'column', or Np-by-Ns if **State Orientation** is specified as 'row'. Also, param\_i represents optional input arguments you may specify. For more information on optional input arguments, see "StateTransitionFcnInputs" on page 3-0 .

You create the state transition function and specify the function name in **Function**. For example, if vdpParticleFilterStateFcn.m is the state transition function that you created and saved, specify **Function** as 'vdpParticleFilterStateFcn'.

You can create **Function** using a Simulink Function block or as a MATLAB function (.m file).

#### **Programmatic Use**

**Block Parameter:** StateTransitionFcn

**Type:** character vector, string

Default: 'vdpParticleFilterStateFcn'

#### Initialization

#### Number of Particles — Number of particles used in the filter

1000 (default) | positive scalar integer

Number of particles used in the filter, specified as a positive scalar integer. Each particle represents a state hypothesis in the system. A higher number of particles increases the state estimation accuracy, but also increases the computational effort required to run the filter.

#### **Programmatic Use**

Block Parameter: NumberOfParticles

**Type:** positive scalar integer

Default: 1000

#### Distribution — Initial distribution of particles

'Gaussian' (default) | 'Uniform' | 'Custom'

Initial distribution of particles, specified as 'Gaussian', 'Uniform', or 'Custom'.

If you choose 'Gaussian', the initial set of particles or state hypotheses are distributed per the multivariate Gaussian distribution, where you specify the **Mean** and **Covariance**. The initial weight of all particles is assumed to be equal.

If you choose 'Uniform', the initial set of particles are distributed per the uniform distribution, where you specify the upper and lower **State bounds**. The initial weight of all particles is assumed to be equal.

'Custom' allows you to specify your own set of initial particles and their weights. You can use arbitrary probability distributions for **Particles** and **Weights** to initialize the filter.

#### **Programmatic Use**

Block Parameter: InitialDistribution

**Type:** character vector

Values: 'Gaussian', 'Uniform', 'Custom'

Default: 'Gaussian'

#### Mean — Initial mean value of particles

[0,0] (default) | vector

Initial mean value of particles, specified as a vector. The number of states to be estimated defines the length of the vector.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Distribution** parameter is set to Gaussian.

#### **Programmatic Use**

**Block Parameter:** InitialMean

Type: array
Default: [0,0]

#### Covariance — Initial covariance of particles

1 (default) | scalar | vector | matrix

Initial covariance of particles, specified as a scalar, vector, or matrix.

#### If **Covariance** is specified as:

- A scalar, then it must be positive. The covariance is assumed to be a [*Ns Ns*] matrix with this scalar on the diagonals. Here, *Ns* is the number of states.
- A vector, then each element must be positive. The covariance is assumed to be a [Ns Ns] matrix with the elements of the vector on the diagonals.
- A matrix, then it must be positive semidefinite.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Distribution** parameter is set to Gaussian.

#### **Programmatic Use**

**Block Parameter:** InitialCovariance

**Type:** scalar, vector, or matrix

Default: 1

#### Circular Variables — Circular variables used for state estimation

0 (default) | scalar | vector

Circular variables used for state estimation, specified as a scalar, or Ns-element vector, where Ns is the number of states.

If **Circular Variables** is specified as a scalar, the software extends it to a vector where each element is equal to this scalar. Circular (or angular) distributions use a probability density function with a range of  $[-\pi \pi]$ . Use circular variables if some of the states in your system represent angular quantities like the orientation of an object.

#### **Programmatic Use**

**Block Parameter:** CircularVariables

**Type:** scalar, vector

Default: 0

#### State Orientation — Orientation of input system states

```
'column' (default) | 'row'
```

Orientation of system states, specified as 'column' or 'row'.

#### If **State Orientation** is specified as:

- 'column', then the first input argument to the state transition and measurement likelihood function is  $[Ns\ Np]$ . In this case,  $i^{th}$  column of this matrix is the  $i^{th}$  particle (state hypothesis). Also, the states estimates **xhat** is output as a  $[Ns\ 1]$  vector. Here, Ns is the number of states, and Np is the number of particles.
- 'row', then the first input argument to the state transition and measurement likelihood function is  $[Np\ Ns]$ , and each row of this matrix contains a particle. Also, the states estimates **xhat** is output as a  $[1\ Ns]$  vector.

#### **Programmatic Use**

Block Parameter: StateOrientation

Type: character vector
Values: 'column', 'row'
Default: 'column'

#### State bounds — Initial bounds on system states

[-3 3;-3 3] (default) | array

Initial bounds on system states, specified as an Ns-by-2 array, where Ns is the number of states.

The  $i^{th}$  row lists the lower and upper bound of the uniform distribution for the initial distribution of particles of the  $i^{th}$  state.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Distribution** parameter is set to **Uniform**.

#### **Programmatic Use**

**Block Parameter:** InitialStateBounds

**Type:** array

**Default:** [-3 3;-3 3]

### Particles — Custom particle distribution for state estimation

[] (default) | array

Custom particle distribution for state estimation, specified as an Ns-by-Np or Np-by-Ns array. Ns is the number of states of the system, and Np is the number of particles.

• If the StateOrientation parameter is specified as 'column', then **Particles** is an *Ns*-by-*Np* array.

 If the StateOrientation parameter is specified as 'row', then Particles is an Npby-Ns array.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Distribution** parameter is set to **Custom**.

#### **Programmatic Use**

**Block Parameter:** InitialParticles

Type: array Default: []

#### Weights — Custom particle weight values for state estimation

[] (default) | positive vector

Custom particle weight values for state estimation, specified as a 1-by-Np or Np-by-1 positive vector, where Np is the number of particles used for state estimation.

- If the StateOrientation parameter is specified as 'column', then **Weights** is a 1-by-Np vector. Each weight in the vector is associated with the particle in the same column in the Particles array.
- If the StateOrientation parameter is specified as 'row', then **Weights** is a *Np*-by-1 vector. Each weight in the vector is associated with the particle in the same row in the Particles array.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Distribution** parameter is set to **Custom**.

#### **Programmatic Use**

**Block Parameter:** InitialWeights

**Type:** positive vector

Default: []

#### Measurement

#### Function — Measurement likelihood function name

'vdpMeasurementLikelihoodFcn' (default) | function name

The measurement likelihood function calculates the likelihood of particles (state hypotheses) using the sensor measurements. For each state hypothesis (particle), the

function first calculates an *Nm*-element measurement hypothesis vector. Then the likelihood of each measurement hypothesis is calculated based on the sensor measurement and the measurement noise probability distribution. this function has the syntax:

```
likelihood = h(particles, measurement, param1, param2, ...)
```

where, likelihood is an Np-element vector, where Np is the number of particles. particles have dimensions Ns-by-Np if **State Orientation** is specified as 'column', or Np-by-Ns if **State Orientation** is specified as 'row'. measurement is an Nm-element vector where, Nm is the number of measurements your sensor provides.  $param_i$  represents optional input arguments you may specify. For more information on optional input arguments, see "MeasurementLikelihoodFcn1Inputs,...,MeasurementLikelihoodFcn5Inputs" on page 3-0.

You create the measurement likelihood function and specify the function name in **Function**. For example, if vdpMeasurementLikelihoodFcn.m is the measurement likelihood function that you created and saved, specify **Function** as 'vdpMeasurementLikelihoodFcn'.

You can create **Function** using a Simulink Function block or as a MATLAB function (.m file).

 You can use a MATLAB function only if h has zero or one additional input argument param\_i other than Particles and Measurement.

The software generates an additional input port **MeasurementLikelihoodFcniInputs** to specify this argument for the  $i^{th}$  measurement likelihood function, and click **Apply**.

• If you are using a Simulink Function block, specify x and y using Argument Inport blocks and the additional inputs param\_i using Inport blocks in the Simulink Function block. You do not provide param\_i to the Particle Filter block.

If you have multiple sensors in your system, you can specify multiple measurement likelihood functions. You can specify up to five measurement likelihood functions using the **Add Measurement** button. To remove measurement likelihood functions, use **Remove Measurement**.

#### **Programmatic Use**

**Block Parameter:** MeasurementLikelihoodFcn1, MeasurementLikelihoodFcn2,

MeasurementLikelihoodFcn3, MeasurementLikelihoodFcn4,

MeasurementLikelihoodFcn5 **Type:** character vector, string

Default: 'vdpMeasurementLikelihoodFcn'

# Add Enable Port — Enable correction of estimated states only when measured data is available

off (default) | on

Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement likelihood function. To generate an input port **Enable1**, select **Add Enable port**. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the i<sup>th</sup> measurement likelihood function, select the corresponding **Add Enable port**.

#### **Programmatic Use**

**Block Parameter:** HasMeasurementEnablePort1, HasMeasurementEnablePort2, HasMeasurementEnablePort3, HasMeasurementEnablePort4,

HasMeasurementEnablePort5

Type: character vector Values: 'off', 'on' Default: 'off'

#### Resampling

### Resampling method — Method used for particle resampling

```
'Multinomial' (default) | 'Systemic' | 'Stratified'
```

Method used for particle resampling, specified as one of the following:

- 'Multinomial'
- 'Systematic'
- 'Stratified'

#### **Programmatic Use**

**Block Parameter:** ResamplingMethod

**Type:** character vector

Values: 'Multinomial', 'Systemic', 'Stratified'

#### Default: 'Multinomial'

#### Trigger method — Method to determine when resampling occurs

'Ratio' (default) | 'Interval'

Method to determine when resampling occurs, specified as either 'Ratio' or 'Interval'. The 'Ratio' value triggers resampling based on the ratio of effective total particles. The 'Interval' value triggers resampling at regular time steps of the particle filter operation.

#### **Programmatic Use**

**Block Parameter:** TriggerMethod

**Type:** character vector

Values: 'Ratio', 'Interval'

Default: 'Ratio'

# Minimum effective particle ratio — Minimum desired ratio of the effective number of particles to the total number of particles

0.5 (default) | positive scalar

Minimum desired ratio of the effective number of particles to the total number of particles, specified as a positive scalar. The effective number of particles is a measure of how well the current set of particles approximates the posterior distribution. A lower effective particle ratio implies that a lower number of particles are contributing to the estimation and resampling is required.

If the ratio of the effective number of particles to the total number of particles falls below the minimum effective particle ratio, a resampling step is triggered.

Specify minimum effective particle ratio as any value from 0 through 1.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Trigger method** parameter is set to Ratio.

#### **Programmatic Use**

Block Parameter: MinEffectiveParticleRatio

**Type:** scalar

Values: Range [0,1]

Default: 0.5

#### Sampling Interval — Fixed interval between resampling

1 (default) | positive scalar integer

Fixed interval between resampling, specified as a positive scalar integer. The sampling interval determines during which correction steps the resampling is executed. For example, a value of two means the resampling is executed every second correction step. A value of inf means that resampling is never executed.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Trigger method** parameter is set to **Interval**.

#### **Programmatic Use**

**Block Parameter:** SamplingInterval

**Type:** positive scalar integer

Default: 1

#### **Random Number Generator Options**

#### Randomness — Whether the random numbers are repeatable

'Repeatable' (default) | 'Not repeatable'

Whether the random numbers are repeatable, specified as either 'Repeatable' or 'Not repeatable'. If you want to be able to produce the same result more than once, set **Randomness** to 'Repeatable', and specify the same random number generator seed value in **Seed**.

#### **Programmatic Use**

**Block Parameter:** Randomness

**Type:** character vector

Values: 'Repeatable', 'Not repeatable'

Default: 'Repeatable'

#### Seed — Seed value for repeatable random numbers

0 (default) | scalar

Seed value for repeatable random numbers, specified as a scalar.

#### **Dependencies**

This parameter is available if in the **System model** tab, the **Randomness** parameter is set to 'Repeatable'.

#### **Programmatic Use**

**Block Parameter: Seed** 

**Type:** scalar

#### **Default:** 0

#### **Settings**

#### Sample time — Block sample time

1 (default) | positive scalar

Block sample time, specified as a positive scalar.

Use the **Sample time** parameter if your state transition and all measurement likelihood functions have the same sample time. Otherwise, select the **Enable multirate operation** option in the **Multirate** tab, and specify sample times in the same tab.

#### **Dependencies**

This parameter is available if in the **Block output, Multirate** tab, the **Enable multirate** operation parameter is off.

#### **Programmatic Use**

**Block Parameter:** SampleTime **Type:** character vector, string

Default: '1'

# **Block Outputs, Multirate Tab**

#### **Outputs**

# State Estimation Method — Method used for extracting a state estimate from particles

```
'Mean' (default) | 'MaxWeight' | 'None'
```

Method used for extracting a state estimate from particles, specified as one of the following:

- 'Mean' The Particle Filter block outputs the weighted mean of the particles, depending on the parameters **Weights** and **Particles**, as the state estimate.
- 'Maxweight' The Particle Filter block outputs the particle with the highest weight as the state estimate.
- 'None' Use this option to implement a custom state estimation method by accessing all particles using the **Output all particles** parameter from the **Block outputs**, **Multirate** tab.

#### **Programmatic Use**

**Block Parameter:** StateEstimationMethod

**Type:** character vector, string

Values: 'Mean', 'MaxWeight', 'None'

Default: 'Mean'

#### Output all particles — Output all particles

```
'off' (default) | 'on'
```

If you select this parameter, an output port for particles used in the estimation, **Particles** is generated in the block.

- If the StateOrientation parameter is specified as 'column', then the particles are output as an Ns-by-Np array. Ns is the number of states of the system, and Np is the number of particles.
- If the StateOrientation parameter is specified as 'row', then the particles are output as an *Np*-by-*Ns* array.

#### **Programmatic Use**

**Block Parameter:** OutputParticles

Type: character vector Values: 'off', 'on' Default: 'off'

### Output weights — Output particle weights

```
'off' (default) | 'on'
```

If you select this parameter, an output port for particle weights used in the estimation, **Weights** is generated in the block.

- If the StateOrientation parameter is specified as 'column', then the particle weights are output as a 1-by-Np vector. Here, where each weight is associated with the particle in the same column in the Particles array. Np is the number of particles used for state estimation.
- If the StateOrientation parameter is specified as 'row', then the particle weights are output as a Np-by-1 vector.

#### **Programmatic Use**

**Block Parameter:** OutputWeights

Type: character vector Values: 'off', 'on'

#### Default: 'off'

# Output state estimation error covariance — Output state estimation error covariance

```
'off' (default) | 'on'
```

If you select this parameter, a state estimation error covariance output port,  ${\bf P}$  is generated in the block.

#### **Dependencies**

This parameter is available if in the **Block outputs**, **Multirate** tab, the **State estimation method** parameter is set to 'Mean'.

#### **Programmatic Use**

Block Parameter: OutputStateCovariance

Type: character vector Values: 'off', 'on' Default: 'off'

# Use the current measurements to improve state estimates — Option to use current measurements for state estimation

```
'on' (default) | 'off'
```

When this parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

#### **Programmatic Use**

**Block Parameter:** UseCurrentEstimator

Type: character vector Values: 'on', 'off' Default: 'on'

#### Multirate

# Enable multirate operation — Enable specification of different sample times for state transition and measurement likelihood functions

```
'off' (default) | 'on'
```

Select this parameter if the sample times of the state transition or any of the measurement likelihood functions differ from the rest. You specify the sample times in the **Multirate** tab, in **Sample time**.

#### **Programmatic Use**

**Block Parameter:** EnableMultirate

Type: character vector Values: 'off', 'on' Default: 'off'

# Sample time — State transition and measurement likelihood function sample times

positive scalar

If the sample times for state transition and measurement likelihood functions are different, specify **Sample time**. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function Additional input to state transition function **StateTransitionFcnInputs**. The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement likelihood functions.
- Ports corresponding to i<sup>th</sup> measurement likelihood function Measured output yi, additional input to measurement likelihood function
   MeasurementLikelihoodFcniInputs, enable signal at port Enablei. The sample times of these ports for the same measurement likelihood function must always be the same, but can differ from the sample time for the state transition function and other measurement likelihood functions.

#### **Dependencies**

This parameter is available if in the **Block outputs**, **Multirate** tab, the **Enable multirate operation** parameter is on.

#### **Programmatic Use**

Block Parameter: StateTransitionFcnSampleTime, MeasurementLikelihoodFcn1SampleTime1, MeasurementLikelihoodFcn1SampleTime2, MeasurementLikelihoodFcn1SampleTime3,

MeasurementLikelihoodFcn1SampleTime4, MeasurementLikelihoodFcn1SampleTime5

**Type:** character vector, string

Default: '1'

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

The state transition and measurement likelihood functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

# See Also

#### **Blocks**

Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

#### **Functions**

extendedKalmanFilter|kalman|kalmd|particleFilter|
unscentedKalmanFilter

# **Topics**

- "Parameter and State Estimation in Simulink Using Particle Filter Block"
- "Validate Online State Estimation in Simulink"
- "Troubleshoot Online State Estimation"
- "Estimate States of Nonlinear System with Multiple, Multirate Sensors"

#### Introduced in R2018a

# **Unscented Kalman Filter**

Estimate states of discrete-time nonlinear system using unscented Kalman filter

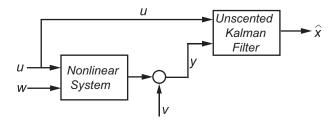
**Library:** Control System Toolbox / State Estimation System Identification Toolbox / Estimators

y1 xhat

# **Description**

The Unscented Kalman Filter block estimates the states of a discrete-time nonlinear system using the discrete-time unscented Kalman filter algorithm.

Consider a plant with states x, input u, output y, process noise w, and measurement noise v. Assume that you can represent the plant as a nonlinear system.



Using the state transition and measurement functions of the system and the unscented Kalman filter algorithm, the block produces state estimates  $\widehat{x}$  for the current time step. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You create the nonlinear state transition function and measurement functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement functions, each corresponding to a sensor in the system. For more information, see "State Transition and Measurement Functions" on page 3-108.

### **Ports**

# Input

y1,y2,y3,y4,y5 — Measured system outputs

vector

Measured system outputs corresponding to each measurement function that you specify in the block. The number of ports equals the number of measurement functions in your system. You can specify up to five measurement functions. For example, if your system has two sensors, you specify two measurement functions in the block. The first port **y1** is available by default. When you click **Apply**, the software generates port **y2** corresponding to the second measurement function.

Specify the ports as N-dimensional vectors, where N is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and velocity of an object, then there is only one port y1. The port is specified as a 2-dimensional vector with values corresponding to position and velocity.

#### **Dependencies**

The first port y1 is available by default. Ports y2 to y5 are generated when you click **Add Measurement**, and click **Apply**.

Data Types: single | double

# StateTransitionFcnInputs — Additional optional input argument to state transition function

scalar | vector | matrix

Additional optional input argument to the state transition function f other than the state x and process noise w. For information about state transition functions see, "State Transition and Measurement Functions" on page 3-108.

Suppose that your system has nonadditive process noise, and the state transition function f has the following form:

x(k+1) = f(x(k), w(k), StateTransitionFcnInputs).

Here k is the time step, and  ${\tt StateTransitionFcnInputs}$  is an additional input argument other than x and w.

If you create f using a MATLAB function (.m file), the software generates the port **StateTransitionFcnInputs** when you click **Apply**. You can specify the inputs to this port as a scalar, vector, or matrix.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Unscented Kalman Filter block.

#### **Dependencies**

This port is generated only if both of the following conditions are satisfied:

- You specify f in Function using a MATLAB function, and f is on the MATLAB path.
- f requires only one additional input argument apart from x and w.

Data Types: single | double

 $\label{lem:measurement} Measurement Fcn 2 Inputs \, , Measurement Fcn 3 Inputs \, , Measurement Fcn 4 Inputs \, , Measurement Fcn 5 Inputs \, - \, Additional \, optional \, input \, argument \, to \, each \, measurement \, function$ 

```
scalar | vector | matrix
```

Additional optional inputs to the measurement functions other than the state x and measurement noise v. For information about measurement functions see, "State Transition and Measurement Functions" on page 3-108.

**MeasurementFcn1Inputs** corresponds to the first measurement function that you specify, and so on. For example, suppose that your system has three sensors and nonadditive measurement noise, and the three measurement functions h1, h2, and h3 have the following form:

```
y1[k] = h1(x[k],v[k],MeasurementFcn1Inputs)

y2[k] = h2(x[k],v[k],MeasurementFcn2Inputs)

y3[k] = h3(x[k],v[k])
```

Here k is the time step, and MeasurementFcn1Inputs and MeasurementFcn2Inputs are the additional input arguments to h1 and h2.

If you specify h1, h2, and h3 using MATLAB functions (.m files) in **Function**, the software generates ports **MeasurementFcn1Inputs** and **MeasurementFcn2Inputs** when you click **Apply**. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Unscented Kalman Filter block.

#### **Dependencies**

A port corresponding to a measurement function h is generated only if both of the following conditions are satisfied:

- You specify h in Function using a MATLAB function, and h is on the MATLAB path.
- h requires only one additional input argument apart from x and v.

Data Types: single | double

### Q — Time-varying process noise covariance

scalar | vector | matrix

Time-varying process noise covariance, specified as a scalar, vector, or matrix depending on the value of the **Process noise** parameter:

- **Process noise** is Additive Specify the covariance as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. Specify a vector of length *Ns*, if there is no cross-correlation between process noise terms, but all the terms have different variances.
- **Process noise** is **Nonadditive** Specify the covariance as a *W*-by-*W* matrix, where *W* is the number of process noise terms in the state transition function.

#### **Dependencies**

This port is generated if you specify the process noise covariance as **Time-Varying**. The port appears when you click **Apply**.

Data Types: single | double

# R1,R2,R3,R4,R5 — Time-varying measurement noise covariance matrix

Time-varying measurement noise covariances for up to five measurement functions of the system, specified as matrices. The sizes of the matrices depend on the value of the **Measurement noise** parameter for the corresponding measurement function:

- **Measurement noise** is Additive Specify the covariance as an *N*-by-*N* matrix, where *N* is the number of measurements of the system.
- Measurement noise is Nonadditive Specify the covariance as a V-by-V matrix, where V is the number of measurement noise terms in the corresponding measurement function.

#### **Dependencies**

A port is generated if you specify the measurement noise covariance as **Time-Varying** for the corresponding measurement function. The port appears when you click **Apply**.

Data Types: single | double

# Enable1, Enable2, Enable3, Enable4, Enable5 — Enable correction of estimated states when measured data is available

scalar

Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement function. Use a signal value other than 0 at the **Enable1** port to enable the correction of estimated states when measured data is available. Specify the port value as 0 when measured data is not available. Similarly, if measured output data is not available at all time points at the port yi for the i<sup>th</sup> measurement function, specify the corresponding port **Enable**i as a value other than 0.

#### **Dependencies**

A port corresponding to a measurement function is generated if you select **Add Enable port** for that measurement function. The port appears when you click **Apply**.

Data Types: single | double | Boolean

## **Output**

#### xhat — Estimated states

vector

Estimated states, returned as a vector of size Ns, where Ns is the number of states of the system. To access the individual states, use the Selector block.

When the **Use the current measurements to improve state estimates** parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

Data Types: single | double

#### P — State estimation error covariance

matrix

State estimation error covariance, returned as an Ns-by-Ns matrix, where Ns is the number of states of the system. To access the individual covariances, use the Selector block.

#### **Dependencies**

This port is generated if you select **Output state estimation error covariance** in the **System Model** tab, and click **Apply**.

Data Types: single | double

## **Parameters**

## **System Model Tab**

#### **State Transition**

#### Function — State transition function name

myStateTransitionFcn (default) | function name

The state transition function calculates the Ns-element state vector of the system at time step k+1, given the state vector at time step k. Ns is the number of states of the nonlinear system. You create the state transition function and specify the function name in **Function**. For example, if vdpStateFcn.m is the state transition function that you created and saved, specify **Function** as vdpStateFcn.

The inputs to the function you create depend on whether you specify the process noise as additive or nonadditive in **Process noise**.

• **Process noise** is Additive — The state transition function *f* specifies how the states evolve as a function of state values at previous time step:

```
x(k+1) = f(x(k), Us1(k), \dots, Usn(k)),
```

where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function, such as system inputs or the sample time. To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

 Process noise is Nonadditive — The state transition function also specifies how the states evolve as a function of the process noise w:

$$x(k+1) = f(x(k),w(k),Us1(k),...,Usn(k)).$$

For more information, see "State Transition and Measurement Functions" on page 3-108.

You can create f using a Simulink Function block or as a MATLAB function (.m file).

 You can use a MATLAB function only if f has one additional input argument Us1 other than x and w.

```
x(k+1) = f(x(k),w(k),Us1(k))
```

The software generates an additional input port **StateTransitionFcnInputs** to specify this argument.

• If you are using a Simulink Function block, specify x and w using Argument Inport blocks and the additional inputs Us1,...,Usn using Inport blocks in the Simulink Function block. You do not provide Us1,...,Usn to the Unscented Kalman Filter block.

#### **Programmatic Use**

**Block Parameter:** StateTransitionFcn

**Type:** character vector, string

**Default:** 'myStateTransitionFcn'

#### Process noise — Process noise characteristics

Additive (default) | Nonadditive

Process noise characteristics, specified as one of the following values:

Additive — Process noise w is additive, and the state transition function f that you specify in Function has the following form:

```
x(k+1) = f(x(k), Us1(k), \dots, Usn(k)),
```

where x(k) is the estimated state at time k, and  $Us1, \ldots, Usn$  are any additional input arguments required by your state transition function.

 Nonadditive — Process noise is nonadditive, and the state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:

```
x(k+1) = f(x(k),w(k),Us1(k),...,Usn(k)).
```

### **Programmatic Use**

Block Parameter: HasAdditiveProcessNoise

**Type:** character vector

Values: 'Additive', 'Nonadditive'

Default: 'Additive'

### **Covariance** — Time-invariant process noise covariance

1 (default) | scalar | vector | matrix

Time-invariant process noise covariance, specified as a scalar, vector, or matrix depending on the value of the **Process noise** parameter:

- **Process noise** is Additive Specify the covariance as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms and all the terms have the same variance. Specify a vector of length *Ns*, if there is no cross-correlation between process noise terms but all the terms have different variances.
- **Process noise** is **Nonadditive** Specify the covariance as a *W*-by-*W* matrix, where *W* is the number of process noise terms.

If the process noise covariance is time-varying, select  $\mathbf{Time\text{-}varying}$ . The block generates input port  $\mathbf{Q}$  to specify the time-varying covariance.

#### **Dependencies**

This parameter is enabled if you do not specify the process noise as **Time-Varying**.

#### **Programmatic Use**

**Block Parameter:** ProcessNoise

**Type:** character vector, string

Default: '1'

### Time-varying — Time-varying process noise covariance

```
'off' (default) | 'on'
```

If you select this parameter, the block includes an additional input port  $\mathbf{Q}$  to specify the time-varying process noise covariance.

#### **Programmatic Use**

**Block Parameter:** HasTimeVaryingProcessNoise

Type: character vector Values: 'off', 'on' Default: 'off'

#### Initialization

#### Initial state — Initial state estimate

0 (default) | vector

Initial state estimate value, specified as an Ns-element vector, where Ns is the number of states in the system. Specify the initial state values based on your knowledge of the system.

#### **Programmatic Use**

**Block Parameter:** InitialState **Type:** character vector, string

Default: '0'

#### Initial covariance — State estimation error covariance

1 (default) | scalar | vector | matrix

State estimation error covariance, specified as a scalar, an *Ns*-element vector, or an *Ns*-by-*Ns* matrix, where *Ns* is the number of states of the system. If you specify a scalar or vector, the software creates an *Ns*-by-*Ns* diagonal matrix with the scalar or vector elements on the diagonal.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in **Initial state**.

#### **Programmatic Use**

Block Parameter: InitialStateCovariance

**Type:** character vector, string

Default: '1'

#### **Unscented Transformation Parameters**

### Alpha — Spread of sigma points

1e-3 (default) | scalar value between 0 and 1

The unscented Kalman filter algorithm treats the state of the system as a random variable with a mean state value and variance. To compute the state and its statistical properties at the next time step, the algorithm first generates a set of state values distributed around the mean value by using the unscented transformation. These generated state values are called sigma points. The algorithm uses each of the sigma points as an input to the state transition and measurement functions to get a new set of transformed state points and measurements. The transformed points are used to compute the state and state estimation error covariance value at the next time step.

The spread of the sigma points around the mean state value is controlled by two parameters **Alpha** and **Kappa**. A third parameter, **Beta**, impacts the weights of the transformed points during state and measurement covariance calculations:

- **Alpha** Determines the spread of the sigma points around the mean state value. Specify as a scalar value between 0 and 1 (0 < **Alpha** <= 1). It is usually a small positive value. The spread of sigma points is proportional to **Alpha**. Smaller values correspond to sigma points closer to the mean state.
- **Kappa** A second scaling parameter that is typically set to 0. Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square-root of **Kappa**.
- **Beta** Incorporates prior knowledge of the distribution of the state. For Gaussian distributions, **Beta** = 2 is optimal.

If you know the distribution of state and state covariance, you can adjust these parameters to capture the transformation of higher-order moments of the distribution. The algorithm can track only a single peak in the probability distribution of the state. If there are multiple peaks in the state distribution of your system, you can adjust these parameters so that the sigma points stay around a single peak. For example, choose a small **Alpha** to generate sigma points close to the mean state value.

For more information, see "Unscented Kalman Filter Algorithm".

**Programmatic Use** 

**Block Parameter:** Alpha **Type:** character vector, string

Default: '1e-3'

#### **Beta** — Characterization of state distribution

2 (default) | scalar value greater than or equal to 0

Characterization of the state distribution that is used to adjust weights of transformed sigma points, specified as a scalar value greater than or equal to 0. For Gaussian distributions, Beta = 2 is the optimal choice.

For more information, see the description for **Alpha**.

#### **Programmatic Use**

**Block Parameter:** Beta **Type:** character vector, string

Default: '2'

### Kappa — Spread of sigma points

0 (default) | scalar value between 0 and 3

Spread of sigma points around mean state value, specified as a scalar value between 0 and 3 ( $0 \le \text{Kappa} \le 3$ ). Kappa is typically specified as 0. Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square root of Kappa. For more information, see the description for Alpha.

#### **Programmatic Use**

**Block Parameter:** Kappa **Type:** character vector, string

Default: '0'

#### Measurement

#### Function — Measurement function name

myMeasurementFcn (default) | function name

The measurement function calculates the N-element output measurement vector of the nonlinear system at time step k, given the state vector at time step k. You create the measurement function and specify the function name in **Function**. For example, if vdpMeasurementFcn.m is the measurement function that you created and saved, specify **Function** as vdpMeasurementFcn.

The inputs to the function you create depend on whether you specify the measurement noise as additive or nonadditive in **Measurement noise**.

Measurement noise is Additive — The measurement function h specifies how the
measurements evolve as a function of state Values:

```
y(k) = h(x(k), Um1(k), ..., Umn(k)),
```

where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function. For example, if you are using a sensor for tracking an object, an additional input could be the sensor position.

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line.

• **Measurement noise** is **Nonadditive**— The measurement function also specifies how the output measurement evolves as a function of the measurement noise v:

```
y(k) = h(x(k), v(k), Um1(k), \dots, Umn(k)).
```

To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

For more information, see "State Transition and Measurement Functions" on page 3-108.

You can create h using a Simulink Function block or as a MATLAB function (.m file).

 You can use a MATLAB function only if h has one additional input argument Um1 other than x and v.

```
y[k] = h(x[k],v[k],Um1(k))
```

The software generates an additional input port **MeasurementFcnInput** to specify this argument.

• If you are using a Simulink Function block, specify x and v using Argument Inport blocks and the additional inputs Um1,...,Umn using Inport blocks in the Simulink Function block. You do not provide Um1,...,Umn to the Unscented Kalman Filter block.

If you have multiple sensors in your system, you can specify multiple measurement functions. You can specify up to five measurement functions using the **Add Measurement** button. To remove measurement functions, use **Remove Measurement**.

### **Programmatic Use**

**Block Parameter:** MeasurementFcn1, MeasurementFcn2, MeasurementFcn3, MeasurementFcn4, MeasurementFcn5

**Type:** character vector, string

**Default:** 'myMeasurementFcn'

#### Measurement noise — Measurement noise characteristics

Additive (default) | Nonadditive

Measurement noise characteristics, specified as one of the following values:

• Additive — Measurement noise v is additive, and the measurement function h that you specify in **Function** has the following form:

```
y(k) = h(x(k), Um1(k), ..., Umn(k)),
```

where y(k) and x(k) are the estimated output and estimated state at time k, and  $Um1, \ldots, Umn$  are any optional input arguments required by your measurement function.

 Nonadditive — Measurement noise is nonadditive, and the measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

```
y(k) = h(x(k), v(k), Um1(k), \dots, Umn(k)).
```

### **Programmatic Use**

**Block Parameter:** HasAdditiveMeasurementNoise1.

HasAdditiveMeasurementNoise2, HasAdditiveMeasurementNoise3, HasAdditiveMeasurementNoise4, HasAdditiveMeasurementNoise5

**Type:** character vector

Values: 'Additive', 'Nonadditive'

Default: 'Additive'

#### Covariance — Time-invariant measurement noise covariance

1 (default) | matrix

Time-invariant measurement noise covariance, specified as a matrix. The size of the matrix depends on the value of the **Measurement noise** parameter:

- **Measurement noise** is Additive Specify the covariance as an *N*-by-*N* matrix, where *N* is the number of measurements of the system.
- **Measurement noise** is **Nonadditive** Specify the covariance as a *V*-by-*V* matrix, where *V* is the number of measurement noise terms.

If the measurement noise covariance is time-varying, select **Time-varying**. The block generates input port  $\mathbf{R}\mathbf{i}$  to specify the time-varying covariance for the  $i^{th}$  measurement function.

#### **Dependencies**

This parameter is enabled if you do not specify the process noise as **Time-Varying**.

#### **Programmatic Use**

Block Parameter: MeasurementNoise1, MeasurementNoise2, MeasurementNoise3,

MeasurementNoise4, MeasurementNoise5

**Type:** character vector, string

Default: '1'

## Time-varying — Time-varying measurement noise covariance

off (default) | on

If you select this parameter for the measurement noise covariance of the first measurement function, the block includes an additional input port  $\mathbf{R1}$ . You specify the time-varying measurement noise covariance in  $\mathbf{R1}$ . Similarly, if you select  $\mathbf{Time-varying}$  for the  $i^{th}$  measurement function, the block includes an additional input port  $\mathbf{Ri}$  to specify the time-varying measurement noise covariance for that function.

#### **Programmatic Use**

**Block Parameter:** HasTimeVaryingMeasurementNoise1,

HasTimeVaryingMeasurementNoise2, HasTimeVaryingMeasurementNoise3, HasTimeVaryingMeasurementNoise4, HasTimeVaryingMeasurementNoise5

Type: character vector Values: 'off', 'on'

Default: 'off'

# Add Enable Port — Enable correction of estimated states only when measured data is available

off (default) | on

Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement function. Select **Add Enable port** to generate an input port **Enable1**. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the i<sup>th</sup> measurement function, select the corresponding **Add Enable port**.

#### **Programmatic Use**

Block Parameter: HasMeasurementEnablePort1, HasMeasurementEnablePort2,

HasMeasurementEnablePort3, HasMeasurementEnablePort4,

HasMeasurementEnablePort5

Type: character vector Values: 'off', 'on' Default: 'off'

#### **Settings**

# Use the current measurements to improve state estimates — Choose between corrected or predicted state estimate

on (default) | off

When this parameter is selected, the block outputs the corrected state estimate  $\widehat{x}[k|k]$  at time step k, estimated using measured outputs until time k. If you clear this parameter, the block returns the predicted state estimate  $\widehat{x}[k|k-1]$  for time k, estimated using measured output until a previous time k-1. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

### **Programmatic Use**

**Block Parameter:** UseCurrentEstimator

Type: character vector Values: 'off', 'on'

Default: 'on'

# Output state estimation error covariance — Output state estimation error covariance

off (default) | on

If you select this parameter, a state estimation error covariance output port  ${\bf P}$  is generated in the block.

### **Programmatic Use**

Block Parameter: OutputStateCovariance

Type: character vector Values: 'off','on' Default: 'off'

## Data type — Data type for block parameters

double (default) | single

Use this parameter to specify the data type for all block parameters.

### **Programmatic Use**

**Block Parameter:** DataType

**Type:** character vector

Values: 'single', 'double'

Default: 'double'

### Sample time — Block sample time

1 (default) | positive scalar

Block sample time, specified as a positive scalar. If the sample times of your state transition and measurement functions are different, select **Enable multirate operation** in the **Multirate** tab, and specify the sample times in the **Multirate** tab instead.

#### **Dependencies**

This parameter is available if in the **Multirate** tab, the **Enable multirate operation** parameter is off.

#### **Programmatic Use**

**Block Parameter:** SampleTime **Type:** character vector, string

Default: '1'

### **Multirate Tab**

# Enable multirate operation — Enable specification of different sample times for state transition and measurement functions

off (default) | on

Select this parameter if the sample times of the state transition and measurement functions are different. You specify the sample times in the **Multirate** tab, in **Sample time**.

#### **Programmatic Use**

**Block Parameter:** EnableMultirate

Type: character vector Values: 'off', 'on' Default: 'off'

## Sample time — State transition and measurement function sample times

positive scalar

If the sample times for state transition and measurement functions are different, specify **Sample time**. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function Additional input to state transition function StateTransitionFcnInputs and time-varying process noise covariance Q.
   The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement functions.
- Ports corresponding to i<sup>th</sup> measurement function Measured output yi, additional input to measurement function MeasurementFcniInputs, enable signal at port Enablei, and time-varying measurement noise covariance Ri. The sample times of these ports for the same measurement function must always be the same, but can differ from the sample time for the state transition function and other measurement functions.

#### **Dependencies**

This parameter is available if in the **Multirate** tab, the **Enable multirate operation** parameter is on.

#### **Programmatic Use**

**Block Parameter:** StateTransitionFcnSampleTime, MeasurementFcn1SampleTime1, MeasurementFcn1SampleTime2, MeasurementFcn1SampleTime3, MeasurementFcn1SampleTime4, MeasurementFcn1SampleTime5

**Type:** character vector, string

Default: '1'

## **More About**

## **State Transition and Measurement Functions**

The algorithm computes the state estimates  $\widehat{x}$  of the nonlinear system using state transition and measurement functions specified by you. You can specify up to five measurement functions, each corresponding to a sensor in the system. The software lets you specify the noise in these functions as additive or nonadditive.

 Additive Noise Terms — The state transition and measurements equations have the following form:

$$x[k+1] = f(x[k], u_s[k]) + w[k]$$
  
 $y[k] = h(x[k], u_m[k]) + v[k]$ 

Here f is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function h relates x to the measurements y at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional optional input arguments that are denoted by  $u_s$  and  $u_m$  in the equations. For example, the additional arguments could be time step k or the inputs u to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, x(k+1) is linearly related to the process noise w(k), and y(k) is linearly related to the measurement noise v(k). For additive noise terms, you do not need to specify the noise terms in the state transition and measurement functions. The software adds the terms to the output of the functions.

• Nonadditive Noise Terms — The software also supports more complex state transition and measurement functions where the state x[k] and measurement y[k] are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$x[k+1] = f(x[k], w[k], u_s[k])$$
  
 $y[k] = h(x[k], v[k], u_m[k])$ 

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

The state transition and measurement functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

## See Also

#### **Blocks**

Extended Kalman Filter | Kalman Filter | Particle Filter

#### **Functions**

extendedKalmanFilter | kalman | kalmd | particleFilter |
unscentedKalmanFilter

## **Topics**

"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

## Introduced in R2017a

<sup>&</sup>quot;Validate Online State Estimation in Simulink"

<sup>&</sup>quot;Troubleshoot Online State Estimation"

# **Varying Lowpass Filter**

Butterworth filter with varying coefficients **Library:** Control System Toolbox / Linear Parameter Varying



Varying Lowpass Filter

# **Description**

The block implements an analog  $N^{\text{th}}$ -order Butterworth filter with unit DC gain and varying cutoff frequency.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

## u — Filter input

scalar

Lowpass filter input signal.

## freq — Cutoff frequency

scalar

Filter cutoff frequency, specified in rad/s.

## **Output**

## y — Filter output

scalar

Lowpass filter output signal.

## **Parameters**

## Filter order — Lowpass filter order

1 (default) | positive integer

Lowpass filter order, specified as a positive integer.

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

## See Also

Discrete Varying Lowpass

## **Topics**

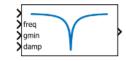
"Model Gain-Scheduled Control Systems in Simulink"

Introduced in R2017b

# **Varying Notch Filter**

Notch filter with varying coefficients

**Library:** Control System Toolbox / Linear Parameter Varying



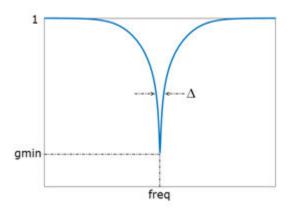
Varying Notch Filter

# **Description**

The block implements a continuous-time notch filter with varying coefficients. The instantaneous transfer function of the filter is given by:

$$N(s) = \frac{s^2 + 2*gmin*damp*freq*s + freq^2}{s^2 + 2*damp*freq*s + freq^2},$$

where gmin, damp, and freq are the values supplied at the corresponding input ports. These parameters control the notch depth and frequency as shown in the following illustration. The damping ratio damp controls the notch width  $\Delta$ ; larger damp means larger  $\Delta$ .



Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

### u - Filter input

scalar

Notch filter input signal

## freq — Notch frequency

scalar

Value of the notch frequency, specified in rad/s.

## gmin — Gain at notch frequency

scalar

Value of the gain at notch frequency, in absolute units. This value controls the notch depth. The notch filter has unit gain at low and high frequency. The gain is lowest at the notch frequency, the value at the **freq** port.

## damp — Damping ratio of the filter poles

scalar

Value of the damping ratio, specified as a positive scalar value. The damping ratio controls the notch width; the closer to 0, the steeper the notch.

## Output

## y — Filter output

scalar

Notch filter output signal.

# **Extended Capabilities**

# **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder $^{\text{\tiny TM}}$ .

## **See Also**

Discrete Varying Notch

# **Topics**

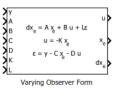
"Model Gain-Scheduled Control Systems in Simulink"

Introduced in R2017b

# **Varying Observer Form**

Observer-form state-space model with varying matrix values

Library: Control System Toolbox / Linear Parameter Varying



# **Description**

Use this block to implement a continuous-time varying state-space model in observer form. The system matrices A, B, C, and D describe the plant dynamics, and the matrices K and L specify the state-feedback and state-observer gains, respectively. Feed the instantaneous values of these matrices to the corresponding input ports. The observer form is given by:

$$dx_e = Ax_e + Bu + L\varepsilon$$
$$u = -Kx_e$$
$$\varepsilon = y - Cx_e - Du,$$

where u is the plant input, y is the plant output,  $x_e$  is the estimated state, and  $\varepsilon$  is the innovation, the difference between the predicted and measured plant output. The observer form works well for gain scheduling of state-space controllers. In particular, the state  $x_e$  tracks the plant state, and all controllers are expressed with the same state coordinates.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

## y — Measurement signal

scalar | vector

Measured plant output signal.

#### A — Plant state matrix

matrix

Plant state matrix of dimensions  $N_x$ -by- $N_x$ , where  $N_x$  is the number of plant states.

### **B** — Plant input matrix

matrix

Plant input matrix of dimensions  $N_x$ -by- $N_u$ , where  $N_u$  is the number of plant inputs.

## C — Plant output matrix

matrix

Plant output matrix  $N_v$ -by- $N_x$ , where  $N_v$  is the number of plant outputs.

#### D — Plant feedforward matrix

matrix

Plant feedforward matrix of dimensions  $N_y$ -by- $N_u$ .

#### K — State-feedback matrix

matrix

State-feedback matrix of dimensions  $N_u$ -by- $N_x$ .

### L — State-observer matrix

matrix

State-observer matrix of dimensions  $N_x$ -by- $N_y$ .

## **Output**

### u — Control signal

scalar | vector

Control signal (plant input).

## $x_e$ — Estimated plant state vector

vector

Vector of estimated plant states.

#### **Dependencies**

To enable this port, select the **Output states** parameter.

## dx<sub>e</sub> — Estimated state derivatives

vector

Derivatives of the corresponding estimated states in  $\mathbf{x}_{\mathbf{e}}$ .

## **Dependencies**

To enable this port, select the **Output state updates** parameter.

## **Parameters**

## Initial conditions — System initial conditions

0 (default) | scalar | vector

Initial state values, specified as a scalar or a vector whose length is the number of plant states.

## State names (e.g., 'position') — Plant state names

' ' (default) | character vector | cell array

To identify plant states, specify state names as a:

- character vector, for a one-state plant.
- Cell array of character vectors, for a multistate plant.

## Output states — Provide state output

on (default) | off

Select to enable the estimated states output port,  $x_e$ .

# **Output state derivatives — Provide state derivatives** on (default) | off

Select to enable the estimated state derivatives output port,  $dx_e$ .

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

## See Also

Discrete Varying Observer Form | Varying State Space

## **Topics**

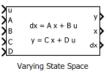
"Model Gain-Scheduled Control Systems in Simulink"

Introduced in R2017b

# **Varying State Space**

State-space model with varying matrix values

**Library:** Control System Toolbox / Linear Parameter Varying



# **Description**

Use this block to implement a continuous-time state-space model with varying matrices. Feed the instantaneous values of the state matrix A, input matrix B, output matrix C, and feedforward matrix D to the corresponding input ports. The system response is given by:

$$dx = Ax + Bu$$
$$y = Cx + Du,$$

where u is the system input, y is the system output, and x and dx are the state vector and state derivatives, respectively.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

u — System input

scalar | vector

System input signal.

A — State matrix

matrix

State matrix of dimensions  $N_x$ -by- $N_x$ , where  $N_x$  is the number of system states.

### **B** — Input matrix

matrix

Input matrix of dimensions  $N_x$ -by- $N_u$ , where  $N_u$  is the number of system inputs.

### C — Output matrix

matrix

Output matrix  $N_{v}$ -by- $N_{x}$ , where  $N_{v}$  is the number of system outputs.

#### D — Feedforward matrix

matrix

Feedforward matrix of dimensions  $N_v$ -by- $N_u$ .

## **Output**

### y — System output

scalar | vector

System output signal.

#### x — Current state vector

vector

Current state values.

#### **Dependencies**

To enable this port, select the **Output states** parameter.

#### dx — State derivatives

vector

Current derivatives of the corresponding states in  $\mathbf{x}$ .

### **Dependencies**

To enable this port, select the **Output state derivatives** parameter.

## **Parameters**

## Initial conditions — System initial conditions

0 (default) | scalar | vector

Initial state values, specified as a scalar or a vector whose length is the number of system states.

### State names — System state names

' ' (default) | character vector | cell array

To identify system states, specify state names as a:

- character vector, for a one-state plant.
- Cell array of character vectors, for a multistate plant.

## **Output states — Provide state output**

on (default) | off

Select to enable the state values output port,  $\mathbf{x}$ .

## Output state derivatives — Provide state derivatives

on (default) | off

Select to enable the state derivatives output port, dx.

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder™.

## **See Also**

Discrete Varying State Space

## **Topics**

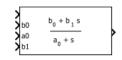
"Model Gain-Scheduled Control Systems in Simulink"

## Introduced in R2017b

# **Varying Transfer Function**

Transfer function with varying coefficients

**Library:** Control System Toolbox / Linear Parameter Varying



Varying Transfer Function

# **Description**

This block implements a continuous-time transfer function with varying coefficients. The instantaneous transfer function is given by:

$$H(s) = \frac{b_0 + b_1 s + \dots b_N s^N}{a_0 + a_1 s + \dots a_{N-1} s^{N-1} + s^N},$$

where N is number of poles, specified with the **Transfer function order** parameter. Feed the values of the coefficients  $a_0$ ,  $a_1$ ,...,  $a_{N-1}$  and  $b_0$ ,  $b_1$ ,...,  $b_N$  to the corresponding block input ports.

Use this block and the other blocks in the Linear Parameter Varying library to implement common control elements with variable parameters or coefficients. For more information, see "Model Gain-Scheduled Control Systems in Simulink".

## **Ports**

## Input

u — Transfer function input

scalar

Transfer function input signal.

**b0, b1, ... — Numerator coefficients** scalar

Transfer function numerator coefficients. The number of coefficient ports is determined by the **Transfer function order** parameter.

## a0, a1, ... — Denominator coefficients

scalar

Transfer function denominator coefficients. The number of coefficient ports is determined by the **Transfer function order** parameter. The coefficient on the highest-order term is fixed to 1.

## Output

## y — Transfer function output

scalar

Transfer function output signal.

## **Parameters**

## Transfer function order — Number of poles

1 (default) | positive integer

Transfer function (number of poles), specified as a positive integer. This parameter determines the number of coefficient input ports on the block.

# **Extended Capabilities**

## **C/C++ Code Generation**

Generate C and C++ code using Simulink® Coder  $^{\text{\tiny TM}}$ .

## See Also

Discrete Varying Transfer Function

# **Topics**

"Model Gain-Scheduled Control Systems in Simulink"

## Introduced in R2017b